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Research Article

Novel Complex T-Spherical Fuzzy 2-Tuple Linguistic Muirhead Mean Aggregation Operators and Their Application to Multi-Attribute Decision-Making

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ABSTRACT

Complex T-spherical fuzzy 2-tuple linguistic set (CTSF2-TLS), which is a combination of complex fuzzy set (CFS), T-spherical fuzzy set (TSFS), and 2-tulpe linguistic variable set (2-TLVS), is a proficient technique to express uncertain and awkward information in real decision-making. CTSF2-TLS contains 2-tuple linguistic variable, truth, abstinence, and falsity grades, which gives an extensive freedom for decision-makers to express the uncertain information compared to other existing notions. In this article, we firstly propose a new concept of CTSF2-TLS by using the CFS, TSFS, and 2-TLVS, and the operational laws and comparison methods for CTSF2-TLSs are established, then the complex T-spherical fuzzy 2-tuple linguistic Muirhead mean (CTSF2-TLMM) operator and the complex T-spherical fuzzy 2-tuple linguistic dual Muirhead mean (CTSF2-TLDMM) operator are explored, and some special cases and the desirable properties of the explored operators are also studied. Moreover, we establish a method to solve the multi-attribute decision-making (MADM) problems, in which the evaluation information is described by CTSF2-TLSs. Finally, we use some numerical examples to explain the advantages of the explored method by comparing with other existing methods.

1. INTRODUCTION

Decision-making is a significant activity for picking the ideal option from a set of schemes based on the data acquired from specialists. We are confronted with numerous decision-making issues in different life circumstances [1], vulnerability and ambiguity should be considered. For adapting such sorts of issues, the theory of fuzzy set (FS) was presented by Zadeh [2] which can express the fuzzy information by the truth grade. But main problem of the FS is that it deals only with truth grade. Further, Atanassov [3] propounded the intuitionistic FS (IFS), which contains the truth and falsity grades, whose sum is belonging to the unit interval. Under this theory, a lot of scholars has explored their theories [4]. However, when a decision-maker (DM) gives the information in the form of pair (0.6,0.5) for truth and falsity grades, the condition of IFS is exceeded from unit interval. For coping with this kind of information, Yager [5] modified the IFS, and proposed the Pythagorean FS (PFS) with a condition that the sum of squares of the truth grade and falsity grade is not exceeded from unit interval. PFS has received more attention [6]. But there was still a problem, when a DM gives the information in the form of pair (0.9, 0.8) for truth and falsity grades, then the condition of IFS and the condition of PFS are exceeded from unit interval. For coping this kind of information, Yager [7] again modified the PFS, and explored the q-rung orthopair FS (q-ROFS) with a condition that the sum of q-powers of the truth grade and falsity grade is not exceeded from unit interval. Because there is an extensive information expression, since it was set up, it has been used in some fields of aggregation operators, similarity measures (SM), hybrid aggregation operators, and so on [8,9].

In real decision-making, DMs may offer their feelings with more responses for a decision index, such as positive, negative, neutral, and abstinence. In order to express this information, Cuong [10] investigated the picture FS (PFS) which contains positive, abstinence, and negative grades, whose sum is bounded to [0, 1]. However, in some cases, for a DM, it is very difficult to face some limitations. So, the spherical FS (SFS) was established by Mahmood *et al.* [11], which is more powerful compared to IFS and PFS. However, the constraint of SFS is that the sum of squares of positive, abstinence, and negative grades belongs to [0, 1]. When a DM gives 0.9 for positive grade, 0.85 for abstinence grade, and 0.8 for negative grade, then the PFS and SFS are not able to cope it. So, the idea of T-spherical fuzzy set (TSFS) was established by Mahmood *et al.* [11], in which the sum of q-powers of positive, abstinence, and negative grades belongs to [0, 1]. The TSFS is more powerful compared to PFS and SFS, and since it was established, it has received the attention of many researchers and it is utilized

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in the environment of aggregation operators, SMs, hybrid aggregation operators, and so on. The various existing works based on PFS, SFS, and TSFS are elaborated as follow as:

1. **Operators based Approaches.** Many scholars have successfully discussed the aggregation operators in the environment of PFS, SFS, TSFS. For instance, Ullah *et al.* [12] evaluated investment policy based on interval-valued T-SFS. Ullah *et al.* [13] explored geometric aggregation operator based on T-SFS. Guleria and Bajaj [14] explored the novel of T-spherical fuzzy soft set with their aggregation operators. Quek *et al.* [15] presented the generalized T-spherical fuzzy weighted aggregation operators. Ullah *et al.* [16] proposed the notion of the averaging aggregation operators based on T-SFS. Ullah *et al.* [17] presented the evaluation of investment performance based on T-spherical fuzzy Hamacher aggregation operators.
2. **Measures based Approaches.** SM is a proficient technique to accurately examine the degree between any two objects. Many scholars have developed some SMs for the different FSs. For example, Ullah *et al.* [18] explored the correlation co-efficient based on T-SFS and their application in medical diagnosis. The SMsbased on T-SFS was presented by Ullah *et al.* [19].
3. **Hybrid Operators based Approaches.** To find the interrelationships between two objects, the hybrid aggregation operators play an essential role in the environment of realistic decision-making. Some scholars explored different hybrid aggregation operators using the T-SFSs. For example, Liu *et al.* [20] explored the power Muirhead mean (MM) operators, Munir *et al.* [21] explored the Einstein hybrid aggregation operators, and Zeng *et al.* [22] presented the Probabilistic interactive aggregation operators based on T-SFSs.

The above studies are based on FS, IFS, PFS, QROFS, and all parts of them are expressed by real numbers. In order to process the complex situations, Ramot *et al.* [23] proposed complex fuzzy set (CFS), which contains complex-valued truth grade in the form of polar coordinates belonging to unit disc in a complex plane. But in some situations, the CFS cannot process the complicated and awkward information effectively. To resolve this kind of problems, Alkouri and Salleh [24] developed the complex IFS (CIFS), characterized by the complex-valued truth grade and complex-valued falsity grade with a condition that the sum of the real parts (also for the imaginary part) of the truth and falsity grades is bounded to the unit interval. Now, CIFS has received extensively attractions. For instance, Garg and Rani [25] established a new generalized Bonferroni mean aggregation operators for CIFS based on Archimedean t-norm and t-conorm. Rani and Garg [26] developed the distance measures for CIFSs and applied them to multi-attribute decision making (MADM) Process. When a DM gives the information in the form of pair $\left(0.6e^{i2\pi 0.6}, 0.5e^{i2\pi 0.5}\right)$ for complex-valued truth and complex-valued falsity grades, the conditions of CIFS is exceeded from unit interval. For coping these kinds of problems, Ullah *et al.* [27] modified the CIFS to explore the complex PFS (CPFS) with conditions that the sum of squares of the real part (also for the imaginary part) of the truth grade and the real part (also for the imaginary part) of the falsity grade is not exceeded from unit interval. CPFS has received more attention [28]. But there was still a problem, when a DM gives the information in the form of pair $\left(0.9e^{i2\pi 0.9}, 0.8e^{i2\pi 0.8}\right)$ for truth and falsity grades, the conditions of CIFS and the conditions of CPFS are exceeded from unit interval. For coping these kinds of problems, Liu *et al.* [29,30] modified the CPFS to explore the complex q-ROFS (Cq-ROFS) with a condition that the sum of q-powers of the real part (also for imaginary part) of the truth grade and the real part (also for imaginary part) of the falsity grade are not exceeded from unit interval, and now Cq-ROFS has received more attention [31].

Linguistic variable (LV) proposed by Zadeh [32] is an important tool to express the qualitative information, and many specialists have investigated the linguistic MADM issues, including linguistic approximation [33], uncertain LV [34], and linguistic 2-tuple information [35], and so on. In some practical problems, the single linguistic term set cannot be involved in those cases which contains two terms like truth and falsity grades. For dealing with such kinds of problems, Wang and Li [36] established the intuitionistic linguistic number (ILN), which contains a linguistic term, truth grade, and falsity grade. The ILN is more powerful idea to cope with uncertainty and vagueness. Further, Liu and Chen [37] explored the intuitionistic 2-tuple linguistic terms set, Peng and Yang [38] established Pythagorean fuzzy linguistic set, Wei *et al.* [39] explored the Pythagorean 2-tuple linguistic aggregation operators, Li *et al.* [40] proposed q-rung orthopair linguistic Heronian mean operators, Ju *et al.* [41] explored the q-rung orthopair fuzzy 2-tuple linguistic MM operators.

The interrelationship among the different attributes in real decision-making is ever-present. Muirhead [42] explored the MM operator, as an effective method to evaluate perfectly the interrelationship among the attributes, then its extensions for the different FSs were made. For instance, Liu and Li [43] explored the intuitionistic fuzzy MM operators. Zhu and Li [44] presented the Pythagorean fuzzy MM operators, Wang *et al.* [45] established q-rung orthopair fuzzy MM operators. On addition, there are some general aggregation operators for q-ROFS, such as the averaging aggregation operator based on q-ROFS [8], Partitioned Bonferroni mean operator (BMO) based on q-ROFS [46], and Maclaurin symmetric mean operator (MSMO) based on q-ROFS [47]. These operators are useful in genuine choice hypothesis. But, the two-dimensional information in a single set cannot be discussed in IFS, PyFS, q-ROFS, PFS, and TSFS, and only is discussed in the environment of CIFS, CPyFS, and Cq-ROFS, but these notions cannot contain the neutral grade, which is used in many real-life scenarios. For example, when a DM provides $\left(\left(s_{\left(S_{(LT-3)}\right)}, 0.03\right), \left(0.5e^{i2\pi 0.3}, 0.4e^{i2\pi 0.5}, 0.3e^{i2\pi 0.4}\right)\right)$, for linguistic term, truth, abstinence, and falsity grades, the existing notions cannot deal with such kind of problems. So, we need to propose a new concept, that is, complex spherical fuzzy 2-tuple linguistic set. We can see that $0.5^2 + 0.4^2 + 0.3^2 = 0.25 + 0.16 + 0.09 = 0.49 \leq 1$, and $0.3^2 + 0.5^2 + 0.4^2 = 0.09 + 0.25 + 0.16 = 0.49 \leq 1$. But, there is one other complication, when a DM gives $\left(\left(s_{\left(S_{(LT-3)}\right)}, 0.03\right), \left(0.9e^{i2\pi 0.8}, 0.8e^{i2\pi 0.7}, 0.7e^{i2\pi 0.6}\right)\right)$, for linguistic term, truth, abstinence, and falsity grades, the existing notions cannot deal with such kind of problems. For addressing with sun kind of issues, we explore the idea of complex T-spherical fuzzy 2-tuple linguistic sets (CTSF2-TLSs) with a condition that the sum of q-powers of the real

parts of the truth, abstinence, and falsity grades is not exceeded form unit interval. So, for $q = 7$, the above problem is solved effectively. Considering the intricacy in the real circumstances and maintaining the benefits of the MM operators and CTSF2-TLSs, the goals of this research are shown as follows.

1. To investigate the novel concept of CTSF2-TLSs and furthermore depict their operational laws.
2. To develop the MM operator and dual MM (DMM) operator based on the CTSF2-TLSs, and discuss some properties and special cases.
3. To investigate the MADM method utilizing the complex T-spherical fuzzy 2-tuple linguistic Muirhead mean (CTSF2-TLMM) operator and complex T-spherical fuzzy 2-tuple linguistic dual Muirhead mean (CTSF2-TLDMM) operator.
4. To show the advantages of the proposed method by some examples.

So we give the following structure. In Section 2, we review some concepts of FSs, CFSs, q-ROFSs, Tuple linguistic function (2-TLFs), inverse 2-TLFs, q-ROF2-TLSs and their operational laws. Further, the MM operator and DMM operator are also discussed. In Section 3, the notion of CTSF2-TLS using CFS, TSFS, and 2-tulpe linguistic variable set (2-TLVS) is defined. In Section 4, based on the established operational laws and comparison methods for CTSF2-TLSs, the CTSF2-TLMM aggregation operator and CTSF2-TLDMM aggregation operator are explored. Some special cases and the desirable properties are also studied. In Section 5, we establish a method to solve the multi-attribute group decision-making (MAGDM) problems, in which the evaluation information is expressed as CTSF2-TLSs. Finally, some numerical examples are given to explain the effectiveness and superiority of the explored method by comparing with other methods. The conclusion of this paper is discussed in Section 6.

2. PRELIMINARIES

In this part, we concisely review some useful notions of 2-TLF [35], inverse 2-TLF [35], TSFS [11] and their operational laws. Further, the MM operator and DMM operator are also discussed. The symbols U_{UNI}, μ, ξ and η are represented the universal, the grade of truth, the grade of abstinence, and the grade of falsity. Where $a_{SC}, \delta_{SC}, q_{SC} \geq 1$.

Definition 1. [35] For a linguistic term set $S_{LT} = \left\{s_{S_{LT-0}}, s_{S_{LT-1}}, ..., s_{S_{LT-g}}\right\}$ with $\beta_{SC} \in [0,1]$, the 2-tuple linguistic function Δ_{LT} is given by:

$$\Delta_{LT} : [0,1] \to S_{S_{LT-j}} \times \left[-\frac{1}{2g}, \frac{1}{2g}\right\} \tag{1}$$

$$\Delta_{LT}(\beta_{SC}) = \left(s_{S_{LT-j}}, a_{SC}\right) \text{ with } \begin{cases} s_{S_{LT-j}} & j = round\left(\beta_{SC}, g\right) \\ a_{SC} = \beta_{SC} - \dfrac{j}{g} & a_{SC} \in \left[-\dfrac{1}{2g}, \dfrac{1}{2g}\right\} \end{cases} \tag{2}$$

The 2-tuple linguistic inverse function Δ_{LT}^{-1} is given by:

$$\Delta_{LT}^{-1} : S_{S_{LT-j}} \times \left[-\frac{1}{2g}, \frac{1}{2g}\right\} \to [0,1] \tag{3}$$

$$\Delta_{LT}^{-1}\left(s_{S_{LT-j}}, a_{SC}\right) = \frac{j}{g} + a_{SC} = \beta_{SC} \tag{4}$$

Definition 2. [11] A TSFS is given by

$$\mathfrak{T}_{TS} = \left\{\left(\mu_{\mathfrak{T}_{TS}}(u), \xi_{\mathfrak{T}_{TS}}(u), \eta_{\mathfrak{T}_{TS}}(u)\right) : u \in U_{UNI}\right\} \tag{5}$$

where $\mu_{\mathfrak{T}_{TS}} : U_{UNI} \to [0,1], \xi_{\mathfrak{T}_{TS}} : U_{UNI} \to [0,1]$ and $\eta_{\mathfrak{T}_{TS}} : U_{UNI} \to [0,1]$, with a condition: $0 \leq \mu_{\mathfrak{T}_{TS}}^{q_{SC}}(u) + \xi_{\mathfrak{T}_{TS}}^{q_{SC}}(u) + \eta_{\mathfrak{T}_{TS}}^{q_{SC}}(u) \leq 1$. Moreover, $\zeta_{\mathfrak{T}_{TS}}(u) = 1 - \left(\mu_{\mathfrak{T}_{TS}}^{q_{SC}}(u) + \xi_{\mathfrak{T}_{TS}}^{q_{SC}}(u) + \eta_{\mathfrak{T}_{TS}}^{q_{SC}}(u)\right)^{\frac{1}{q_{SC}}}$ is called refusal grade, the T-spherical fuzzy number (TSFN) is represented by $\mathfrak{T}_{TS} = \left(\mu_{\mathfrak{T}_{TS}}(u), \xi_{\mathfrak{T}_{TS}}(u), \eta_{\mathfrak{T}_{TS}}(u)\right)$.

Definition 3. [11] For any two TSFNs $\mathfrak{T}_{TS-1} = \left(\mu_{\mathfrak{T}_{TS-1}}(u), \xi_{\mathfrak{T}_{TS-1}}(u), \eta_{\mathfrak{T}_{TS-1}}(u)\right)$ and $\mathfrak{T}_{TS-2} = \left(\mu_{\mathfrak{T}_{TS-2}}(u), \xi_{\mathfrak{T}_{TS-2}}(u), \eta_{\mathfrak{T}_{TS-2}}(u)\right)$, then

1. $\mathfrak{T}_{TS-1} \oplus_{TS} \mathfrak{T}_{TS-2} = \left(\left(\mu_{\mathfrak{T}_{TS-1}}^{q_{SC}}(u) + \mu_{\mathfrak{T}_{TS-2}}^{q_{SC}}(u) - \mu_{\mathfrak{T}_{TS-1}}^{q_{SC}}(u)\mu_{\mathfrak{T}_{TS-2}}^{q_{SC}}(u)\right)^{\frac{1}{q_{SC}}}, \xi_{\mathfrak{T}_{TS-1}}(u)\xi_{\mathfrak{T}_{TS-2}}(u), \eta_{\mathfrak{T}_{TS-1}}(u)\eta_{\mathfrak{T}_{TS-2}}(u)\right)$;

2. $$\mathfrak{F}_{TS-1} \otimes_{TS} \mathfrak{F}_{TS-2} = \left(\begin{array}{l} \mu_{\mathfrak{F}_{TS-1}}(u)\mu_{\mathfrak{F}_{TS-2}}(u), \left(\xi^{q_{SC}}_{\mathfrak{F}_{TS-1}}(u) + \xi^{q_{SC}}_{\mathfrak{F}_{TS-2}}(u) - \xi^{q_{SC}}_{\mathfrak{F}_{TS-1}}(u)\xi^{q_{SC}}_{\mathfrak{F}_{TS-2}}(u)\right)^{\frac{1}{q_{SC}}}, \\ \left(\eta^{q_{SC}}_{\mathfrak{F}_{TS-1}}(u) + \eta^{q_{SC}}_{\mathfrak{F}_{TS-2}}(u) - \eta^{q_{SC}}_{\mathfrak{F}_{TS-1}}(u)\eta^{q_{SC}}_{\mathfrak{F}_{TS-2}}(u)\right)^{\frac{1}{q_{SC}}} \end{array} \right);$$

3. $$\mathfrak{F}^{\delta_{SC}}_{TS-1} = \left(\mu^{\delta_{SC}}_{\mathfrak{F}_{TS-1}}(u), \left(1 - \left(1 - \xi^{q_{SC}}_{\mathfrak{F}_{TS-1}}(u)\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}}, \left(1 - \left(1 - \eta^{q_{SC}}_{\mathfrak{F}_{TS-1}}(u)\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}} \right);$$

4. $$\delta_{SC}\mathfrak{F}_{TS-1} = \left(\left(1 - \left(1 - \mu^{q_{SC}}_{\mathfrak{F}_{TS-1}}(u)\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}}, \xi^{\delta_{SC}}_{\mathfrak{F}_{TS-1}}(u), \eta^{\delta_{SC}}_{\mathfrak{F}_{TS-1}}(u) \right).$$

Definition 4. [11] For any two TSFNs $\mathfrak{F}_{TS-1} = \left(\mu_{\mathfrak{F}_{TS-1}}(u), \xi_{\mathfrak{F}_{TS-1}}(u), \eta_{\mathfrak{F}_{TS-1}}(u)\right)$ and $\mathfrak{F}_{TS-2} = \left(\mu_{\mathfrak{F}_{TS-2}}(u), \xi_{\mathfrak{F}_{TS-2}}(u), \eta_{\mathfrak{F}_{TS-2}}(u)\right)$, the score and accuracy function are given by:

$$S(\mathfrak{F}_{TS-1}) = \mu^{q_{SC}}_{\mathfrak{F}_{TS}}(u) - \xi^{q_{SC}}_{\mathfrak{F}_{TS}}(u) - \eta^{q_{SC}}_{\mathfrak{F}_{TS}}(u) \tag{6}$$

$$H(\mathfrak{F}_{TS-1}) = \mu^{q_{SC}}_{\mathfrak{F}_{TS}}(u) + \xi^{q_{SC}}_{\mathfrak{F}_{TS}}(u) + \eta^{q_{SC}}_{\mathfrak{F}_{TS}}(u) \tag{7}$$

Based on the above two notions, the compassion between two TSFNs is given by:

1. If $S(\mathfrak{F}_{TS-1}) > S(\mathfrak{F}_{TS-2})$, then $\mathfrak{F}_{TS-1} \succ \mathfrak{F}_{TS-2}$;
2. If $S(\mathfrak{F}_{TS-1}) = S(\mathfrak{F}_{TS-2})$, then:
   (a) If $H(\mathfrak{F}_{TS-1}) > H(\mathfrak{F}_{TS-2})$, then $\mathfrak{F}_{TS-1} \succ \mathfrak{F}_{TS-2}$;
   (b) If $H(\mathfrak{F}_{TS-1}) = H(\mathfrak{F}_{TS-2})$, then $\mathfrak{F}_{TS-1} = \mathfrak{F}_{TS-2}$.

Definition 5. [42] Choose the family of positive numbers $\mathfrak{F}_{TS-j}(j = 1, 2, 3, ..., n)$ with the vector of parameters $t_{VP} = \{t_{VP-1}, t_{VP-2}, ..., t_{VP-n}\} \in R^n$, the MM operator is given by:

$$MM^{t_{VP}}(\mathfrak{F}_{TS-1}, \mathfrak{F}_{TS-2}, ..., \mathfrak{F}_{TS-n}) = \left(\frac{1}{n!} \sum_{\vartheta(j) \in s_{SC-n}} \prod_{j=1}^{n} \mathfrak{F}^{t_{VP-j}}_{TS-\vartheta(j)} \right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \tag{8}$$

The DMM is given by:

$$DMM^{t_{VP}}(\mathfrak{F}_{TS-1}, \mathfrak{F}_{TS-2}, ..., \mathfrak{F}_{TS-n}) = \frac{1}{\sum_{j=1}^{n} t_{VP-j}} \left(\prod_{\vartheta(j) \in s_{SC-n}} \sum_{j=1}^{n} t_{VP-j}\mathfrak{F}_{TS-\vartheta(j)} \right)^{\frac{1}{n!}} \tag{9}$$

where $\vartheta(j), (j = 1, 2, 3, ..., n)$ is an n permutation and the set of all permutation of 1 to n is denoted by S_{SC-n}.

2.1. Complex T-Spherical Fuzzy 2-Tuple Linguistic Sets

In this part, we propose the novel concept of CTSF2-TLS, which is the mixture of CFS, TSFS, and 2-TLS. Some important fundamental operational laws of the CTSF2-TLS are also established.

Definition 6. A CTSF2-TLS is given by:

$$\mathfrak{F}_{CTS} = \left\{ \left(\left(s_{S_{LT}(u)}, a_{SC}\right), \left(\mu_{\mathfrak{F}_{CTS}}(u), \xi_{\mathfrak{F}_{CTS}}(u), \eta_{\mathfrak{F}_{CTS}}(u)\right) \right) : u \in U_{UNI} \right\} \tag{10}$$

where $\mu_{\mathfrak{I}_{CTS}} = \mu_{\mathfrak{I}_{RPTL}} e^{i2\pi W_{\mu_{\mathfrak{I}_{IPTL}}}}, \xi_{\mathfrak{I}_{CTS}} = \xi_{\mathfrak{I}_{RPTL}} e^{i2\pi W_{\xi_{\mathfrak{I}_{IPTL}}}}$ and $\eta_{\mathfrak{I}_{CTS}} = \eta_{\mathfrak{I}_{RPTL}} e^{i2\pi W_{\eta_{\mathfrak{I}_{IPTL}}}}$, with a condition: $0 \leq \mu^{q_{SC}}_{\mathfrak{I}_{RPTL}}(u) + \xi^{q_{SC}}_{\mathfrak{I}_{RPTL}}(u) + \eta^{q_{SC}}_{\mathfrak{I}_{RPTL}}(u) \leq 1, 0 \leq W^{q_{SC}}_{\mu_{\mathfrak{I}_{IPTL}}}(u) + W^{q_{SC}}_{\xi_{\mathfrak{I}_{IPTL}}}(u) + W^{q_{SC}}_{\eta_{\mathfrak{I}_{IPTL}}}(u) \leq 1$ and the pair $\left(s_{S_{LT}(u)}, a_{SC}\right)$ is called 2-tulpe linguistic variable with $a_{SC} \in \left[-\frac{1}{2g}, \frac{1}{2g}\right\}$ and $s_{S_{LT}(u)} \in S_{LT}$. Moreover, $\zeta_{\mathfrak{I}_{TS}}(u) = \zeta_{\mathfrak{I}_{RPTL}} e^{i2\pi W_{\zeta_{\mathfrak{I}_{IPTL}}}} = \left(1 - \left(\mu^{q_{SC}}_{\mathfrak{I}_{RPTL}}(u) + \xi^{q_{SC}}_{\mathfrak{I}_{RPTL}}(u) + \eta^{q_{SC}}_{\mathfrak{I}_{RPTL}}(u)\right)\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(1-\left(W^{q_{SC}}_{\mu_{\mathfrak{I}_{IPTL}}}(u)+W^{q_{SC}}_{\xi_{\mathfrak{I}_{IPTL}}}(u)+W^{q_{SC}}_{\eta_{\mathfrak{I}_{IPTL}}}(u)\right)\right)^{\frac{1}{q_{SC}}}}$ is called refusal grade, the complex T-spherical fuzzy 2-tuple linguistic number (CTSF2-TLN) is represented by:

$$\mathfrak{I}_{CTS} = \left(\left(s_{S_{LT}(u)}, a_{SC}\right), \left(\mu_{\mathfrak{I}_{CTS}}(u), \xi_{\mathfrak{I}_{CTS}}(u), \eta_{\mathfrak{I}_{CTS}}(u)\right)\right) \text{ or}$$

$$\mathfrak{I}_{CTS} = \left(\left(s_{S_{LT}}, a_{SC}\right), \left(\mu_{\mathfrak{I}_{RPTL}} e^{i2\pi W_{\mu_{\mathfrak{I}_{IPTL}}}}, \xi_{\mathfrak{I}_{RPTL}} e^{i2\pi W_{\xi_{\mathfrak{I}_{IPTL}}}}, \eta_{\mathfrak{I}_{RPTL}} e^{i2\pi W_{\eta_{\mathfrak{I}_{IPTL}}}}\right)\right).$$

An example of CTSF2-TLS is given as follows:

$$\mathfrak{I}_{CTS} = \begin{pmatrix} \left(\left(s_{S_{LT-2}}, 0.01\right), \left(0.8e^{i2\pi(0.8)}, 0.1e^{i2\pi(0.1)}, 0.3e^{i2\pi(0.3)}\right)\right), \\ \left(\left(s_{S_{LT-2}}, 0.02\right), \left(0.18e^{i2\pi(0.81)}, 0.11e^{i2\pi(0.11)}, 0.31e^{i2\pi(0.31)}\right)\right), \\ \left(\left(s_{S_{LT-3}}, 0.03\right), \left(0.81e^{i2\pi(0.18)}, 0.01e^{i2\pi(0.1)}, 0.03e^{i2\pi(0.3)}\right)\right), \\ \left(\left(s_{S_{LT-3}}, 0.04\right), \left(0.018e^{i2\pi(0.081)}, 0.011e^{i2\pi(0.11)}, 0.031e^{i2\pi(0.31)}\right)\right) \end{pmatrix}$$

Moreover, we can give some special cases. If we ignore the terms of 2-tuple linguistic sets, complex-valued abstinence, and complex-valued nonmembership, then the CTSF2-TLS (Eq. (10)) will be converted to CFS, similarly, if we ignore the terms of 2-tuple linguistic sets and the imaginary parts of the complex-valued membership, complex-valued abstinence, and complex-valued nonmembership, then the CTSF2-TLS (Eq. (10)) will be converted to TSFSs. Finally, if we ignore the complex-valued membership, complex-valued abstinence, and complex-valued nonmembership, then the idea of CTSF2-TLS (Eq. (10)) will be converted to 2-tuple linguistic sets.

Definition 7. For any two CTSF2-TLNs $\mathfrak{I}_{CTS-1} = \left(\left(s_{S_{LT-1}}, a_{SC-1}\right), \left(\mu_{\mathfrak{I}_{RPTL-1}} e^{i2\pi W_{\mu_{\mathfrak{I}_{IPTL-1}}}}, \xi_{\mathfrak{I}_{RPTL-1}} e^{i2\pi W_{\xi_{\mathfrak{I}_{IPTL-1}}}}, \eta_{\mathfrak{I}_{RPTL-1}} e^{i2\pi W_{\eta_{\mathfrak{I}_{IPTL-1}}}}\right)\right)$ and $\mathfrak{I}_{CTS-2} = \left(\left(s_{S_{LT-2}}, a_{SC-2}\right), \left(\mu_{\mathfrak{I}_{RPTL-2}} e^{i2\pi W_{\mu_{\mathfrak{I}_{IPTL-2}}}}, \xi_{\mathfrak{I}_{RPTL-2}} e^{i2\pi W_{\xi_{\mathfrak{I}_{IPTL-2}}}}, \eta_{\mathfrak{I}_{RPTL-2}} e^{i2\pi W_{\eta_{\mathfrak{I}_{IPTL-2}}}}\right)\right)$, then

1. $$\mathfrak{I}_{CTS-1} \oplus_{CTS} \mathfrak{I}_{CTS-2} = \begin{pmatrix} \triangle_{LT}\left(\Delta^{-1}_{LT}\left(s_{S_{LT-1}}, a_{SC-1}\right) + \Delta^{-1}_{LT}\left(s_{S_{LT-2}}, a_{SC-2}\right)\right), \\ \left(\mu^{q_{SC}}_{\mathfrak{I}_{RPTL-1}} + \mu^{q_{SC}}_{\mathfrak{I}_{RPTL-2}} - \mu^{q_{SC}}_{\mathfrak{I}_{RPTL-1}} \mu^{q_{SC}}_{\mathfrak{I}_{RPTL-2}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(W^{q_{SC}}_{\mu_{\mathfrak{I}_{IPTL-1}}} + W^{q_{SC}}_{\mu_{\mathfrak{I}_{IPTL-2}}} - W^{q_{SC}}_{\mu_{\mathfrak{I}_{IPTL-1}}} W^{q_{SC}}_{\mu_{\mathfrak{I}_{IPTL-2}}}\right)^{\frac{1}{q_{SC}}}}, \\ \xi_{\mathfrak{I}_{RPTL-1}} \xi_{\mathfrak{I}_{RPTL-2}} e^{i2\pi\left(W_{\xi_{\mathfrak{I}_{IPTL-1}}} W_{\xi_{\mathfrak{I}_{IPTL-2}}}\right)}, \eta_{\mathfrak{I}_{RPTL-1}} \eta_{\mathfrak{I}_{RPTL-2}} e^{i2\pi\left(W_{\eta_{\mathfrak{I}_{IPTL-1}}} W_{\eta_{\mathfrak{I}_{IPTL-2}}}\right)} \end{pmatrix};$$
2. $$\mathfrak{I}_{CTS-1} \otimes_{CTS} \mathfrak{I}_{CTS-2} = \begin{pmatrix} \triangle_{LT}\left(\Delta^{-1}_{LT}\left(s_{S_{LT-1}}, a_{SC-1}\right) \times \Delta^{-1}_{LT}\left(s_{S_{LT-2}}, a_{SC-2}\right)\right), \mu_{\mathfrak{I}_{RPTL-1}} \mu_{\mathfrak{I}_{RPTL-2}} e^{i2\pi\left(W_{\mu_{\mathfrak{I}_{IPTL-1}}} W_{\mu_{\mathfrak{I}_{IPTL-2}}}\right)}, \\ \left(\xi^{q_{SC}}_{\mathfrak{I}_{RPTL-1}} + \xi^{q_{SC}}_{\mathfrak{I}_{RPTL-2}} - \xi^{q_{SC}}_{\mathfrak{I}_{RPTL-1}} \xi^{q_{SC}}_{\mathfrak{I}_{RPTL-2}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(W^{q_{SC}}_{\xi_{\mathfrak{I}_{IPTL-1}}} + W^{q_{SC}}_{\xi_{\mathfrak{I}_{IPTL-2}}} - W^{q_{SC}}_{\xi_{\mathfrak{I}_{IPTL-1}}} W^{q_{SC}}_{\xi_{\mathfrak{I}_{IPTL-2}}}\right)^{\frac{1}{q_{SC}}}}, \\ \left(\eta^{q_{SC}}_{\mathfrak{I}_{RPTL-1}} + \eta^{q_{SC}}_{\mathfrak{I}_{RPTL-2}} - \eta^{q_{SC}}_{\mathfrak{I}_{RPTL-1}} \eta^{q_{SC}}_{\mathfrak{I}_{RPTL-2}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(W^{q_{SC}}_{\eta_{\mathfrak{I}_{IPTL-1}}} + W^{q_{SC}}_{\eta_{\mathfrak{I}_{IPTL-2}}} - W^{q_{SC}}_{\eta_{\mathfrak{I}_{IPTL-1}}} W^{q_{SC}}_{\eta_{\mathfrak{I}_{IPTL-2}}}\right)^{\frac{1}{q_{SC}}}} \end{pmatrix};$$

3. $$\mathfrak{F}_{CTS-1}^{\delta_{SC}} = \begin{pmatrix} \triangle_{LT}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-1}}, a_{SC-1}\right)^{\delta_{SC}}\right), \mu_{\mathfrak{F}_{RPTL-1}}^{\delta_{SC}} e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-1}}}^{\delta_{SC}}}, \\ \left(1-\left(1-\xi_{\mathfrak{F}_{RPTL-1}}^{q_{SC}}\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(1-\left(1-W_{\xi_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}}\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}}}, \\ \left(1-\left(1-\eta_{\mathfrak{F}_{RPTL-1}}^{q_{SC}}\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(1-\left(1-W_{\eta_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}}\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}}} \end{pmatrix};$$

4. $$\delta_{SC}\mathfrak{F}_{CTS-1} = \begin{pmatrix} \triangle_{LT}\left(\delta_{SC} \times \Delta_{LT}^{-1}\left(s_{S_{LT-1}}, a_{SC-1}\right)\right), \left(1-\left(1-\mu_{\mathfrak{F}_{RPTL-1}}^{q_{SC}}\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(1-\left(1-W_{\mu_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}}\right)^{\delta_{SC}}\right)^{\frac{1}{q_{SC}}}}, \\ \xi_{\mathfrak{F}_{RPTL-1}}^{\delta_{SC}} e^{i2\pi W_{\xi_{\mathfrak{F}_{IPTL-1}}}^{\delta_{SC}}}, \eta_{\mathfrak{F}_{RPTL-1}}^{\delta_{SC}} e^{i2\pi W_{\eta_{\mathfrak{F}_{IPTL-1}}}^{\delta_{SC}}} \end{pmatrix}.$$

Definition 8. For any two CTSF2-TLNs $\mathfrak{F}_{CTS-1} = \left(\left(s_{S_{LT-1}}, a_{SC-1}\right), \left(\mu_{\mathfrak{F}_{RPTL-1}} e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-1}}}}, \xi_{\mathfrak{F}_{RPTL-1}} e^{i2\pi W_{\xi_{\mathfrak{F}_{IPTL-1}}}}, \eta_{\mathfrak{F}_{RPTL-1}} e^{i2\pi W_{\eta_{\mathfrak{F}_{IPTL-1}}}}\right)\right)$ and $\mathfrak{F}_{CTS-2} = \left(\left(s_{S_{LT-2}}, a_{SC-2}\right), \left(\mu_{\mathfrak{F}_{RPTL-2}} e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-2}}}}, \xi_{\mathfrak{F}_{RPTL-2}} e^{i2\pi W_{\xi_{\mathfrak{F}_{IPTL-2}}}}, \eta_{\mathfrak{F}_{RPTL-2}} e^{i2\pi W_{\eta_{\mathfrak{F}_{IPTL-2}}}}\right)\right)$, the score and accuracy function are given by:

$$S(\mathfrak{F}_{CTS-1}) = \frac{\Delta_{LT}^{-1}\left(s_{S_{LT-1}}, a_{SC-1}\right) \times \left(1 + \mu_{\mathfrak{F}_{TS-1}}^{q_{SC}} + W_{\mu_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}} - \xi_{\mathfrak{F}_{TS-1}}^{q_{SC}} - W_{\xi_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}} - \eta_{\mathfrak{F}_{TS-1}}^{q_{SC}} - W_{\eta_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}}\right)}{4} \tag{11}$$

$$H(\mathfrak{F}_{CTS-1}) = \frac{\Delta_{LT}^{-1}\left(s_{S_{LT-1}}, a_{SC-1}\right) \times \left(\mu_{\mathfrak{F}_{TS-1}}^{q_{SC}} + W_{\mu_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}} + \xi_{\mathfrak{F}_{TS-1}}^{q_{SC}} + W_{\xi_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}} + \eta_{\mathfrak{F}_{TS-1}}^{q_{SC}} + W_{\eta_{\mathfrak{F}_{IPTL-1}}}^{q_{SC}}\right)}{4} \tag{12}$$

Based on the above two notions, the compassion between two CTSF2-TLNs is given by:

1. If $S(\mathfrak{F}_{CTS-1}) > S(\mathfrak{F}_{CTS-2})$, then $\mathfrak{F}_{CTS-1} \succ \mathfrak{F}_{CTS-2}$;
2. If $S(\mathfrak{F}_{CTS-1}) = S(\mathfrak{F}_{CTS-2})$, then:
   (a) If $H(\mathfrak{F}_{CTS-1}) > H(\mathfrak{F}_{CTS-2})$, then $\mathfrak{F}_{CTS-1} \succ \mathfrak{F}_{CTS-2}$;
   (b) If $H(\mathfrak{F}_{CTS-1}) = H(\mathfrak{F}_{CTS-2})$, then $\mathfrak{F}_{CTS-1} = \mathfrak{F}_{CTS-2}$.

Example 1. For any two CTSF2-TLNs $\mathfrak{F}_{CTS-1} = \left(\left(s_{S_{LT-2}}, 0.01\right), \left(0.8e^{i2\pi(0.8)}, 0.1e^{i2\pi(0.1)}, 0.3e^{i2\pi(0.3)}\right)\right)$ and $\mathfrak{F}_{CTS-2} = \left(\left(s_{S_{LT-4}}, -0.02\right), \left(0.9e^{i2\pi(0.9)}, 0.1e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.2)}\right)\right)$, and for $q_{SC} = \delta_{SC} = 2$. Then

1. $$\mathfrak{F}_{CTS-1} \oplus_{CTS} \mathfrak{F}_{CTS-2}$$
$$= \begin{pmatrix} \Delta_{LT}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-2}}, 0.01\right) + \Delta_{LT}^{-1}\left(s_{S_{LT-4}}, -0.02\right)\right), \\ \left(\left(0.8^2 + 0.9^2 - 0.8^2 \times 0.9^2\right)^{(1/2)} e^{i2\pi\left(0.8^2+0.9^2-0.8^2\times0.9^2\right)^{(1/2)}}, (0.1 \times 0.1)e^{i2\pi(0.1\times0.1)}, (0.3 \times 0.2)e^{i2\pi(0.3\times0.2)}\right) \end{pmatrix}$$
$$= \left(\Delta_{LT}\left(\left(\tfrac{2}{4} + 0.01\right) + \left(\tfrac{4}{4} - 0.02\right)\right), \left(0.9652e^{i2\pi(0.9652)}, 0.01e^{i2\pi(0.01)}, 0.06e^{i2\pi(0.06)}\right)\right)$$
$$= \left(\Delta_{LT}(1.49), \left(0.9652e^{i2\pi(0.9652)}, 0.01e^{i2\pi(0.01)}, 0.06e^{i2\pi(0.06)}\right)\right)$$
$$= \left(\left(s_{S_{LT-4}}, 0.49\right), \left(0.9652e^{i2\pi(0.9652)}, 0.01e^{i2\pi(0.01)}, 0.06e^{i2\pi(0.06)}\right)\right).$$

2. $\mathfrak{F}_{CTS-1} \otimes_{CTS} \mathfrak{F}_{CTS-2}$

$$= \left(\begin{array}{c} \Delta_{LT}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-2}}, 0.01\right) \times \Delta_{LT}^{-1}\left(s_{S_{LT-4}}, -0.02\right)\right), \\ \left(\begin{array}{c} (0.8 \times 0.9) e^{i2\pi(0.8\times 0.9)}, \\ \left(0.1^2 + 0.1^2 - 0.1^2 \times 0.1^2\right)^{(1/2)} e^{i2\pi\left(0.1^2+0.1^2-0.1^2\times 0.1^2\right)^{(1/2)}} \\ \left(0.3^2 + 0.2^2 - 0.3^2 \times 0.2^2\right)^{(1/2)} e^{i2\pi\left(0.3^2+0.2^2-0.3^2\times 0.2^2\right)^{(1/2)}} \end{array} \right) \end{array} \right)$$

$$= \left(\Delta_{LT}\left(\left(\tfrac{2}{4} + 0.01\right) \times \left(\tfrac{4}{4} - 0.02\right)\right), \left(0.72e^{i2\pi(0.72)}, 0.1e^{i2\pi(0.1)}, 0.36e^{i2\pi(0.36)}\right)\right)$$

$$= \left(\Delta_{LT}(0.4998), \left(0.72e^{i2\pi(0.72)}, 0.1e^{i2\pi(0.1)}, 0.36e^{i2\pi(0.36)}\right)\right)$$

$$= \left(\left(s_{S_{LT-3}}, 0.1665\right), \left(0.72e^{i2\pi(0.72)}, 0.1e^{i2\pi(0.1)}, 0.36e^{i2\pi(0.36)}\right)\right).$$

3. $\mathfrak{F}_{CTS-1}^2 = \left(\begin{array}{c} \Delta_{LT}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-2}}, 0.01\right)^2\right), \\ \left(\begin{array}{c} 0.8^2 e^{i2\pi(0.8^2)}, \left(1 - \left(1 - 0.1^2\right)^2\right)^{(1/2)} e^{i2\pi\left(1-\left(1-0.1^2\right)^2\right)^{(1/2)}}, \\ \left(1 - \left(1 - 0.3^2\right)^2\right)^{(1/2)} e^{i2\pi\left(1-\left(1-0.3^2\right)^2\right)^{(1/2)}} \end{array} \right) \end{array} \right)$

$$= \left(\Delta_{LT}\left(0.51^2\right), \left(0.64e^{i2\pi(0.64)}, \left(1 - (1 - 0.01)^2\right)^{(1/2)} e^{i2\pi\left(1-(1-0.01)^2\right)^{(1/2)}}, \left(1 - (1 - 0.09)^2\right)^{(1/2)} e^{i2\pi\left(1-(1-0.09)^2\right)^{(1/2)}}\right)\right)$$

$$= \left(\Delta_{LT}(0.2601), \left(0.64e^{i2\pi(0.64)}, 0.14e^{i2\pi(0.14)}, 0.42e^{i2\pi(0.42)}\right)\right)$$

$$= \left(\left(s_{S_{LT-1}}, 0.0101\right), \left(0.64e^{i2\pi(0.64)}, 0.14e^{i2\pi(0.14)}, 0.42e^{i2\pi(0.42)}\right)\right).$$

4. $2 \times \mathfrak{F}_{CTS-1} = \left(\Delta_{LT}\left(2 \times \Delta_{LT}^{-1}\left(s_{S_{LT-2}}, 0.01\right)\right), \left(\left(1 - \left(1 - 0.8^2\right)^2\right)^{(1/2)} e^{i2\pi\left(1-\left(1-0.8^2\right)^2\right)^{(1/2)}}, 0.1^2 e^{i2\pi(0.1)^2}, 0.3^2 e^{i2\pi(0.3)^2}\right)\right)$

$$= \left(\Delta_{LT}(2 \times 0.51), \left(\left(1 - (1 - 0.64)^2\right)^{(1/2)} e^{i2\pi\left(1-(1-0.64)^2\right)^{(1/2)}}, 0.01e^{i2\pi(0.01)}, 0.09e^{i2\pi(0.09)}\right)\right)$$

$$= \left(\Delta_{LT}(1.02), \left(0.9330e^{i2\pi(0.9330)}, 0.01e^{i2\pi(0.01)}, 0.09e^{i2\pi(0.09)}\right)\right)$$

$$= \left(\left(s_{S_{LT-4}}, 0.02\right), \left(0.9330e^{i2\pi(0.9330)}, 0.01e^{i2\pi(0.01)}, 0.09e^{i2\pi(0.09)}\right)\right).$$

Further, we examine the interrelationship between two CTSF2-TLNs based on the score functions, such that

$S(\mathfrak{F}_{CTS-1}) = \left(\Delta_{LT}^{-1}\left(s_{S_{LT-2}}, 0.01\right) \times \left(1 + 0.8^2 + 0.8^2 - 0.1^2 - 0.1^2 - 0.3^2 - 0.3^2\right)\right)/4 = ((0.51) \times (2.08))/4 = 0.2652$, $S(\mathfrak{F}_{CTS-2}) = \left(\Delta_{LT}^{-1}\left(s_{S_{LT-4}}, -0.02\right) \times \left(1 + 0.9^2 + 0.9^2 - 0.1^2 - 0.1^2 - 0.2^2 - 0.2^2\right)\right)/4 = ((0.98) \times (2.52))/4 = 0.6174$,

So, $S(\mathfrak{F}_{CTS-2}) \geq S(\mathfrak{F}_{CTS-1})$.

3. MM AGGREGATION OPERATORS FOR CTSF2-TLSs

In this part, we investigate the MM and DMM operators based on a CTSF2-TLS, and they are called CTSF2-TLMM operator and CTSF2-TLDMM operator. Their advantages are that they are more generalized than averaging operator (AO), geometric operator (GO), BMO, and MSMO which are the special cases of the explored operators.

Definition 9. Choose the family of CTSF2-TLNs $\mathfrak{F}_{CTS-j} = \left(\left(s_{S_{LT-j}}, a_{SC-j}\right), \left(\mu_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-j}}}}, \xi_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\xi_{\mathfrak{F}_{IPTL-j}}}}, \eta_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\eta_{\mathfrak{F}_{IPTL-j}}}}\right)\right)$, $(j = 1, 2, 3, .., n)$ with vector of parameters $t_{VP} = \{t_{VP-1}, t_{VP-2}, ..., t_{VP-n}\} \in R^n$, the CTSF2-TLMM operator is given by:

$$CTSF2 - TLMM^{t_{VP}}(\mathfrak{F}_{CTS-1}, \mathfrak{F}_{CTS-2}, ..., \mathfrak{F}_{CTS-n}) = \left(\frac{1}{n!} \underset{\vartheta(j)\in s_{SC-n}}{\oplus_{CTS}} \left(\underset{j=1}{\overset{n}{\otimes_{CTS}}} \left(\mathfrak{F}_{TS-\vartheta(j)}^{t_{VP-j}}\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \tag{13}$$

where $\vartheta(j), (j = 1, 2, 3, ..., n)$ is an n permutation and the set of all permutation of 1 to n is denoted by S_{SC-n}.

Based on the operational laws, we establish the following result.

Theorem 1. *Suppose the family of CTSF2-TLNs is* $\mathfrak{T}_{CTS-j} = \left(\left(s_{S_{LT-j}}, a_{SC-j}\right), \left(\mu_{\mathfrak{T}_{RPTL-j}} e^{i2\pi W_{\mu_{\mathfrak{T}_{IPTL-j}}}}, \xi_{\mathfrak{T}_{RPTL-j}} e^{i2\pi W_{\xi_{\mathfrak{T}_{IPTL-j}}}}, \eta_{\mathfrak{T}_{RPTL-j}} e^{i2\pi W_{\eta_{\mathfrak{T}_{IPTL-j}}}}\right)\right)$, $(j = 1, 2, 3, .., n)$ *with vector of parameters* $t_{VP} = \{t_{VP-1}, t_{VP-2}, ..., t_{VP-n}\} \in R^n$. *Then the aggregated value of CTSF2-TLNs is again a CTSF2-TLN, and*

$$
\begin{aligned}
&CTSF2-TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1}, \mathfrak{T}_{CTS-2}, ..., \mathfrak{T}_{CTS-n}\right) \\
&= \left(\begin{array}{l}
\Delta_{LT}\left(\frac{1}{n!}\sum_{\vartheta(j)\in s_{SC-n}}\prod_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}, \\
\left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\mu_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} e^{i2\pi\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n} W_{\mu_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}}, \\
\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\xi_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}\right)^{\frac{1}{q_{SC}}} \\
e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\xi_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}, \\
\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\eta_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}\right)^{\frac{1}{q_{SC}}} \\
e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\eta_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}
\end{array} \right)
\end{aligned}
$$

Proof: Using the Definition 7, we have

$$\mathfrak{F}_{CTS-\vartheta(j)}^{t_{VP-j}}=\left(\begin{array}{l}\triangle_{LT}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)^{t_{VP-j}}\right),\mu_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{t_{VP-j}}e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{t_{VP-j}}},\\ \left(1-\left(1-\xi_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)^{\frac{1}{q_{SC}}}e^{i2\pi\left(1-\left(1-W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)^{\frac{1}{q_{SC}}}},\\ \left(1-\left(1-\eta_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)^{\frac{1}{q_{SC}}}e^{i2\pi\left(1-\left(1-W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)^{\frac{1}{q_{SC}}}}\end{array}\right)$$

$$\otimes_{j=1}^{n}{}_{CTS}\mathfrak{F}_{CTS-\vartheta(j)}^{t_{VP-j}}=\left(\begin{array}{l}\triangle_{LT}\left(\prod_{j=1}^{n}\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)^{t_{VP-j}}\right),\prod_{j=1}^{n}\mu_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{t_{VP-j}}e^{i2\pi\prod_{j=1}^{n}W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{t_{VP-j}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\xi_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)^{\frac{1}{q_{SC}}}e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)^{\frac{1}{q_{SC}}}},\\ \left(1-\prod_{j=1}^{n}\left(1-\eta_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)^{\frac{1}{q_{SC}}}e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)^{\frac{1}{q_{SC}}}}\end{array}\right)$$

$$\oplus_{\vartheta(j)\in s_{SC-n}}{}_{CTS}\otimes_{j=1}^{n}{}_{CTS}\mathfrak{F}_{CTS-\vartheta(j)}^{t_{VP-j}}=\left(\begin{array}{l}\Delta_{LT}\left(\sum_{\vartheta(j)\in s_{SC-n}}\prod_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right),\\ \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\mu_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)\right)^{\frac{1}{q_{SC}}}\right)e^{i2\pi\left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)\right)^{\frac{1}{q_{SC}}}\right)},\\ \left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\xi_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{q_{SC}}}e^{i2\pi\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{q_{SC}}}},\\ \left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\eta_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{q_{SC}}}e^{i2\pi\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{q_{SC}}}}\end{array}\right)$$

$$\left(\frac{1}{n!}\oplus_{\substack{CTS\\ \vartheta(j)\in s_{SC-n}}}\left(\otimes_{\substack{CTS\\ j=1}}^{n}\left(\mathfrak{T}_{TS-\vartheta(j)}^{t_{VP-j}}\right)\right)\right)=$$

$$\left(\begin{array}{l}\Delta_{LT}\left(\frac{1}{n!}\sum_{\vartheta(j)\in s_{SC-n}}\prod_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right),\\ \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\mu_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)e^{i2\pi\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}W_{\mu_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)},\\ \left(\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\xi_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\\ \qquad e^{i2\pi\left(\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\xi_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}},\\ \left(\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\eta_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\\ \qquad e^{i2\pi\left(\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\eta_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}}\end{array}\right)$$

$$\left(\frac{1}{n!}\underset{\vartheta(j)\in s_{SC-n}}{\oplus_{CTS}}\left(\otimes_{j=1}^{n}{}_{CTS}\left(\mathfrak{T}_{TS-\vartheta(j)}^{t_{VP-j}}\right)\right)\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}$$

$$=\left(\begin{array}{l}\Delta_{LT}\left(\frac{1}{n!}\sum_{\vartheta(j)\in s_{SC-n}}\prod_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}},\\ \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\mu_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\\ e^{i2\pi\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}W_{\mu_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}},\\ \left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\xi_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}\\ e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\xi_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}},\\ \left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\eta_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}\\ e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\eta_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}\end{array}\right)$$

Hence, the result is proved.

Example 2. For any three CTSF2-TLNs $\mathfrak{T}_{CTS-1}=\left(\left(s_{S_{LT-2}},0.01\right),\left(0.8e^{i2\pi(0.8)},0.0e^{i2\pi(0.0)},0.6e^{i2\pi(0.6)}\right)\right)$, $\mathfrak{T}_{CTS-2}=\left(\left(s_{S_{LT-3}},0.05\right),\left(0.7e^{i2\pi(0.7)},0.1e^{i2\pi(0.1)},0.5e^{i2\pi(0.5)}\right)\right)$, and $\mathfrak{T}_{CTS-3}=\left(\left(s_{S_{LT-4}},-0.02\right),\left(0.9e^{i2\pi(0.9)},0.01e^{i2\pi(0.01)},0.4e^{i2\pi(0.4)}\right)\right)$, and their linguistic term set $S_{LT}=\left\{s_{S_{LT-0}},s_{S_{LT-1}},s_{S_{LT-2}},s_{S_{LT-3}},s_{S_{LT-4}}\right\}$ with the value of parameter $t_{VP}=(1,2,1)$ for $q_{SC}=2$. Then by CTSF2-TLMM operator, we get

$$\Delta_{LT}\left(\frac{1}{3!}\sum_{\vartheta(j)\in S_{SC-3}}\prod_{j=1}^{3}\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)\right)^{t_{VP-j}}\right)^{\frac{1}{1+2+1}}$$

$$=\Delta_{LT}\left(\frac{1}{6}\begin{pmatrix}\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-2}},0.01\right)\right)\right)^{1}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-3}},0.05\right)\right)\right)^{2}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-4}},-0.02\right)\right)\right)^{1}\\+\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-2}},0.01\right)\right)\right)^{1}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-4}},-0.02\right)\right)\right)^{2}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-3}},0.05\right)\right)\right)^{1}\\+\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-3}},0.05\right)\right)\right)^{1}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-2}},0.01\right)\right)\right)^{2}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-4}},-0.02\right)\right)\right)^{1}\\+\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-3}},0.05\right)\right)\right)^{1}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-4}},-0.02\right)\right)\right)^{2}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-2}},0.01\right)\right)\right)^{1}\\+\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-4}},-0.02\right)\right)\right)^{1}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-2}},0.01\right)\right)\right)^{2}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-3}},0.05\right)\right)\right)^{1}\\+\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-4}},-0.02\right)\right)\right)^{1}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-3}},0.05\right)\right)\right)^{2}\times\left(\Delta_{LT}^{-1}\left(\left(s_{S_{LT-2}},0.01\right)\right)\right)^{1}\end{pmatrix}^{\frac{1}{4}}\right)$$

$$=\Delta_{LT}(0.7649)=\left(\left(s_{S_{LT-3}},0.0149\right)\right);$$

Further, we calculate the value of the complex-valued truth grade of the CTSF2-TLMM operator, as follows:

$$\left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{3}\mu_{\Im_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{6}}\right)^{\frac{1}{2}}\right)^{\frac{1}{4}}e^{i2\pi\left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{3}W_{\mu_{\Im_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{6}}\right)^{\frac{1}{2}}\right)^{\frac{1}{4}}}$$

$$=\left(\left(1-\begin{pmatrix}\left(1-0.8^{2\times1}\times0.7^{2\times2}\times0.9^{2\times1}\right)\\\times\left(1-0.8^{2\times1}\times0.9^{2\times2}\times0.7^{2\times1}\right)\\\times\left(1-0.7^{2\times1}\times0.8^{2\times2}\times0.9^{2\times1}\right)\\\times\left(1-0.7^{2\times1}\times0.9^{2\times2}\times0.8^{2\times1}\right)\\\times\left(1-0.9^{2\times1}\times0.7^{2\times2}\times0.8^{2\times1}\right)\\\times\left(1-0.9^{2\times1}\times0.8^{2\times2}\times0.7^{2\times1}\right)\end{pmatrix}^{\frac{1}{6}}\right)^{\frac{1}{2}}\right)^{\frac{1}{4}}e^{i2\pi\left(1-\begin{pmatrix}\left(1-0.8^{2\times1}\times0.7^{2\times2}\times0.9^{2\times1}\right)\\\times\left(1-0.8^{2\times1}\times0.9^{2\times2}\times0.7^{2\times1}\right)\\\times\left(1-0.7^{2\times1}\times0.8^{2\times2}\times0.9^{2\times1}\right)\\\times\left(1-0.7^{2\times1}\times0.9^{2\times2}\times0.8^{2\times1}\right)\\\times\left(1-0.9^{2\times1}\times0.7^{2\times2}\times0.8^{2\times1}\right)\\\times\left(1-0.9^{2\times1}\times0.8^{2\times2}\times0.7^{2\times1}\right)\end{pmatrix}^{\frac{1}{6}}\right)^{\frac{1}{2}}\right)^{\frac{1}{4}}}$$

$$=0.7983e^{i2\pi(0.7983)};$$

Next, we get the value of the complex-valued abstinence grade of the CTSF2-TLMM operator, such that

$$\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{3}\left(1-\xi_{\Im_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{3!}}\right)^{\frac{1}{4}}\right)^{\frac{1}{2}} e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\xi_{\Im_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{3!}}\right)^{\frac{1}{4}}\right)^{\frac{1}{2}}}$$

$$=\left(1-\left(1-\left(\begin{array}{l}\left(1-\left(1-0.0^{2}\right)^{1}\times\left(1-0.1^{2}\right)^{2}\times\left(1-0.01^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.0^{2}\right)^{1}\times\left(1-0.01^{2}\right)^{2}\times\left(1-0.1^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.1^{2}\right)^{1}\times\left(1-0.0^{2}\right)^{2}\times\left(1-0.01^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.1^{2}\right)^{1}\times\left(1-0.01^{2}\right)^{2}\times\left(1-0.0^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.01^{2}\right)^{1}\times\left(1-0.0^{2}\right)^{2}\times\left(1-0.1^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.01^{2}\right)^{1}\times\left(1-0.1^{2}\right)^{2}\times\left(1-0.0^{2}\right)^{1}\right)\end{array}\right)^{\frac{1}{3!}}\right)^{\frac{1}{4}}\right)^{\frac{1}{2}}$$

$$e^{i2\pi\left(1-\left(1-\left(\begin{array}{l}\left(1-\left(1-0.0^{2}\right)^{1}\times\left(1-0.1^{2}\right)^{2}\times\left(1-0.01^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.0^{2}\right)^{1}\times\left(1-0.01^{2}\right)^{2}\times\left(1-0.1^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.1^{2}\right)^{1}\times\left(1-0.0^{2}\right)^{2}\times\left(1-0.01^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.1^{2}\right)^{1}\times\left(1-0.01^{2}\right)^{2}\times\left(1-0.0^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.01^{2}\right)^{1}\times\left(1-0.0^{2}\right)^{2}\times\left(1-0.1^{2}\right)^{1}\right)\\ \times\left(1-\left(1-0.01^{2}\right)^{1}\times\left(1-0.1^{2}\right)^{2}\times\left(1-0.0^{2}\right)^{1}\right)\end{array}\right)^{\frac{1}{3!}}\right)^{\frac{1}{4}}\right)^{\frac{1}{2}}}$$

$$=0.0e^{i2\pi(0.0)}$$

In last, we obtain the value of the complex-valued falsity grade of the CTSF2-TLMM operator, such that

$$\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{3}\left(1-\eta_{\Im_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{3!}}\right)^{\frac{1}{4}}\right)^{\frac{1}{2}} e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\eta_{\Im_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{3!}}\right)^{\frac{1}{4}}\right)^{\frac{1}{2}}}$$

$$= \left(1-\left(1-\left(\begin{array}{l}\left(1-\left(1-0.6^2\right)^1\times\left(1-0.5^2\right)^2\times\left(1-0.4^2\right)^1\right)\\ \times\left(1-\left(1-0.6^2\right)^1\times\left(1-0.4^2\right)^2\times\left(1-0.5^2\right)^1\right)\\ \times\left(1-\left(1-0.5^2\right)^1\times\left(1-0.6^2\right)^2\times\left(1-0.4^2\right)^1\right)\\ \times\left(1-\left(1-0.5^2\right)^1\times\left(1-0.4^2\right)^2\times\left(1-0.6^2\right)^1\right)\\ \times\left(1-\left(1-0.4^2\right)^1\times\left(1-0.6^2\right)^2\times\left(1-0.5^2\right)^1\right)\\ \times\left(1-\left(1-0.4^2\right)^1\times\left(1-0.5^2\right)^2\times\left(1-0.6^2\right)^1\right)\end{array}\right)^{\frac{1}{3!}}\right)^{\frac{1}{4}}\right)^{\frac{1}{2}} e^{i2\pi\left(1-\left(1-\left(\begin{array}{l}\left(1-\left(1-0.6^2\right)^1\times\left(1-0.5^2\right)^2\times\left(1-0.4^2\right)^1\right)\\ \times\left(1-\left(1-0.6^2\right)^1\times\left(1-0.4^2\right)^2\times\left(1-0.5^2\right)^1\right)\\ \times\left(1-\left(1-0.5^2\right)^1\times\left(1-0.6^2\right)^2\times\left(1-0.4^2\right)^1\right)\\ \times\left(1-\left(1-0.5^2\right)^1\times\left(1-0.4^2\right)^2\times\left(1-0.6^2\right)^1\right)\\ \times\left(1-\left(1-0.4^2\right)^1\times\left(1-0.6^2\right)^2\times\left(1-0.5^2\right)^1\right)\\ \times\left(1-\left(1-0.4^2\right)^1\times\left(1-0.5^2\right)^2\times\left(1-0.6^2\right)^1\right)\end{array}\right)^{\frac{1}{3!}}\right)^{\frac{1}{4}}\right)^{\frac{1}{2}}}$$

$$= 0.5095e^{i2\pi(0.5095)};$$

then $\left(\frac{1}{3!}\bigoplus_{\substack{CTS\\ \vartheta(j)\in s_{SC-n}}}\left(\bigotimes_{j=1}^{3}{}_{CTS}\left(\mathfrak{T}_{TS-\vartheta(j)}^{t_{VP-j}}\right)\right)\right)^{\frac{1}{(1+2+1)}} = \left(\left(s_{S_{LT-3}}, 0.0149\right), \left(0.7983e^{i2\pi(0.7983)}, 0.0e^{i2\pi(0.0)}, 0.5095e^{i2\pi(0.5095)}\right)\right)$.

If we set zero to the imaginary parts of the complex-valued truth and falsity grades, then the Example 2 is converted for T-spherical fuzzy 2-tuple linguistic variables, which is a special case of the established operator.

Theorem 2. *Suppose the families of two CTSF2-TLNs are* $\mathfrak{T}_{CTS-j} = \left(\left(s_{S_{LT-j}}, a_{SC-j}\right), \left(\mu_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W_{\mu_{\mathfrak{T}_{IPTL-j}}}}, \xi_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W_{\xi_{\mathfrak{T}_{IPTL-j}}}}, \eta_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W_{\eta_{\mathfrak{T}_{IPTL-j}}}}\right)\right)$, $(j = 1, 2, 3, .., n)$ *and* $\mathfrak{T}_{CTS-*j} = \left(\left(s_{S_{LT-*j}}, a_{SC-*j}\right), \left(\mu_{\mathfrak{T}_{RPTL-*j}}e^{i2\pi W_{\mu_{\mathfrak{T}_{IPTL-*j}}}}, \xi_{\mathfrak{T}_{RPTL-*j}}e^{i2\pi W_{\xi_{\mathfrak{T}_{IPTL-*j}}}}, \eta_{\mathfrak{T}_{RPTL-*j}}e^{i2\pi W_{\eta_{\mathfrak{T}_{IPTL-*j}}}}\right)\right)$, $(j = 1, 2, 3, .., n)$ *with vector of parameters* $t_{VP} = \{t_{VP-1}, t_{VP-2}, ..., t_{VP-n}\} \in R^n$. *Then the idempotency, monotonicity, and boundedness are shown as follows:*

1. *If* $\mathfrak{T}_{CTS-j} = \left(\left(s_{S_{LT-j}}, a_{SC-j}\right), \left(\mu_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W_{\mu_{\mathfrak{T}_{IPTL-j}}}}, \xi_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W_{\xi_{\mathfrak{T}_{IPTL-j}}}}, \eta_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W_{\eta_{\mathfrak{T}_{IPTL-j}}}}\right)\right)$, $(j = 1, 2, 3, .., n)$ *all are equal if and only if* $\mathfrak{T}_{CTS-j} = \mathfrak{T}_{CTS}$, *then*
   $CTSF2 - TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1}, \mathfrak{T}_{CTS-2}, ..., \mathfrak{T}_{CTS-n}\right) = \mathfrak{T}_{CTS}$
2. *If* $\left(s_{S_{LT-j}}, a_{SC-j}\right) \geq \left(\hat{s}_{S_{LT-j}}, a_{SC-j}\right)$, $\mu_{\mathfrak{T}_{RPTL-j}} \geq \mu_{\mathfrak{T}_{RPTL-j}}$, $W_{\mu_{\mathfrak{T}_{IPTL-j}}} \geq W_{\mu_{\mathfrak{T}_{IPTL-j}}}$, $\xi_{\mathfrak{T}_{RPTL-j}} \leq \xi_{\mathfrak{T}_{RPTL-j}}$, $W_{\xi_{\mathfrak{T}_{IPTL-j}}} \leq W_{\xi_{\mathfrak{T}_{IPTL-j}}}$, *and* $\eta_{\mathfrak{T}_{RPTL-j}} \leq \hat{\eta}_{\mathfrak{T}_{RPTL-j}}$, $W_{\eta_{\mathfrak{T}_{IPTL-j}}} \leq W_{\eta_{\mathfrak{T}_{IPTL-j}}}$, *then*
   $CTSF2 - TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1}, \mathfrak{T}_{CTS-2}, ..., \mathfrak{T}_{CTS-n}\right) \geq CTSF2 - TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1}, \mathfrak{T}_{CTS-2}, ..., \mathfrak{T}_{CTS-n}\right)$.
3. *If* $\left(s_{S_{LT-j}}^{-}, a_{SC-j}^{-}\right) = \min_{1\leq j\leq n}\left(s_{S_{LT-j}}, a_{SC-j}\right)$, $\left(s_{S_{LT-j}}^{+}, a_{SC-j}^{+}\right) = \max_{1\leq j\leq n}\left(s_{S_{LT-j}}, a_{SC-j}\right)$,
   $\mu_{\mathfrak{T}_{RPTL-j}}^{+}e^{i2\pi W_{\mu_{\mathfrak{T}_{IPTL-j}}}^{+}} = \max_{1\leq j\leq n}\mu_{\mathfrak{T}_{RPTL-j}}e^{i2\pi\max_{1\leq j\leq n}W_{\mu_{\mathfrak{T}_{IPTL-j}}}}, \mu_{\mathfrak{T}_{RPTL-j}}^{-}e^{i2\pi W_{\mu_{\mathfrak{T}_{IPTL-j}}}^{-}} = \min_{1\leq j\leq n}\mu_{\mathfrak{T}_{RPTL-j}}e^{i2\pi\min_{1\leq j\leq n}W_{\mu_{\mathfrak{T}_{IPTL-j}}}}$,
   $\xi_{\mathfrak{T}_{RPTL-j}}^{+}e^{i2\pi W_{\xi_{\mathfrak{T}_{IPTL-j}}}^{+}} = \max_{1\leq j\leq n}\xi_{\mathfrak{T}_{RPTL-j}}e^{i2\pi\max_{1\leq j\leq n}W_{\xi_{\mathfrak{T}_{IPTL-j}}}}, \xi_{\mathfrak{T}_{RPTL-j}}^{-}e^{i2\pi W_{\xi_{\mathfrak{T}_{IPTL-j}}}^{-}} = \min_{1\leq j\leq n}\xi_{\mathfrak{T}_{RPTL-j}}e^{i2\pi\min_{1\leq j\leq n}W_{\xi_{\mathfrak{T}_{IPTL-j}}}}$,

$$\eta^{+}_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^{+}_{\eta_{\mathfrak{I}_{IPTL-j}}}} = \max_{1\le j\le n} \eta_{\mathfrak{I}_{RPTL-j}} e^{i2\pi \max\limits_{1\le j\le n} W_{\eta_{\mathfrak{I}_{IPTL-j}}}}, \eta^{-}_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^{-}_{\eta_{\mathfrak{I}_{IPTL-j}}}} = \min_{1\le j\le n} \eta_{\mathfrak{I}_{RPTL-j}} e^{i2\pi \min\limits_{1\le j\le n} W_{\eta_{\mathfrak{I}_{IPTL-j}}}}, \text{ then}$$

$$\left(\left(s^{-}_{S_{LT-j}}, a^{-}_{SC-j}\right), \left(\mu^{-}_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^{-}_{\mu_{\mathfrak{I}_{IPTL-j}}}}, \xi^{-}_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^{-}_{\xi_{\mathfrak{I}_{IPTL-j}}}}, \eta^{-}_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^{-}_{\eta_{\mathfrak{I}_{IPTL-j}}}}\right)\right)$$

$$\le CTSF2 - TLMM^{t_{VP}}\left(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n}\right)$$

$$\le \left(\left(s^{+}_{S_{LT-j}}, a^{+}_{SC-j}\right), \left(\mu^{+}_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^{+}_{\mu_{\mathfrak{I}_{IPTL-j}}}}, \xi^{+}_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^{+}_{\xi_{\mathfrak{I}_{IPTL-j}}}}, \eta^{+}_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^{+}_{\eta_{\mathfrak{I}_{IPTL-j}}}}\right)\right)$$

Proof: The proofs of the above three properties are shown as follows:

1. $CTSF2 - TLMM^{t_{VP}}\left(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n}\right) =$

$$\left(\begin{array}{l}
\Delta_{LT}\left(\frac{1}{n!}\sum\limits_{\vartheta(j)\in s_{SC-n}}\prod\limits_{j=1}^{n}\left(\Delta^{-1}_{LT}\left(s_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}, \\
\left(\left(1-\left(\prod\limits_{\vartheta(j)\in s_{SC-n}}\left(1-\prod\limits_{j=1}^{n}\mu^{q_{SC}t_{VP-j}}_{\mathfrak{I}_{RPTL-\vartheta(j)}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} e^{i2\pi\left(1-\left(\prod\limits_{\vartheta(j)\in s_{SC-n}}\left(1-\prod\limits_{j=1}^{n} W^{q_{SC}t_{VP-j}}_{\mu_{\mathfrak{I}_{IPTL-\vartheta(j)}}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}}, \\
\left(1-\left(1-\left(\prod\limits_{\vartheta(j)\in s_{SC-n}}\left(1-\prod\limits_{j=1}^{n}\left(1-\xi^{q_{SC}}_{\mathfrak{I}_{RPTL-\vartheta(j)}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}\right)^{\frac{1}{q_{SC}}} \\
\qquad e^{i2\pi\left(1-\left(1-\left(\prod\limits_{\vartheta(j)\in s_{SC-n}}\left(1-\prod\limits_{j=1}^{n}\left(1-W^{q_{SC}}_{\xi_{\mathfrak{I}_{IPTL-\vartheta(j)}}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}, \\
\left(1-\left(1-\left(\prod\limits_{\vartheta(j)\in s_{SC-n}}\left(1-\prod\limits_{j=1}^{n}\left(1-\eta^{q_{SC}}_{\mathfrak{I}_{RPTL-\vartheta(j)}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}\right)^{\frac{1}{q_{SC}}} \\
\qquad e^{i2\pi\left(1-\left(1-\left(\prod\limits_{\vartheta(j)\in s_{SC-n}}\left(1-\prod\limits_{j=1}^{n}\left(1-W^{q_{SC}}_{\eta_{\mathfrak{I}_{IPTL-\vartheta(j)}}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}
\end{array}\right)$$

$$
\begin{aligned}
&= \left(\begin{array}{l}
\Delta_{LT}\left(\frac{1}{n!} \sum_{\vartheta(j)\in s_{SC-n}} \prod_{j=1}^{n} \left(\Delta_{LT}^{-1}\left(s_{S_{LT}}, a_{SC} \right) \right)^{t_{VP-j}} \right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}, \\
\left(\begin{array}{l}
\left(1 - \left(\prod_{\vartheta(j)\in s_{SC-n}} \left(1 - \prod_{j=1}^{n} \mu_{\mathfrak{T}_{RPTL}}^{q_{SC} t_{VP-j}} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{q_{SC}}} \right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \\
e^{i2\pi \left(1 - \left(\prod_{\vartheta(j)\in s_{SC-n}} \left(1 - \prod_{j=1}^{n} W_{\mu_{\mathfrak{T}_{IPTL}}}^{q_{SC} t_{VP-j}} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{q_{SC}}} \left(\frac{1}{\sum_{j=1}^{n} t_{VP-j}} \right)}
\end{array} \right), \\
\left(\begin{array}{l}
1 - \left(1 - \left(\prod_{\vartheta(j)\in s_{SC-n}} \left(1 - \prod_{j=1}^{n} \left(1 - \xi_{\mathfrak{T}_{RPTL}}^{q_{SC}} \right)^{t_{VP-j}} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \right)^{\frac{1}{q_{SC}}} \\
e^{i2\pi \left(1 - \left(1 - \left(\prod_{\vartheta(j)\in s_{SC-n}} \left(1 - \prod_{j=1}^{n} \left(1 - W_{\xi_{\mathfrak{T}_{IPTL}}}^{q_{SC}} \right)^{t_{VP-j}} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \right)^{\frac{1}{q_{SC}}}}, \\
\left(\begin{array}{l}
1 - \left(1 - \left(\prod_{\vartheta(j)\in s_{SC-n}} \left(1 - \prod_{j=1}^{n} \left(1 - \eta_{\mathfrak{T}_{RPTL}}^{q_{SC}} \right)^{t_{VP-j}} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \right)^{\frac{1}{q_{SC}}} \\
e^{i2\pi \left(1 - \left(1 - \left(\prod_{\vartheta(j)\in s_{SC-n}} \left(1 - \prod_{j=1}^{n} \left(1 - W_{\eta_{\mathfrak{T}_{IPTL}}}^{q_{SC}} \right)^{t_{VP-j}} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \right)^{\frac{1}{q_{SC}}}}
\end{array} \right) \\
&= \left(\left(s_{S_{LT}}, a_{SC} \right), \left(\mu_{\mathfrak{T}_{RPTL}} e^{i2\pi W_{\mu_{\mathfrak{T}_{IPTL}}}}, \xi_{\mathfrak{T}_{RPTL}} e^{i2\pi W_{\xi_{\mathfrak{T}_{IPTL}}}}, \eta_{\mathfrak{T}_{RPTL}} e^{i2\pi W_{\eta_{\mathfrak{T}_{IPTL}}}} \right) \right) = \mathfrak{T}_{CTS}
\end{aligned}
$$

Hence, $CTSF2 - TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1}, \mathfrak{T}_{CTS-2}, ..., \mathfrak{T}_{CTS-n} \right) = \mathfrak{T}_{CTS}$.

2. We know that

$$CTSF2-TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1},\mathfrak{T}_{CTS-2},...,\mathfrak{T}_{CTS-n}\right)=\mathfrak{T}_{CTS-j}$$

$$=\left(\begin{array}{l}\Delta_{LT}\left(\frac{1}{n!}\sum_{\vartheta(j)\in s_{SC-n}}\prod_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}},\\ \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\mu_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}e^{i2\pi\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}W_{\mu_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}},\\ \left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\xi_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}\\ e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\xi_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}},\\ \left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\eta_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}\\ e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\eta_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}\end{array}\right)$$

and

$$CTSF2-TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1},\mathfrak{T}_{CTS-2},...,\mathfrak{T}_{CTS-n}\right)=\hat{\mathfrak{T}}_{CTS-j}$$

$$=\left(\begin{array}{l}\Delta_{LT}\left(\frac{1}{n!}\sum_{\vartheta(j)\in s_{SC-n}}\prod_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(\hat{s}_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}},\\ \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\mu_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}e^{i2\pi\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}W_{\mu_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\left(\sum_{j=1}^{n}t_{VP-j}\right)^{\frac{1}{}}},\\ \left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\xi_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\sum_{j=1}^{n}t_{VP-j}}\right)^{\frac{1}{q_{SC}}}\\ e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\xi_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\sum_{j=1}^{n}t_{VP-j}}\right)^{\frac{1}{q_{SC}}}},\\ \left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\hat{\eta}_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\sum_{j=1}^{n}t_{VP-j}}\right)^{\frac{1}{q_{SC}}}\\ e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\eta_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\sum_{j=1}^{n}t_{VP-j}}\right)^{\frac{1}{q_{SC}}}}\end{array}\right)$$

where $\left(s_{S_{LT-j}}, a_{SC-j}\right) \geq \left(\hat{s}_{S_{LT-j}}, a_{SC-j}\right)$

$\Rightarrow \left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}} \geq \left(\Delta_{LT}^{-1}\left(\hat{s}_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}$

$$\Rightarrow \Delta_{LT}\left(\frac{1}{n!}\sum_{\vartheta(j)\in s_{SC-n}}\prod_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \geq \Delta_{LT}\left(\frac{1}{n!}\sum_{\vartheta(j)\in s_{SC-n}}\prod_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(\hat{s}_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}$$ Further,

we check the real part of the complex-valued truth grade, such that $\mu_{\mathfrak{I}_{RPTL-j}} \geq \mu_{\mathfrak{I}_{RPTL-j}}$

$$\Rightarrow \mu_{\mathfrak{I}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}} \geq \hat{\mu}_{\mathfrak{I}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}$$

$$\Rightarrow 1-\prod_{j=1}^{n}\mu_{\mathfrak{I}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}} \leq 1-\prod_{j=1}^{n}\mu_{\mathfrak{I}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}$$

$$\Rightarrow \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\mu_{\mathfrak{I}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}} \geq \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\mu_{\mathfrak{I}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n} t_{VP-j}}}$$

Similarly we can prove the imaginary part of the complex-valued truth grade, real and imaginary parts of the complex-valued abstinence and falsity grades, such that
$W_{\mu_{\mathfrak{I}_{IPTL-j}}} \geq W_{\mu_{\mathfrak{I}_{IPTL-j}}}, \xi_{\mathfrak{I}_{RPTL-j}} \leq \xi_{\mathfrak{I}_{RPTL-j}}, W_{\xi_{\mathfrak{I}_{IPTL-j}}} \leq W_{\xi_{\mathfrak{I}_{IPTL-j}}}$, and $\eta_{\mathfrak{I}_{RPTL-j}} \leq \hat{\eta}_{\mathfrak{I}_{RPTL-j}}, W_{\eta_{\mathfrak{I}_{IPTL-j}}} \leq W_{\eta_{\mathfrak{I}_{IPTL-j}}}$, then it is clear that If $S\left(\mathfrak{I}_{CTS-j}\right) \succ S\left(\mathfrak{I}_{CTS-j}\right) \Rightarrow \mathfrak{I}_{CTS-j} \succ \mathfrak{I}_{CTS-j}$;
If $S\left(\mathfrak{I}_{CTS-j}\right) = S\left(\mathfrak{I}_{CTS-j}\right)$, then we also use the accuracy function, such that if $H\left(\mathfrak{I}_{CTS-j}\right) \succ H\left(\mathfrak{I}_{CTS-j}\right) \Rightarrow \mathfrak{I}_{CTS-j} \succ \mathfrak{I}_{CTS-j}$; if $H\left(\mathfrak{I}_{CTS-j}\right) = H\left(\mathfrak{I}_{CTS-j}\right) \Rightarrow \mathfrak{I}_{CTS-j} = \mathfrak{I}_{CTS-j}$. Hence the inequality is holds true.

$$CTSF2-TLMM^{t_{VP}}\left(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n}\right) \geq CTSF2-TLMM^{t_{VP}}\left(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n}\right).$$

The results has been completed.

3. By using the result 1 and result 2, we get the following result, such that

$$\left(\left(s_{S_{LT-j}}^{-}, a_{SC-j}^{-}\right), \left(\mu_{\mathfrak{I}_{RPTL-j}}^{-} e^{i2\pi W_{\mu_{\mathfrak{I}_{IPTL-j}}}^{-}}, \xi_{\mathfrak{I}_{RPTL-j}}^{-} e^{i2\pi W_{\xi_{\mathfrak{I}_{IPTL-j}}}^{-}}, \eta_{\mathfrak{I}_{RPTL-j}}^{-} e^{i2\pi W_{\eta_{\mathfrak{I}_{IPTL-j}}}^{-}}\right)\right)$$

$$\leq CTSF2-TLMM^{t_{VP}}\left(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n}\right)$$

and

$$CTSF2-TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1},\mathfrak{T}_{CTS-2},...,\mathfrak{T}_{CTS-n}\right)$$

$$\leq\left(\left(s^{+}_{S_{LT-j}},a^{+}_{SC-j}\right),\left(\mu^{+}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{+}_{\mu_{\mathfrak{T}_{IPTL-j}}}},\xi^{+}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{+}_{\xi_{\mathfrak{T}_{IPTL-j}}}},\eta^{+}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{+}_{\eta_{\mathfrak{T}_{IPTL-j}}}}\right)\right)$$

then

$$\left(\left(s^{-}_{S_{LT-j}},a^{-}_{SC-j}\right),\left(\mu^{-}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{-}_{\mu_{\mathfrak{T}_{IPTL-j}}}},\xi^{-}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{-}_{\xi_{\mathfrak{T}_{IPTL-j}}}},\eta^{-}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{-}_{\eta_{\mathfrak{T}_{IPTL-j}}}}\right)\right)$$

$$\leq CTSF2-TLMM^{t_{VP}}\left(\mathfrak{T}_{CTS-1},\mathfrak{T}_{CTS-2},...,\mathfrak{T}_{CTS-n}\right)$$

$$\leq\left(\left(s^{+}_{S_{LT-j}},a^{+}_{SC-j}\right),\left(\mu^{+}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{+}_{\mu_{\mathfrak{T}_{IPTL-j}}}},\xi^{+}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{+}_{\xi_{\mathfrak{T}_{IPTL-j}}}},\eta^{+}_{\mathfrak{T}_{RPTL-j}}e^{i2\pi W^{+}_{\eta_{\mathfrak{T}_{IPTL-j}}}}\right)\right).$$

Hence the result is proved.
Further, the special cases of the explored operator is discussed based on t_{VP}.

Case 1: If we choose $t_{VP}=(1,0,0,....,0)$, then the CTSF2-TLMM operator is reduced to the complex T-spherical fuzzy 2-tuple linguistic arithmetic averaging operator shown as

$$CTSF2-TLAA^{t_{VP}}\left(\mathfrak{T}_{CTS-1},\mathfrak{T}_{CTS-2},...,\mathfrak{T}_{CTS-n}\right)$$

$$=\left\{\begin{array}{l}\Delta_{LT}\left(\sum_{j=1}^{n}\left(\frac{1}{n}\times\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)\right),\\ \left(1-\prod_{j=1}^{n}\left(1-\mu^{q_{SC}}_{\mathfrak{T}_{RPTL-\vartheta(j)}}\right)^{\frac{1}{n}}\right)^{\frac{1}{q_{SC}}}e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-W^{q_{SC}}_{\mu_{\mathfrak{T}_{IPTL-\vartheta(j)}}}\right)^{\frac{1}{n}}\right)^{\frac{1}{q_{SC}}}},\\ \prod_{j=1}^{n}\left(\xi^{\frac{1}{n}}_{\mathfrak{T}_{RPTL-\vartheta(j)}}\right)e^{i2\pi\prod_{j=1}^{n}\left(W^{\frac{1}{n}}_{\xi_{\mathfrak{T}_{IPTL-\vartheta(j)}}}\right)},\prod_{j=1}^{n}\left(\eta^{\frac{1}{n}}_{\mathfrak{T}_{RPTL-\vartheta(j)}}\right)e^{i2\pi\prod_{j=1}^{n}\left(W^{\frac{1}{n}}_{\eta_{\mathfrak{T}_{IPTL-\vartheta(j)}}}\right)}\end{array}\right\}$$

Case 2: If we choose $t_{VP} = (1, 1, 0, ..., 0)$, then the CTSF2-TLMM operator is reduced to the complex T-spherical fuzzy 2-tuple linguistic BMO given by:

$$CTSF2 - TLBM^{t_{VP}}\left(\mathfrak{F}_{CTS-1}, \mathfrak{F}_{CTS-2}, ..., \mathfrak{F}_{CTS-n}\right)$$

$$= \left(\begin{array}{l} \Delta_{LT}\left(\frac{1}{n(n-1)}\sum_{j\neq k=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right)\times\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(k)}}, a_{SC-\vartheta(k)}\right)\right)\right)^{\frac{1}{2}}, \\ \left(\left(1-\prod_{j\neq k=1}^{n}\left(1-\mu_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\times\mu_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{2}} e^{i2\pi\left(1-\prod_{j\neq k=1}^{n}\left(1-W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\times W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{q_{SC}}}}, \\ \left(1-\left(1-\left(\left(\prod_{j=1}^{n}\left(1-\left(1-\xi_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)\times\left(1-\xi_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{SC}}} \\ \qquad e^{i2\pi\left(1-\left(1-\left(\left(\prod_{j=1}^{n}\left(1-\left(1-W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)\times\left(1-W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{SC}}}}, \\ \left(1-\left(1-\left(\left(\prod_{j=1}^{n}\left(1-\left(1-\eta_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)\times\left(1-\eta_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{SC}}} \\ \qquad e^{i2\pi\left(1-\left(1-\left(\left(\prod_{j=1}^{n}\left(1-\left(1-W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)\times\left(1-W_{\xi_{\eta_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{SC}}}} \end{array} \right)$$

Case 3: If we choose $t_{VP} = \left(\overbrace{1, 11, 1, 1}^{n}, \ldots., \overbrace{0, 0, 0, 0}^{n-k}\right)$, then the CTSF2-TLMM operator is reduced to the complex T-spherical fuzzy 2-tuple linguistic MSMO given by:

$$CTSF2 - TLMSM^{t_{VP}}\left(\mathfrak{F}_{CTS-1}, \mathfrak{F}_{CTS-2}, ..., \mathfrak{F}_{CTS-n}\right)$$

$$= \left(\begin{array}{l} \Delta_{LT}\left(\frac{1}{C_n^k} \sum\limits_{1\le j_1\le j_2\le \ldots\le j_k\le n} \prod\limits_{k=1}^{n} \left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(k)}}, a_{SC-\vartheta(k)}\right)\right)\right)^{\frac{1}{k}}, \\ \left(\left(1 - \left(\prod\limits_{\le j_1\le j_2\le \ldots\le j_k\le n} \left(1 - \prod\limits_{k=1}^{n} \mu_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(1-\left(\prod\limits_{\le j_1\le j_2\le \ldots\le j_k\le n} \left(1 - \prod\limits_{k=1}^{n} W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q_{SC}}}}\right)^{\frac{1}{k}}, \\ \left(1 - \left(1 - \left(\prod\limits_{\le j_1\le j_2\le \ldots\le j_k\le n} \left(1 - \prod\limits_{k=1}^{n} \left(1 - \xi_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q_{SC}}} \\ \qquad e^{i2\pi\left(1-\left(1-\left(\prod\limits_{\le j_1\le j_2\le \ldots\le j_k\le n} \left(1 - \prod\limits_{k=1}^{n} \left(1 - W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\sum\limits_{j=1}^{n}\frac{1}{t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}, \\ \left(1 - \left(1 - \left(\prod\limits_{\le j_1\le j_2\le \ldots\le j_k\le n} \left(1 - \prod\limits_{k=1}^{n} \left(1 - \eta_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q_{SC}}} \\ \qquad e^{i2\pi\left(1-\left(1-\left(\prod\limits_{\le j_1\le j_2\le \ldots\le j_k\le n} \left(1 - \prod\limits_{k=1}^{n} \left(1 - W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q_{SC}}}} \end{array} \right)$$

Case 4: If we choose $t_{VP} = (1, 1, 1, ..., 1)$, then the CTSF2-TLMM operator is reduced to the complex T-spherical fuzzy 2-tuple linguistic geometric AO is given by:

$$CTSF2 - TLGA^{t_{VP}}(\mathfrak{F}_{CTS-1}, \mathfrak{F}_{CTS-2}, ..., \mathfrak{F}_{CTS-n})$$

$$= \left(\Delta_{LT}\left(\prod_{j=1}^{n} \left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)} \right) \right) \right)^{\frac{1}{n}}, \left(\prod_{j=1}^{n} \left(\mu_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{\frac{1}{n}} \right) e^{i2\pi \prod_{j=1}^{n} \left(W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{\frac{1}{n}} \right)}, \right. \right.$$

$$\left(1 - \prod_{j=1}^{n} \left(1 - \xi_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}} \right)^{\frac{1}{n}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left(1 - \prod_{j=1}^{n} \left(1 - W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}} \right)^{\frac{1}{n}} \right)^{\frac{1}{q_{SC}}}},$$

$$\left. \left. \left(1 - \prod_{j=1}^{n} \left(1 - \eta_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}} \right)^{\frac{1}{n}} \right)^{\frac{1}{q_{SC}}} e^{i2\pi \left(1 - \prod_{j=1}^{n} \left(1 - W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}} \right)^{\frac{1}{n}} \right)^{\frac{1}{q_{SC}}}} \right) \right)$$

Further, we investigate the DMM operator based on a CTSF2-TLS which is called CTSF2-TLDMM operator.

Definition 10. Choose the family of CTSF2-TLNs $\mathfrak{F}_{CTS-j} = \left(\left(s_{S_{LT-j}}, a_{SC-j} \right), \left(\mu_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-j}}}}, \xi_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\xi_{\mathfrak{F}_{IPTL-j}}}}, \eta_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\eta_{\mathfrak{F}_{IPTL-j}}}} \right) \right), (j = 1, 2, 3, .., n)$ with vector of parameters $t_{VP} = \{t_{VP-1}, t_{VP-2}, ..., t_{VP-n}\} \in R^n$, the CTSF2-TLDMM operator is given by:

$$CTSF2 - TLDMM^{t_{VP}}(\mathfrak{F}_{CTS-1}, \mathfrak{F}_{CTS-2}, ..., \mathfrak{F}_{CTS-n}) = \frac{1}{\sum_{j=1}^{n} t_{VP-j}} \left(\bigotimes_{\substack{CTS \\ \vartheta(j) \in s_{SC-n}}} \left(\bigoplus_{j=1}^{n}{}_{CTS} \left(t_{VP-j} \mathfrak{F}_{TS-\vartheta(j)} \right) \right) \right)^{\frac{1}{n!}} \tag{14}$$

where $\vartheta(j), (j = 1, 2, 3, ..., n)$ is n permutation, and the set of all permutation of 1 to n is denoted by S_{SC-n}.

Based on the above analysis related to operational laws and Definition 10, we establish the following results.

Theorem 3. *Suppose the family of CTSF2-TLNs* $\mathfrak{F}_{CTS-j} = \left(\left(s_{S_{LT-j}}, a_{SC-j} \right), \left(\mu_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-j}}}}, \xi_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\xi_{\mathfrak{F}_{IPTL-j}}}}, \eta_{\mathfrak{F}_{RPTL-j}} e^{i2\pi W_{\eta_{\mathfrak{F}_{IPTL-j}}}} \right) \right)$, $(j = 1, 2, 3, .., n)$ *with vector of parameters* $t_{VP} = \{t_{VP-1}, t_{VP-2}, ..., t_{VP-n}\} \in R^n$. *Then the aggregated value of CTSF2-TLNs is a CTSF2-TLN,*

and

$$CTSF2-TLDMM^{t_{VP}}\left(\mathfrak{F}_{CTS-1},\mathfrak{F}_{CTS-2},...,\mathfrak{F}_{CTS-n}\right)$$

$$=\left(\begin{array}{l}\Delta_{LT}\left(\frac{1}{\sum_{j=1}^{n}t_{VP-j}}\prod_{\vartheta(j)\in s_{SC-n}}\sum_{j=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)^{t_{VP-j}}\right)^{\frac{1}{n!}},\\ \left(\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-\mu_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}\right.\right.\\ \left.\left.e^{i2\pi\left(1-\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\left(1-W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}\right),\\ \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\xi_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}e^{i2\pi\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\left(\frac{1}{\sum_{j=1}^{n}t_{VP-j}}\right)},\\ \left(\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}\eta_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}e^{i2\pi\left(1-\left(\prod_{\vartheta(j)\in s_{SC-n}}\left(1-\prod_{j=1}^{n}W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}t_{VP-j}}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{q_{SC}}}\left(\frac{1}{\sum_{j=1}^{n}t_{VP-j}}\right)}\end{array}\right)$$

Proof: Straightforward. (The proof of this theorem is similar to Theorem 1).

Example 3. Based on the information in Example 2, we can get the aggregated value by the CTSF2-TLDMM operator, such that

$$\frac{1}{\sum_{j=1}^{n}t_{VP-j}}\left(\otimes_{\substack{CTS\\ \vartheta(j)\in s_{SC-n}}}\left(\oplus_{j=1}^{n}{}_{CTS}\left(t_{VP-j}\mathfrak{F}_{TS-\vartheta(j)}\right)\right)\right)^{\frac{1}{n!}}=\left(\left(s_{S_{LT-3}},0.0118\right),\left(0.8165e^{i2\pi(0.8165)},0.0e^{i2\pi(0.0)},0.4966e^{i2\pi(0.4966)}\right)\right).$$

If we set to zero to the imaginary parts of the complex-valued truth and falsity grades, then the Example 3 is converted to T-spherical fuzzy 2-tuple linguistic variables, which is the special cases of the established operator.

Theorem 4. *Suppose the families of two CTSF2-TLNs are* $\mathfrak{F}_{CTS-j}=\left(\left(s_{S_{LT-j}},a_{SC-j}\right),\left(\mu_{\mathfrak{F}_{RPTL-j}}e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-j}}}},\xi_{\mathfrak{F}_{RPTL-j}}e^{i2\pi W_{\xi_{\mathfrak{F}_{IPTL-j}}}},\eta_{\mathfrak{F}_{RPTL-j}}e^{i2\pi W_{\eta_{\mathfrak{F}_{IPTL-j}}}}\right)\right),(j=1,2,3,..,n)$ *and* $\mathfrak{F}_{CTS-*j}=\left(\left(s_{S_{LT-*j}},a_{SC-*j}\right),\left(\mu_{\mathfrak{F}_{RPTL-*j}}e^{i2\pi W_{\mu_{\mathfrak{F}_{IPTL-*j}}}},\xi_{\mathfrak{F}_{RPTL-*j}}e^{i2\pi W_{\xi_{\mathfrak{F}_{IPTL-*j}}}},\right.$

$\eta_{\mathfrak{I}_{RPTL-*j}} e^{i2\pi W_{\eta_{\mathfrak{I}_{IPTL-*j}}}}\Big)\Big)$, $(j = 1, 2, 3, .., n)$ *with vector of parameters* $t_{VP} = \{t_{VP-1}, t_{VP-2}, ..., t_{VP-n}\} \in R^n$. *Then the idempotency, monotonicity, and boundedness are shown as follows:*

1. *If* $\mathfrak{I}_{CTS-j} = \left(\left(s_{S_{LT-j}}, a_{SC-j}\right), \left(\mu_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W_{\mu_{\mathfrak{I}_{IPTL-j}}}}, \xi_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W_{\xi_{\mathfrak{I}_{IPTL-j}}}}, \eta_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W_{\eta_{\mathfrak{I}_{IPTL-j}}}}\right)\right)$, $(j = 1, 2, 3, .., n)$ *are all equal if and only if* $\mathfrak{I}_{CTS-j} = \mathfrak{I}_{CTS}$, *then* $CTSF2 - TLDMM^{t_{VP}}(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n}) = \mathfrak{I}_{CTS}$
2. *If* $\left(s_{S_{LT-j}}, a_{SC-j}\right) \geq \left(\hat{s}_{S_{LT-j}}, a_{SC-j}\right)$, $\mu_{\mathfrak{I}_{RPTL-j}} \geq \mu_{\mathfrak{I}_{RPTL-j}}$, $W_{\mu_{\mathfrak{I}_{IPTL-j}}} \geq W_{\mu_{\mathfrak{I}_{IPTL-j}}}$, $\xi_{\mathfrak{I}_{RPTL-j}} \leq \xi_{\mathfrak{I}_{RPTL-j}}$, $W_{\xi_{\mathfrak{I}_{IPTL-j}}} \leq W_{\xi_{\mathfrak{I}_{IPTL-j}}}$, *and* $\eta_{\mathfrak{I}_{RPTL-j}} \leq \hat{\eta}_{\mathfrak{I}_{RPTL-j}}$, $W_{\eta_{\mathfrak{I}_{IPTL-j}}} \leq W_{\eta_{\mathfrak{I}_{IPTL-j}}}$, *then*
$CTSF2 - TLDMM^{t_{VP}}(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n}) \geq CTSF2 - TLDMM^{t_{VP}}(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n})$.
3. *If* $\left(s^-_{S_{LT-j}}, a^-_{SC-j}\right) = \min_{1\leq j\leq n}\left(s_{S_{LT-j}}, a_{SC-j}\right)$, $\left(s^+_{S_{LT-j}}, a^+_{SC-j}\right) = \max_{1\leq j\leq n}\left(s_{S_{LT-j}}, a_{SC-j}\right)$,
$\mu^+_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^+_{\mu_{\mathfrak{I}_{IPTL-j}}}} = \max_{1\leq j\leq n} \mu_{\mathfrak{I}_{RPTL-j}} e^{i2\pi \max_{1\leq j\leq n} W_{\mu_{\mathfrak{I}_{IPTL-j}}}}, \mu^-_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^-_{\mu_{\mathfrak{I}_{IPTL-j}}}} = \min_{1\leq j\leq n} \mu_{\mathfrak{I}_{RPTL-j}} e^{i2\pi \min_{1\leq j\leq n} W_{\mu_{\mathfrak{I}_{IPTL-j}}}}$,
$\xi^+_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^+_{\xi_{\mathfrak{I}_{IPTL-j}}}} = \max_{1\leq j\leq n} \xi_{\mathfrak{I}_{RPTL-j}} e^{i2\pi \max_{1\leq j\leq n} W_{\xi_{\mathfrak{I}_{IPTL-j}}}}, \xi^-_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^-_{\xi_{\mathfrak{I}_{IPTL-j}}}} = \min_{1\leq j\leq n} \xi_{\mathfrak{I}_{RPTL-j}} e^{i2\pi \min_{1\leq j\leq n} W_{\xi_{\mathfrak{I}_{IPTL-j}}}}$,
$\eta^+_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^+_{\eta_{\mathfrak{I}_{IPTL-j}}}} = \max_{1\leq j\leq n} \eta_{\mathfrak{I}_{RPTL-j}} e^{i2\pi \max_{1\leq j\leq n} W_{\eta_{\mathfrak{I}_{IPTL-j}}}}, \eta^-_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^-_{\eta_{\mathfrak{I}_{IPTL-j}}}} = \min_{1\leq j\leq n} \eta_{\mathfrak{I}_{RPTL-j}} e^{i2\pi \min_{1\leq j\leq n} W_{\eta_{\mathfrak{I}_{IPTL-j}}}}$, *then*
$\left(\left(s^-_{S_{LT-j}}, a^-_{SC-j}\right), \left(\mu^-_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^-_{\mu_{\mathfrak{I}_{IPTL-j}}}}, \xi^-_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^-_{\xi_{\mathfrak{I}_{IPTL-j}}}}, \eta^-_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^-_{\eta_{\mathfrak{I}_{IPTL-j}}}}\right)\right)$
$\leq CTSF2 - TLDMM^{t_{VP}}(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n})$
$\leq \left(\left(s^+_{S_{LT-j}}, a^+_{SC-j}\right), \left(\mu^+_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^+_{\mu_{\mathfrak{I}_{IPTL-j}}}}, \xi^+_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^+_{\xi_{\mathfrak{I}_{IPTL-j}}}}, \eta^+_{\mathfrak{I}_{RPTL-j}} e^{i2\pi W^+_{\eta_{\mathfrak{I}_{IPTL-j}}}}\right)\right)$

Proof: Straightforward. (The proof of this theorem is similar to the Theorem 2).

Further, the special cases of the explored operator is discussed based on the value of t_{VP}.

Case 1: If we choose $t_{VP} = (0, 0, 0, ..., 0)$, then the CTSF2-TLDMM operator is reduced to the complex T-spherical fuzzy 2-tuple linguistic geometric AO given by:

$$
\begin{aligned}
&CTSF2 - TLGA^{t_{VP}}(\mathfrak{I}_{CTS-1}, \mathfrak{I}_{CTS-2}, ..., \mathfrak{I}_{CTS-n}) \\
&= \left(\Delta_{LT}\left(\prod_{j=1}^{n}\left(\Delta^{-1}_{LT}\left(s_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right)\right)\right)^{\frac{1}{n}}, \left(\prod_{j=1}^{n}\left(\mu^{\frac{1}{n}}_{\mathfrak{I}_{RPTL-\vartheta(j)}}\right) e^{i2\pi\prod_{j=1}^{n}\left(W^{\frac{1}{n}}_{\mu_{\mathfrak{I}_{IPTL-\vartheta(j)}}}\right)}, \right.\right. \\
&\left(1 - \prod_{j=1}^{n}\left(1 - \xi^{q_{SC}}_{\mathfrak{I}_{RPTL-\vartheta(j)}}\right)^{\frac{1}{n}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(1 - \prod_{j=1}^{n}\left(1 - W^{q_{SC}}_{\xi_{\mathfrak{I}_{IPTL-\vartheta(j)}}}\right)^{\frac{1}{n}}\right)^{\frac{1}{q_{SC}}}}, \\
&\left.\left.\left(1 - \prod_{j=1}^{n}\left(1 - \eta^{q_{SC}}_{\mathfrak{I}_{RPTL-\vartheta(j)}}\right)^{\frac{1}{n}}\right)^{\frac{1}{q_{SC}}} e^{i2\pi\left(1 - \prod_{j=1}^{n}\left(1 - W^{q_{SC}}_{\eta_{\mathfrak{I}_{IPTL-\vartheta(j)}}}\right)^{\frac{1}{n}}\right)^{\frac{1}{q_{SC}}}}\right)\right).
\end{aligned}
$$

Case 2: If we choose $t_{VP} = (1, 1, 0, ..., 0)$, then the CTSF2-TLDMM operator is reduced to the complex T-spherical fuzzy 2-tuple linguistic geometric BMO given by:

$$CTSF2 - TLGBM^{t_{VP}}(\mathfrak{F}_{CTS-1}, \mathfrak{F}_{CTS-2}, ..., \mathfrak{F}_{CTS-n})$$

$$= \left(\begin{array}{l} \Delta_{LT}\left(\frac{1}{2}\prod_{j\neq k=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}}, a_{SC-\vartheta(j)}\right) + \Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(k)}}, a_{SC-\vartheta(k)}\right)\right)\right)^{\frac{1}{n(n-1)}}, \\ \left(\left(1 - \left(1 - \left(\left(\left(\prod_{j=1}^{n}\left(1 - \left(1 - \mu_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}}\right) \times \left(1 - \mu_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{SC}}}\right. \\ \qquad e^{i2\pi\left(1-\left(1-\left(\left(\prod_{j=1}^{n}\left(1-\left(1-W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)\times\left(1-W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{SC}}}}, \\ \left(\left(1 - \prod_{j\neq k=1}^{n}\left(1 - \xi_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}} \times \xi_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{2}} e^{i2\pi\left(1-\prod_{j\neq k=1}^{n}\left(1-W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\times W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{2}}}, \\ \left(\left(1 - \prod_{j\neq k=1}^{n}\left(1 - \eta_{\mathfrak{F}_{RPTL-\vartheta(j)}}^{q_{SC}} \times \eta_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{2}} e^{i2\pi\left(1-\prod_{j\neq k=1}^{n}\left(1-W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(j)}}}^{q_{SC}}\times W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{2}}} \end{array} \right)$$

Case 3: If we choose $t_{VP} = \left(\overbrace{1, 1, 1, 1, 1, 1}^{k}, ..., \overbrace{0, 0, 0, 0, 0, 0}^{n-k}\right)$, then the CTSF2-TLDMM operator is reduced to the complex T-spherical

fuzzy 2-tuple linguistic geometric MSMO given by:

$$CTSF2-TLGMSM^{t_{VP}}\left(\mathfrak{F}_{CTS-1},\mathfrak{F}_{CTS-2},...,\mathfrak{F}_{CTS-n}\right)$$

$$=\left(\begin{array}{l}\Delta_{LT}\left(\frac{1}{k}\prod_{1\le j_1\le j_2\le\ldots\le j_k\le n}\sum_{k=1}^{n}\left(\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(k)}},a_{SC-\vartheta(k)}\right)\right)\right)^{\frac{1}{C_n^k}},\\ \left(\left(1-\left(1-\left(\prod_{\le j_1\le j_2\le\ldots\le j_k\le n}\left(1-\prod_{k=1}^{n}\left(1-\mu_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{q_{SC}}}\right.\\ \left.e^{i2\pi\left(1-\left(1-\left(\prod_{\le j_1\le j_2\le\ldots\le j_k\le n}\left(1-\prod_{k=1}^{n}\left(1-W_{\mu_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{\sum_{j=1}^{n}t_{VP-j}}}\right)^{\frac{1}{q_{SC}}}}\right),\\ \left(\left(1-\left(\prod_{\le j_1\le j_2\le\ldots\le j_k\le n}\left(1-\prod_{k=1}^{n}\xi_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{k}}e^{i2\pi\left(\left(1-\left(\prod_{\le j_1\le j_2\le\ldots\le j_k\le n}\left(1-\prod_{k=1}^{n}W_{\xi_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{k}}},\\ \left(\left(1-\left(\prod_{\le j_1\le j_2\le\ldots\le j_k\le n}\left(1-\prod_{k=1}^{n}\eta_{\mathfrak{F}_{RPTL-\vartheta(k)}}^{q_{SC}}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{k}}e^{i2\pi\left(\left(1-\left(\prod_{\le j_1\le j_2\le\ldots\le j_k\le n}\left(1-\prod_{k=1}^{n}W_{\eta_{\mathfrak{F}_{IPTL-\vartheta(k)}}}^{q_{SC}}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q_{SC}}}\right)^{\frac{1}{k}}}\end{array}\right)$$

Case 4: If we choose $t_{VP}=(1,1,1,....,1)$, then the CTSF2-TLDMM operator is reduced to the complex T-spherical fuzzy 2-tuple linguistic

arithmetic AO given by:

$$CTSF2-TLAA^{t_{VP}}\left(\mathfrak{T}_{CTS-1},\mathfrak{T}_{CTS-2},...,\mathfrak{T}_{CTS-n}\right)$$
$$=\left(\begin{array}{l}\Delta_{LT}\left(\sum_{j=1}^{n}\left(\frac{1}{n}\times\Delta_{LT}^{-1}\left(s_{S_{LT-\vartheta(j)}},a_{SC-\vartheta(j)}\right)\right)\right),\left(1-\prod_{j=1}^{n}\left(1-\mu_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{q_{SC}}\right)^{\frac{1}{n}}\right)^{\frac{1}{q_{SC}}}e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-W_{\mu_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{q_{SC}}\right)^{\frac{1}{n}}\right)^{\frac{1}{q_{SC}}}},\\ \prod_{j=1}^{n}\left(\xi_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{\frac{1}{n}}\right)e^{i2\pi\prod_{j=1}^{n}\left(W_{\xi_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{\frac{1}{n}}\right)},\prod_{j=1}^{n}\left(\eta_{\mathfrak{T}_{RPTL-\vartheta(j)}}^{\frac{1}{n}}\right)e^{i2\pi\prod_{j=1}^{n}\left(W_{\eta_{\mathfrak{T}_{IPTL-\vartheta(j)}}}^{\frac{1}{n}}\right)}\end{array}\right)$$

4. MADM METHOD BASED ON COMPLEX T-SPHERICAL FUZZY 2-TUPLE LINGUISTIC INFORMATION

For a MADM problem based on complex T-spherical fuzzy 2-tuple linguistic information, we consider the families of the alternatives and attributes, which are stated as: $A_{Al}=\{A_{Al-1},A_{Al-2},...,A_{Al-m}\}$, $C_{AT}=\{C_{AT-1},C_{AT-2},...,C_{AT-n}\}$, and then construct the decision-making matrix $R_{DM}=\left(r_{jk}\right)_{m\times n}$, where $r_{jk}=\left(\left(s_{S_{LT-jk}},a_{SC-jk}\right),\left(\mu_{\mathfrak{T}_{RPTL-jk}}e^{i2\pi W_{\mu_{\mathfrak{T}_{IPTL-jk}}}},\xi_{\mathfrak{T}_{RPTL-jk}}e^{i2\pi W_{\xi_{\mathfrak{T}_{IPTL-jk}}}},\eta_{\mathfrak{T}_{RPTL-jk}}e^{i2\pi W_{\eta_{\mathfrak{T}_{IPTL-jk}}}}\right)\right)$ is in the form of CTSF2-TLNs for alternative $A_{AL-j}(j=1,2,3,...,m)$ under the attribute $C_{AT-k}(k=1,2,3,...,n)$, then the steps of this MADM problem based on CTSF2-TLNs are as follow:

1. By CTSF2-TLMM operator or CTSF2-TLDMM operator to get the aggregated result.
2. By Definition 4, we get the score values of the aggregated values.
3. By Definition 4, we get the ranking results, and then obtain the best one alternative.
4. The end.

Example 4. The purpose of this example is to select the emergency alternative. Suppose there are four alternatives shown as: $A_{Al}=\{A_{Al-1},A_{Al-2},A_{Al-3},A_{Al-4}\}$, and there are four attributes which are explained as C_{AT-1} = Preparing Capacity, C_{AT-2} = Rescuing Capacity, C_{AT-3} = Recovering Capacity, and C_{AT-4} = Responding Time. Further, the linguistic term set $S_{LT}=\left\{s_{S_{LT-0}}=very-poor,s_{S_{LT-1}}=poor,s_{S_{LT-2}}=fair,s_{S_{LT-3}}=good,s_{S_{LT-4}}=very-good\right\}$ is adopted, and the decision matrix $R_{DM}=\left(r_{jk}\right)_{4\times4}$ is build up which is in the form CTSF2-TLNs shown in the Table 1. The steps are shown as follows:

1. By the CTSF2-TLMM operator, we can get the aggregated values for four alternatives, where, we select the parameter $t_{VP}=(1,1,1,1)$ and $q_{SC}=3$, then
$$A_{Al-1}=CTSF2-TLMM^{t_{VP}}\left(C_{Al-1,AT-1},C_{Al-1,AT-2},C_{Al-1,AT-3},C_{Al-1,AT-4}\right)$$
$$=\left(\left(s_{S_{LT-2}},0.097669,\left(0.0005125e^{i2\pi(0.0005125)},0.32e^{i2\pi(0.32)},0.23e^{i2\pi(0.23)}\right)\right)\right);$$
$$A_{Al-2}=CTSF2-TLMM^{t_{VP}}\left(C_{Al-2,AT-1},C_{Al-2,AT-2},C_{Al-2,AT-3},C_{Al-2,AT-4}\right)$$
$$=\left(\left(s_{S_{LT-1}},0.22658,\left(0.0e^{i2\pi(0.0)},0.16e^{i2\pi(0.16)},0.36e^{i2\pi(0.36)}\right)\right)\right);$$
$$A_{Al-3}=CTSF2-TLMM^{t_{VP}}\left(C_{Al-3,AT-1},C_{Al-3,AT-2},C_{Al-3,AT-3},C_{Al-3,AT-4}\right)$$
$$=\left(\left(s_{S_{LT-2}},0.94927,\left(0.0e^{i2\pi(0.0)},0.22e^{i2\pi(0.22)},0.18e^{i2\pi(0.18)}\right)\right)\right);$$
$$A_{Al-4}=CTSF2-TLMM^{t_{VP}}\left(C_{Al-4,AT-1},C_{Al-4,AT-2},C_{Al-4,AT-3},C_{Al-4,AT-4}\right)$$
$$=\left(\left(s_{S_{LT-1}},0.24592,\left(0.0e^{i2\pi(0.0)},0.184e^{i2\pi(0.184)},0.144e^{i2\pi(0.144)}\right)\right)\right).$$
2. By Definition 4, we get the score values of the aggregated values as follows.
$S(A_{Al-1})=-0.22135, S(A_{Al-2})=-0.167, S(A_{Al-3})=-0.160, S(A_{Al-4})=-0.4385.$

3. By Definition 4, we obtain the ranking result and the best one alternative.
$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
and then A_{Al-3} is the best alternative for emergency preplan.

4. The end.

4.1. Further Discussion

To evaluate the influence of the parameter t_{VP}, based on the Example 4, we set the different values to the parameter t_{VP} in the established approaches, that is, based on CTSF2-TLMM operator and CTSF2-TLDMM operator, then the ranking results are shown in Table 2.

Table 1 | Decision matrix in the form of complex picture fuzzy 2-tuple linguistic numbers.

Symbols	C_{AT-1}	C_{AT-2}	C_{AT-3}	C_{AT-4}
A_{Al-1}	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.03\end{pmatrix}, \begin{pmatrix}0.5e^{i2\pi(0.5)}, \\ 0.1e^{i2\pi(0.1)}, \\ 0.2e^{i2\pi(0.2)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-4}}, 0.13\end{pmatrix}, \begin{pmatrix}0.9e^{i2\pi(0.9)}, \\ 0.01e^{i2\pi(0.01)}, \\ 0.01e^{i2\pi(0.01)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.11\end{pmatrix}, \begin{pmatrix}0.8e^{i2\pi(0.8)}, \\ 0.1e^{i2\pi(0.1)}, \\ 0.1e^{i2\pi(0.1)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-4}}, 0.01\end{pmatrix}, \begin{pmatrix}0.19e^{i2\pi(0.19)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.23e^{i2\pi(0.23)}\end{pmatrix}\right)$
A_{Al-2}	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.01\end{pmatrix}, \begin{pmatrix}0.51e^{i2\pi(0.51)}, \\ 0.11e^{i2\pi(0.11)}, \\ 0.12e^{i2\pi(0.12)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.14\end{pmatrix}, \begin{pmatrix}0.6e^{i2\pi(0.6)}, \\ 0.11e^{i2\pi(0.11)}, \\ 0.15e^{i2\pi(0.15)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.13\end{pmatrix}, \begin{pmatrix}0.4e^{i2\pi(0.4)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.2e^{i2\pi(0.2)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-2}}, 0.02\end{pmatrix}, \begin{pmatrix}0.36e^{i2\pi(0.36)}, \\ 0.3e^{i2\pi(0.3)}, \\ 0.13e^{i2\pi(0.13)}\end{pmatrix}\right)$
A_{Al-3}	$\left(\begin{pmatrix}s_{S_{LT-4}}, 0.0101\end{pmatrix}, \begin{pmatrix}0.7e^{i2\pi(0.7)}, \\ 0.1e^{i2\pi(0.1)}, \\ 0.1e^{i2\pi(0.1)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.15\end{pmatrix}, \begin{pmatrix}0.19e^{i2\pi(0.19)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.2e^{i2\pi(0.2)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-4}}, 0.06\end{pmatrix}, \begin{pmatrix}0.3e^{i2\pi(0.3)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.12e^{i2\pi(0.12)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.15\end{pmatrix}, \begin{pmatrix}0.19e^{i2\pi(0.19)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.2e^{i2\pi(0.2)}\end{pmatrix}\right)$
A_{Al-4}	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.12\end{pmatrix}, \begin{pmatrix}0.7e^{i2\pi(0.7)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.01e^{i2\pi(0.01)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-2}}, 0.16\end{pmatrix}, \begin{pmatrix}0.38e^{i2\pi(0.38)}, \\ 0.13e^{i2\pi(0.13)}, \\ 0.3e^{i2\pi(0.3)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.05\end{pmatrix}, \begin{pmatrix}0.28e^{i2\pi(0.28)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.22e^{i2\pi(0.22)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-2}}, 0.16\end{pmatrix}, \begin{pmatrix}0.38e^{i2\pi(0.38)}, \\ 0.13e^{i2\pi(0.13)}, \\ 0.3e^{i2\pi(0.3)}\end{pmatrix}\right)$

Table 2 | Ranking results for different value of parameter t_{VP}.

Parameter Vector	Operators	Score Values	Ranking Result
$t_{VP} = (1, 1, 1, 1)$	MM	$S(A_{AL-1}) = -0.22135, S(A_{AL-2}) = -0.167,$ $S(A_{AL-3}) = -0.160, S(A_{AL-4}) = -0.4385$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.135, S(A_{AL-2}) = -0.127,$ $S(A_{AL-3}) = -0.110, S(A_{AL-4}) = -0.246$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
$t_{VP} = (1, 0, 0, 0)$	MM	$S(A_{AL-1}) = -0.1395, S(A_{AL-2}) = -0.137,$ $S(A_{AL-3}) = -0.130, S(A_{AL-4}) = -0.285$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.235, S(A_{AL-2}) = -0.178,$ $S(A_{AL-3}) = -0.170, S(A_{AL-4}) = -0.453$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
$t_{VP} = (1, 1, 0, 0)$	MM	$S(A_{AL-1}) = -0.254, S(A_{AL-2}) = -0.247,$ $S(A_{AL-3}) = -0.240, S(A_{AL-4}) = -0.558$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.227, S(A_{AL-2}) = -0.169,$ $S(A_{AL-3}) = -0.164, S(A_{AL-4}) = -0.445$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
$t_{VP} = (1, 1, 1, 0)$	MM	$S(A_{AL-1}) = -0.223, S(A_{AL-2}) = -0.168,$ $S(A_{AL-3}) = -0.161, S(A_{AL-4}) = -0.4402$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.221, S(A_{AL-2}) = -0.173,$ $S(A_{AL-3}) = -0.163, S(A_{AL-4}) = -0.438$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
$t_{VP} = (2, 2, 2, 2)$	MM	$S(A_{AL-1}) = -0.203, S(A_{AL-2}) = -0.147,$ $S(A_{AL-3}) = -0.135, S(A_{AL-4}) = -0.425$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.214, S(A_{AL-2}) = -0.154,$ $S(A_{AL-3}) = -0.134, S(A_{AL-4}) = -0.430$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
$t_{VP} = (3, 3, 3, 3)$	MM	$S(A_{AL-1}) = -0.189, S(A_{AL-2}) = -0.1137,$ $S(A_{AL-3}) = -0.113, S(A_{AL-4}) = -0.378$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.188, S(A_{AL-2}) = -0.127,$ $S(A_{AL-3}) = -0.117, S(A_{AL-4}) = -0.376$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$

DMM, dual Muirhead mean; MM, Muirhead mean.

From the Table 1, we get the ranking results from the both methods with different values of the parameters, obviously, there is the same ranking result, and the best alternative is A_{Al-3}. When we change the value of parameter t_{VP}, the result is still remain same result.

4.2. Comparative Analysis

The purpose of this sub-section is to prove that the established operators in this manuscript are effective, based on Example 4, we make a comparison between explored operators with existing operators, such as averaging aggregation operator, geometric aggregation operator, geometric BMO, geometric MSMO based on the complex T-spherical fuzzy 2-tuple linguistic information, complex spherical fuzzy 2-tuple linguistic information, and complex picture fuzzy 2-tuple linguistic information. The comparisons between established operators with some existing operators are discussed in Table 3.

From the Table 3, we obtain that some existing methods [48,49] cannot deal with this decision-making problem and the proposed method get the same ranking result $A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$ and the best alternative is A_{AL-3}. Obviously, the proposed method is more general than the methods [48,49]. The graphical interpretation based on Table 3 is shown in Figure 1.

In the Figure 1, we have discussed four different series, which denote the alternatives A_{AL-1} to A_{AL-4}. From the Figure 1, it is clear that the series 3 gives greater values compared to other values in different series.

Example 5. In this example, the meanings of alternatives and attributes are the same as Example 4, we only consider the complex spherical fuzzy 2-tuple linguistic information, which is shown in Table 4. The explored operators are compared with some existing operators to examine the reliability and proficiency of the established operators.

Table 3 Comparative analysis between established operators with existing operators by using Example 4.

Methods	Operators	Score Values	Ranking Values
Wei *et al.* [48]	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Ju *et al.* [49]	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Spherical fuzzy 2-tuple linguistic variables	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
T-spherical fuzzy 2-tuple linguistic variables	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Proposed method (q = 1)	MM	$S(A_{AL-1}) = -0.235, S(A_{AL-2}) = -0.157, S(A_{AL-3}) = -0.150, S(A_{AL-4}) = -0.385$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.271, S(A_{AL-2}) = -0.172, S(A_{AL-3}) = -0.156, S(A_{AL-4}) = -0.431$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.205, S(A_{AL-2}) = -0.137, S(A_{AL-3}) = -0.118, S(A_{AL-4}) = -0.296$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.143, S(A_{AL-2}) = -0.132, S(A_{AL-3}) = -0.107, S(A_{AL-4}) = -0.253$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
Proposed method (q = 2)	MM	$S(A_{AL-1}) = -0.243, S(A_{AL-2}) = -0.127, S(A_{AL-3}) = -0.107, S(A_{AL-4}) = -0.345$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.271, S(A_{AL-2}) = -0.122, S(A_{AL-3}) = -0.102, S(A_{AL-4}) = -0.337$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.144, S(A_{AL-2}) = -0.113, S(A_{AL-3}) = -0.102, S(A_{AL-4}) = -0.346$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.143, S(A_{AL-2}) = -0.114, S(A_{AL-3}) = -0.104, S(A_{AL-4}) = -0.363$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
Proposed method (q = 3)	MM	$S(A_{AL-1}) = -0.22135, S(A_{AL-2}) = -0.167, S(A_{AL-3}) = -0.160, S(A_{AL-4}) = -0.438$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.2371, S(A_{AL-2}) = -0.1662, S(A_{AL-3}) = -0.152, S(A_{AL-4}) = -0.437$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.135, S(A_{AL-2}) = -0.127, S(A_{AL-3}) = -0.110, S(A_{AL-4}) = -0.246$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.133, S(A_{AL-2}) = -0.128, S(A_{AL-3}) = -0.104, S(A_{AL-4}) = -0.263$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$

DMM, dual Muirhead mean; MM, Muirhead mean: WDMM, weighted dual Muirhead mean; WMM, weighted Muirhead mean.

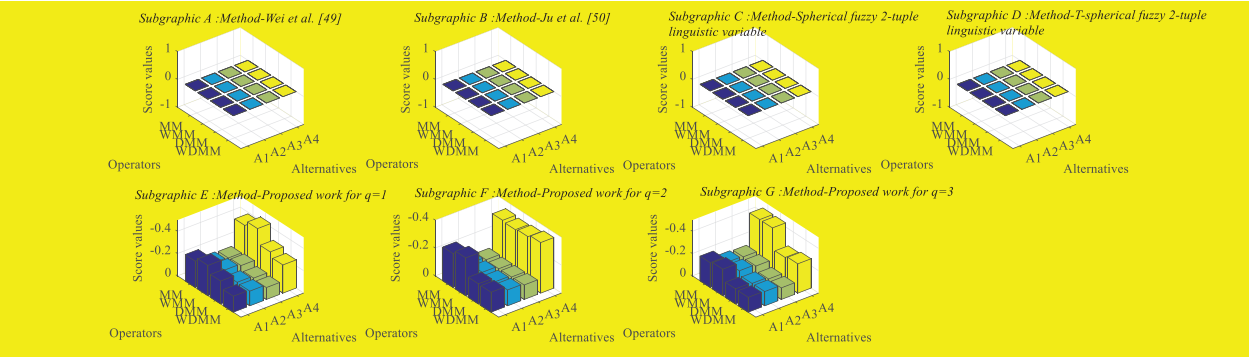

Figure 1. Geometrical representation from Table 3.

Table 4 Decision matrix in the form of complex spherical fuzzy 2-tuple linguistic numbers.

Symbols	C_{AT-1}	C_{AT-2}	C_{AT-3}	C_{AT-4}
A_{Al-1}	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.03\right),\\ \left(\begin{array}{c}0.5e^{i2\pi(0.5)},\\ 0.4e^{i2\pi(0.4)},\\ 0.2e^{i2\pi(0.2)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-4}}, 0.13\right),\\ \left(\begin{array}{c}0.9e^{i2\pi(0.9)},\\ 0.1e^{i2\pi(0.1)},\\ 0.01e^{i2\pi(0.01)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.11\right),\\ \left(\begin{array}{c}0.8e^{i2\pi(0.8)},\\ 0.1e^{i2\pi(0.1)},\\ 0.1e^{i2\pi(0.1)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-4}}, 0.01\right),\\ \left(\begin{array}{c}0.19e^{i2\pi(0.19)},\\ 0.2e^{i2\pi(0.2)},\\ 0.23e^{i2\pi(0.23)}\end{array}\right)\end{array}\right)$
A_{Al-2}	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.01\right),\\ \left(\begin{array}{c}0.51e^{i2\pi(0.51)},\\ 0.41e^{i2\pi(0.41)},\\ 0.12e^{i2\pi(0.12)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.14\right),\\ \left(\begin{array}{c}0.6e^{i2\pi(0.6)},\\ 0.31e^{i2\pi(0.31)},\\ 0.15e^{i2\pi(0.15)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.13\right),\\ \left(\begin{array}{c}0.7e^{i2\pi(0.7)},\\ 0.2e^{i2\pi(0.2)},\\ 0.2e^{i2\pi(0.2)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-2}}, 0.02\right),\\ \left(\begin{array}{c}0.36e^{i2\pi(0.36)},\\ 0.3e^{i2\pi(0.3)},\\ 0.13e^{i2\pi(0.13)}\end{array}\right)\end{array}\right)$
A_{Al-3}	$\left(\begin{array}{c}\left(s_{S_{LT-4}}, 0.0101\right),\\ \left(\begin{array}{c}0.7e^{i2\pi(0.7)},\\ 0.3e^{i2\pi(0.3)},\\ 0.1e^{i2\pi(0.1)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.15\right),\\ \left(\begin{array}{c}0.7e^{i2\pi(0.7)},\\ 0.2e^{i2\pi(0.2)},\\ 0.2e^{i2\pi(0.2)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-4}}, 0.06\right),\\ \left(\begin{array}{c}0.8e^{i2\pi(0.8)},\\ 0.2e^{i2\pi(0.2)},\\ 0.12e^{i2\pi(0.12)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.15\right),\\ \left(\begin{array}{c}0.19e^{i2\pi(0.19)},\\ 0.2e^{i2\pi(0.2)},\\ 0.2e^{i2\pi(0.2)}\end{array}\right)\end{array}\right)$
A_{Al-4}	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.12\right),\\ \left(\begin{array}{c}0.7e^{i2\pi(0.7)},\\ 0.4e^{i2\pi(0.4)},\\ 0.01e^{i2\pi(0.01)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-2}}, 0.16\right),\\ \left(\begin{array}{c}0.38e^{i2\pi(0.38)},\\ 0.13e^{i2\pi(0.13)},\\ 0.3e^{i2\pi(0.3)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-3}}, 0.05\right),\\ \left(\begin{array}{c}0.8e^{i2\pi(0.8)},\\ 0.1e^{i2\pi(0.1)},\\ 0.22e^{i2\pi(0.22)}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(s_{S_{LT-2}}, 0.16\right),\\ \left(\begin{array}{c}0.38e^{i2\pi(0.38)},\\ 0.13e^{i2\pi(0.13)},\\ 0.3e^{i2\pi(0.3)}\end{array}\right)\end{array}\right)$

By using the above steps of the algorithm, the comparison results between the established approach and some existing operators are shown in Table 5 and Figure 2.

From the Table 5, we can see that some existing methods [48,49] cannot deal with this decision-making problem and the proposed method get the same ranking result $A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$ and the best alternative is A_{AL-3}. Obviously, the proposed method is more general than the methods [48,49]. The graphical interpretation based on Table 5 is shown in Figure 2.

Example 6. In this example, the meanings of alternatives and attributes are the same as Example 4, we only consider the complex T-spherical fuzzy 2-tuple linguistic information which is shown in Table 6, and then the explored operators are compared with some existing operators to show the reliability and proficiency of the established operators.

By using the above steps of the algorithm, the comparison results between the established operators and some existing operators are shown in Table 7 and Figure 3.

From the Table 7, we can see that some existing methods [48,49] cannot deal with this decision-making problem and the proposed method get the same ranking result $A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$ and the best alternative is A_{AL-3}. Obviously, the proposed method is more general than the methods [48,49]. The graphical interpretation based on Table 7 is shown in Figure 3.

Example 7. In this example, the meanings of alternatives and attributes are the same as Example 4, we only consider the picture fuzzy 2-tuple linguistic information which is shown in Table 8, and then the explored operators are compared with some existing operators to show the validity of the established operators.

Table 5 Comparative analysis between established operators with existing operators by using Example 5.

Methods	Operators	Score Values	Ranking Values
Wei *et al.* [48]	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Ju *et al.* [49]	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Spherical fuzzy 2-tuple linguistic variables	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
T-spherical fuzzy 2-tuple linguistic variables	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Proposed method (q = 1)	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Proposed method (q = 2)	MM	$S(A_{AL-1}) = -0.354, S(A_{AL-2}) = -0.204, S(A_{AL-3}) = -0.172, S(A_{AL-4}) = -0.554$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.371, S(A_{AL-2}) = -0.222, S(A_{AL-3}) = -0.187, S(A_{AL-4}) = -0.537$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.251, S(A_{AL-2}) = -0.178, S(A_{AL-3}) = -0.142, S(A_{AL-4}) = -0.343$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.255, S(A_{AL-2}) = -0.174, S(A_{AL-3}) = -0.144, S(A_{AL-4}) = -0.353$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
Proposed method (q = 3)	MM	$S(A_{AL-1}) = -0.322, S(A_{AL-2}) = -0.184, S(A_{AL-3}) = -0.163, S(A_{AL-4}) = -0.424$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.321, S(A_{AL-2}) = -0.192, S(A_{AL-3}) = -0.167, S(A_{AL-4}) = -0.437$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.351, S(A_{AL-2}) = -0.188, S(A_{AL-3}) = -0.148, S(A_{AL-4}) = -0.356$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.349, S(A_{AL-2}) = -0.164, S(A_{AL-3}) = -0.143, S(A_{AL-4}) = -0.361$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$

DMM, dual Muirhead mean; MM, Muirhead mean.

Table 6 Decision matrix in the form of complex T-spherical fuzzy 2-tuple linguistic numbers.

Symbols	C_{AT-1}	C_{AT-2}	C_{AT-3}	C_{AT-4}
A_{Al-1}	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.03\end{pmatrix}, \begin{pmatrix}0.8e^{i2\pi(0.5)}, \\ 0.7e^{i2\pi(0.1)}, \\ 0.2e^{i2\pi(0.2)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-4}}, 0.13\end{pmatrix}, \begin{pmatrix}0.9e^{i2\pi(0.9)}, \\ 0.9e^{i2\pi(0.01)}, \\ 0.9e^{i2\pi(0.01)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.11\end{pmatrix}, \begin{pmatrix}0.8e^{i2\pi(0.8)}, \\ 0.79e^{i2\pi(0.1)}, \\ 0.71e^{i2\pi(0.1)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-4}}, 0.01\end{pmatrix}, \begin{pmatrix}0.9e^{i2\pi(0.19)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.63e^{i2\pi(0.23)}\end{pmatrix}\right)$
A_{Al-2}	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.01\end{pmatrix}, \begin{pmatrix}0.59e^{i2\pi(0.51)}, \\ 0.91e^{i2\pi(0.11)}, \\ 0.12e^{i2\pi(0.12)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.14\end{pmatrix}, \begin{pmatrix}0.6e^{i2\pi(0.6)}, \\ 0.81e^{i2\pi(0.11)}, \\ 0.15e^{i2\pi(0.15)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.13\end{pmatrix}, \begin{pmatrix}0.84e^{i2\pi(0.4)}, \\ 0.82e^{i2\pi(0.2)}, \\ 0.82e^{i2\pi(0.2)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-2}}, 0.02\end{pmatrix}, \begin{pmatrix}0.36e^{i2\pi(0.36)}, \\ 0.3e^{i2\pi(0.3)}, \\ 0.13e^{i2\pi(0.13)}\end{pmatrix}\right)$
A_{Al-3}	$\left(\begin{pmatrix}s_{S_{LT-4}}, 0.0101\end{pmatrix}, \begin{pmatrix}0.7e^{i2\pi(0.7)}, \\ 0.6e^{i2\pi(0.1)}, \\ 0.5e^{i2\pi(0.1)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.15\end{pmatrix}, \begin{pmatrix}0.89e^{i2\pi(0.19)}, \\ 0.72e^{i2\pi(0.2)}, \\ 0.2e^{i2\pi(0.2)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-4}}, 0.06\end{pmatrix}, \begin{pmatrix}0.3e^{i2\pi(0.3)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.12e^{i2\pi(0.12)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.15\end{pmatrix}, \begin{pmatrix}0.19e^{i2\pi(0.19)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.2e^{i2\pi(0.2)}\end{pmatrix}\right)$
A_{Al-4}	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.12\end{pmatrix}, \begin{pmatrix}0.7e^{i2\pi(0.7)}, \\ 0.62e^{i2\pi(0.2)}, \\ 0.71e^{i2\pi(0.01)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-2}}, 0.16\end{pmatrix}, \begin{pmatrix}0.88e^{i2\pi(0.38)}, \\ 0.83e^{i2\pi(0.13)}, \\ 0.39e^{i2\pi(0.3)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-3}}, 0.05\end{pmatrix}, \begin{pmatrix}0.28e^{i2\pi(0.28)}, \\ 0.2e^{i2\pi(0.2)}, \\ 0.22e^{i2\pi(0.22)}\end{pmatrix}\right)$	$\left(\begin{pmatrix}s_{S_{LT-2}}, 0.16\end{pmatrix}, \begin{pmatrix}0.38e^{i2\pi(0.38)}, \\ 0.13e^{i2\pi(0.13)}, \\ 0.3e^{i2\pi(0.3)}\end{pmatrix}\right)$

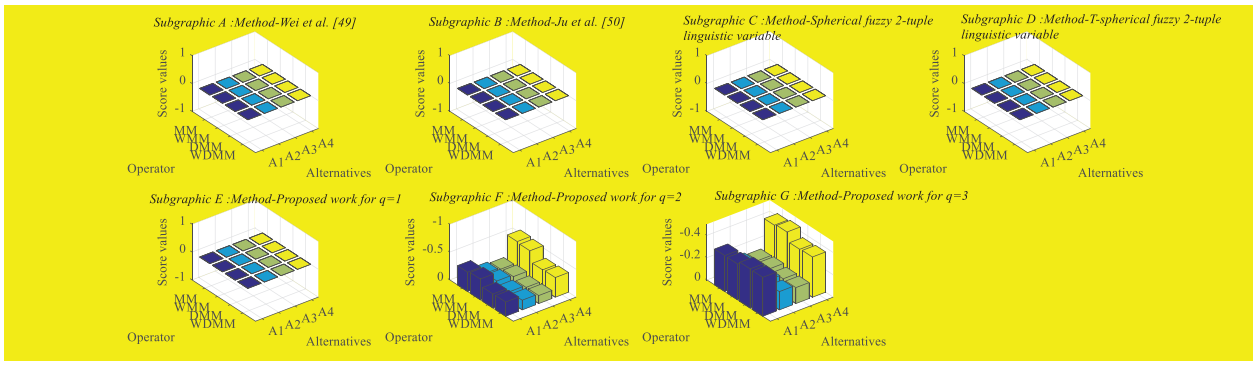

Figure 2 | Geometrical interpretation from Table 5.

Table 7 | Comparative analysis between established operators with existing operators by using Example 6.

Methods	Operators	Score Values	Ranking Values
Wei *et al.* [48]	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Ju *et al.* [49]	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Spherical fuzzy 2-tuple linguistic variables	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
T-spherical fuzzy 2-tuple linguistic variables	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Proposed method (q = 1)	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Proposed method (q = 2)	MM	*Cannot be classified*	–
	WMM	*Cannot be classified*	–
	DMM	*Cannot be classified*	–
	WDMM	*Cannot be classified*	–
Proposed method (q = 12)	MM	$S(A_{AL}-1) = -0.955, S(A_{AL}-2) = -0.857,$ $S(A_{AL}-3) = -0.781, S(A_{AL}-4) = -0.985$	$A_{Al}-3 > A_{Al}-2 > A_{Al}-1 > A_{Al}-4$
	WMM	$S(A_{AL}-1) = -0.975, S(A_{AL}-2) = -0.853,$ $S(A_{AL}-3) = -0.762, S(A_{AL}-4) = -0.982$	$A_{Al}-3 > A_{Al}-2 > A_{Al}-1 > A_{Al}-4$
	DMM	$S(A_{AL}-1) = -0.948, S(A_{AL}-2) = -0.849,$ $S(A_{AL}-3) = -0.734, S(A_{AL}-4) = -0.978$	$A_{Al}-3 > A_{Al}-2 > A_{Al}-1 > A_{Al}-4$
	WDMM	$S(A_{AL}-1) = -0.955, S(A_{AL}-2) = -0.851,$ $S(A_{AL}-3) = -0.799, S(A_{AL}-4) = -0.984$	$A_{Al}-3 > A_{Al}-2 > A_{Al}-1 > A_{Al}-4$

DMM, dual Muirhead mean; MM, Muirhead mean.

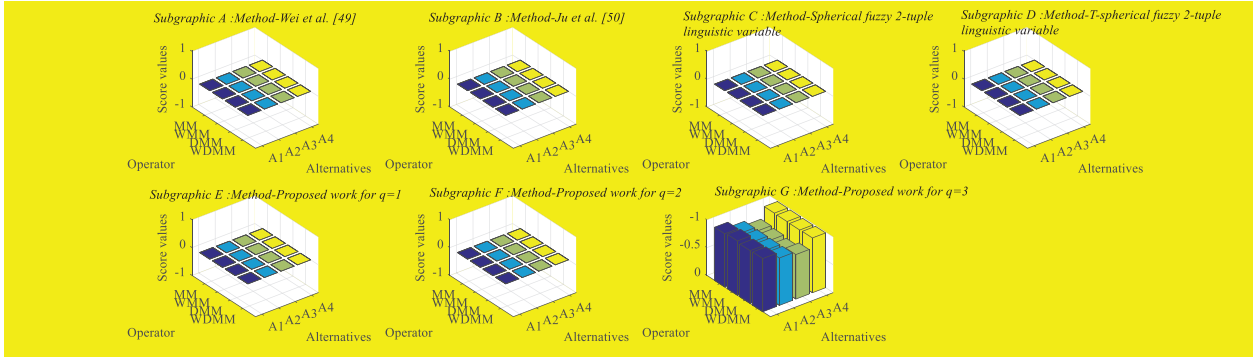

Figure 3 | Geometrical interpretation from Table 7.

Table 8 | Decision matrix in the form of picture fuzzy 2-tuple linguistic numbers.

Symbols	C_{AT-1}	C_{AT-2}	C_{AT-3}	C_{AT-4}
A_{Al-1}	$\left(\left(s_{S_{LT-3}}, 0.03\right), \begin{pmatrix} 0.5e^{i2\pi(0.0)}, \\ 0.1e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-4}}, 0.13\right), \begin{pmatrix} 0.9e^{i2\pi(0.0)}, \\ 0.01e^{i2\pi(0.0)}, \\ 0.01e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-3}}, 0.11\right), \begin{pmatrix} 0.8e^{i2\pi(0.0)}, \\ 0.1e^{i2\pi(0.0)}, \\ 0.1e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-4}}, 0.01\right), \begin{pmatrix} 0.19e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)}, \\ 0.23e^{i2\pi(0.0)} \end{pmatrix}\right)$
A_{Al-2}	$\left(\left(s_{S_{LT-3}}, 0.01\right), \begin{pmatrix} 0.51e^{i2\pi(0.0)}, \\ 0.11e^{i2\pi(0.0)}, \\ 0.12e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-3}}, 0.14\right), \begin{pmatrix} 0.6e^{i2\pi(0.0)}, \\ 0.11e^{i2\pi(0.0)}, \\ 0.15e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-3}}, 0.13\right), \begin{pmatrix} 0.4e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-2}}, 0.02\right), \begin{pmatrix} 0.36e^{i2\pi(0.0)}, \\ 0.3e^{i2\pi(0.0)}, \\ 0.13e^{i2\pi(0.0)} \end{pmatrix}\right)$
A_{Al-3}	$\left(\left(s_{S_{LT-4}}, 0.0101\right), \begin{pmatrix} 0.7e^{i2\pi(0.0)}, \\ 0.1e^{i2\pi(0.0)}, \\ 0.1e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-3}}, 0.15\right), \begin{pmatrix} 0.19e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-4}}, 0.06\right), \begin{pmatrix} 0.3e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)}, \\ 0.12e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-3}}, 0.15\right), \begin{pmatrix} 0.19e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)} \end{pmatrix}\right)$
A_{Al-4}	$\left(\left(s_{S_{LT-3}}, 0.12\right), \begin{pmatrix} 0.7e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)}, \\ 0.01e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-2}}, 0.16\right), \begin{pmatrix} 0.38e^{i2\pi(0.0)}, \\ 0.13e^{i2\pi(0.0)}, \\ 0.3e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-3}}, 0.05\right), \begin{pmatrix} 0.28e^{i2\pi(0.0)}, \\ 0.2e^{i2\pi(0.0)}, \\ 0.22e^{i2\pi(0.0)} \end{pmatrix}\right)$	$\left(\left(s_{S_{LT-2}}, 0.16\right), \begin{pmatrix} 0.38e^{i2\pi(0.0)}, \\ 0.13e^{i2\pi(0.0)}, \\ 0.3e^{i2\pi(0.0)} \end{pmatrix}\right)$

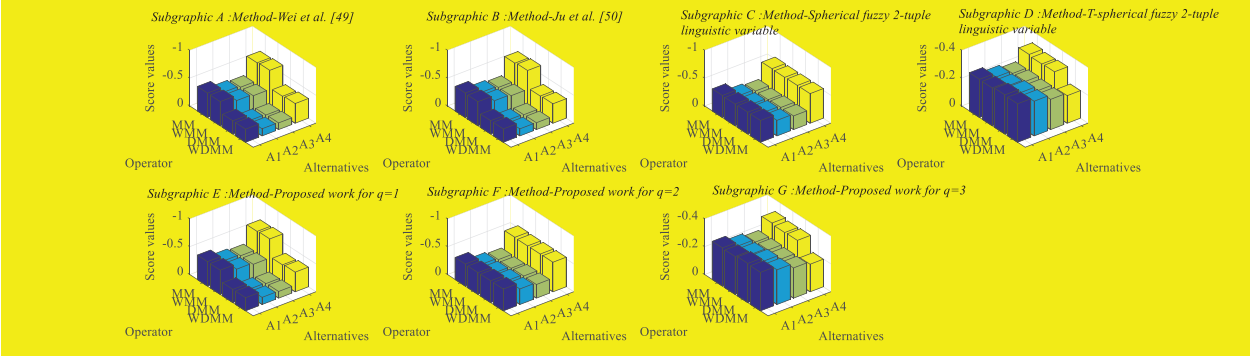

Figure 4 | Geometrical interpretation from Table 9.

By using the above steps of the algorithm, the comparison results between the established operators and some existing operators are shown in Table 9 and Figure 4.

From the Table 9 and Figure 4, we can see that all methods get the same ranking result $A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$ and the best alternative is A_{AL-3}. Obviously, this can explain the validity of the proposed method.

4.3. Advantages

The explored MM operator and DMM operator using the novel concept of CTSF2-TLSs are more powerful and more superior than existing operators which are discussed in Tables 3, 5, and 7, based on the complex picture fuzzy 2-tuple linguistic information and complex spherical fuzzy 2-tuple linguistic information. So the established approaches in this manuscript is more reliable and more efficient than existing methods [48,49].

5. CONCLUSIONS

CTSF2-TLS combined from CFS, TSFS, and 2-TLVS is a proficient technique to express uncertain and awkward information in real decision-making, which contains 2-tuple linguistic variable, truth, abstinence, and falsity grades, and gives more extensive freedom than some existing information expressions due to its constraint that the sum of q-powers of the real parts of the truth, abstinence, and falsity grades is not exceeded form unit interval. Based on the established operational laws and comparison methods for CTSF2-TLSs, the CTSF2-TLMM aggregation operator and CTSF2-TLDMM aggregation operator are explored. Some special cases and the desirable properties of the explored

Table 9 Comparative analysis between established operators with existing operators by using Example 7.

Methods	Operators	Score Values	Ranking Values
Wei *et al.* [48]	MM	$S(A_{AL-1}) = -0.438, S(A_{AL-2}) = -0.342, S(A_{AL-3}) = -0.316, S(A_{AL-4}) = -0.640$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.433, S(A_{AL-2}) = -0.338, S(A_{AL-3}) = -0.311, S(A_{AL-4}) = -0.653$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.237, S(A_{AL-2}) = -0.127, S(A_{AL-3}) = -0.113, S(A_{AL-4}) = -0.336$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.235, S(A_{AL-2}) = -0.128, S(A_{AL-3}) = -0.120, S(A_{AL-4}) = -0.353$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
Ju *et al.* [49]	MM	$S(A_{AL-1}) = -0.439, S(A_{AL-2}) = -0.340, S(A_{AL-3}) = -0.312, S(A_{AL-4}) = -0.634$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.431, S(A_{AL-2}) = -0.335, S(A_{AL-3}) = -0.311, S(A_{AL-4}) = -0.650$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.239, S(A_{AL-2}) = -0.130, S(A_{AL-3}) = -0.118, S(A_{AL-4}) = -0.340$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.239, S(A_{AL-2}) = -0.127, S(A_{AL-3}) = -0.123, S(A_{AL-4}) = -0.349$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
Spherical fuzzy 2-tuple linguistic variables	MM	$S(A_{AL-1}) = -0.395, S(A_{AL-2}) = -0.278, S(A_{AL-3}) = -0.247, S(A_{AL-4}) = -0.534$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.389, S(A_{AL-2}) = -0.272, S(A_{AL-3}) = -0.240, S(A_{AL-4}) = -0.527$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.391, S(A_{AL-2}) = -0.279, S(A_{AL-3}) = -0.249, S(A_{AL-4}) = -0.530$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.390, S(A_{AL-2}) = -0.279, S(A_{AL-3}) = -0.245, S(A_{AL-4}) = -0.531$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
T-spherical fuzzy 2-tuple linguistic variables	MM	$S(A_{AL-1}) = -0.274, S(A_{AL-2}) = -0.245, S(A_{AL-3}) = -0.225, S(A_{AL-4}) = -0.316$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.271, S(A_{AL-2}) = -0.243, S(A_{AL-3}) = -0.227, S(A_{AL-4}) = -0.307$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.281, S(A_{AL-2}) = -0.254, S(A_{AL-3}) = -0.221, S(A_{AL-4}) = -0.310$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.276, S(A_{AL-2}) = -0.249, S(A_{AL-3}) = -0.217, S(A_{AL-4}) = -0.198$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
Proposed method (q = 1)	MM	$S(A_{AL-1}) = -0.438, S(A_{AL-2}) = -0.342, S(A_{AL-3}) = -0.316, S(A_{AL-4}) = -0.640$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.433, S(A_{AL-2}) = -0.338, S(A_{AL-3}) = -0.311, S(A_{AL-4}) = -0.653$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.237, S(A_{AL-2}) = -0.127, S(A_{AL-3}) = -0.113, S(A_{AL-4}) = -0.336$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.235, S(A_{AL-2}) = -0.128, S(A_{AL-3}) = -0.120, S(A_{AL-4}) = -0.353$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
Proposed method (q = 2)	MM	$S(A_{AL-1}) = -0.395, S(A_{AL-2}) = -0.278, S(A_{AL-3}) = -0.247, S(A_{AL-4}) = -0.534$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.389, S(A_{AL-2}) = -0.272, S(A_{AL-3}) = -0.240, S(A_{AL-4}) = -0.527$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.391, S(A_{AL-2}) = -0.279, S(A_{AL-3}) = -0.249, S(A_{AL-4}) = -0.530$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.390, S(A_{AL-2}) = -0.279, S(A_{AL-3}) = -0.245, S(A_{AL-4}) = -0.531$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
Proposed method (q = 3)	MM	$S(A_{AL-1}) = -0.274, S(A_{AL-2}) = -0.245, S(A_{AL-3}) = -0.225, S(A_{AL-4}) = -0.316$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WMM	$S(A_{AL-1}) = -0.271, S(A_{AL-2}) = -0.243, S(A_{AL-3}) = -0.227, S(A_{AL-4}) = -0.307$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	DMM	$S(A_{AL-1}) = -0.281, S(A_{AL-2}) = -0.254, S(A_{AL-3}) = -0.221, S(A_{AL-4}) = -0.310$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$
	WDMM	$S(A_{AL-1}) = -0.276, S(A_{AL-2}) = -0.249, S(A_{AL-3}) = -0.217, S(A_{AL-4}) = -0.198$	$A_{Al-3} > A_{Al-2} > A_{Al-1} > A_{Al-4}$

DMM, dual Muirhead mean; MM, Muirhead mean.

operators are also established and studied. Moreover, we establish a method to solve the MADM problems, in which the evaluation information is expressed by CTSF2-TLNs. Finally, we solve some numerical examples to explain the validity and advantaged of the explored method by comparing with some other methods. In a word, the proposed operators are a generalization of some existing operators such as averaging aggregation operator, geometric aggregation operator, geometric BMO, geometric MSMO based on the complex picture fuzzy 2-tuple linguistic information. In the future researches, we will explore some real applications based on the proposed operators, or some new operators based on CTSF2-TLNs.

CONFLICTS OF INTERESTS

The authors declare that there are no conflicts of interests regarding the publication of this article.

AUTHORS' CONTRIBUTIONS

All authors contributed equally.

DATA AVAILABILITY STATEMENT

The data used to support the findings of this study are included within the article.

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