Metamodelling in a Ontology Network

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Abstract. When designing ontology networks, ontologies can be combined through mapping or metamodelling relationships, among others. The coexistence of metamodelling with other kind of relationships in a ontology network leads to identify a set of problems, which are the focus of the present work.

Keywords. Ontology network, Metamodelling, Well-founded model

Introduction

Haase et al. [1] define an *ontology network* as "a collection of ontologies related together via a variety of different relationships such as mapping, modularization, version, and dependency relationships", denominating the component ontologies as "networked ontologies".

Regarding ontology engineering, in different case studies the need of linking ontologies of different domains arises. In general, the way how these ontologies are related is not always the same. For instance, sometimes it is required to align concepts of two ontologies, while in other cases reusing a whole ontology and extending it can be the most suitable alternative. Sometimes it is necessary to link individuals of two ontologies through a new role and in other cases, one of the ontologies is the metamodel for the other.

Like the modular software development in software engineering, the fenomenon of reusing and linking existing ontologies have also been studied. There exist several proposals of logics which define a formal semantics of integration of modular ontologies. Serafini et al. [2] compare these formalisms, which define a new syntax and semantics to control the interaction between the modules. These set of formalisms are refered to as "Modular Ontology Languages" by Cuenca et al. [3]. The most prominent ones are *ε-connections* [4,5], *Package-based Description Logics* [6], *Distributed Description Logics* [7] and *Integrated Distributed Description Logics* [8]. Besides, there is a different approach that establishes some non-standard reasoning services over the union of a set of ontologies, testing different aspects of the semantics of the union ontology and their components. This approach, called "Modular Reuse of Ontologies" provides a mechanism to ensure a "safe" combination of the involved domain ontologies [9].

In the present work we identify a set of possible relationships to relate ontologies, preliminarily presented in [10], which from the point of view of ontology engineering result naturally distinct at the moment of building an ontology network. This set of relationships allows the ontology engineer to conceptualize the ontology network at a higher level of granularity, visualizing the interaction among the involved ontologies.

Among the set of selected ontology relationships, we pay special attention to the *metamodelling relationship*. The remainder relationships have been broadly studied, but up to our knowledge, the scenario in which metamodelling coexists with other kind of relationships in a ontology network has not been studied from the point of view of what inconsistencies or contradictions can arise, which can not be detected by a standard Description Logics (DL) reasoner.

There exist some works that define a different semantics for a knowledge base when there is metamodelling, either specifying different "layers" or "stratums" [11], or treating a symbol of the signature which is both an instance and a concept, as two independent elements when reasoning [12]. But, up to our knowledge none of them adress the problem of what contradictions can arise when there are also different axioms involving these symbols.

In our work, we redefine the semantics of the metamodelling relationship to interpret an instance and its corresponding concept or role in the metamodelling relationship as the same element in the domain of discourse. Moreover, regarding the coexistence of metamodelling and other relationships, we propose an additional condition which must be satisfied in the ontology network, when there are metamodelling relationships.

The remainder of this paper is organized as follows. Section 1 defines an ontology network semantics to contemplate the metamodelling relationship. Section 2 addresses the issue of having metamodelling along with other relationships in a ontology network. Section 3 define a condition that must be satisfied in order to avoid a poor design choice when metamodelling. Section 4 gives an overview of the existing literature about metamodelling in the ontology design and the theoretical background regarding metamodelling semantics. Finally, Section 5 analizes some conclusions and proposes some future work.

1. Ontology Network

In this section we define four conceptually different relationships between ontologies. For three of them, *mapping*, *link* and *extension*, we relied on existing approaches of the literature [6,4,3]. For the remaining relationship, the *metamodelling relationship*, we redefine the semantics of the interpretation of an ontology network. We base our definitions on the foundations of Description Logics syntax and semantics, which can be reviewed by the reader in [13,14].

Definition 1 (Relationship between Ontologies) *$\mathcal{R}$ is a relationship between the ontologies $\mathcal{O}_1$ and $\mathcal{O}_2$ if $\mathcal{R}$ is a relation (or set of relations) between the signatures of $\mathcal{O}_1$ and $\mathcal{O}_2$.*

Definition 2 (Extension) [3] *$\mathcal{R}$ is an* extension relationship *between $\mathcal{O}_1$ and $\mathcal{O}_2$ if the signature and axioms of $\mathcal{O}_1$ are included in the ones of $\mathcal{O}_2$.*

Definition 3 (Mapping) [6] *$\mathcal{R}$ is a* mapping relationship *between $\mathcal{O}_1$ and $\mathcal{O}_2$ if there exists a set $\mathcal{RA}$ of axioms that have one of the following forms:*

$$C \sqsubseteq D \quad C \equiv D \quad C \sqcap D \sqsubseteq \bot \quad D(a) \quad \{a\} \equiv \{b\}$$

where $C, a \in \mathcal{O}_1$ and $D, b \in \mathcal{O}_2$.

Definition 4 (Link) [4] *$\mathcal{R}$ is a* link relationship *between $\mathcal{O}_1$ and $\mathcal{O}_2$ if:*

1. *there exists a set $\mathcal{RL}$ of new roles called* linking roles. *Given $\mathcal{RL}$, we define the languages $\mathcal{C}_1$ and $\mathcal{C}_2$ of concepts generated from $\mathcal{RL}$ by simmultaneous induction. The rules for $\mathcal{C}_1$ are as follows.*

   (a) *Any basic concept of $\mathcal{O}_1$ is in $\mathcal{C}_1$,*
   (b) *$\mathcal{C}_1$ is closed under $\sqcup$, $\neg$,*
   (c) *If $C \in \mathcal{C}_1$ and $R \in \mathcal{O}_1$ then $\exists R.C$, and $\geq nR.C$ belong to $\mathcal{C}_1$,*
   (d) *If $C \in \mathcal{C}_2$ and $L \in \mathcal{RL}$ then $\exists L.C$, and $\geq nL.C$ belong to $\mathcal{C}_1$.*

   The rules for $\mathcal{C}_2$ are as follows.

   (a) *Any basic concept of $\mathcal{O}_2$ is in $\mathcal{C}_2$,*
   (b) *$\mathcal{C}_2$ is closed under $\sqcup$, $\neg$,*
   (c) *If $C \in \mathcal{C}_2$ and $R \in \mathcal{O}_2$ then $\exists R.C$, and $\geq nR.C$ belong to $\mathcal{C}_2$,*
   (d) *If $C \in \mathcal{C}_1$ and $L \in \mathcal{RL}$ then $\exists L^{-1}.C$, and $\geq nL^{-1}.C$ belong to $\mathcal{C}_2$.*

2. *there exists a set $\mathcal{RA}$ of axioms that have one of the following forms:*

$$C_1 \sqsubseteq D_1 \quad C_2 \sqsubseteq D_2 \quad C_1(a_1) \quad C_2(a_2) \quad L(a_1, a_2) \quad L^{-1}(a_2, a_1)$$

   where all C_1, D_1, a_1 belong to $\mathcal{C}_1$, C_2, D_2, a_2 belong to $\mathcal{C}_2$ and L is a linking role belonging to $\mathcal{RL}$.

Definition 5 (Metamodeling) *We say that $\mathcal{R}$ is a* metamodelling relationship *between $\mathcal{O}_1$ and $\mathcal{O}_2$ if there exists a partial function* m *from the set of individuals of $\mathcal{O}_1$ to the set of atomic concepts and roles (primitives and defined) of $\mathcal{O}_2$. The set $\mathcal{RA}$ for metamodelling relationships is the empty set.*

Figure 1 illustrates *link* and *metamodelling* relationships.

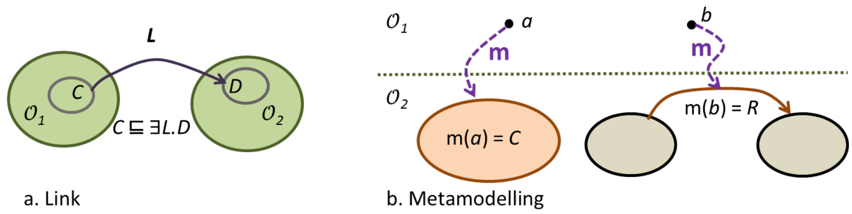

Figure 1. Link and Metamodelling Relationships

Definition 6 (Ontology Network) *An* ontology network *is a pair* $(\mathbb{O}, \mathbb{R})$ *such that* $\mathbb{O} = \{\mathcal{O}_1, \ldots, \mathcal{O}_n\}$ *is a set of ontologies and* $\mathbb{R} = \{\mathcal{R}_1, \ldots, \mathcal{R}_m\}$ *is a set of ontology relationships between them. If we name* m_i *the partial function associated to a metamodelling relationship* $\mathcal{R}_i$ *and there exist* m_i*,* m_j *such that* $a \in dom(\mathsf{m}_i)$ *and* $a \in dom(\mathsf{m}_j)$*, (i) if* $\mathsf{m}_i(a)$ *belongs to the set of atomic concepts* (AC) *then* $\mathsf{m}_j(a)$ *also belongs to it and (ii) if* $\mathsf{m}_i(a)$ *belongs to the set of atomic roles* (AR) *then* $\mathsf{m}_j(a)$ *also belongs to it. The ontology associated to an ontology network is denoted by* $\mathsf{Ont}(\mathbb{O}, \mathbb{R})$ *and defined as* $\bigcup_{i=1}^{n} \mathcal{O}_i \;\cup\; \bigcup_{i=1}^{m} \mathcal{RA}_i$ *where* $\mathcal{RA}_i$ *are the set of DL axioms associated to the relationship* $\mathcal{R}_i$ *for all* $1 \leq i \leq m$*.*

The *semantics* of a DL is based on *interpretations* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where the *domain* $\Delta^{\mathcal{I}}$ is a non-empty set, and $\cdot^{\mathcal{I}}$ is the *interpretation function* [13]. That is, the semantics of a single ontology is defined in terms of a single interpretation domain $\Delta^{\mathcal{I}}$. When more than one ontology interact through different relationships, as in a ontology network, regarding their semantics, there are two main approaches already mentioned in the introduction:"Modular Ontology Languages" (MOL) [2] and "Modular Reuse of Ontologies" [9]. In the former each ontology or "module" is represented through a local language and a local semantics. The meaning of local symbols is interpreted within each ontology, whereas the meaning of symbols which are external to an ontology is given by a special semantics, specific for each formalism in this approach. That is, a different interpretation domain is considered for each ontology in the network. Unlike this, the "Modular Reuse of Ontologies" approach specifies a single domain of interpretation for the union of all ontology axioms, that is, a single domain for the ontology network. The discussion about which approach to take is not the focus of the present work, but we chose the second one because the "Modular Reuse of Ontologies" approach is based on the standard DL formalism without defining a special syntax and the specification of a single interpretation domain appears as a simpler approach, which seems the right when a developer combines different ontologies to describe a particular application.

Having adopted the approach of a single interpretation domain, we will define the semantics of a ontology network taking into account the relationships: *mapping*, *link*, *extension* and *metamodelling*. For the last one, in Definition 6 we established that an individual a in an ontology $\mathcal{O}_1$ has a corresponding concept or role $\mathsf{m}(a)$ in another ontology $\mathcal{O}_2$. This means that, in the ontology network, the individual a must be interpreted as a set of domain elements, the interpretation of the concept $\mathsf{m}(a)$, or as a binary relation, the interpretation of the role $\mathsf{m}(a)$, respectively. Then, to support this new semantics, we redefine the interpretation domain. Before giving a formal definition of it, we introduce the idea through a very simple example.

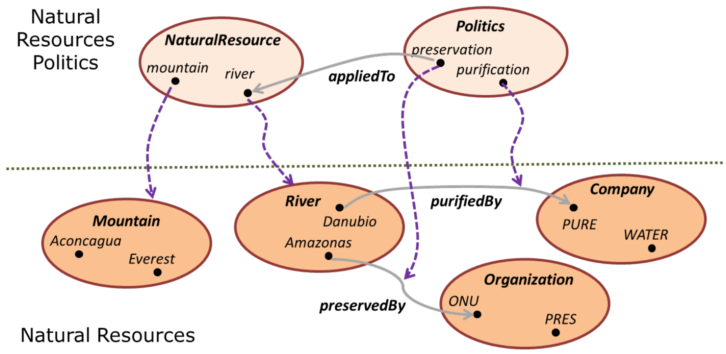

Figure 2. Metamodelling Example

Figure 2 shows an ontology network about politics over natural resources. The ontology "Natural Resources Politics" conceptualize the politics (preservation, purification) applied to different natural resources (mountains, rivers). The ontology "Natural Resources" conceptualize at a lower abstraction level the existing natural resources, and moreover companies and organizations that implement the particular politics. So at an upper abstraction level, mountains and rivers are individuals, while they are concepts at a lower level. Moreover, preservation and purification are individuals in the ontology "Natural Resources Politics", corresponding to roles (preservedBy, purifiedBy) in the ontology "Natural Resources". Then, in this example our interpretation domain is composed by: (i) atomic elements: $Aconcagua$, $Everest$, $Danubio$, $Amazonas$, ONU, $PRES$, $PURE$, $WATER$), (ii) sets of elements: $Mountain = \{Aconcagua, Everest\}$ and $River = \{Danubio, Amazonas\}$ corresponding to the individuals $mountain$ and $river$ in the upper level and (iii) binary relations of sets of elements: $purifiedBy = \{\langle Danubio, PURE\rangle\}$, $preservedBy = \{\langle Amazonas, ONU\rangle\}$ corresponding to the individuals $purification$ and $preservation$ in the upper level.

First we assume at ground level that we have a set $\Delta_0^{\mathcal{I}}$ for our interpretation domain $\Delta^{\mathcal{I}}$ that contains some atomic objects. Then, for the first level of metamodelling we introduce $\Delta_1^{\mathcal{I}}$ as the set that besides containing all elements in $\Delta_0^{\mathcal{I}}$ it also contains all subsets and relations on $\Delta_0^{\mathcal{I}}$.

Definition 7 (Ontology Network Domain of Interpretation) *Given a non empty set* $\Delta_0^{\mathcal{I}}$ *of atomic objects, we define the* ontology network domain $\Delta^{\mathcal{I}}$ of interpretation *as follows:*

$\Delta^{\mathcal{I}} = \bigcup_{n>0} \Delta_n^{\mathcal{I}}$, *where* $\Delta_n^{\mathcal{I}}$ *is inductively defined as:*

$$\Delta_n^{\mathcal{I}} = \Delta_{n-1}^{\mathcal{I}} \cup \mathcal{P}(\Delta_{n-1}^{\mathcal{I}}) \cup \mathcal{P}(\Delta_{n-1}^{\mathcal{I}} \times \Delta_{n-1}^{\mathcal{I}})$$

From Definition 7, we can see that the domain of a ontology network is a well-founded set. [1]

Figure 3 shows a concrete example of the defined domain of interpretation.

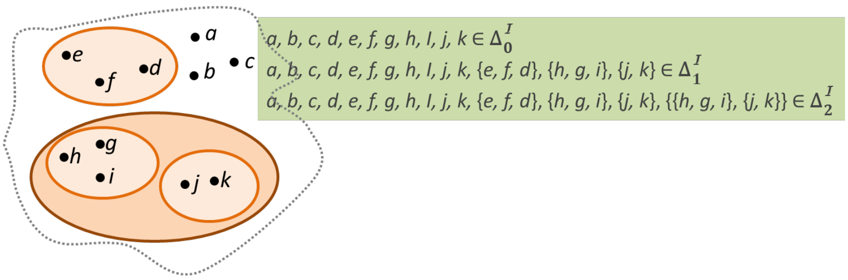

Figure 3. Domain of Interpretation - Example

Definition 8 (Ontology Network Interpretation) *An* ontology network Interpretation $\mathcal{I}$ *is a pair* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$*, where* $\Delta^{\mathcal{I}}$ *is a ontology network domain of interpretation, and* $\cdot^{\mathcal{I}}$ *is the* interpretation function *that assigns:*

- *to every concept* A *a subset* $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
- *to every role* R *a subset* $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- *to every individual* a *an element* $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$

In the usual way, the interpretation function $\cdot^{\mathcal{I}}$ *is extended to complex concepts and roles via DL-constructors, see* [14].

Considering standard DL [13,14], $\mathcal{I}$ is a *model* if it satisfies all standard DL axioms in the ontology network, which are basically the TBox axioms of concept subsumption, $C \sqsubseteq D$, and ABox axioms of concept and role assertions, $C(a)$ and $R(a, b)$. But now we also have metamodelling relationships, in which an instance corresponds to a concept or a role. Then, the instance interpretation will coincide with the concept or role interpretation. This lead to the following definition of model of a ontology network.

Definition 9 (Model of a Ontology Network) *An interpretation* $\mathcal{I}$ *of a ontology network is a* model *if the following holds:*

1. $\mathcal{I}$ *is a model of* $\mathsf{Ont}(\mathbb{O}, \mathbb{R})$ *where* $\mathsf{Ont}(\mathbb{O}, \mathbb{R})$ *is the ontology associated to the ontology network without the metamodelling.*
2. *Moreover, for all* $1 \leq i \leq m$*, if* $\mathcal{R}_i$ *is a metamodelling relationship given by a partial function* m*, for all* $a \in dom(\mathsf{m})$*:*

   a. $a^{\mathcal{I}} = C^{\mathcal{I}}$ *when* $\mathsf{m}(a) = C$, $C \in AC$
   b. $a^{\mathcal{I}} = R^{\mathcal{I}}$ *when* $\mathsf{m}(a) = R$, $C \in AR$

[1] A relation S is well-founded if every non-empty subset S' has a minimal element. In set theory, a set X is called a well-founded set if the set membership relation is well-founded on the transitive closure of X.

Definition 10 (Consistency of an Ontology Network) *We say that an ontology network* $(\mathbb{O}, \mathbb{R})$ *is* consistent *if there exists a model of* $(\mathbb{O}, \mathbb{R})$.

The first part of Definition 9 refers to a model which satisfy the union of the axioms of all ontologies in the network plus the axioms expressing the relationships: *mapping*, *link* and *extension*, which are expressed in standard DL. In the second part of the definition, we add another condition that the model must satisfy considering the metamodelling relationships. This condition restricts the interpretation of an individual that has a corresponding concept or role in a metamodelling relationship to be equal to the concept or role interpretation. In the example of Figure 2 we have a metamodelling relationship with the following correspondences:

$\mathsf{m}(mountain) = Mountain$
$\mathsf{m}(river) = River$
$\mathsf{m}(preservation) = preservedBy$
$\mathsf{m}(purification) = purifiedBy$

The correspoding interpretation of the individuals which belong to the domain of the partial function m is:

$mountain^{\mathcal{I}} = Mountain^{\mathcal{I}} = \{Aconcagua, Everest\}$
$river^{\mathcal{I}} = River^{\mathcal{I}} = \{Amazonas, Danubio\}$
$preservation^{\mathcal{I}} = preservedBy^{\mathcal{I}} = \{\langle Amazonas, ONU\rangle\}$
$purification^{\mathcal{I}} = purifiedBy^{\mathcal{I}} = \{\langle Danubio, PURE\rangle\}$

If we also had the axioms $mountain = river$ and $Mountain \sqcap River \sqsubseteq \bot$, and there was not a metamodelling relationship, the ontology $\mathsf{Ont}(\mathbb{O}, \mathbb{R})$ associated to the ontology network would be consistent. On the other hand, if we have the metamodelling relationship given by $\mathsf{m}(mountain) = Mountain$, $\mathsf{m}(river) = River$, the ontology $\mathsf{Ont}(\mathbb{O}, \mathbb{R})$ is no longer consistent.

We extend the notion of logical consequence to ontology networks in the obvious way [14].

Definition 11 (Logical Consequence) *We say that* $\mathcal{J}$ *is a* logical consequence of $(\mathbb{O}, \mathbb{R})$ *(denoted as* $(\mathbb{O}, \mathbb{R}) \models \mathcal{J}$*) if all models of* $(\mathbb{O}, \mathbb{R})$ *are also models of* $\mathcal{J}$ *where* $\mathcal{J}$ *may be any typical DL judgement (depending on the DL language of choice) such as:* $C \sqsubseteq D$, $a \in C$, $R \sqsubseteq S$ *or* $\langle a, b\rangle \in R$.

2. Metamodelling coexisting with other Relationships

Given the definition of model of a ontology network, which take into account the existence of metamodelling relationships, we pose two questions: (i) is it possible to infer new knowledge in the ontology network, from the metamodelling relationships? (ii) are the standard mechanisms of reasoning enough when there exist metamodelling relationships?

The answer to the first question is obviously positive and shown by the previous example where $River \equiv Mountain$ is a semantic consequence of the ontology network only when we take into account the metamodelling relationships.

The standard services of reasoning provided for DL make it possible to check the consistency of a knowledge base and to infer new axioms that are not explicitly declared. For the first part of the Definition 9 a model can be obtained using the standard mechanisms of reasoning, which comprise all axioms expressed in DL, so including the relationships *mapping*, *link* and *extension*. However, the Tableau algorithm does not consider the metamodelling relationships, that is, does not "know" that the interpretation of an individual in one ontology coincides with the interpretation of a concept or role in another ontology, because of a metamodelling relationship.

Then, in order to try to answer the second question, we will extend the ontology $\mathsf{Ont}(\mathbb{O}, \mathbb{R})$, which is the ontology associated to the ontology network $(\mathbb{O}, \mathbb{R})$ without metamodelling, with new axioms. At first, the new extended ontology $\mathsf{Ont}^*(\mathbb{O}, \mathbb{R})$ is set equal to $\mathsf{Ont}(\mathbb{O}, \mathbb{R})$, and then we extend it as follows [2].

1. If $\mathsf{Ont}^*(\mathbb{O}, \mathbb{R}) \models a = b$ and we have the metamodelling relationships $\mathsf{m}_i(a) = X$ and $\mathsf{m}_j(b) = Y$, with a and b individuals, X and Y both atomic concepts or roles, and does not exists a TBox axiom $X \equiv Y$, we add this axiom to $\mathsf{Ont}^*(\mathbb{O}, \mathbb{R})$.
2. If $\mathsf{Ont}^*(\mathbb{O}, \mathbb{R}) \models X \equiv Y$ and we have the metamodelling relationships $\mathsf{m}_i(a) = X$ and $\mathsf{m}_j(b) = Y$, with X and Y both atomic concepts or roles, and does not exist an ABox axiom $a = b$, we add this axiom.

In the above rules, we execute a DL-reasoner to obtain the entailments of the form: $X \equiv Y$ and $a = b$, for X and Y both atomic concepts or roles, a and b both individuals. These rules have to be applied several times for each entailment of the form $X \equiv Y$ and $a = b$ until no more rules 1 and 2 can be applied. This process always terminate since we are considering an ontology network with a finite set of atomic concepts, atomic roles and individuals. In spite of the fact that the set of entailments may increase for we are adding new axioms each time, from some point on it should stabilize.

We consider the example of Figure 2 with the axioms $mountain = river$ and $Mountain \sqcap River \sqsubseteq \bot$. Using the rule 1, the axiom $Mountain \equiv River$ is added. Now we can apply any DL-reasoner to the extended ontology and this will return that the new knowledge base is inconsistent.

We also show an example where the set of inferences of the form $a = b$ or $X = Y$ increases and new rules need to be applied that were not visible before having those new inferences. For this, we consider the example of Figure 2 with the axiom $Mountain = River$ and a functional property $hasPolitics$ such

[2] Similar inference rules as the ones in Section 2 appear in Jekjantuk et al. [11], who analized metamodelling in a single ontology and defined the interpretation domain fragmented in layers.

that $hasPolitics(mountain, preservation)$ and $hasPolitics(river, purification)$. We apply first rule 2 and add that $mountain = river$ to the ontology. Now, since $hasPolitics$ is functional, we have a new inference that we did not have before, which is $preservation = purification$. Then, we apply rule 1 and add that $purifiedby = preservedby$.

The following lemma is very easy to prove:

Lemma 1 *If* $(\mathbb{O}, \mathbb{R})$ *is consistent then so is* $\mathsf{Ont}^*(\mathbb{O}, \mathbb{R})$.

However, the converse does not hold as the following counterexample shows.

We add a mapping relationship to the ontology network of Figure 2:

$\mathcal{RA}_2 = \{NaturalResource \sqsubseteq Mountain\}$

It is easy to see that $\mathsf{Ont}^*(\mathbb{O}, \mathbb{R})$ is consistent. However, $(\mathbb{O}, \mathbb{R})$ is inconsistent because now for any model $\mathcal{I}$ of $(\mathbb{O}, \mathbb{R})$ we have that:

$$Mountain^{\mathcal{I}} = mountain^{\mathcal{I}} \in NaturalResource^{\mathcal{I}} \subseteq Mountain^{\mathcal{I}}$$

That is, the set $Mountain^{\mathcal{I}}$ is a non well-founded set, since belongs to itself. This contradicts one of our basic prerequisite for being a model of an ontology network: *the domain of the interpretation should be well-founded* (see Definition 7).

The following lemma is also easy to prove:

Lemma 2 *If* $\mathsf{Ont}^*(\mathbb{O}, \mathbb{R}) \models \mathcal{J}$ *then* $(\mathbb{O}, \mathbb{R}) \models \mathcal{J}$.

However, the converse does not hold. To see this, we can apply the counterexample given after the previous lemma.

3. Stratified Ontology Network

In order to avoid interpretations that have sets with cyclic definitions, we have introduced the notion of domain of an ontology network (Definition 7) which is well-founded. However, we think that in order to ensure that our ontology network has a sensible design we need to require a stronger condition on our sets. For this, we define the notion of stratified set.

Definition 12 (Meta Membership) *Let X, Y be sets. We define that X is* a meta-member *of Y (denoted as $X \in_M Y$) by induction as follows:*

1. $X \in_M Y$ *if* $X \in Y$*;*
2. $X \in_M Y$ *if* $\langle X, Z\rangle \in Y$ *or* $\langle Z, X\rangle \in Y$*;*
3. $X \in_M Y$ *if there exists Z such that* $X \in_M Z$ *and* $Z \in_M Y$*,*

For example, $river \in_M Y = \{\{\{river\}\}\}$.

Note that the above definition can be applied to relations and pairs. If $\langle a, b\rangle \in R$ and $\langle R, b\rangle \in S$ then $\langle a, b\rangle \in_M S$ where R and S are binary relations on sets.

Definition 13 (Stratified set) *We say that a set X is* stratified *if for all x, $y \in X$, we have that $x \notin_M y$.*

Note that $A^{\mathcal{I}} = \{Peter, \{Simon\}\}$ is a stratified set. This situation arises in a ontology that has a concept with two elements and only one element has a meta-modelling.

In the example of Figure 3, we have that:
$\{a, b, c, d, e, f, g, h, i, j, k\}$ and $\{a, b, e, f, \{\{h, g, i\}, \{j, k\}\}\}$ are stratified sets, but $\{a, e, f, i, k, \{e, f, g\}, \{j, k\}\}$ and $\{a, b, e, f, h, k, \{\{h, g, i\}, \{j, k\}\}\}$ are not stratified sets.

Definition 14 (Stratified Interpretation) *An interpretation $\mathcal{I}$ of $(\mathbb{O}, \mathbb{R})$ is* stratified *if for atomic concepts A, atomic roles R and individuals a, we have that $A^{\mathcal{I}}$ is a stratified set, $R^{\mathcal{I}} \subseteq X \times X$ for some stratified set X, and if $a^{\mathcal{I}}$ is not an atomic object then it is a stratified set or if $a^{\mathcal{I}}$ is a relation then $a^{\mathcal{I}} \subseteq X \times X$ for some stratified set X.*

Definition 15 (Stratified Ontology Network) *An ontology network $(\mathbb{O}, \mathbb{R})$ is* stratified *if there exists a model of $(\mathbb{O}, \mathbb{R})$ which is a stratified interpretation.*

Now we add a different mapping relationship to the example of Figure 2:

$\mathcal{RA}_3 = \{Mountain \sqsubseteq NaturalResource\}$

None of the models of this ontology network is stratified. To see this, suppose we have a model $\mathcal{I}$ of this ontology network then:

$$Aconcagua^{\mathcal{I}} \in Mountain^{\mathcal{I}} \subseteq NaturalResource^{\mathcal{I}}$$
$$Mountain^{\mathcal{I}} \in NaturalResource^{\mathcal{I}}$$

In the above, there is nothing that contradicts the condition of well-foundness. However, $NaturalResource^{\mathcal{I}}$ is not a stratified set since it contains two elements $Aconcagua$ and $Mountain^{\mathcal{I}}$ where the first element belongs to the second one.

We add the following link relationship in the ontology network of Figure 2:

$\mathcal{RA}_4 = \{handledBy(purification, PURE)\}$ where $handledBy$ is a new role.

None of the models of this ontology network is stratified. This is because in any model $\mathcal{I}$ of this ontology network we have that if $handledBy^{\mathcal{I}} \subseteq X \times X$ then X is not a stratified set since $purification^{\mathcal{I}}$ and $PURE^{\mathcal{I}}$ should belong to X, but $PURE^{\mathcal{I}} \in_M purification^{\mathcal{I}}$.

4. Related Work

Up to our knowledge, the works that address metamodelling in depth, consider the issue for a single ontology. De Giacomo et al. [12] specifies a new formalism, "Higher/Order Description Logics", that allows to treat the same symbol of the signature as an instance, a concept and a role. With respect to the semantics, in principle they associate a domain element to each symbol of the signature, and then, if it is treated as a concept or a role, a set of domain elements or a binary relation is also associated to the symbol through a pair of functions $\mathcal{I}_C$ and $\mathcal{I}_R$. Regarding reasoning, given a DL $\mathcal{L}$ they define a "high-order" version of it, $\mathcal{H}i(\mathcal{L})$, mapping the same symbol of the signature to three different symbols, which represent an instance, a concept or a role. This makes it possible to use the standard mechanisms of reasoning for the DL $\mathcal{L}$, which treat the three new symbols as independent elements. Unlike this, in our approach we define metamodelling between two different ontologies, keeping the individual in one ontology and the concept or role in the other one as different symbols of the signature.

In [11], metamodelling is addressed defining different "layers" or "stratums" within a knowledge base, in such a way that instances in each layer belong to the lower layer. They propose an algorithm to infer new axioms that arise from metamodelling, but do not allow axioms different from metamodelling involving elements of different layers. So, the problem of coexistence of metamodelling and other relations among different layers is not addressed.

Glimm et al. [15] define two layers within the knowledge base, the "instance layer" and the "metalayer". Then, they add special roles to map concepts and roles in the model ("instance layer") to instances in the metamodel ("metalayer"), as well as additional axioms to constrain the roles being introduced. That is, they only consider two layers and, although they study the coexistence of metamodelling and other axioms, they introduce additional elements in order to represent metamodelling through DL and do not use another formalism.

5. Conclusion and Future Work

In the present work we study the metamodelling relationship in the context of a ontology network, when there are other relationships such as mapping, link or extension along with metamodelling relationships. We specify a semantics for the ontology network, redefining the domain of interpretation in such a way the interpretation of an individual can coincide with that of a concept or a role. Moreover, in our metamodelling semantics definition, we associate the same interpretation to both symbols, since as we explained through examples, it is important to "know" that they are the same domain element. If not, when they are in certain axioms combined with other elements, an ontology network that is consistent without metamodelling can becomes inconsistent.

We know that metamodelling combined with expressive DL can become undecidable [16]. We plan to study the problem that checks whether an ontology network is stratified or not. We think that studying metamodelling in the context of ontology networks is an interesting and very challenging issue.

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