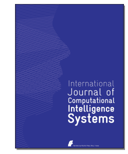
International Journal of Computational Intelligence Systems
Vol. 14(1), 2021, pp. 1653–1671
DOI: https://doi.org/10.2991/ijcis.d.210528.002; ISSN: 1875-6891; eISSN: 1875-6883
https://www.atlantis-press.com/journals/ijcis/



Research Article

Some Cosine Similarity Measures and Distance Measures between Complex q-Rung Orthopair Fuzzy Sets and Their Applications

Peide Liu[1,*], Zeeshan Ali[2], Tahir Mahmood[2]

[1] *School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan, 250015, China*
[2] *Department of Mathematics and Statistics, International Islamic University, Islamabad, 44000, Pakistan*

ARTICLE INFO

Article History
Received 21 Mar 2021
Accepted 25 May 2021

Keywords
Complex q-rung orthopair fuzzy sets
Cosine similarity measures
Cosine distance measures
Technique for an order of preference by similarity to ideal solution

ABSTRACT

As a modification of the q-rung orthopair fuzzy sets (QROFSs), complex QROFSs (CQROFSs) can describe the inaccurate information by complex-valued truth grades with an additional term, named as phase term. Cosine similarity measures (CSMs) and distance measures (DMs) are important tools to verify the grades of discrimination between the two sets. In this manuscript, we develop some CSMs and DMs for CQROFSs. Firstly, the CSMs and Euclidean DMs (EDMs) for CQROFSs and their properties are investigated. Because the CSMs do not keep the axiom of similarity measure (SM), we investigate a technique to develop other SMs based on CQROFSs, and they meet the axiom of the SMs. Moreover, we propose a cosine DM (CDM) based on CQROFSs by considering the interrelationship among the SMs and DMs, then we propose an extended TOPSIS method to solve the multi-attribute decision-making problems. Finally, we provide some sensible examples to demonstrate the practicality and efficiency of the suggested procedure, at the same time, the graphical representations of the developed measures are also utilized in this manuscript.

1. INTRODUCTION

For a real example, when an institute chooses whether to enroll a tutoring team, a ten-representative committee of authorities evaluated the selected persons, seven of them approved to employ these persons, two of them gave negative opinion, and the additional one did not give any judgment. To characterize this result, an intuitionistic fuzzy set (IFS) was presented by Atanassov [1] to express this kind of information by including a falsity grade based on the fuzzy set (FS) [2] The truth and the falsity in IFS meet a rule that the sum of both of them is restricted to [0, 1]. Now IFS has received extensive attentions from many scholars and has been widely utilized in the different decision areas [3–7]. Due to some complications of decision environment, sometimes, it is difficult for IFS to describe some daily life issues, for instance, if a person gives 0.6 for truth grade and 0.5 for falsity, then the sum of both values is beyond the scope of [0, 1], the IFS is not able to express this type of information accurately. Therefore, Yager [8] proposed the Pythagorean FS (PFS) which is a proficient and capable technique to express complex information for the decision-making problems. The truth and falsity in PFS meet a rule that the sum of the squares of them is in [0, 1]. The PFS has been widely utilized in the different decision making areas [9–14]. Similarly, if a person gives 0.9 for truth grade and 0.8 for falsity, then the sum of the squares of both values is not in [0, 1], the PFS is not able to express this type of information accurately. Therefore, Yager [15] proposed the q-rung orthopair FS (QROFS) to solve this issue. The truth and falsity in QROFS meet a rule that the sum of the q-powers of them is restricted to [0, 1]. Now the QROFS has received extensive attentions from many scholars and has been widely utilized in the different areas [16–19].

To process complex fuzzy information, the truth and falsity degrees are modified from a real subset to the unit disc of the complex plane, and then Alkouri and Salleh [20] established the complex IFS (CIFS) by including the complex-valued falsity on the basis of complex FS (CFS) [21] to handle complex information. The truth and falsity in CIFS meet the rule that the sum of the real parts (also for imaginary parts) of them is restricted to [0, 1]. The CIFS has received extensive attentions from many scholars and has been widely utilized in the different areas [22–25]. However, the CIFS is not able to process some problems, for instance, if a person gives $0.6e^{i2\pi(7)}$ for truth grade and $0.5e^{i2\pi(6)}$ for falsity, then the sum of the real parts (also for imaginary parts) of both values is beyond the scope of [0, 1]. Therefore, Ullah *et al.* [26] proposed the complex PFS (CPFS) in which the truth and falsity meet the rule that the sum of the squares of the real parts (also

* *Corresponding author. Email: peide.liu@gmail.com*

for imaginary parts) of them is restricted to [0, 1]. The CPFS has received extensive attentions from scholars and has been widely utilized in the different areas [27]. Similarly, if a person gives $0.9e^{i2\pi(8)}$ for truth grade and $0.8e^{i2\pi(7)}$ for falsity, then the sum of the squares of the real parts (also for imaginary parts) of them is beyond the scope of [0, 1], the CPFS is not able to describe this type of information accurately. Therefore, Liu *et al.* [28,29] proposed the complex QROFS (CQROFS) in which the truth and falsity meet the rule that the sum of the q-powers of the real parts (also for imaginary parts) of them is restricted to [0, 1]. The CQROFS has received extensive attentions from many scholars and has been widely utilized in the different areas [30–36].

In real decision problems, we go over numerous circumstances where we need to measure the vulnerability existing in the information to get one ideal choice. Data measures are significant tools for taking care of uncertain information presented in our day-to-day life issues. Different measures of information, such as similarity, distance, entropy, and inclusion, can process the uncertain information and facilitate us to reach some conclusions. Recently, these measures have gained much attention from many scholars due to their wide applications in various fields, such as pattern recognition, medical diagnosis, clustering analysis, and image segment. All the prevailing approaches of decision-making, based on information measures for PFS and QROFS, can only deal with the real-valued truth and falsity grades. In CQROFS, truth and falsity grades are complex-values and are represented in polar coordinates. The amplitudes corresponding to truth and falsity degrees give the extents of membership and nonmembership of an object in a CQROFS with a rule that the sum of the q-powers of the real and unreal parts of both grades is restricted to the unit interval. The phase parts are novel parameters of the truth and falsity degrees added from traditional QROFS. QROFS can deal with only one dimension at a time, which results in information loss in some instances. However, in real life, we come across complex natural phenomena where only one dimensional information cannot express fully the evaluation value, and the second dimensional information is needed to express the truth and falsity grades. By adding the second dimension, the complete information can be projected in one set, and hence, loss of information can be avoided. To illustrate the significance of the phase term, we give an example. Assume XYZ organization chooses to set up biometric-based participation gadgets (BBPGs) in the entirety of its workplaces spread everywhere in the country. For this, the organization counsels a specialist who gives the data concerning (i) demonstrates of BBPGs and (ii) creation dates of BBPGs. The organization needs to choose the most ideal model of BBPGs with its creation date all the while. Here, this issue is two-dimensional, to be specific, the model of BBPGs and the creation date of BBPGs. This kind of issue cannot be expressed precisely by the conventional QROFS. The most ideal approach to address this problem is by utilizing the CQROFS. The amplitudes in CQROFS might be utilized to give the organization's choice regarding the model of BBPGs and the phase parts might be utilized to address the organization's judgment concerning the creation date of BBPGs.

In addition, cosine similarity is one of the most important measures, which can not only compare one data entity with others but also show the extents of association between them and their direction. Also, CQROFSs have a powerful ability to model the imprecise and ambiguous information in real-world applications than the existing information expressions such as CFSs, CIFSs, CPFSs. Besides, the SM is a valid tool to examine the interrelationships among any number of CQROFSs, and it has been utilized to different areas [34]. Rani and Garg [23] investigated the distance similarity by using CIFS. Garg and Rani [37] proposed some information measures based on CIFS. Garg and Rani [24] developed the robust correlation coefficient based on CIFS. But up to date, the SMs for CQROFSs have not been investigated. Because the CQROFSs are a reliable technique to express complex fuzzy information, and the SM is an important tool for decision-making problems, it is necessary to develop some SMs for CQROFSs. Therefore, keeping the advantages of SMs and CQROFSs, the main investigations of this manuscript are summarized as follows:

1. The cosine similarity measures (CSMs) and Euclidean distance measures (EDMs) for CQROFSs and their properties are investigated.
2. Considering that the CSMs do not meet the axiom of similarity measure (SM), some new SMs based on CQROFSs using the explored CSMs and EDMs are developed, which meets the axiom of the SMs.
3. Cosine DMs (CDMs) based on CQROFSs by considering the interrelationship among the SM and DMs are proposed and an extended TOPSIS method is developed.
4. Some examples are given to demonstrate the practicality and efficiency of the suggested procedure.
5. The graphical representations of the developed measures are also given in this manuscript.

This manuscript is summarized as follows: In Section 2, we briefly recall the concept of CIFSs, CPFSs, CQROFSs, and their fundamental laws. In Section 3, we develop the CSMs and DMs by using CQROFNs. In Section 4, we develop the TOPSIS method based on the investigated measures. In Section 5, we give a comparative analysis of the proposed work with some existing approaches. The conclusion of this manuscript is discussed in Section 6.

2. PRELIMINARIES

In this work, we recall the main ideas of CIFSs, CPFSs, CQROFSs, and their fundamental laws. We use the symbol $\breve{\mho}$ for universal sets and the truth and falsity degrees are shown by $\mathfrak{M}_{\mathfrak{C}_{CQ}}$ and $\mathfrak{N}_{\mathfrak{C}_{CQ}}$, where $\mathfrak{M}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathfrak{M}_{\mathfrak{C}_{RP}}(\breve{\sigma})e^{i2\pi(\mathfrak{M}_{\mathfrak{C}_{IP}}(\breve{\sigma}))}$ and $\mathfrak{N}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathfrak{N}_{\mathfrak{C}_{RP}}(\breve{\sigma})e^{i2\pi(\mathfrak{N}_{\mathfrak{C}_{IP}}(\breve{\sigma}))}$.

Definition 1. [20] A CIFS $\mathfrak{C}_{CQ}$ is demonstrated by

$$\mathfrak{C}_{CQ} = \left\{\left(\mathfrak{M}_{\mathfrak{C}_{CQ}}(\breve{\sigma}), \mathfrak{N}_{\mathfrak{C}_{CQ}}(\breve{\sigma})\right) : \breve{\sigma} \in \breve{O}\right\} \tag{1}$$

where $\mathfrak{M}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathfrak{M}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)}$ and $\mathfrak{N}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathfrak{N}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)}$ express the truth degree and the falsity degree with $0 \leq \mathfrak{M}_{\mathfrak{C}_{RP}}(\breve{\sigma}) + \mathfrak{N}_{\mathfrak{C}_{RP}}(\breve{\sigma}) \leq 1$ and $0 \leq \mathfrak{M}_{\mathfrak{C}_{IP}}(\breve{\sigma}) + \mathfrak{N}_{\mathfrak{C}_{IP}}(\breve{\sigma}) \leq 1$. Moreover, the term $\mathcal{J}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathcal{J}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathcal{J}_{\mathfrak{C}_{IP}}\right)} = \left(1 - \mathfrak{M}_{\mathfrak{C}_{RP}}(\breve{\sigma}) - \mathfrak{N}_{\mathfrak{C}_{RP}}(\breve{\sigma})\right) e^{i2\pi\left(1 - \mathfrak{M}_{\mathfrak{C}_{IP}}(\breve{\sigma}) - \mathfrak{N}_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)}$ expresses the degree of indeterminacy.

Definition 2. [26] A CPFS $\mathfrak{C}_{CQ}$ is demonstrated by

$$\mathfrak{C}_{CQ} = \left\{\left(\mathfrak{M}_{\mathfrak{C}_{CQ}}(\breve{\sigma}), \mathfrak{N}_{\mathfrak{C}_{CQ}}(\breve{\sigma})\right) : \breve{\sigma} \in \breve{O}\right\} \tag{2}$$

where $\mathfrak{M}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathfrak{M}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)}$ and $\mathfrak{N}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathfrak{N}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)}$ express the truth degree and the falsity degree with $0 \leq \mathfrak{M}^2_{\mathfrak{C}_{RP}}(\breve{\sigma}) + \mathfrak{N}^2_{\mathfrak{C}_{RP}}(\breve{\sigma}) \leq 1$ and $0 \leq \mathfrak{M}^2_{\mathfrak{C}_{IP}}(\breve{\sigma}) + \mathfrak{N}^2_{\mathfrak{C}_{IP}}(\breve{\sigma}) \leq 1$. Moreover, the term $\mathcal{J}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathcal{J}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathcal{J}_{\mathfrak{C}_{IP}}\right)} = \begin{pmatrix} 1 - \mathfrak{M}^2_{\mathfrak{C}_{RP}}(\breve{\sigma}) - \\ \mathfrak{N}^2_{\mathfrak{C}_{RP}}(\breve{\sigma}) \end{pmatrix}^{\frac{1}{2}} e^{i2\pi\left(1 - \mathfrak{M}^2_{\mathfrak{C}_{IP}}(\breve{\sigma}) - \mathfrak{N}^2_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)^{\frac{1}{2}}}$ expresses the degree of indeterminacy.

Definition 3. [28,29] A CQROFS $\mathfrak{C}_{CQ}$ is demonstrated by

$$\mathfrak{C}_{CQ} = \left\{\left(\mathfrak{M}_{\mathfrak{C}_{CQ}}(\breve{\sigma}), \mathfrak{N}_{\mathfrak{C}_{CQ}}(\breve{\sigma})\right) : \breve{\sigma} \in \breve{O}\right\} \tag{3}$$

where $\mathfrak{M}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathfrak{M}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)}$ and $\mathfrak{N}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathfrak{N}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)}$ express the truth degree and the falsity degree with $0 \leq \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP}}(\breve{\sigma}) + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP}}(\breve{\sigma}) \leq 1$ and $0 \leq \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP}}(\breve{\sigma}) + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP}}(\breve{\sigma}) \leq 1, q_{CQ} \geq 1$. Moreover, the term $\mathcal{J}_{\mathfrak{C}_{CQ}}(\breve{\sigma}) = \mathcal{J}_{\mathfrak{C}_{RP}}(\breve{\sigma}) e^{i2\pi\left(\mathcal{J}_{\mathfrak{C}_{IP}}\right)} = \begin{pmatrix} 1 - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP}}(\breve{\sigma}) - \\ \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP}}(\breve{\sigma}) \end{pmatrix}^{\frac{1}{q_{CQ}}} e^{i2\pi\left(1 - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP}}(\breve{\sigma}) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP}}(\breve{\sigma})\right)^{\frac{1}{q_{CQ}}}}$ expresses the degree of indeterminacy. Throughout, this manuscript, the complex q-rung orthopair fuzzy numbers (CQROFNs) are shown by $\mathfrak{C}_{CQ} = \left(\mathfrak{M}_{\mathfrak{C}_{RP}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP}}\right)}, \mathfrak{N}_{\mathfrak{C}_{RP}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP}}\right)}\right)$. Further, we define the score and accuracy values such that

$$\mathfrak{S}_{CQ}\left(\mathfrak{C}_{CQ}\right) = \frac{1}{2}\left(\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP}} - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP}} - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP}}\right), \mathfrak{S}_{CQ}\left(\mathfrak{C}_{CQ}\right) \in [-1, 1] \tag{4}$$

$$\mathfrak{H}_{CQ}\left(\mathfrak{C}_{CQ}\right) = \frac{1}{2}\left(\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP}} + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP}}\right), \mathfrak{H}_{CQ}\left(\mathfrak{C}_{CQ}\right) \in [0, 1] \tag{5}$$

To find the relationships between any two CQROFNs $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\right)}\right)$ and $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\right)}\right)$, we use the following rules:

1. If $\mathfrak{S}_{CQ}\left(\mathfrak{C}_{CQ-1}\right) > \mathfrak{S}_{CQ}\left(\mathfrak{C}_{CQ-2}\right) \Rightarrow \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-2}$;
2. If $\mathfrak{S}_{CQ}\left(\mathfrak{C}_{CQ-1}\right) < \mathfrak{S}_{CQ}\left(\mathfrak{C}_{CQ-2}\right) \Rightarrow \mathfrak{C}_{CQ-1} < \mathfrak{C}_{CQ-2}$;
3. If $\mathfrak{S}_{CQ}\left(\mathfrak{C}_{CQ-1}\right) = \mathfrak{S}_{CQ}\left(\mathfrak{C}_{CQ-2}\right) \Rightarrow$;
   1) If $\mathfrak{H}_{CQ}\left(\mathfrak{C}_{CQ-1}\right) > \mathfrak{H}_{CQ}\left(\mathfrak{C}_{CQ-2}\right) \Rightarrow \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-2}$;
   2) If $\mathfrak{H}_{CQ}\left(\mathfrak{C}_{CQ-1}\right) < \mathfrak{H}_{CQ}\left(\mathfrak{C}_{CQ-2}\right) \Rightarrow \mathfrak{C}_{CQ-1} < \mathfrak{C}_{CQ-2}$.

3. CSMs AND DMs BETWEEN CQROFSs

In this part, some CSMs and DMs for CQROFSs are proposed.

Definition 4. For any two CQROFNs $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}\right)$ and $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}\right), i = 1, 2, , ..., \tilde{n}$, based on a universal set $\widetilde{\mathcal{O}} = \left\{\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_{\tilde{n}}\right\}$, then the CSM $CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)$ is demonstrated by

$$CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(\frac{\begin{pmatrix} \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right) \\ + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right) \end{pmatrix}}{\begin{pmatrix} \sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)} \\ \times \sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)} \end{pmatrix}} \right) \tag{6}$$

Theorem 1. *For any two CQROFNs* $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}\right)$ *and* $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}\right), i = 1, 2, , ..., \tilde{n}$, *based on a universal set* $\widetilde{\mathcal{O}} = \left\{\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_{\tilde{n}}\right\}$, *then the CSM* $CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)$ *holds the following conditions:*

1. $0 \leq CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) \leq 1$;
2. $CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = CSM_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-1}\right)$;
3. $CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1$ if $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$ that is $\mathfrak{M}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}_{\mathfrak{C}_{IP-2}} \mathfrak{N}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}_{\mathfrak{C}_{IP-2}}$.

Proof: Based on Definition 4, conditions (1) and (2) are straightforward. Moreover, if we choose the $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$, that is, $\mathfrak{M}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}_{\mathfrak{C}_{IP-2}} \mathfrak{N}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}_{\mathfrak{C}_{IP-2}}$, then

$$\begin{aligned} CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) &= \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(\frac{\begin{pmatrix} \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right) \\ + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right) \end{pmatrix}}{\begin{pmatrix} \sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)} \\ \times \sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)} \end{pmatrix}} \right) \\ &= \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(\frac{\begin{pmatrix} \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \\ + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \end{pmatrix}}{\begin{pmatrix} \sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)} \\ \times \sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)} \end{pmatrix}} \right) \\ &= \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(\frac{\begin{pmatrix} \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \\ + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) \end{pmatrix}}{\left(\left(\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)^{\frac{1}{2}+\frac{1}{2}}\right)} \right) \\ &= 1. \end{aligned}$$

Hence, we obtain $CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1$.

By using the weight vector $\Omega_{WV} = \left\{\Omega_{WV-1}, \Omega_{WV-2}, \ldots, \Omega_{WV-\tilde{n}}\right\}$ with $\sum_{i=1}^{\tilde{n}} \Omega_{WV-i} = 1, \Omega_{WV-i} \in [0, 1]$, then the WCSM is given by

Definition 5. For any two CQROFNs $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}\right)$ and $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}\right), i = 1, 2, , ..., \tilde{n}$, based on a universal set $\widetilde{\mathcal{O}} = \left\{\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_{\tilde{n}}\right\}$, then the WCSM

$WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ is defined by

$$WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\sum_{i=1}^{\tilde{n}}\Omega_{WV-i}\left(\frac{\begin{pmatrix}\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)+\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\\+\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)+\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\end{pmatrix}}{\begin{pmatrix}\sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)+\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)+\mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)+\mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)}\\\times\sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)+\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)+\mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)+\mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)}\end{pmatrix}}\right) \tag{7}$$

For any two CQROFNs $\mathfrak{C}_{CQ-1}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}\right)$ and $\mathfrak{C}_{CQ-2}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}\right)$, $i=1,2,,\ldots,\tilde{n}$, based on a universal set $\widetilde{\mathcal{O}}$, if we choose the weight vector $\Omega_{WV}=\{\Omega_{WV-1},\Omega_{WV-2},\ldots,\Omega_{WV-\tilde{n}}\}=\left(\frac{1}{\tilde{n}},\frac{1}{\tilde{n}},\ldots,\frac{1}{\tilde{n}}\right)$, then the $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ is reduced to $CSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$.

Theorem 2. *For any two CQROFNs* $\mathfrak{C}_{CQ-1}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}\right)$ *and* $\mathfrak{C}_{CQ-2}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}\right)$, $i=1,2,,\ldots,\tilde{n}$, *based on a universal set* $\widetilde{\mathcal{O}}=\{\widetilde{\sigma}_1,\widetilde{\sigma}_2,\ldots,\widetilde{\sigma}_{\tilde{n}}\}$, *then the WCSM* $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ *holds the following conditions:*

1. $0\leq WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)\leq 1$;
2. $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=WCSM_{CQ}\left(\mathfrak{C}_{CQ-2},\mathfrak{C}_{CQ-1}\right)$;
3. $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=1$ if $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$ that is $\mathfrak{M}_{\mathfrak{C}_{RP-1}}=\mathfrak{M}_{\mathfrak{C}_{RP-2}},\mathfrak{M}_{\mathfrak{C}_{IP-1}}=\mathfrak{M}_{\mathfrak{C}_{IP-2}}\mathfrak{N}_{\mathfrak{C}_{RP-1}}=\mathfrak{N}_{\mathfrak{C}_{RP-2}},\mathfrak{N}_{\mathfrak{C}_{IP-1}}=\mathfrak{N}_{\mathfrak{C}_{IP-2}}$.

Proof: All are omitted.

Example 1.

Based on the universal set $\widetilde{\mathcal{O}}=\{\widetilde{\sigma}_1,\widetilde{\sigma}_2,\widetilde{\sigma}_3,\widetilde{\sigma}_4,\widetilde{\sigma}_5\}$, two CQROFNs are $\mathfrak{C}_{CQ-1}=\left\{\begin{matrix}\left(\widetilde{\sigma}_1,\left(0.2e^{i2\pi(0.21)},0.5e^{i2\pi(0.51)}\right)\right),\left(\widetilde{\sigma}_2,\left(0.4e^{i2\pi(0.41)},0.2e^{i2\pi(0.21)}\right)\right),\left(\widetilde{\sigma}_3,\left(0.5e^{i2\pi(0.51)},0.4e^{i2\pi(0.41)}\right)\right),\\\left(\widetilde{\sigma}_4,\left(0.3e^{i2\pi(0.31)},0.3e^{i2\pi(0.31)}\right)\right),\left(\widetilde{\sigma}_5,\left(0.7e^{i2\pi(0.71)},0.1e^{i2\pi(0.11)}\right)\right)\end{matrix}\right\}$ and $\mathfrak{C}_{CQ-2}=$ $\left\{\begin{matrix}\left(\widetilde{\sigma}_1,\left(0.2e^{i2\pi(0.21)},0.7e^{i2\pi(0.71)}\right)\right),\left(\widetilde{\sigma}_2,\left(0.6e^{i2\pi(0.61)},0.3e^{i2\pi(0.31)}\right)\right),\left(\widetilde{\sigma}_3,\left(0.4e^{i2\pi(0.41)},0.3e^{i2\pi(0.31)}\right)\right),\\\left(\widetilde{\sigma}_4,\left(0.4e^{i2\pi(0.41)},0.4e^{i2\pi(0.41)}\right)\right),\left(\widetilde{\sigma}_5,\left(0.6e^{i2\pi(0.61)},0.1e^{i2\pi(0.11)}\right)\right)\end{matrix}\right\}$, further, suppose $q_{CQ}=3$, and $\Omega_{WV}=\{\Omega_{WV-1},\Omega_{WV-2},\Omega_{WV-3},\Omega_{WV-4},\Omega_{WV-5}\}=(0.35,0.2,0.1,0.15,0.2)$, then we can get $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=0.99938$. If we ignore the imaginary parts in all the above information, then we get $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=0.999438$, which is discussed in Ref. [38]. When an SM holds the conditions of SMs, then it is called the original SM.

Lemma 1. For any two FSs $\mathfrak{C}_{CQ-1}$ and $\mathfrak{C}_{CQ-2}$, if an SM $SM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ holds the following axioms:

1. $0\leq SM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)\leq 1$;
2. $SM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=SM_{CQ}\left(\mathfrak{C}_{CQ-2},\mathfrak{C}_{CQ-1}\right)$;
3. $SM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=1$ if $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$.

Then, we say that the $SM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ is called the original SM. Where the DM is given by $DM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=1-SM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ based on SM. Moreover, we develop the EDM $EDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$, which is demonstrated below.

Definition 6. For any two CQROFNs $\mathfrak{C}_{CQ-1}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)\right)}\right)$ and $\mathfrak{C}_{CQ-2}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right)}\right)$, $i=1,2,,\ldots,\tilde{n}$, based on a universal set $\widetilde{\mathcal{O}}=\{\widetilde{\sigma}_1,\widetilde{\sigma}_2,\ldots,\widetilde{\sigma}_{\tilde{n}}\}$, then the EDM $EDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ is defined by

$$EDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\left(\frac{1}{4\tilde{n}}\sum_{\widetilde{\sigma}_i\in\widetilde{\mathcal{O}}}\begin{pmatrix}\left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)-\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)\right|^2+\left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)-\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right|^2\\+\left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\sigma}_i\right)-\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\sigma}_i\right)\right|^2+\left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\sigma}_i\right)-\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\sigma}_i\right)\right|^2\end{pmatrix}\right)^{\frac{1}{2}} \tag{8}$$

By using the weight vector $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \dots, \Omega_{WV-\tilde{n}}\}$ meeting $\sum_{i=1}^{\tilde{n}} \Omega_{WV-i} = 1, \Omega_{WV-i} \in [0,1]$, then the WEDM $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is defined below.

$$WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = \left(\frac{1}{4}\sum_{\tilde{\sigma}_i \in \tilde{O}} \Omega_{WV-i}\left(\begin{array}{c} \left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i)\right|^2 + \left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right|^2 \\ + \left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i)\right|^2 + \left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right|^2 \end{array}\right)\right)^{\frac{1}{2}} \tag{9}$$

Theorem 3. *For any two CQROFNs* $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i)\right)}\right)$ *and* $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right)}\right), i = 1, 2, , \dots, \tilde{n}$, *based on a universal set* $\tilde{O} = \{\tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_{\tilde{n}}\}$, *then the* $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ *holds the following conditions:*

1. $0 \le WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) \le 1$;
2. $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = WEDM_{CQ}(\mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-1})$;
3. $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = 0$ if $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$ that is $\mathfrak{M}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}_{\mathfrak{C}_{IP-2}} \mathfrak{N}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}_{\mathfrak{C}_{IP-2}}$.

Proof:

1. Based on Definition 6, we know that $0 \le \mathfrak{M}_{\mathfrak{C}_{RP-1}}, \mathfrak{M}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}_{\mathfrak{C}_{IP-1}}, \mathfrak{M}_{\mathfrak{C}_{IP-2}} \mathfrak{N}_{\mathfrak{C}_{RP-1}}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}_{\mathfrak{C}_{IP-1}}, \mathfrak{N}_{\mathfrak{C}_{IP-2}} \le 1$ and the parameter $q_{CQ} > 0$, then $0 \le \left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i)\right|^2 \le 1, 0 \le \left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right|^2 \le 1, 0 \le \left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i)\right|^2 \le 1$ and $0 \le \left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right|^2 \le 1$. Therefore, $0 \le WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) \le \left(\frac{1}{4}\right)^{\frac{1}{2}}\left(4\sum_{\tilde{\sigma}_i \in \tilde{O}} \Omega_{WV-i}\right)^{\frac{1}{2}} = 1$.
2. By using Definition 6, we easily obtain the $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = WEDM_{CQ}(\mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-1})$.
3. $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = 0 \Leftrightarrow \left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i)\right|^2 = 0, \left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right|^2 = 0, \left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i)\right|^2 = 0, \left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right|^2 = 0$ that is $\mathfrak{M}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}_{\mathfrak{C}_{IP-2}} \mathfrak{N}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}_{\mathfrak{C}_{IP-2}} \Leftrightarrow \mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$.

Definition 7. For any two CQROFNs $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i)\right)}\right)$ and $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i) e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i) e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right)}\right), i = 1, 2, , \dots, \tilde{n}$, based on a universal set $\tilde{O} = \{\tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_{\tilde{n}}\}$, then the new SM $NSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is demonstrated by

$$NSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = \frac{CSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) + 1 - EDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})}{2} \tag{10}$$

where

$$CSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = \frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}}\left(\frac{\left(\begin{array}{c} \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i)\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i) + \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i)\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i) \\ + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i)\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i) + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i)\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i) \end{array}\right)}{\left(\begin{array}{c} \sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i)} \\ \times\sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)} \end{array}\right)}\right)$$

$$EDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = \left(\frac{1}{4\tilde{n}}\sum_{\tilde{\sigma}_i \in \tilde{O}}\left(\begin{array}{c} \left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i)\right|^2 + \left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right|^2 \\ + \left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\sigma}_i) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\sigma}_i)\right|^2 + \left|\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\sigma}_i) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\sigma}_i)\right|^2 \end{array}\right)\right)^{\frac{1}{2}}$$

By using the weight vector $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \dots, \Omega_{WV-\tilde{n}}\}$ meeting $\sum_{i=1}^{\tilde{n}} \Omega_{WV-i} = 1, \Omega_{WV-i} \in [0,1]$, then the weighted new SM $WNSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is defined as follows.

Definition 8. For any two CQROFNs $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)\right)}\right)$ and $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)}\right), i = 1, 2, , \ldots, \tilde{n}$, based on a universal set $\widetilde{\widetilde{O}} = \{\widetilde{\widetilde{o}}_1, \widetilde{\widetilde{o}}_2, \ldots, \widetilde{\widetilde{o}}_{\tilde{n}}\}$, then the $WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)$ is demonstrated by

$$NSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \frac{WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) + 1 - WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)}{2} \tag{11}$$

where

$$WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \sum_{i=1}^{\tilde{n}} \Omega_{WV-i}\left(\frac{\begin{pmatrix}\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right)\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right) + \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\\ +\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right)\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right) + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\end{pmatrix}}{\begin{pmatrix}\sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)}\\ \times\sqrt{\mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right) + \mathfrak{M}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right) + \mathfrak{N}^{2q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)}\end{pmatrix}}\right)$$

$$WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \left(\frac{1}{4}\sum_{\widetilde{\widetilde{o}}_i \in \widetilde{\widetilde{O}}} \Omega_{WV-i}\begin{pmatrix}\left|\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right|^2 + \left(\mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right) - \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)^2 + \\ \left(\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)^2 + \left(\mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right) - \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)^2\end{pmatrix}\right)^{\frac{1}{2}}$$

If we choose the vector $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \ldots, \Omega_{WV-\tilde{n}}\} = \left(\frac{1}{\tilde{n}}, \frac{1}{\tilde{n}}, \ldots, \frac{1}{\tilde{n}}\right)$, then the $WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)$ is reduced to $NSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)$.

Theorem 4. *For any two CQROFNs* $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)\right)}\right)$ *and* $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)}\right), i = 1, 2, , \ldots, \tilde{n}$, *based on a universal set* $\widetilde{\widetilde{O}} = \{\widetilde{\widetilde{o}}_1, \widetilde{\widetilde{o}}_2, \ldots, \widetilde{\widetilde{o}}_{\tilde{n}}\}$, *then the WNSM* $WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)$ *holds the following conditions:*

1. $0 \le WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) \le 1$;
2. $WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = WNSM_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-1}\right)$;
3. $WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1$ iff $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$ that is $\mathfrak{M}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}_{\mathfrak{C}_{IP-2}}\mathfrak{N}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}_{\mathfrak{C}_{IP-2}}$.

Proof:

1. Based on Definition 8 and Theorem 2, we know that $0 \le WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) \le 1$ for the parameter $q_{CQ} > 0$, then $0 \le WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) \le 1$, then by using Lemma 1, we obtain $0 \le \frac{WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)+1-WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)}{2} \le 1$ which implies that $0 \le WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) \le 1$.
2. By using Definition 6, Theorem 2, and Theorem 3, we easily obtain the $WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = WNSM_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-1}\right)$.
3. When $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$, we know that $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1$ and $WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 0$, then $WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1$. In contrast, we have $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1$, then $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) + 1 - WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1 + 1 - 0 = 2$, such that $CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1 - WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)$. For all CQROFNs $0 \le WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) \le 1$ and $0 \le WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) \le 1$ exists continuously, then $WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1$ and $WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 0$, by using Theorem 3, if $WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 0$, then it is obviously $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$. Hence $WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1$ iff $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$ that is $\mathfrak{M}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}_{\mathfrak{C}_{IP-2}}\mathfrak{N}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}_{\mathfrak{C}_{IP-2}}$.

Definition 9. For any two CQROFNs $\mathfrak{C}_{CQ-1} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\widetilde{\widetilde{o}}_i\right)\right)}\right)$ and $\mathfrak{C}_{CQ-2} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\widetilde{\widetilde{o}}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\widetilde{\widetilde{o}}_i\right)\right)}\right), i = 1, 2, , \ldots, \tilde{n}$, based on a universal set $\widetilde{\widetilde{O}} = \{\widetilde{\widetilde{o}}_1, \widetilde{\widetilde{o}}_2, \ldots, \widetilde{\widetilde{o}}_{\tilde{n}}\}$, then the weighted DM $WDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)$ is expressed by:

$$WDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1 - WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \frac{1 - WCSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) + WEDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)}{2} \tag{12}$$

where

$$WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\sum_{i=1}^{\tilde{n}}\Omega_{WV-i}\left(\frac{\left(\begin{array}{c}\mathfrak{M}_{\mathfrak{C}_{RP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)\mathfrak{M}_{\mathfrak{C}_{RP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{M}_{\mathfrak{C}_{IP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)\mathfrak{M}_{\mathfrak{C}_{IP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\\+\mathfrak{N}_{\mathfrak{C}_{RP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)\mathfrak{N}_{\mathfrak{C}_{RP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{IP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)\mathfrak{N}_{\mathfrak{C}_{IP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\end{array}\right)}{\left(\begin{array}{c}\sqrt{\mathfrak{M}_{\mathfrak{C}_{RP-1}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{M}_{\mathfrak{C}_{IP-1}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{RP-1}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{IP-1}}^{2q_{CQ}}\left(\tilde{o}_i\right)}\\\times\sqrt{\mathfrak{M}_{\mathfrak{C}_{RP-2}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{M}_{\mathfrak{C}_{IP-2}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{RP-2}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{IP-2}}^{2q_{CQ}}\left(\tilde{o}_i\right)}\end{array}\right)}\right)$$

$$WEDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\left(\frac{1}{4}\sum_{\tilde{o}_i\in\tilde{O}}\Omega_{WV-i}\left(\begin{array}{c}\left|\mathfrak{M}_{\mathfrak{C}_{RP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)-\mathfrak{M}_{\mathfrak{C}_{RP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\right|^2+\left|\mathfrak{M}_{\mathfrak{C}_{IP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)-\mathfrak{M}_{\mathfrak{C}_{IP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\right|^2\\+\left|\mathfrak{N}_{\mathfrak{C}_{RP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)-\mathfrak{N}_{\mathfrak{C}_{RP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\right|^2+\left|\mathfrak{N}_{\mathfrak{C}_{IP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)-\mathfrak{N}_{\mathfrak{C}_{IP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\right|^2\end{array}\right)\right)^{\frac{1}{2}}$$

If we choose the weight vector $\Omega_{WV}=\{\Omega_{WV-1},\Omega_{WV-2},\ldots,\Omega_{WV-\tilde{n}}\}=\left(\frac{1}{\tilde{n}},\frac{1}{\tilde{n}},\ldots,\frac{1}{\tilde{n}}\right)$, then the $WDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ is reduced to $DM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$.

Definition 10. For any two CQROFNs $\mathfrak{C}_{CQ-1}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\tilde{o}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\tilde{o}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\tilde{o}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\tilde{o}_i\right)\right)}\right)$ and $\mathfrak{C}_{CQ-2}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\tilde{o}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\tilde{o}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\tilde{o}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\tilde{o}_i\right)\right)}\right)$, $i=1,2,,\ldots,\tilde{n}$, based on a universal set $\tilde{O}=\{\tilde{o}_1,\tilde{o}_2,\ldots,\tilde{o}_{\tilde{n}}\}$, then the weighted DM $WDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ is defined by

$$WDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=1-NSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\frac{1-CSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)+EDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)}{2}\tag{13}$$

where

$$CSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\frac{1}{\tilde{n}}\sum_{i=1}^{\tilde{n}}\left(\frac{\left(\begin{array}{c}\mathfrak{M}_{\mathfrak{C}_{RP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)\mathfrak{M}_{\mathfrak{C}_{RP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{M}_{\mathfrak{C}_{IP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)\mathfrak{M}_{\mathfrak{C}_{IP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\\+\mathfrak{N}_{\mathfrak{C}_{RP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)\mathfrak{N}_{\mathfrak{C}_{RP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{IP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)\mathfrak{N}_{\mathfrak{C}_{IP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\end{array}\right)}{\left(\begin{array}{c}\sqrt{\mathfrak{M}_{\mathfrak{C}_{RP-1}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{M}_{\mathfrak{C}_{IP-1}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{RP-1}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{IP-1}}^{2q_{CQ}}\left(\tilde{o}_i\right)}\\\times\sqrt{\mathfrak{M}_{\mathfrak{C}_{RP-2}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{M}_{\mathfrak{C}_{IP-2}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{RP-2}}^{2q_{CQ}}\left(\tilde{o}_i\right)+\mathfrak{N}_{\mathfrak{C}_{IP-2}}^{2q_{CQ}}\left(\tilde{o}_i\right)}\end{array}\right)}\right)$$

$$EDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\left(\frac{1}{4\tilde{n}}\sum_{\tilde{o}_i\in\tilde{O}}\left(\begin{array}{c}\left|\mathfrak{M}_{\mathfrak{C}_{RP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)-\mathfrak{M}_{\mathfrak{C}_{RP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\right|^2+\left|\mathfrak{M}_{\mathfrak{C}_{IP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)-\mathfrak{M}_{\mathfrak{C}_{IP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\right|^2\\+\left|\mathfrak{N}_{\mathfrak{C}_{RP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)-\mathfrak{N}_{\mathfrak{C}_{RP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\right|^2+\left|\mathfrak{N}_{\mathfrak{C}_{IP-1}}^{q_{CQ}}\left(\tilde{o}_i\right)-\mathfrak{N}_{\mathfrak{C}_{IP-2}}^{q_{CQ}}\left(\tilde{o}_i\right)\right|^2\end{array}\right)\right)^{\frac{1}{2}}$$

Theorem 5. *For any two CQROFNs* $\mathfrak{C}_{CQ-1}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-1}}\left(\tilde{o}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-1}}\left(\tilde{o}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-1}}\left(\tilde{o}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-1}}\left(\tilde{o}_i\right)\right)}\right)$ *and* $\mathfrak{C}_{CQ-2}=\left(\mathfrak{M}_{\mathfrak{C}_{RP-2}}\left(\tilde{o}_i\right)e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-2}}\left(\tilde{o}_i\right)\right)},\mathfrak{N}_{\mathfrak{C}_{RP-2}}\left(\tilde{o}_i\right)e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-2}}\left(\tilde{o}_i\right)\right)}\right)$, $i=1,2,,\ldots,\tilde{n}$, *based on a universal set* $\tilde{O}=\{\tilde{o}_1,\tilde{o}_2,\ldots,\tilde{o}_{\tilde{n}}\}$, *then the* $WDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$ *holds the following conditions:*

1. $0\leq WDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)\leq 1$;
2. $WDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=WDM_{CQ}\left(\mathfrak{C}_{CQ-2},\mathfrak{C}_{CQ-1}\right)$;
3. $WDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=1$ iff $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$ that is $\mathfrak{M}_{\mathfrak{C}_{RP-1}}=\mathfrak{M}_{\mathfrak{C}_{RP-2}},\mathfrak{M}_{\mathfrak{C}_{IP-1}}=\mathfrak{M}_{\mathfrak{C}_{IP-2}}\mathfrak{N}_{\mathfrak{C}_{RP-1}}=\mathfrak{N}_{\mathfrak{C}_{RP-2}},\mathfrak{N}_{\mathfrak{C}_{IP-1}}=\mathfrak{N}_{\mathfrak{C}_{IP-2}}$.

Proof: Based on Theorem 4, we obtain $WDM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=1-WNSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)$, by Theorem 4, we easily obtain the proof of Theorem 5.

4. EXTENDED TOPSIS METHOD WITH CQROFSs

TOPSIS method is a useful tool for MADM problems, and many researches on extended TOPSIS for the different FSs are done, for example, Chen *et al.* [39] proposed an extended TOPSIS method for PHFLTS; Chen *et al.* [40] proposed a proportional interval type-2 hesitant fuzzy TOPSIS approach based on Hamacher aggregation operators and andness optimization models. Now there are no extensions of TOPSIS for CQROFSs, so it is necessary to develop TOPSIS method for CQROFSs.

In this part, we develop the extended TOPSIS method for CQROFSs. Suppose the family of alternatives is $\mathfrak{C}_{Al} = \{\mathfrak{C}_{Al-1}, \mathfrak{C}_{Al-2}, \dots, \mathfrak{C}_{Al-\tilde{m}}\}$, which is evaluated by the decision-maker concerning the attributes $\mathfrak{P}_{At} = \{\mathfrak{P}_{At-1}, \mathfrak{P}_{At-2}, \dots, \mathfrak{P}_{At-\tilde{n}}\}$ by using CQROFNs. $\mathfrak{C}_{CQ-ij} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-ij}}\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-ij}}\right)}\right)$ is an evaluation value of alternative $\mathfrak{C}_{Al-i}$ for attribute $\mathfrak{P}_{At-j}$ meeting $0 \le \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{RP-ij}} + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{RP-ij}} \le 1$ and $0 \le \mathfrak{M}^{q_{CQ}}_{\mathfrak{C}_{IP-ij}} + \mathfrak{N}^{q_{CQ}}_{\mathfrak{C}_{IP-ij}} \le 1$, $q_{CQ} \ge 1$ with $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \dots, \Omega_{WV-\tilde{n}}\}$. Then the complex q-rung orthopair fuzzy decision matrix (CQROFDM) $\mathcal{Q}_{DM} = \left(\mathfrak{C}_{Al-ij}\right)_{\tilde{m}\times\tilde{n}} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-ij}}\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-ij}}\right)}\right)_{\tilde{m}\times\tilde{n}}$ is expressed as follows:

$$\mathcal{Q}_{DM} = \begin{bmatrix} \mathfrak{C}_{Al-11} & \mathfrak{C}_{Al-12} & \mathfrak{C}_{Al-12} & \cdots & \mathfrak{C}_{Al-1\tilde{n}} \\ \mathfrak{C}_{Al-21} & \mathfrak{C}_{Al-22} & \mathfrak{C}_{Al-23} & \cdots & \mathfrak{C}_{Al-2\tilde{n}} \\ \mathfrak{C}_{Al-31} & \mathfrak{C}_{Al-32} & \mathfrak{C}_{Al-33} & \cdots & \mathfrak{C}_{Al-3\tilde{n}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathfrak{C}_{Al-\tilde{m}1} & \mathfrak{C}_{Al-\tilde{m}2} & \mathfrak{C}_{Al-\tilde{m}3} & \cdots & \mathfrak{C}_{Al-\tilde{m}\tilde{n}} \end{bmatrix}$$

Based on the investigated CSMs, the steps of the developed decision-making procedure are as follows:

Step 1: The CQROFDM $\mathcal{Q}_{DM} = \left(\mathfrak{C}_{Al-ij}\right)_{\tilde{m}\times\tilde{n}} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-ij}}\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-ij}}\right)}\right)_{\tilde{m}\times\tilde{n}}$ is normalized. If all criteria are benefits, then we cannot do anything, but, if one criterion is cost type, then we convert the cost criteria into benefits, by

$$\tilde{\mathfrak{C}}_{Al-ij} = \left(\tilde{\mathfrak{M}}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\tilde{\mathfrak{M}}_{\mathfrak{C}_{IP-ij}}\right)}, \tilde{\mathfrak{N}}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\tilde{\mathfrak{N}}_{\mathfrak{C}_{IP-ij}}\right)}\right) = \begin{cases} \left(\mathfrak{M}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-ij}}\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-ij}}\right)}\right) & for\ benefit\ types \\ \left(\mathfrak{N}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-ij}}\right)}, \mathfrak{M}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-ij}}\right)}\right) & for\ cost\ types \end{cases} \tag{14}$$

Step 2: the positive ideal solution (PIS) $\mathfrak{C}^{+}_{Al} = \left\{\mathfrak{C}^{+}_{Al-1}, \mathfrak{C}^{+}_{Al-2}, \dots, \mathfrak{C}^{+}_{Al-\tilde{n}}\right\}$ and negative ideal solution (NIS) $\mathfrak{C}^{-}_{Al} = \left\{\mathfrak{C}^{-}_{Al-1}, \mathfrak{C}^{-}_{Al-2}, \dots, \mathfrak{C}^{-}_{Al-\tilde{n}}\right\}$ are obtained by score values, which are shown as

$$\mathfrak{C}^{+}_{Al-j} = \max\left\{\mathfrak{S}_{CQ}\left(\mathfrak{C}_{Al-1j}\right), \mathfrak{S}_{CQ}\left(\mathfrak{C}_{Al-2j}\right), \dots, \mathfrak{S}_{CQ}\left(\mathfrak{C}_{Al-\tilde{m}j}\right)\right\}, j = 1, 2, \dots, \tilde{n} \tag{15}$$

$$\mathfrak{C}^{-}_{Al-j} = \min\left\{\mathfrak{S}_{CQ}\left(\mathfrak{C}_{Al-1j}\right), \mathfrak{S}_{CQ}\left(\mathfrak{C}_{Al-2j}\right), \dots, \mathfrak{S}_{CQ}\left(\mathfrak{C}_{Al-\tilde{m}j}\right)\right\}, j = 1, 2, \dots, \tilde{n} \tag{16}$$

Step 3: the closeness indexes Ψ_{CI-i} and Ψ'_{CI-i}, can be calculated by

$$\Psi_{CI-i} = \frac{WDM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{+}_{Al}\right)}{WDM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{+}_{Al}\right) + WDM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{-}_{Al}\right)}, i = 1, 2, \dots, \tilde{m} \tag{17}$$

$$\Psi'_{CI-i} = \frac{WNSM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{+}_{Al}\right)}{WNsM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{+}_{Al}\right) + WNSM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{-}_{Al}\right)}, i = 1, 2, \dots, \tilde{m} \tag{18}$$

Step 3: rank all alternatives by the closeness indexes Ψ_{CI-i} and Ψ'_{CI-i}.

Because the DM between the alternative $\mathfrak{C}_{Al-i}$ and PIS $\mathfrak{C}^{+}_{Al}$ is smaller and the CM between the alternative $\mathfrak{C}_{Al-i}$ and PIS $\mathfrak{C}^{+}_{Al}$ is bigger, the alternative $\mathfrak{C}_{Al-i}$ is better. So we can rank the Ψ_{CI-i} from smallest to biggest, or rank the Ψ'_{CI-i} from biggest to smallest, and we can get the ranking orders of all alternatives from the best to worst.

Example 2.

To show the application of the investigated method, we choose the real MADM example from Ref. [38]. To increase monthly income, an enterprise wants to invest money in the market. For this, we choose four potential companies denoted by $\left\{\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-3}, \mathfrak{C}_{CQ-4}\right\}$ as alternatives, which are evaluated by the family of attributes shown as follows:

$\mathfrak{P}_{At-1}$: Risk analysis.

$\mathfrak{P}_{At-2}$: Growth analysis.

Table 1 Orignal decision matrix by complex q-rung orthopair fuzzy numbers.

Alternatives\Attributes	$\mathfrak{P}_{At-1}$	$\mathfrak{P}_{At-2}$	$\mathfrak{P}_{At-3}$	$\mathfrak{P}_{At-4}$
$\mathfrak{C}_{CQ-1}$	$\left(0.7e^{i2\pi(0.6)}, 0.9e^{i2\pi(0.8)}\right)$	$\left(0.91e^{i2\pi(0.81)}, 0.71e^{i2\pi(0.61)}\right)$	$\left(0.92e^{i2\pi(0.82)}, 0.72e^{i2\pi(0.62)}\right)$	$\left(0.93e^{i2\pi(0.83)}, 0.73e^{i2\pi(0.63)}\right)$
$\mathfrak{C}_{CQ-2}$	$\left(0.8e^{i2\pi(0.7)}, 0.85e^{i2\pi(0.89)}\right)$	$\left(0.86e^{i2\pi(0.9)}, 0.81e^{i2\pi(0.71)}\right)$	$\left(0.87e^{i2\pi(0.91)}, 0.82e^{i2\pi(0.72)}\right)$	$\left(0.88e^{i2\pi(0.92)}, 0.83e^{i2\pi(0.73)}\right)$
$\mathfrak{C}_{CQ-3}$	$\left(0.6e^{i2\pi(0.9)}, 0.7e^{i2\pi(0.8)}\right)$	$\left(0.71e^{i2\pi(0.81)}, 0.61e^{i2\pi(0.91)}\right)$	$\left(0.72e^{i2\pi(0.82)}, 0.62e^{i2\pi(0.92)}\right)$	$\left(0.73e^{i2\pi(0.83)}, 0.63e^{i2\pi(0.93)}\right)$
$\mathfrak{C}_{CQ-4}$	$\left(0.81e^{i2\pi(0.61)}, 0.85e^{i2\pi(0.7)}\right)$	$\left(0.86e^{i2\pi(0.71)}, 0.82e^{i2\pi(0.62)}\right)$	$\left(0.87e^{i2\pi(0.72)}, 0.83e^{i2\pi(0.63)}\right)$	$\left(0.88e^{i2\pi(0.73)}, 0.84e^{i2\pi(0.64)}\right)$

Table 2 Normalized decision matrix.

Alternatives\Attributes	$\mathfrak{P}_{At-1}$	$\mathfrak{P}_{At-2}$	$\mathfrak{P}_{At-3}$	$\mathfrak{P}_{At-4}$
$\mathfrak{C}_{CQ-1}$	$\left(0.9e^{i2\pi(0.8)}, 0.7e^{i2\pi(0.6)}\right)$	$\left(0.91e^{i2\pi(0.81)}, 0.71e^{i2\pi(0.61)}\right)$	$\left(0.92e^{i2\pi(0.82)}, 0.72e^{i2\pi(0.62)}\right)$	$\left(0.93e^{i2\pi(0.83)}, 0.73e^{i2\pi(0.63)}\right)$
$\mathfrak{C}_{CQ-2}$	$\left(0.85e^{i2\pi(0.89)}, 0.8e^{i2\pi(0.7)}\right)$	$\left(0.86e^{i2\pi(0.9)}, 0.81e^{i2\pi(0.71)}\right)$	$\left(0.87e^{i2\pi(0.91)}, 0.82e^{i2\pi(0.72)}\right)$	$\left(0.88e^{i2\pi(0.92)}, 0.83e^{i2\pi(0.73)}\right)$
$\mathfrak{C}_{CQ-3}$	$\left(0.7e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.9)}\right)$	$\left(0.71e^{i2\pi(0.81)}, 0.61e^{i2\pi(0.91)}\right)$	$\left(0.72e^{i2\pi(0.82)}, 0.62e^{i2\pi(0.92)}\right)$	$\left(0.73e^{i2\pi(0.83)}, 0.63e^{i2\pi(0.93)}\right)$
$\mathfrak{C}_{CQ-4}$	$\left(0.85e^{i2\pi(0.7)}, 0.81e^{i2\pi(0.61)}\right)$	$\left(0.86e^{i2\pi(0.71)}, 0.82e^{i2\pi(0.62)}\right)$	$\left(0.87e^{i2\pi(0.72)}, 0.83e^{i2\pi(0.63)}\right)$	$\left(0.88e^{i2\pi(0.73)}, 0.84e^{i2\pi(0.64)}\right)$

$\mathfrak{P}_{At-3}$: Social Impact.

$\mathfrak{P}_{At-4}$: Environment Impact.

where $\mathfrak{P}_{At-1}$ is cost type, and the others are benefit types. To solve this example, suppose the weight vector of the attributes is $(0.4, 0.3, 0.2, 0.1)^T$, then the CQROFDM is expressed shown in Table 1.

The steps of the extended TOPSIS method are shown as follows:

Step 1: The CQROFDM $\mathcal{Q}_{DM} = \left(\mathfrak{C}_{Al-ij}\right)_{\tilde{4}\times\tilde{4}} = \left(\mathfrak{M}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{M}_{\mathfrak{C}_{IP-ij}}\right)}, \mathfrak{N}_{\mathfrak{C}_{RP-ij}} e^{i2\pi\left(\mathfrak{N}_{\mathfrak{C}_{IP-ij}}\right)}\right)_{\tilde{4}\times\tilde{4}}$ is normalized which is shown in Table 2. (only convert the attribute $\mathfrak{P}_{At-1}$).

Step 2: The PIS $\mathfrak{C}^+_{Al} = \{\mathfrak{C}^+_{Al-1}, \mathfrak{C}^+_{Al-2}, \dots, \mathfrak{C}^+_{Al-\tilde{n}}\}$ and NIS $\mathfrak{C}^-_{Al} = \{\mathfrak{C}^-_{Al-1}, \mathfrak{C}^-_{Al-2}, \dots, \mathfrak{C}^-_{Al-\tilde{n}}\}$ are obtained as follows:

$$\mathfrak{C}^+_{Al-j} = \left\{ \begin{array}{c} \left(0.93e^{i2\pi(0.83)}, 0.73e^{i2\pi(0.63)}\right), \left(0.88e^{i2\pi(0.92)}, 0.83e^{i2\pi(0.73)}\right), \\ \left(0.7e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.9)}\right), \left(0.88e^{i2\pi(0.73)}, 0.84e^{i2\pi(0.64)}\right) \end{array} \right\}$$

$$\mathfrak{C}^-_{Al-j} = \left\{ \begin{array}{c} \left(0.9e^{i2\pi(0.8)}, 0.7e^{i2\pi(0.6)}\right), \left(0.85e^{i2\pi(0.89)}, 0.8e^{i2\pi(0.7)}\right), \\ \left(0.73e^{i2\pi(0.83)}, 0.63e^{i2\pi(0.93)}\right), \left(0.85e^{i2\pi(0.7)}, 0.81e^{i2\pi(0.61)}\right) \end{array} \right\}$$

Step 3: $WDM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^+_{Al}\right)$, $WNSM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^+_{Al}\right)$ and $WDM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^-_{Al}\right)$, $WNSM_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{C}^-_{Al}\right)$ are calculated shown as ($q_{CQ} = 6$).

$$WDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^+_{Al}\right) = 0.5871\, WDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^-_{Al}\right) = 0.5871$$

$$WDM_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}^+_{Al}\right) = 0.5835\, WDM_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}^-_{Al}\right) = 0.5867$$

$$WDM_{CQ}\left(\mathfrak{C}_{CQ-3}, \mathfrak{C}^+_{Al}\right) = 0.649\, WDM_{CQ}\left(\mathfrak{C}_{CQ-3}, \mathfrak{C}^-_{Al}\right) = 0.6326$$

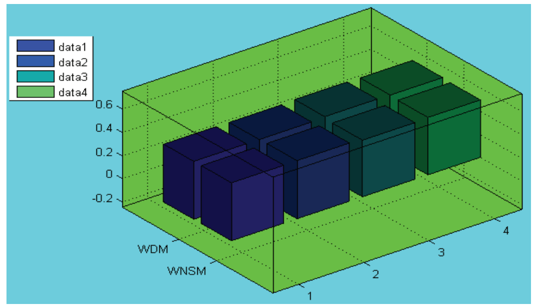


Figure 1 Geometrical expressions of the Example 2.

$$WDM_{CQ}\left(\mathfrak{C}_{CQ-4}, \mathfrak{C}^{+}_{Al}\right) = 0.6037\, WDM_{CQ}\left(\mathfrak{C}_{CQ-4}, \mathfrak{C}^{-}_{Al}\right) = 0.5963$$

and

$$WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^{+}_{Al}\right) = 0.4129\, WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^{-}_{Al}\right) = 0.4129$$

$$WNsM_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}^{+}_{Al}\right) = 0.4165\, WNSM_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}^{-}_{Al}\right) = 0.4133$$

$$WDM_{CQ}\left(\mathfrak{C}_{CQ-3}, \mathfrak{C}^{+}_{Al}\right) = 0.351\, WNSM_{CQ}\left(\mathfrak{C}_{CQ-3}, \mathfrak{C}^{-}_{Al}\right) = 0.3674$$

$$WNSM_{CQ}\left(\mathfrak{C}_{CQ-4}, \mathfrak{C}^{+}_{Al}\right) = 0.3963\, WNSM_{CQ}\left(\mathfrak{C}_{CQ-4}, \mathfrak{C}^{-}_{Al}\right) = 0.4037$$

Then the closeness indexes Ψ_{CI-i} and Ψ'_{CI-i} are gotten as follows:

$$\Psi_{CI-1} = 0.5, \Psi_{CI-2} = 0.4986, \Psi_{CI-3} = 0.5064, \Psi_{CI-4} = 0.5031$$

$$\widehat{\Psi}'_{CI-1}CI - 1 = 0.5, \widehat{\Psi}'_{CI-2} = 0.5019, \widehat{\Psi}'_{CI-3} = 0.4886, \widehat{\Psi}'_{CI-4} = 0.4954$$

The graphical shows the closeness indexes in Figure 1.

Step 3: The ranking results can be obtained as follows:

Because

$$\Psi_{CI-3} > \Psi_{CI-4} > \Psi_{CI-1} > \Psi_{CI-2}$$

$$\widehat{\Psi}'_{CI-2} > \hat{\Psi}'_{CI-1} > \widehat{\Psi}'_{CI-4} > \hat{\Psi}'_{CI-3}$$

So we can get the ranking orders of four alternatives shown as $\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3}$.

From this ranking result, the TOPSIS based on WDM and WNSM obtained the same ranking result. In Example 2, the CQROFNs are used to express the evaluation information. Moreover, we choose the complex Pythagorean fuzzy information (CPFIs) and complex intuitionistic fuzzy information (CIFIs) to solve it by using the investigated measures. To discuss the above issues, we use the following examples.

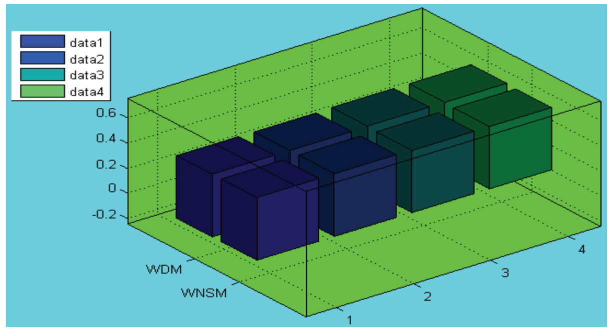


Figure 2 Geometrical expressions of Example 3.

Example 3.

To show the application of the investigated procedure in the environment of the MADM technique, we choose the real MADM example from Ref. [38]. Moreover, the needed information is discussed in Example 2. To resolve the above issue, we considered the weight vector for the attributes is demonstrated by: $(0.4, 0.3, 0.2, 0.1)^T$, then the CPFIs are expressed shown in Table 3 (which are normalized). Based on the proposed TOPSIS, the steps of the developed decision-making procedure are given as follows.

Then by the investigated measures, the closeness indexes Ψ_{CI-i} and Ψ'_{CI-i} are obtained as follows:

$$\Psi_{CI-1} = 0.5009, \Psi_{CI-2} = 0.4995, \Psi_{CI-3} = 0.5052, \Psi_{CI-4} = 0.5046$$

$$\hat{\Psi}'_{CI-1} = 0.4994, \widehat{\Psi}'_{CI-2} = 0.5003, \widehat{\Psi}'_{CI-3} = 0.4961, \hat{\Psi}'_{CI-4} = 0.4968$$

The calculated values are demonstrated in Figure 2. fig 2

Next, the ranking results can be obtained as follows:

Because

$$\Psi_{CI-3} > \Psi_{CI-4} > \Psi_{CI-1} > \Psi_{CI-2}$$

$$\widehat{\Psi}'_{CI-2} > \hat{\Psi}'_{CI-1} > \widehat{\Psi}'_{CI-4} > \hat{\Psi}'_{CI-3}$$

So we can get the ranking orders of four alternatives shown as

$$\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3}$$

There are the same ranking results by WDM and WNSM, and the best alternative is $\mathfrak{C}_{CQ-2}$. In Example 3, we used the CPFIs to resolve this problem by investigated measures. Moreover, we choose the complex intuitionistic fuzzy information (CIFIs) to resolve this problem.

Table 3 Normalized decision matrix with CPFIs.

Alternatives\Attributes	$\mathfrak{P}_{At-1}$	$\mathfrak{P}_{At-2}$	$\mathfrak{P}_{At-3}$	$\mathfrak{P}_{At-4}$
$\mathfrak{C}_{CQ-1}$	$\begin{pmatrix} 0.9e^{i2\pi(0.8)}, \\ 0.1e^{i2\pi(0.2)} \end{pmatrix}$	$\begin{pmatrix} 0.91e^{i2\pi(0.81)}, \\ 0.11e^{i2\pi(0.21)} \end{pmatrix}$	$\begin{pmatrix} 0.92e^{i2\pi(0.82)}, \\ 0.12e^{i2\pi(0.22)} \end{pmatrix}$	$\begin{pmatrix} 0.93e^{i2\pi(0.83)}, \\ 0.13e^{i2\pi(0.23)} \end{pmatrix}$
$\mathfrak{C}_{CQ-2}$	$\begin{pmatrix} 0.85e^{i2\pi(0.89)}, \\ 0.2e^{i2\pi(0.1)} \end{pmatrix}$	$\begin{pmatrix} 0.86e^{i2\pi(0.9)}, \\ 0.21e^{i2\pi(0.11)} \end{pmatrix}$	$\begin{pmatrix} 0.87e^{i2\pi(0.91)}, \\ 0.22e^{i2\pi(0.12)} \end{pmatrix}$	$\begin{pmatrix} 0.88e^{i2\pi(0.92)}, \\ 0.23e^{i2\pi(0.13)} \end{pmatrix}$
$\mathfrak{C}_{CQ-3}$	$\begin{pmatrix} 0.7e^{i2\pi(0.8)}, \\ 0.3e^{i2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} 0.71e^{i2\pi(0.81)}, \\ 0.31e^{i2\pi(0.31)} \end{pmatrix}$	$\begin{pmatrix} 0.72e^{i2\pi(0.82)}, \\ 0.32e^{i2\pi(0.32)} \end{pmatrix}$	$\begin{pmatrix} 0.73e^{i2\pi(0.83)}, \\ 0.33e^{i2\pi(0.33)} \end{pmatrix}$
$\mathfrak{C}_{CQ-4}$	$\begin{pmatrix} 0.85e^{i2\pi(0.7)}, \\ 0.2e^{i2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} 0.86e^{i2\pi(0.71)}, \\ 0.22e^{i2\pi(0.32)} \end{pmatrix}$	$\begin{pmatrix} 0.87e^{i2\pi(0.72)}, \\ 0.23e^{i2\pi(0.33)} \end{pmatrix}$	$\begin{pmatrix} 0.88e^{i2\pi(0.73)}, \\ 0.24e^{i2\pi(0.34)} \end{pmatrix}$

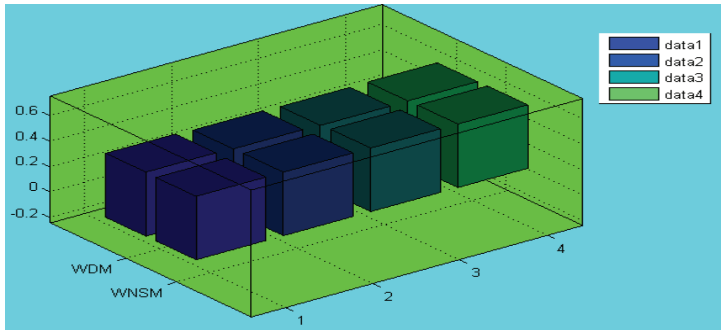


Figure 3 Graphical expressions of Example 4.

Example 4.

To show the application of the investigated procedure in the environment of the MADM technique, we choose the real MADM example from Ref. [38]. Moreover, the needed information is discussed in Example 2. To resolve this problem, we considered the weight vector for the attributes is $(0.4, 0.3, 0.2, 0.1)^T$, then the CIFIs are expressed shown in Table 4 (which are normalized). Based on the proposed TOPSIS, the steps of the developed decision-making procedure are given as follows.

The calculated values are demonstrated in Figure 3.

Then by the investigated measures, the closeness indexes Ψ_{CI-i} and Ψ'_{CI-i} are obtained as follows.

$$\Psi_{CI-1} = 0.5051, \Psi_{CI-2} = 0.4981, \Psi_{CI-3} = 0.4937, \Psi_{CI-4} = 0.4973$$

$$\widehat{\Psi}'_{CI-1} = 0.4996, \hat{\Psi}'_{CI-2} = 0.5002, \widehat{\Psi}'_{CI-3} = 0.5007, \widehat{\Psi}'_{CI-4} = 0.5003$$

Then the ranking results can be obtained as follows:

Because

$$\Psi_{CI-1} > \Psi_{CI-2} > \Psi_{CI-4} > \Psi_{CI-3}$$

$$\widehat{\Psi}'_{CI-3} > \hat{\Psi}'_{CI-4} > \widehat{\Psi}'_{CI-2} > \hat{\Psi}'_{CI-1}$$

So we can get the ranking orders of four alternatives shown as

$$\mathfrak{C}_{CQ-3} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1}$$

There are the same ranking results by WDM and WNSM, and the best alternative is $\mathfrak{C}_{CQ-3}$. Therefore, the investigated measures based on CQROFSs are extensively useful to process complex data.

Table 4 Normalized decision matrix with CIFIs.

Alternatives\Attributes	$\mathfrak{P}_{At-1}$	$\mathfrak{P}_{At-2}$	$\mathfrak{P}_{At-3}$	$\mathfrak{P}_{At-4}$
$\mathfrak{C}_{CQ-1}$	$\begin{pmatrix} 0.7e^{i2\pi(0.6)}, \\ 0.1e^{i2\pi(0.2)} \end{pmatrix}$	$\begin{pmatrix} 0.71e^{i2\pi(0.61)}, \\ 0.11e^{i2\pi(0.21)} \end{pmatrix}$	$\begin{pmatrix} 0.72e^{i2\pi(0.62)}, \\ 0.12e^{i2\pi(0.22)} \end{pmatrix}$	$\begin{pmatrix} 0.73e^{i2\pi(0.63)}, \\ 0.13e^{i2\pi(0.23)} \end{pmatrix}$
$\mathfrak{C}_{CQ-2}$	$\begin{pmatrix} 0.6e^{i2\pi(0.8)}, \\ 0.2e^{i2\pi(0.1)} \end{pmatrix}$	$\begin{pmatrix} 0.61e^{i2\pi(0.81)}, \\ 0.21e^{i2\pi(0.11)} \end{pmatrix}$	$\begin{pmatrix} 0.62e^{i2\pi(0.82)}, \\ 0.22e^{i2\pi(0.12)} \end{pmatrix}$	$\begin{pmatrix} 0.63e^{i2\pi(0.83)}, \\ 0.23e^{i2\pi(0.13)} \end{pmatrix}$
$\mathfrak{C}_{CQ-3}$	$\begin{pmatrix} 0.5e^{i2\pi(0.5)}, \\ 0.3e^{i2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} 0.51e^{i2\pi(0.51)}, \\ 0.31e^{i2\pi(0.31)} \end{pmatrix}$	$\begin{pmatrix} 0.52e^{i2\pi(0.52)}, \\ 0.32e^{i2\pi(0.32)} \end{pmatrix}$	$\begin{pmatrix} 0.53e^{i2\pi(0.53)}, \\ 0.33e^{i2\pi(0.33)} \end{pmatrix}$
$\mathfrak{C}_{CQ-4}$	$\begin{pmatrix} 0.7e^{i2\pi(0.4)}, \\ 0.2e^{i2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} 0.71e^{i2\pi(0.41)}, \\ 0.22e^{i2\pi(0.32)} \end{pmatrix}$	$\begin{pmatrix} 0.72e^{i2\pi(0.42)}, \\ 0.23e^{i2\pi(0.33)} \end{pmatrix}$	$\begin{pmatrix} 0.73e^{i2\pi(0.43)}, \\ 0.24e^{i2\pi(0.34)} \end{pmatrix}$

Table 5 Comparative analysis of the proposed and existing distance measures.

Methods	Score Values/Measures Values	Ranking Values
Ye [41]	*Cannot resolve it*	*Cannot resolve it*
Mohd and Abdullah [42]	*Cannot resolve it*	*Cannot resolve it*
Liu *et al.* [38]	*Cannot resolve it*	*Cannot resolve it*
Garg and Rani [37]	*Cannot resolve it*	*Cannot resolve it*
Ullah *et al.* [27]	*Cannot resolve it*	*Cannot resolve it*
Proposed WDM	$\Psi_{CI-1} = 0.5, \Psi_{CI-2} = 0.4986, \Psi_{CI-3} = 0.5064, \Psi_{CI-4} = 0.5031$	$\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3}$

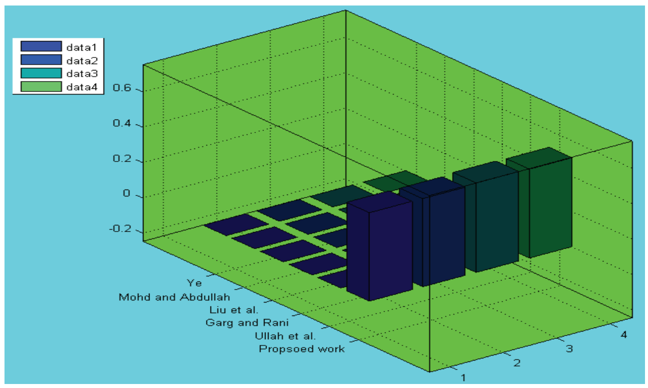


Figure 4 Geometrical expressions of Table 5.

Table 6 Comparative analysis of the proposed and existing ideas for similarity measures.

Methods	Score Values/Measures Values	Ranking Values
Ye [41]	*Cannot resolve it*	*Cannot resolve it*
Mohd and Abdullah [42]	..	*Cannot resolve it*
Liu *et al.* [38]	*Cannot resolve it*	*Cannot resolve it*
Garg and Rani [37]	*Cannot resolve it*	*Cannot resolve it*
Ullah *et al.* [27]	*Cannot resolve it*	*Cannot resolve it*
Proposed WNSM	$\widehat{\Psi}'_{CI-1} = 0.5, \widehat{\Psi}'_{CI-2} = 0.5019, \widehat{\Psi}'_{CI-3} = 0.4886, \widehat{\Psi}'_{CI-4} = 0.4954$	$\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3}$

5. COMPARATIVE ANALYSIS

To show the validity and capability of the presented approach, we can compare it with some existing methods discussed as follows: Ye [41] developed CSMs based on IFSs, Mohd and Abdullah [42] explored CSMs for PFS, Liu *et al.* [38] presented CSMs for QROFSs, Garg and Rani [37] investigated the SMs for CIFSs, and Ullah *et al.* [27] explored DMs for CPFSs. By Example 2, the comparative analysis is shown in Tables 5 and 6.

The calculated values in Tables 5 and 6 are demonstrated in Figures 4 and 5.

Figures 4 and 5 contain graphical expressions of six different types of measures, and each measure contains four alternatives.

Based on the information of Example 3, the comparative analysis of the presented method with some existing methods is discussed in Tables 7 and 8.

For the existing measures, we choose another set: $\mathfrak{C}_{CQ} = \left\{ \begin{array}{l} \left(1e^{i2\pi(1)}, 0.0e^{i2\pi(0.0)}\right), \left(1e^{i2\pi(1)}, 0.0e^{i2\pi(0.0)}\right), \\ \left(1e^{i2\pi(1)}, 0.0e^{i2\pi(0.0)}\right), \left(1e^{i2\pi(1)}, 0.0e^{i2\pi(0.0)}\right) \end{array} \right\}$, then

The calculated values in Table 7 are demonstrated in Figure 6.

The calculated values in Table 8 are demonstrated in Figure 7.

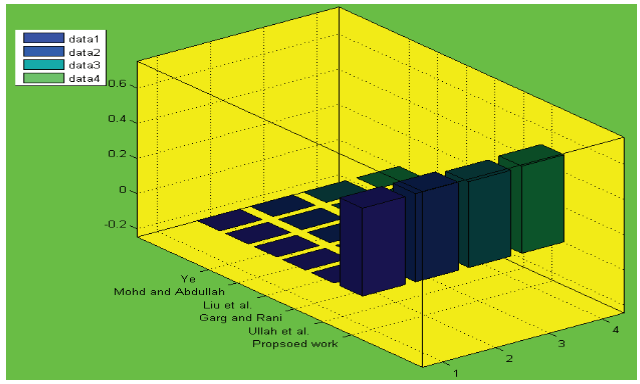


Figure 5 Geometrical expressions of Table 6.

Table 7 Comparative analysis of the proposed and existing distance measures.

Methods	Score Values/Measures Values	Ranking Values
Ye [41]	*Cannot resolve it*	*Cannot resolve it*
Mohd and Abdullah [42]	*Cannot resolve it*	*Cannot resolve it*
Liu *et al.* [38]	*Cannot resolve it*	*Cannot resolve it*
Garg and Rani [37]	*Cannot resolve it*	*Cannot resolve it*
Ullah *et al.* [27]	$\Psi_{CI-1} = 0.6171, \Psi_{CI-2} = 0.6003, \Psi_{CI-3} = 0.6278, \Psi_{CI-4} = 0.6189$	$\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3}$
Proposed WDM	$\Psi_{CI-1} = 0.5009, \Psi_{CI-2} = 0.4995, \Psi_{CI-3} = 0.5052, \Psi_{CI-4} = 0.5046$	$\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3}$

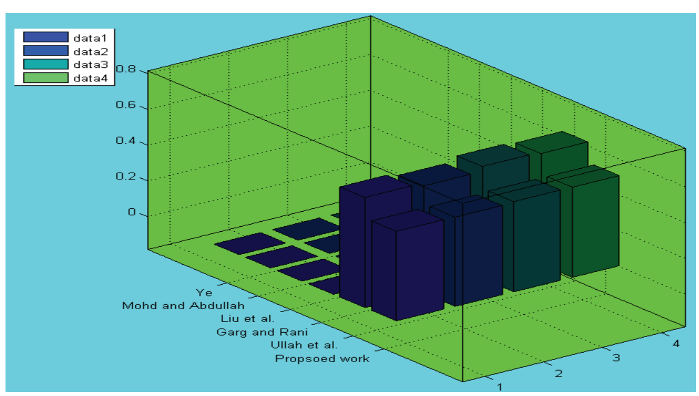


Figure 6 Graphical expressions of Table 7.

Table 8 Comparative analysis of the proposed and existing ideas for similarity measures.

Methods	Score Values/Measures Values	Ranking Values
Ye [41]	*Cannot resolve it*	*Cannot resolve it*
Mohd and Abdullah [42]	*Cannot resolve it*	*Cannot resolve it*
Liu *et al.* [38]	*Cannot resolve it*	*Cannot resolve it*
Garg and Rani [37]	*Cannot resolve it*	*Cannot resolve it*
Ullah *et al.* [27]	$\hat{\Psi}_{CI-1} = 0.3829, \hat{\Psi}_{CI-2} = 0.3997, \hat{\Psi}_{CI-3} = 0.3722, \hat{\Psi}_{CI-4} = 0.3811$	$\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3}$
Proposed WNSM	$\widehat{\Psi'}_{CI-1} = 0.4994, \widehat{\Psi'}_{CI-2} = 0.5003, \widehat{\Psi'}_{CI-3} = 0.4961, \widehat{\Psi'}_{CI-4} = 0.4968$	$\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3}$

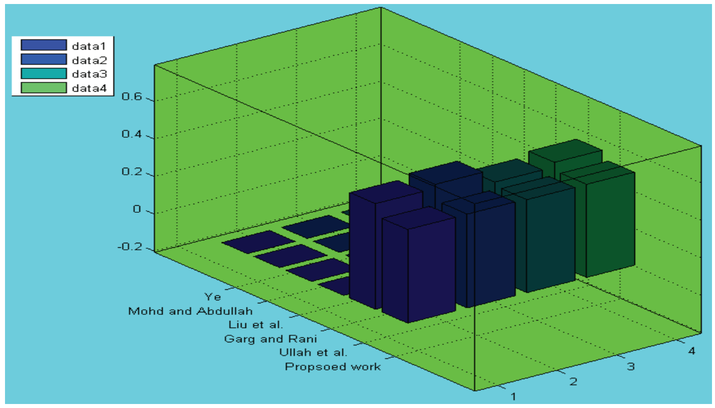


Figure 7 | Graphical expression of Table 8.

Table 9 | Comparative analysis of the proposed and existing distance measures.

Methods	Score Values/Measures Values	Ranking Values
Ye [41]	*Cannot resolve it*	*Cannot resolve it*
Mohd and Abdullah [42]	*Cannot resolve it*	*Cannot resolve it*
Liu *et al.* [38]	*Cannot resolve it*	*Cannot resolve it*
Garg and Rani [37]	$\hat{\Psi}_{CI-1} = 0.5038, \hat{\Psi}_{CI-2} = 0.4978, \hat{\Psi}_{CI-3} = 0.3926, \hat{\Psi}_{CI-4} = 0.4955$	$\mathfrak{C}_{CQ-3} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1}$
Ullah *et al.* [27]	$\hat{\Psi}_{CI-1} = 0.5115, \hat{\Psi}_{CI-2} = 0.4991, \hat{\Psi}_{CI-3} = 0.5043, \hat{\Psi}_{CI-4} = 0.5025$	$\mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-3} > \mathfrak{C}_{CQ-1}$
Proposed WDM	$\Psi_{CI-1} = 0.5051, \Psi_{CI-2} = 0.4981, \Psi_{CI-3} = 0.4937, \Psi_{CI-4} = 0.4973$	$\mathfrak{C}_{CQ-3} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1}$

Table 10 | Comparative analysis of the proposed and existing similarity measures.

Methods	Score Values/Measures Values	Ranking Values
Ye [41]	*Cannot resolve it*	*Cannot resolve it*
Mohd and Abdullah [42]	*Cannot resolve it*	*Cannot resolve it*
Liu *et al.* [38]	*Cannot resolve it*	*Cannot resolve it*
Garg and Rani [37]	$\hat{\Psi}_{CI-1} = 0.4985, \hat{\Psi}_{CI-2} = 0.4991, \hat{\Psi}_{CI-3} = 0.4998, \hat{\Psi}_{CI-4} = 0.4993$	$\mathfrak{C}_{CQ-3} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1}$
Ullah *et al.* [27]	$\hat{\Psi}_{CI-1} = 0.4991, \hat{\Psi}_{CI-2} = 0.4997, \hat{\Psi}_{CI-3} = 0.5002, \hat{\Psi}_{CI-4} = 0.4999$	$\mathfrak{C}_{CQ-3} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1}$
Proposed WNSM	$\widehat{\Psi}'_{CI-1} = 0.4996, \widehat{\Psi}'_{CI-2} = 0.5001, \widehat{\Psi}' = 0.5007, \hat{\Psi}_{CI-4} = 0.5003$	$\mathfrak{C}_{CQ-3} > \mathfrak{C}_{CQ-4} > \mathfrak{C}_{CQ-2} > \mathfrak{C}_{CQ-1}$

Figures 6 and 7 contain graphical expressions of six different types of measures, and each measure contains four alternatives.

Based on the information of Example 4, the comparative analysis of the presented method with some existing methods is discussed in Tables 9 and 10.

The ranking order produced by Ullah *et al.* [27] is different from the others.

The calculated values in Tables 9 and 10 are demonstrated in Figures 8 and 9.

Figures 8 and 9 contain graphical expressions of six different types of measures, and each measure contains four alternatives.

From the above discussions, we obtain that if we choose the CQRIFIs, then the existing measures based on CIFSs, CPFSs are their special cases based on Tables 5–10. Therefore, the investigated measures based on CQROFSs are more general and useful to solve the MADM problem with complex uncertain information.

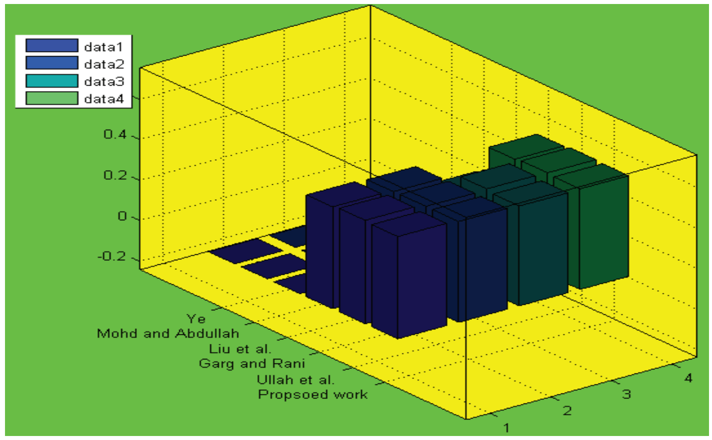


Figure 8 Geometrical expressions of Table 9.

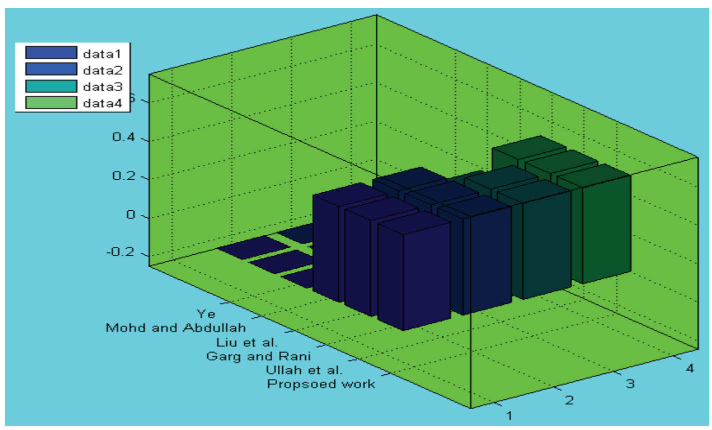


Figure 9 Geometrical expressions of Table 10.

6. CONCLUSION

As a modification of the QROFSs, CQROFSs are an important and useful tool to describe the complex inaccurate information by complex-valued truth grades with an additional term, named as phase term. CSMs and DMs are an important tool to verify the grades of similarity and discrimination between the two sets. In this manuscript, we develop some CSMs and DMs for CQROFSs. Then based on CSMs and EDMs of CQROFSs, we propose an extended TOPSIS method to solve the MADM problems. Finally, we provide some examples to demonstrate the practicality and efficiency of the suggested procedure. The graphical representations of the developed measures are also utilized in this manuscript.

The proposed work is more powerful than the existing ones such as IFSs, CIFSs, PFSs, CPFSs, and QROFSs. In the future, In the future, we will also extend some ideas [39,40,43,44] for complex QROFSs, or for some consensus-based extensions, we will extend the proposed ideas to complex spherical FSs [45] and complex T-spherical FS [46]. We will also develop some new MADM methods based on the proposed CSMs and EDMs for CQROFSs.

CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest.

AUTHORS' CONTRIBUTIONS

Peide Liu: Conceptualization, Formal analysis, Data curation, Fund, Supervision, Writing review & editing. Zeeshan Ali: Conceptualization, Formal analysis, Investigation, Visualization, Project administration, Writing – original draft. Tahir Mahmood: Supervision, Validation, Software, Writing – review & editing.

ACKNOWLEDGMENTS

This paper is supported by the National Natural Science Foundation of China (No. 71771140), Project of cultural masters and ”the four kinds of a batch” talents, the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045), Major bidding projects of National Social Science Fund of China (No. 19ZDA080).

REFERENCES

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1986), 87–96.
[2] L.A. Zadeh, Fuzzy sets, Inf. Control. 8 (1965), 338–353.
[3] K.T. Atanassov, Interval valued intuitionistic fuzzy sets,in Intuitionistic Fuzzy Sets, Physica, Heidelberg, Germany, 1999, pp. 139–177.
[4] H. Garg, K. Kumar, An advanced study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making, Soft Comput. 22 (2018), 4959–4970.
[5] H. Garg, K. Kumar, A novel exponential distance and its based TOPSIS method for interval-valued intuitionistic fuzzy sets using connection number of SPA theory, Artif. Intell. Rev. 53 (2020), 595–624.
[6] H. Garg, A new generalized improved score function of interval-valued intuitionistic fuzzy sets and applications in expert systems, Appl. Soft Comput. 38 (2016), 988–999.
[7] H. Zhang, L. Yu, MADM method based on cross-entropy and extended TOPSIS with interval-valued intuitionistic fuzzy sets, Knowl.-Based Syst. 30 (2012), 115–120.
[8] R.R. Yager, Pythagorean fuzzy subsets, in 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), IEEE, Edmonton, Canada, 2013, pp. 57–61.
[9] H. Garg, A novel improved accuracy function for interval valued Pythagorean fuzzy sets and its applications in the decision-making process, Int. J. Intell. Syst. 32 (2017), 1247–1260.
[10] H. Garg, Linguistic Pythagorean fuzzy sets and its applications in multiattribute decision-making process, Int. J. Intell. Syst. 33 (2018), 1234–1263.
[11] H. Garg, Confidence levels based Pythagorean fuzzy aggregation operators and its application to decision-making process, Comput. Math. Organ. Theory. 23 (2017), 546–571.
[12] H. Garg, Some methods for strategic decision-making problems with immediate probabilities in Pythagorean fuzzy environment, Int. J. Intell. Syst. 33 (2018), 687–712.
[13] H. Garg, New logarithmic operational laws and their aggregation operators for Pythagorean fuzzy set and their applications, Int. J. Intell. Syst. 34 (2019), 82–106.
[14] H. Garg, Hesitant Pythagorean fuzzy Maclaurin symmetric mean operators and its applications to multiattribute decision-making process, Int. J. Intell. Syst. 34 (2019), 601–626.
[15] R.R. Yager, Generalized orthopair fuzzy sets, IEEE Trans. Fuzzy Syst. 25 (2016), 1222–1230.
[16] B.P. Joshi, A. Singh, P.K. Bhatt, K.S. Vaisla, Interval valued q-rung orthopair fuzzy sets and their properties, J. Intell. Fuzzy Syst. 35 (2018), 5225–5230.
[17] P. Liu, P. Wang, Multiple-attribute decision-making based on Archimedean Bonferroni Operators of q-rung orthopair fuzzy numbers, IEEE Trans. Fuzzy Syst. 27 (2018), 834–848.
[18] P. Liu, S.M. Chen, P. Wang, Multiple-attribute group decision-making based on q-rung orthopair fuzzy power Maclaurin symmetric mean operators, IEEE Trans. Syst. Man Cybern. Syst. 50 (2020), 3741–3756.
[19] D. Banerjee, B. Dutta, D. Guha, L. Martínez, SMAA-QUALIFLEX methodology to handle multicriteria decision-making problems based on q-rung fuzzy set with hierarchical structure of criteria using bipolar Choquet integral, Int. J. Intell. Syst. 35 (2020), 401–431.
[20] A.M.D.J.S. Alkouri, A.R. Salleh, Complex intuitionistic fuzzy sets, AIP Conf. Proc. 1482 (2012), 464–470.
[21] D. Ramot, R. Milo, M. Friedman, A. Kandel, Complex fuzzy sets, IEEE Trans. Fuzzy Syst. 10 (2002), 171–186.
[22] H. Garg, D. Rani, Complex interval-valued intuitionistic fuzzy sets and their aggregation operators, Fundam. Inf. 164 (2019), 61–101.
[23] D. Rani, H. Garg, Distance measures between the complex intuitionistic fuzzy sets and their applications to the decision-making process, Int. J. Uncertain. Quan. 7 (2017), 423–439.
[24] H. Garg, D. Rani, A robust correlation coefficient measure of complex intuitionistic fuzzy sets and their applications in decision-making, Appl. Intell. 49 (2019), 496–512.
[25] H. Garg, D. Rani, Novel aggregation operators and ranking method for complex intuitionistic fuzzy sets and their applications to decision-making process, Artif. Intell. Rev. 53 (2020), 3595–3620.
[26] K. Ullah, T. Mahmood, Z. Ali, N. Jan, On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition, Comp. Intell. Syst. 6 (2020), 15–27.
[27] M. Akram, S. Naz, A novel decision-making approach under complex Pythagorean fuzzy environment, Math. Comput. Appl. 24 (2019), 73.
[28] P. Liu, Z. Ali, T. Mahmood, A method to multi-attribute group decision-making problem with complex q-rung orthopair linguistic information based on Heronian mean operators, Int. J. Comput. Intell. Syst. 12 (2019), 1465–1496.
[29] P. Liu, T. Mahmood, Z. Ali, Complex Q-rung orthopair fuzzy aggregation operators and their applications in multi-attribute group decision making, Information. 11 (2020), 5.

[30] H. Garg, Z. Ali, T. Mahmood, Algorithms for complex interval-valued q-rung orthopair fuzzy sets in decision making based on aggregation operators, AHP, and TOPSIS, Expert Syst. 38 (2021), e12609.
[31] H. Garg, J. Gwak, T. Mahmood, Z. Ali, Power aggregation operators and VIKOR methods for complex q-rung orthopair fuzzy sets and their applications, Mathematics. 8 (2020), 538.
[32] Z. Ali, T. Mahmood, Maclaurin symmetric mean operators and their applications in the environment of complex q-rung orthopair fuzzy sets, Comput. Appl. Math. 39 (2020), 161.
[33] P. Liu, Z. Ali, T. Mahmood, N. Hassan, Group decision-making using complex q-rung orthopair fuzzy Bonferroni mean, Int. J. Comput. Intell. Syst. 13 (2020), 822–851.
[34] T. Mahmood, Z. Ali, Entropy measure and TOPSIS method based on correlation coefficient using complex q-rung orthopair fuzzy information and its application to multi-attribute decision making, Soft Comput. 25 (2021), 1249–1275.
[35] P. Liu, Z. Ali, T. Mahmood, Generalized complex q-rung orthopair fuzzy Einstein averaging aggregation operators and their application in multi-attribute decision making, Complex Intell. Syst. 7 (2021), 511–538.
[36] T. Mahmood, Z. Ali, Aggregation operators and VIKOR method based on complex q-rung orthopair uncertain linguistic informations and their applications in multi-attribute decision making, Comput. Appl. Math. 39 (2020), 1–44.
[37] H. Garg, D. Rani, Some results on information measures for complex intuitionistic fuzzy sets, Int. J. Intell. Syst. 34 (2019), 2319–2363.
[38] D. Liu, X. Chen, D. Peng, Some cosine similarity measures and distance measures between q-rung orthopair fuzzy sets, Int. J. Intell. Syst. 34 (2019), 1572–1587.
[39] Z.S. Chen, M. Li, W.T. Kong, K.S. Chin, Evaluation and selection of hazmat transportation alternatives: a PHFLTS-and TOPSIS-integrated multi-perspective approach, Int. J. Environ. Res. Pub. Health. 16 (2019), 4116.
[40] Z.S. Chen, Y. Yang, X.J. Wang, K.S. Chin, K.L. Tsui, Fostering linguistic decision-making under uncertainty: a proportional interval type-2 hesitant fuzzy TOPSIS approach based on Hamacher aggregation operators and andness optimization models, Inf. Sci. 500 (2019), 229–258.
[41] J. Ye, Cosine similarity measures for intuitionistic fuzzy sets and their applications, Math. Comput. Modell. 53 (2011), 91–97.
[42] W.R.W. Mohd, L. Abdullah, Similarity measures of Pythagorean fuzzy sets based on combination of cosine similarity measure and Euclidean distance measure, AIP Conf. Proc. 1974 (2018), 030017.
[43] Z.S. Chen, X. Zhang, R.M. Rodríguez, W. Pedrycz, L. Martínez, Expertise-based bid evaluation for construction-contractor selection with generalized comparative linguistic ELECTRE III, Autom. Constr. 125 (2021), 103578.
[44] R.M. Rodríguez, Á. Labella, M. Sesma-Sara, H. Bustince, L. Martínez, A cohesion-driven consensus reaching process for large scale group decision making under a hesitant fuzzy linguistic term sets environment, Comput. Ind. Eng. 155 (2021), 107158.
[45] Z. Ali, T. Mahmood, M.S. Yang, TOPSIS method based on complex spherical fuzzy sets with Bonferroni mean operators, Mathematics. 8 (2020), 1739.
[46] Z. Ali, T. Mahmood, M.S. Yang, Complex T-spherical fuzzy aggregation operators with application to multi-attribute decision making, Symmetry. 12 (2020), 1311.