A Personal Financial Planning Model Based on Fuzzy Multiple Goals Programming Method

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Abstract

Traditional financial planning procedures began from taking into account the planner's initial financial situation, his/her financial goals, and expectations for the future, etc. and then calculating the future cash flows for different time periods under variant scenarios. If the planning result cannot meet the planner's expectation, then the planner has to adjust the tunable parameters continuously until obtaining an acceptable financial arrangement. Such a "trial-and-error" or so-called "what-if analysis" method does not promise to achieve an optimal planning result, and cannot afford to analyze how the financial plan will be modified when the parameters change. This paper proposes a generalized personal financial planning programming model with fuzzy multiple goals to solve the personal financial planning problem under a different way compared to the traditional methodology.

Keywords: personal financial planning, mathematical programming, fuzzy multiple goals programming.

1. Introduction

Personal financial planning, in regards to the wealth holder as the decision center, tries to manage all money activities during a person's lifetime, including maximizing one's wealth, satisfying one's life goals, and managing different sources of risk. Financial planning begins by measuring personal financial statements which could require that the planner provide financial data, his/her life goals, risk preferences, etc. By trial calculation under different scenarios, the planner can make a better decision for the wealth holder, but it might not be the best decision for non-trivial financial actions. Such a "what-if" analysis might present the following problems.

- Solving the financial planning problem by a "trial-and-error" method might obtain a satisfied suggestion, but may not achieve the best decision in the solution space. Even some numerical analysis methods, for example, goal seeking, could be applied to find the best decision for single decision variable (Crabb, 2003). However, because financial planning includes many different dimensions and decision variables, a traditional tuning method is unable to cope with realistic situations.
- The regulation of possible financial plans might consider multiple goals, but traditional "what-if" analysis cannot handle it.
- Planners might possess various preference structures for different objectives.
- For the achieved level of each specific financial goal, it is not always a crisp binary state, rather a fuzzy continuous space.

As Fortin(1997) stated, the solution to financial planning might not be a closed-form solution. To resolve such difficulties, a mathematical programming method could be applied to meet the essence of planning, which is the main motivation for this research. By applying the mathematical programming method, the above problems could be solved.

This paper selects an integrated financial planning model quoted from[1] as an illustrative example to develop a general financial planning model. In this example, the decision variables include salary, living spending, cost of purchasing a home, raising children, and education expenses, etc. The financial planning mathematical model can cope with the aforementioned problems.

2. Numerical Example of Personal Financial Planning

A numerical example is provided as follows.

<u>*Example 1:(quoted from [1])*</u>
Mr. Chiang, the planner, is 30 years old and his wife is 28 years old. They plan to have their first child after 2 years and have another one after 5 years. Yearly revenue for Mr. Chiang is about NT$600,000, and for Mrs. Chiang it is NT$400,000. Yearly living expenses

for this couple are NT$450,000 and they pay NT$200,000 to rent their home every year before they own their house. They own NT$500,000 in investments. The estimated increasing rates for yearly revenue, living expenses, and home rent are all 5%. Each child will increase living expenses by 25%, but when their child graduates from university at age 22, their living expenses will be cut down by about 20% of their total living expenses. Mr. Chiang wishes they can buy their own house, at a costing of about NT$8,000,000 when he is 35 years old. He will prepare NT$3,000,000 for a down-payment and pay the deficit through mortgage payments. The amortization schedule will last for 20 years. Both of them plan to retire when they are 60 years old. The retirement payment will be double of their yearly salary when they retire. At present, the education expense for entering a university is NT$100,000 per year, but it will increase by 7% every year. The expected return of investment for their capital now is 12% and the loan rate is 8%.

Using spreadsheet software, e.g. Microsoft Excel, we can calculate the yearly cash flows, including revenue, various expenses, and accumulated capital, etc., for this family. The traditional procedure of personal financial planning tries to obtain an acceptable, which may not be the optimal, results by tuning the decision variables. For example, when the rate of return on investment, which cannot be determined by the planner, does not meet the original setting, for example, from 12% down to 6%, and thus the planner should be in debt after 5 years. What should he do to avoid this situation? He might reduce living expenses, earn more money, or delay the timing to purchase his own house, etc. By the “trial-and-error” process, i.e. the “what-if” analysis, the planner might find the optimal decision by tuning a single variable. However, it is hard to consider all trade-offs among all related decision variables, without taking the preference structure into account.

A reasonable approach for solving the above problem is by adapting a mathematical programming method. Considering the essence of the problem, multiple goals and fuzzy characteristics should be tackled in this model.

3. Fuzzy Multiple Goal Programming Model for Personal Financial Planning

The personal financial planning model developed in this section considers: (i) income of salaries and investments of the couple, (ii) the expenses of living, renting a home, purchasing a home, raising children, and education.

Assume the planning duration is t_m and the ages of the planner and his wife at year t are a_t^p and a_t^c at different time periods, respectively. Terms a_0^p and a_0^c represent the ages of the planner at period 0. If the couple wishes to have their first baby after t years, the age of their baby can be represented by $a_0^{k_1} = -t$. Since the age will grow with the planning period, we have:

$$a_{t+1}^p = a_t^p + 1, \quad for \quad t = 0,...,t_m \tag{1}$$

$$a_{t+1}^c = a_t^c + 1, \quad for \quad t = 0,...,t_m \tag{2}$$

$$a_{t+1}^{k_n} = a_t^{k_n} + 1, \quad for \quad n = 1,2 \quad ; t = 0,...,t_m \tag{3}$$

Assume the salaries of the planner and his wife in the initial stage are s_0^p and s_0^c, respectively, and they have a growth rate of r_s until they retire. The retirement age is planned to be t_j and the retirement pension depends on the salary before retirement. From the above data, the yearly salaries of the couple are:

$$s_t^p = s_0^p(1+r_s)^t, \quad for \quad t = 0,...,t_j - a_0^p - 1 \tag{4}$$

$$s_t^p = qs_{t-1}^p, \quad for \quad t = t_j - a_0^p \tag{5}$$

$$s_t^p = 0, \quad for \quad t = t_j - a_0^p + 1,...,t_m \tag{6}$$

$$s_t^c = s_0^c(1+r_s)^t, \quad for \quad t = 0,...,t_j - a_0^c - 1 \tag{7}$$

$$s_t^c = qs_{t-1}^c, \quad for \quad t = t_j - a_0^c \tag{8}$$

$$s_t^c = 0, \quad for \quad t = t_j - a_0^c + 1,...,t_m \tag{9}$$

As to the investment income, s_0^f represents such income at the initial period and it increases every year by r_i. The rate of return on investment at year t will be:

$$s_{t+1}^f = c_t(1+r_i), \quad for \quad t = 0,...,t_m \tag{10}$$

In the begin of the planning, living expenses amount to e_0^l, which keeps a growth rate of r_l in subsequent periods. The upraising for each child will increase the living expenses by a growth rate of r_k until the child has graduated from university at age 22. After that the living expenses will decrease to be r_g percentage of the living expenses:

$$e_t^l = e_0^l(1+r_l)^t, \quad \forall t| \quad a_t^{k_1} < 0, \quad a_t^{k_2} < 0 \tag{11}$$

$$e_t^l = e_0^l(1+r_l)^t(1+r_k), \quad \forall t| \quad 0 \le a_t^{k_1} \le 22, \quad a_t^{k_2} < 0 \tag{12}$$

$$e_t^l = e_0^l(1+r_l)^t(1+r_k)^2, \quad \forall t| \quad 0 \le a_t^{k_1} \le 22, \quad 0 \le a_t^{k_2} \le 22 \tag{13}$$

$$e_t^l = e_0^l(1+r_l)^t(1+r_k)^2 r_g, \quad \forall t| \quad a_t^{k_1} \ge 23, \quad 0 \le a_t^{k_2} \le 22 \tag{14}$$

$$e_t^l = e_0^l(1+r_l)^t(1+r_k)^2 r_g^2, \quad \forall t| \quad a_t^{k_1} \ge 23, \quad a_t^{k_2} \ge 23 \tag{15}$$

In the initial stage, the education fee for entering a university is e_0^u and maintains a growth rate of r_u for each year:

$$e_t^{u_n} = e_0^u(1+r_u)^t, \quad for \quad t = 19 - a_0^{k_n},...,22 - a_0^{k_n}, \quad n = 1,2 \tag{16}$$

Before the planner has bought a house, they shall pay rental fees which amount to h_0^x and have a growth rate of r_h for each year. Assume that after t_h years the planner buys a home valued at h^w and pays h^b for the down-payment, taking a mortgage payment amounting to h^b for t_d years at a mortgage rate of r_d. The yearly mortgage payment for the home can then be calculated to be e_t^d.

$$h_t^x = h_0^x(1+r_x)^t, \quad for \quad t = 0,...t_h \tag{17}$$

$$h_t^x = 0, \quad for \quad t = t_h + 1,...,t_m \tag{18}$$

$$h^d = h^w - h^b \tag{19}$$

$$p = \frac{h^w - h^b}{h^w} = \frac{h^d}{h^w} \tag{20}$$

$$e_t^b = h^b, \quad for \quad t = t_h \tag{21}$$

$$e_t^d = h^d\left(\frac{r_d(1+r_d)^d}{(1+r_d)^d - 1}\right), \quad for \quad t = t_h + 1,...,t_h + t_d. \tag{22}$$

Summarizing the above incomes and expenses, the yearly balance y_t and accumulated capital used for investment c_t can be calculated as:

$$y_t = (i_t^p + i_t^c + i_t^f) - (e_t^l + e_t^u + h_t^x + e_t^b + e_t^d), \quad for \quad t = 1,...,t_m \tag{23}$$

$$c_{t+1} = c_t + y_{t+1}, \quad for \quad t = 1,...,t_m. \tag{24}$$

By the above equations, a multiple goal personal financial planning model can be developed as:

$$Min \quad w_1|s_0^p - s_0^{p*}| + w_2|e_0^l - e_0^{l*}| + w_3|t_j - t_j^*| + w_4|t_h - t_h^*| + w_5|h^w - h^{w*}| + w_6|p - p^*| + w_7|a_0^{k_1} - a_0^{k_1*}|$$

subject to (1)~(24)

$$s_0^p, e_0^l, a_0^{k_1}, t_h, h^w, p, t_j \geq 0, \tag{25}$$

where w_i represents the weights for different planning goals which can be determined by the preference structure of the planner, s_0^{p*} represents the planner's salary in the initial period, e_0^{l*} represents the planner's living expenses in the initial period, t_j^* represents the planner's expected retirement age, t_h^* represents the timing of buying a home, h^{w*} represents the price of the house, p^* represents the percentage of home payment by a mortgage, $a_0^{k_1^*}$ represents the age for the first child, and $s_0^p, e_0^l, a_0^{k_1}, t_h, h^w, p, t_j$ are decision variables of the model

By the Max-Min model proposed by Zimmermann (1978), the fuzzy multiple goal programming model for personal financial planning can be proposed as:

$$Max \quad \lambda \tag{26}$$

subject to $s_0^{p*} \tilde{\geq} s_0^p \Rightarrow \lambda \leq \frac{f_s - g_s}{l_s - g_s}$

$$e_0^l \tilde{\geq} e_0^{l*} \Rightarrow \lambda \leq \frac{f_e - l_e}{g_e - l_e}$$

$$t_j^* \tilde{\geq} t_j \Rightarrow \lambda \leq \frac{f_{tj} - g_{tj}}{l_{tj} - g_{tj}}$$

$$t_h^* \tilde{\geq} t_h \Rightarrow \lambda \leq \frac{f_{tk} - g_{tk}}{l_{tk} - g_{tk}}$$

$$h^w \tilde{\geq} h^{w*} \Rightarrow \lambda \leq \frac{f_h - l_h}{g_h - l_h}$$

$$p^* \tilde{\geq} p \Rightarrow \lambda \leq \frac{f_p - g_p}{l_p - g_p}$$

$$a_0^{k*} \tilde{\geq} a_0^k \Rightarrow \lambda \leq \frac{f_a - g_a}{l_a - g_a},$$

subject to (1)~(24), and $s_0^p, e_0^l, a_0^{k_1}, t_h, h^w, p, t_j \geq 0$.

where g_i represent the satisfied levels for different goals, and l_i represent the minimal acceptable levels for different goals.

4. Numerical Example Analysis

This section provides a numerical example with two tunable decision variables, living expenses and price of a home, and applies the proposed model from the previous section. Assume that the planner insists that he will never be in debt during the planning duration, i.e., his balance account will always be greater than 0. Since the return on investment is down to 6%, from the analysis in Section 2 we know that the desired levels for living expenses and the home price that are initially NT$450,000 per month and NT$8,000,000, respectively, cannot meet his objective, i.e. no debt during the planning duration. Some compromises must be obviously made. The decision might be to buy a cheaper home or reduce their living expenses.

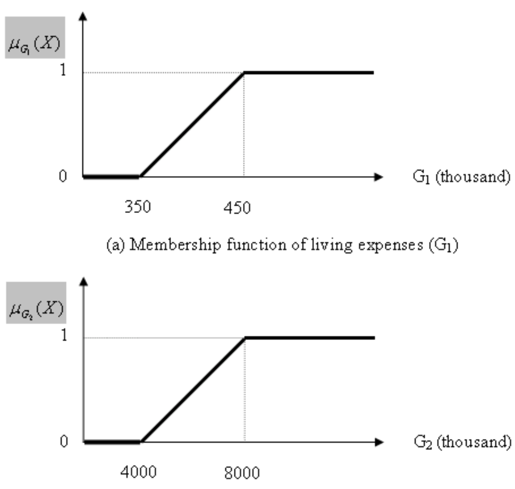
Further assume that the minimal acceptable living expense for the planner is NT$350,000 and the minimum price of a home he will pay is NT$4,000,000. The membership functions of the living expenses and price of the home are depicted as shown in Figure 1.

If only one of the goals is tuned, then by goal-seeking we can achieve the setting goal, i.e. no debt, whereby he should reduce living expenses to be NT$400,000 or buy a house priced at NT$4,920,000, whose satisfaction levels are 0.23 and 0.51, respectively. How can we increase the minimal satisfaction level as much as possible? We apply the model proposed in Section 3, and the resulting maximal satisfaction that the planner can make by trading off between the two objectives is 0.7.

Table 1 shows different scenarios for tuning two goals and their respective λ, i.e. the satisfaction level shown by the membership functions. Figure 2 shows alternative feasible solution spaces and achievement goal values for the two fuzzy goals.

Table 1: Planning results (Unit: NT$1,000)

Expense	340.00	345.00	350.00	355.00	360.00	365.00
House Price	9,900.00	9,801.00	9,696.40	9,564.10	9,431.81	9,299.51
Lamda	-0.10	-0.05	0.00	0.05	0.10	0.15
Expense	370.00	375.00	380.00	385.00	390.00	395.00
House Price	9,167.22	9,034.92	8,902.62	8,770.33	8,637.62	8,364.23
Lamda	0.20	0.25	0.30	0.35	0.40	0.45
Expense	400.81	405.00	410.00	415.00	420.00	425.00
House Price	8,000.00	7,737.37	7,423.94	7,110.51	6,797.08	6,483.66
Lamda	0.51	0.55	0.60	0.65	0.70	0.62
Expense	430.00	435.00	440.00	445.00	450.00	455.00
House Price	6,170.23	5,856.80	5,543.37	5,229.94	4,916.51	4,603.08
Lamda	0.54	0.46	0.39	0.31	0.23	0.15
Expense	460.00	465.00	470.00	475.00	480.00	--
House Price	4,289.65	3,976.23	3,662.80	3,349.37	3,035.94	--
Lamda	0.07	-0.01	-0.08	-0.16	-0.24	--



(a) Membership function of living expenses (G_1)

(b) Membership function of the price of a home (G_2)

Fig. 1: Membership functions of two decision

5. Conclusions and Further Research

The mathematical programming model with fuzzy multiple goals is selected in this paper in order to propose a generalized personal financial planning model which includes the decision variables related to living expenses, revenue, buying a home, education planning, etc. Compared to the traditional "what-if" analysis used in the personal financial planning, the proposed method finds the optimal solution among different conflicting goals and tackles the fuzzy characteristics of the goals.

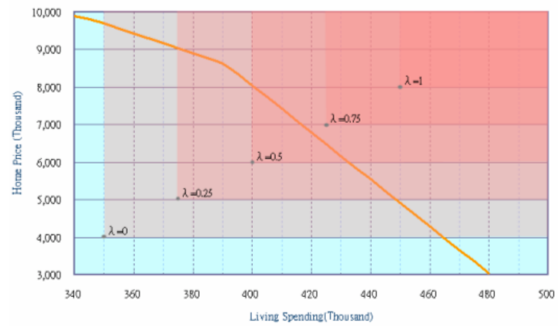


Fig. 2: Alternative feasible solution spaces and achievement goal values for two fuzzy goals

Based upon the model in this work, the following research directions can be suggested.

- Since the proposed framework is a non-linear mathematical programming model, a heuristic algorithm can be developed to find the global optimal solution.
- The preference structure among different goals can be considered in the proposed model and the preference weights for the decision maker can be detected by techniques, such as AHP (Analytic Hierarchy Process)(Saaty, 1980).
- More facets of personal financial planning, such as tax planning, insurance planning, estate planning, etc, can be considered in the model.
- Based on the model proposed in this paper, a decision support system can be developed to facilitate personal financial planning.

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