International Journal of Computational Intelligence Systems
Vol. **13(1)**, 2020, *pp.* 822–851
DOI: https://doi.org/10.2991/ijcis.d.200514.001; ISSN: 1875-6891; eISSN: 1875-6883
https://www.atlantis-press.com/journals/ijcis/

Research Article

Group Decision-Making Using Complex q-Rung Orthopair Fuzzy Bonferroni Mean

Peide Liu[1,*], Zeeshan Ali[2], Tahir Mahmood[2], Nasruddin Hassan[3]

[1] *School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan 250015, China*
[2] *Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan*
[3] *School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, Bangi 43600, Malaysia*

ARTICLE INFO

Article History
Received 03 Apr 2020
Accepted 12 May 2020

Keywords
Complex q-rung orthopair fuzzy sets
Bonferroni mean operators
Multi-attribute group decision-making problems

ABSTRACT

Complex q-rung orthopair fuzzy set (CQROFS), as a modified notion of complex fuzzy set (CFS), is an important tool to cope with awkward and complicated information. CQROFS contains two functions which are called truth grade and falsity grade by the form of complex numbers belonging to unit disc in a complex plane. The condition of CQROFS is that the sum of q-powers of the real part (Also for imaginary part) of the truth grade and real part (Also for imaginary part) of the falsity grade is limited to the unit interval. Bonferroni mean (BM) operator is an important and meaningful concept to examine the interrelationships between the different attributes. Keeping the advantages of the CQROFS and BM operator, in this manuscript, the complex q-rung orthopair fuzzy BM (CQROFBM) operator, complex q-rung orthopair fuzzy weighted BM (CQROFWBM) operator, complex q-rung orthopair fuzzy geometric BM (CQROFGBM) operator, and complex q-rung orthopair fuzzy weighted geometric BM (CQROFWGBM) operator are proposed, and some properties are discussed, further, based on the CQROFWGBM operator, a multi-attribute group decision-making (MAGDM) method is developed, and the ranking results are examined by score function. Finally, we give some numerical examples to verify the rationality of the established method, and show its advantages by comparative analysis with some existing methods.

1. INTRODUCTION

Intuitionistic fuzzy set (IFS) was explored by Atanssove [1] as a modified notion of the fuzzy set (FS) [2], and it contains two functions called as truth grade and falsity grade, whose sum is not exceeded to the unit interval. IFS is an effective tool to describe the complicated fuzzy information, and it has received extensive attention. For example, Garg and Kumar [3] explored a novel exponential distance and TOPSIS methods for interval-valued IFS; Garg and Kaur [4] investigated the extended TOPSIS method using cubic IFS and applied it to multi-attribute group decision-making (MAGDM) problem; Joshi [5] examined a new decision-making method based on IFS and applied it to fault detection in a machine; Kumar [6] explored intuitionistic fuzzy zero point method for solving type-2 intuitionistic fuzzy transportation problem; Alcantud *et al.* [7] aggregated the finite chains of IFSs to deal with temporal IFSs; Kumar [8] evaluated the models for examining the optimization problems using IFSs. Yue [9] applied a projection-based approach based on IFSs to software quality evaluation.

However, the scope of the IFS is narrow because it should satisfy the condition that the sum of truth and falsity grades is bounded to the unit interval. If some decision makers (DMs) provide such kind of information whose sum is not limited to the unit interval, IFS cannot express it. For example, considering the pair (0.6, 0.5) represents the truth grade and the falsity grade which cannot hold the condition of IFS i.e., $0.6 + 0.5 = 1.1 \nleq 1$, the pair $(0.6, 0.5)$ cannot be described by IFS. In order to process these issues, Yager [10] explored pythagorean FS (PYFS) which contains two functions called as truth and falsity grades, whose sum of squares is not exceeded to the unit interval. PYFS is an effective tool to describe the complicated fuzzy information, and it has received extensive attention. For example, Fei and Deng [11] explored pythagorean fuzzy decision-making; Akram *et al.* [12] developed an ELECTRE-1 method for pythagorean fuzzy information; Zhou *et al.* [13] gave a new diverge measure for PYFSs based on belief function and applied it to medical diagnosis; Oztaysi *et al.* [14] and Song *et al.* [15] developed a AHP method for PYFS; Guleria and Bajaj [16] developed pythagorean fuzzy (R, S)-norm discriminant measure.

However, the scope of the PYFS is still narrow because it should satisfy the condition that the sum of squares of truth and falsity grades is bounded to the unit interval. If some DMs provide such kind of information whose sum of squares is not limited to the unit interval, PYFS cannot deal with it. For example, considering the pair (0.9, 0.8) represents the truth grade and the falsity grade, obviously, it cannot hold

*Corresponding author. Email: peide.liu@gmail.com

the condition $0.9^2 + 0.8^2 = 0.81 + 0.64 = 1.45 \nleq 1$. Therefore, in order to deal with these issues, q-rung orthopair fuzzy set (QROFFS) was explored by Yager [17], which contains two functions called as truth and falsity grades, whose sum of q-powers is not exceeded to the unit interval ($q \geq 1$). QROFS is an effective tools to describe the complicated fuzzy information, and it has received extensive attention. For example, Garg and Chen [18] developed neutrality aggregation operators for QROFS; Senapati and Yager [19] restricted the QROFS and gave the Fermatean FS; Darko and Liang [20] established some hamacher aggregation operators for QROFS. Recently, Verma [21] gave the ordered a-diverges and entropy measures for QROFS. Zhang *et al.* [22] explored multiplicative consistency for QROFS. Figure 1 shows the relations of IFS, PYFS, and QROFS.

Further, complex IFS (CIFS) was explored by Alkouri and Salleh [23], as a modified notion of the complex FS (CFS) [24], which contains two functions called as truth and falsity grades by the form of complex numbers from unit disc in a complex plane, whose sum of real parts (Also imaginary parts) is not exceeded to the unit interval. CIFS is an effective tool to describe two-dimensional information in a single set, and it has received extensive attention. For example, Ngan *et al.* [25] represented the CIFS by quaternion numbers; Garg and Rani [26,27] established new generalized Bonferroni mean (BM) operators and robust averaging-geometric operators for CIFS.

However, the scope of the CIFS is narrow because it should satisfy the condition that the sum of the real part (also imaginary part) of truth and the real part (also imaginary part) of the falsity grades is bounded to the unit interval. If some DMs provide such kind of information whose sum of real parts (also imaginary parts) is not limited to the unit interval, CIFS cannot describe it. For example, considering the pair $\left(0.6e^{i2\Pi(0.61)}, 0.5e^{i2\Pi(0.51)}\right)$ represents the truth grade and the falsity grade which cannot hold the condition of CIFS $0.6+0.5 = 1.1 \nleq 1$ and $0.61+0.51 = 1.12 \nleq 1$. Therefore, in order to deal with these issues, Ullah *et al.* [28] explored complex PYFS (CPYFS), which contains two functions called as truth and falsity grades by the form of complex numbers from unit disc in a complex plane, whose sum of squares in real parts (also imaginary parts) is not exceeded to the unit interval. CPYFS is an effective tool to describe the complicated fuzzy information, and it has received extensive attention. Akram and Naz [29] explored the complex pythagorean fuzzy graphs.

However, the scope of the CPYFS is narrow because it should satisfy the condition that the sum of squares of the real part (Also imaginary part) of truth and the real part (also imaginary part) of the falsity grades is bounded to the unit interval. If some DMs provide such kind of information whose sum of squares in the real part (also imaginary part) of truth and the real part (also imaginary part) is not limited to the unit interval, the CPYFS will not deal with it. For example, considering the pair $\left(0.9e^{i2\Pi(0.91)}, 0.8e^{i2\Pi(0.81)}\right)$ represents the truth grade and the falsity grade which cannot hold the condition of CPYFS $0.9^2 + 0.8^2 = 0.81 + 0.64 = 1.45 \nleq 1$ and $0.91^2 + 0.81^2 = 0.8281 + 0.6561 = 1.4842 \nleq 1$. Therefore, in order to deal with these issues, complex q-rung orthopair fuzzy set (CQROFFS) was explored by Liu *et al.* [30,31], which contains two functions called as truth and falsity grades in the form of complex numbers from unit disc in a complex plane, whose sum of q-powers of the real parts (also imaginary parts) is not exceeded to the unit interval. CQROFS is an effective tool to describe the complicated fuzzy information. The comparison of the established work with existing methods [32–36] are also discussed, to examine the reliability and effectiveness of the explored work.

In some real-life decisions, the interrelationships between the attributes are common. For example, in decision-making process of buying a laptop, laptop's performance and its hardware are related. For taking the responsible decision, it is necessary to choose the interrelationships between the attributes. For coping such kind of problems, the BM operators are playing a key role in examining the interrelationships between the attributes, then Xu and Yager [37] explored the intuitionistic fuzzy BM operators; Liang *et al.* [38] established the pythagorean fuzzy BM operators and their application in MAGDM. Liu and Liu [32] explored the q-rung orthopair fuzzy BM operators and their application in MAGDM problems. Further, because the constraint of CQROFS is that the sum of q-powers of the real part (also for imaginary part) of the truth and real part (also for imaginary part) of the falsity grades is limited to the unit interval, the CQROFS can provide a wide range to decision information. From the above discussions, it is clear that the CQROFS is more versatile and more superior to CIFS and

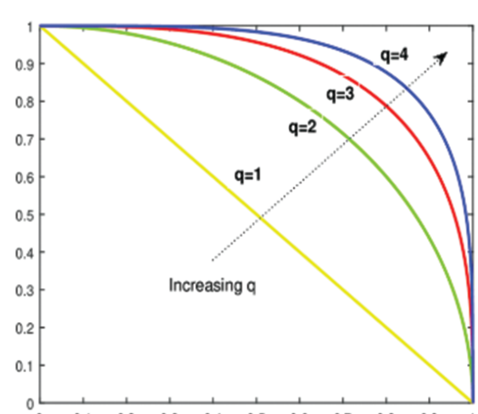

Figure 1 Geometrical interpretation of the intuitionistic fuzzy set (IFS), pythagorean fuzzy set (PYFS), and complex q-rung orthopair fuzzy set (QROFS).

CPFS to describe awkward and complication information in real-decision. In addition, the BM operators based on CQROFS have not been established yet. So the goals and motivations of this article are explained as follows:

1. The BM operators based on QROFS [32] is not able to deal with two-dimensional information in a single set. For coping such type of issues, the BM operator based on CQROFS is an important and meaningful concept to examine the interrelationships between the different attributes and can easily cope with two-dimensional information in a single set. So the goals of this article are to establish the complex q-rung orthopair fuzzy BM (CQROFBM) operator, complex q-rung orthopair fuzzy weighted BM (CQROFWBM) operator, complex q-rung orthopair fuzzy geometric BM (CQROFGBM) operator, and complex q-rung orthopair fuzzy weighted geometric BM (CQROFWGBM) operator and to discuss their properties.
2. Further, we will propose a MAGDM method based on the established operators, which can consider the advantages of BM operators, i.e., considering the interrelationships between the attributes.
3. Moreover, to examine the feasibility and consistency of the established method, we solve some numerical examples to verify the rationality of the explored operators. The advantages, graphical interpretation, and comparative analysis of the established work are also discussed.

For better understanding, we have drawn the flowchart for the proposed approaches, which is shown in Figure 2.

Form Figure 2, it clear that, we propose the BM operator based on CQROFS, which is called complex q-rung orthopair fuzzy BM operator, and discuss its special cases. The proposed technique is more powerful than some other existing operators based on IFS, PFS, QROFS, CIFS, and CPFS. Because the sum of q-powers of the realm parts (also for imaginary parts) of the truth and falsity grades in the CQROFS is not exceeded form unit interval, if we choose the value of parameter q = 1, then the presented work is converted to complex intuitionistic fuzzy BM operator. Similarly if we choose the value of parameter q = 2, then the presented work is converted to complex pythagorean fuzzy BM operator. At the same time, all these operators consider the relationship between two inputs.

The rest of this manuscript is shown as follows: In Section 2, the QROFS, CQROFS, and their operational laws are discussed. In Section 3, the CQROFBM operator, CQROFWBM operator, CQROFGBM operator, and CQROFWGBM operator are explored. In Section 4, we develop the MAGDM method based on the CQROFWGBM operator, and some numerical examples are given to verify the rationality of the explored method. In Section 5, we give the conclusion of this manuscript.

2. PRELIMINARIES

This section is to review some existing notions like QROFSs, CQROFSs, and their operational laws. In this article, we use $\Psi_{Univsersal}$ to represent the fix set. Further, and suppose the symbols keep $s_{CQ}, t_{CQ} \geq 0, q_{CQ} \geq 1$.

Definition 1: [17] A QROFS is stated by

$$\mathcal{C}_Q = \left\{ \left(u, \Phi'_{\mathcal{C}_Q}(u), \xi'_{\mathcal{C}_Q}(u) \right) : u \in \Psi_{Universal} \right\} \quad (1)$$

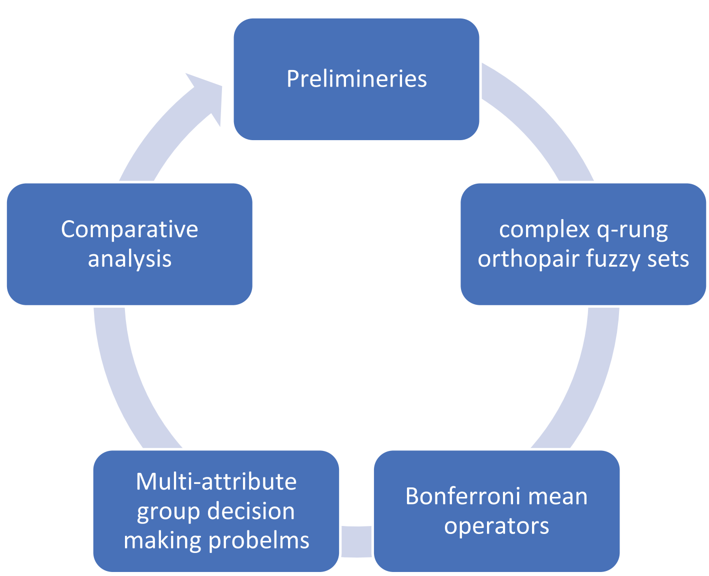

Figure 2 Graphical interpretation of the presented work in this article.

where $\Phi'_{\mathcal{C}_Q}$ and $\xi'_{\mathcal{C}_Q}$ is called truth and falsity grades with a condition: $0 \le \Phi'^{\,q_{CQ}}_{\mathcal{C}_Q}(u) + \xi'^{\,q_{CQ}}_{\mathcal{C}_Q}(u) \le 1$. Further, the symbol $H_{\mathcal{C}_Q}(u) = \left(1 - \left(\Phi'^{\,q_{CQ}}_{\mathcal{C}_Q}(u) + \xi'^{\,q_{CQ}}_{\mathcal{C}_Q}(u)\right)\right)^{\frac{1}{q_{CQ}}}$ represents the hesitancy grade. The q-rung orthopair fuzzy number (QROFN) is denoted by $\mathcal{C}_Q = \left(\Phi'_{\mathcal{C}_Q}(u), \xi'_{\mathcal{C}_Q}(u)\right)$.

Definition 2: [30,31] A CQROFS is stated by

$$\mathcal{C}_{CQ} = \left\{\left(u, \Phi'_{\mathcal{C}_{CQ}}(u), \xi'_{\mathcal{C}_{CQ}}(u)\right) : u \in \mathrm{Ų}_{Universal}\right\} \tag{2}$$

where $\Phi'_{\mathcal{C}_{CQ}} = \Phi_{\mathcal{C}_{RP}} e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP}}}}$ and $\xi'_{\mathcal{C}_{CQ}} = \xi_{\mathcal{C}_{RP}} e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP}}}}$ is called truth and falsity grades in the form of complex number from unit disc in a complex plane with conditions $0 \le \Phi^{q_{CQ}}_{\mathcal{C}_{RP}}(u) + \xi^{q_{CQ}}_{\mathcal{C}_{RP}}(u) \le 1$ and $0 \le \Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP}}}(u) + \Psi^{q_{CQ}}_{\xi_{IP}}(u) \le 1$. Further, the symbol $H_{\mathcal{C}_{CQ}}(u) = \mu_{\mathcal{C}_{RP}} e^{i2\Pi\Psi_{\mu_{\mathcal{C}_{IP}}}} = \left(1 - \left(\begin{matrix}\Phi^{q_{CQ}}_{\mathcal{C}_{RP}}(u) + \\ \xi^{q_{CQ}}_{\mathcal{C}_{RP}}(u)\end{matrix}\right)\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1 - \left(\begin{matrix}\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP}}}(u) + \\ \Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP}}}(u)\end{matrix}\right)\right)^{\frac{1}{q_{CQ}}}}$ represents the hesitancy grade. The complex q-rung orthopair fuzzy number (CQROFN) is denoted by $\mathcal{C}_{CQ} = \left(\Phi'_{\mathcal{C}_{CQ}}(u), \xi'_{\mathcal{C}_{CQ}}(u)\right) = \left(\Phi_{\mathcal{C}_{RP}} e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP}}}}, \xi_{\mathcal{C}_{RP}} e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP}}}}\right)$.

Definition 3: [30,31] For any CQROFS $\mathcal{C}_Q = \left(\Phi_{\mathcal{C}_{RP}} e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP}}}}, \xi_{\mathcal{C}_{RP}} e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP}}}}\right)$, the score function S_{SF} and accuracy function H_{AF} is stated by

$$S_{SF}\left(\mathcal{C}_{CQ}\right) = \frac{1}{2}\left(\left(\Phi_{\mathcal{C}_{RP}} - \xi_{\mathcal{C}_{RP}}\right) + \left(\Psi_{\Phi_{\mathcal{C}_{IP}}} - \Psi_{\xi_{\mathcal{C}_{IP}}}\right)\right) \tag{3}$$

$$H_{AF}\left(\mathcal{C}_{CQ}\right) = \frac{1}{2}\left(\left(\Phi_{\mathcal{C}_{RP}} + \xi_{\mathcal{C}_{RP}}\right) + \left(\Psi_{\Phi_{\mathcal{C}_{IP}}} + \Psi_{\xi_{\mathcal{C}_{IP}}}\right)\right) \tag{4}$$

where $S_{SF}\left(\mathcal{C}_{CQ}\right), H_{AF}\left(\mathcal{C}_{CQ}\right) \in [-1, 1]$. A comparison between CQROFNs $\mathcal{C}_{CQ-1}$ and $\mathcal{C}_{CQ-2}$ is stated by

1. If $S_{SF}\left(\mathcal{C}_{CQ-1}\right) > S_{SF}\left(\mathcal{C}_{CQ-2}\right)$, then $\mathcal{C}_{CQ-1} > \mathcal{C}_{CQ-2}$
2. If $S_{SF}\left(\mathcal{C}_{CQ-1}\right) = S_{SF}\left(\mathcal{C}_{CQ-2}\right)$, then $\mathcal{C}_{CQ-1} = \mathcal{C}_{CQ-2}$, then
   i. If $H_{AF}\left(\mathcal{C}_{CQ-1}\right) > H_{AF}\left(\mathcal{C}_{CQ-2}\right)$, then $\mathcal{C}_{CQ-1} > \mathcal{C}_{CQ-2}$
   ii. If $H_{AF}\left(\mathcal{C}_{CQ-1}\right) = H_{AF}\left(\mathcal{C}_{CQ-2}\right)$, then $\mathcal{C}_{CQ-1} = \mathcal{C}_{CQ-2}$.

Definition 4: [30,31] For any two CQROFNs $\mathcal{C}_{CQ-1}$ and $\mathcal{C}_{CQ-2}$ with s_{CQ}, the operational laws is stated by

1. $\mathcal{C}^c_{CQ-1} = \left(\xi_{\mathcal{C}_{RP-1}} e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-1}}}}, \Phi_{\mathcal{C}_{RP-1}} e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-1}}}}\right)$
2. $\mathcal{C}_{CQ-1} \vee \mathcal{C}_{CQ-2} = \left(\begin{matrix}\max\left(\Phi_{\mathcal{C}_{RP-1}}, \Phi_{\mathcal{C}_{RP-2}}\right) . e^{i2\Pi.\max\left(\Psi_{\Phi_{\mathcal{C}_{IP-1}}}, \Psi_{\Phi_{\mathcal{C}_{IP-2}}}\right)}, \\ \min\left(\xi_{\mathcal{C}_{RP-1}}, \xi_{\mathcal{C}_{RP-2}}\right) . e^{i2\Pi.\min\left(\Psi_{\xi_{\mathcal{C}_{IP-1}}}, \Psi_{\xi_{\mathcal{C}_{IP-2}}}\right)}\end{matrix}\right)$
3. $\mathcal{C}_{CQ-1} \wedge \mathcal{C}_{CQ-2} = \left(\begin{matrix}\min\left(\Phi_{\mathcal{C}_{RP-1}}, \Phi_{\mathcal{C}_{RP-2}}\right) . e^{i2\Pi.\min\left(\Psi_{\Phi_{\mathcal{C}_{IP-1}}}, \Psi_{\Phi_{\mathcal{C}_{IP-2}}}\right)}, \\ \max\left(\xi_{\mathcal{C}_{RP-1}}, \xi_{\mathcal{C}_{RP-2}}\right) . e^{i2\Pi.\max\left(\Psi_{\xi_{\mathcal{C}_{IP-1}}}, \Psi_{\xi_{\mathcal{C}_{IP-2}}}\right)}\end{matrix}\right)$
4. $\mathcal{C}_{CQ-1} \oplus \mathcal{C}_{CQ-2} = \left(\begin{matrix}\left(\begin{matrix}\Phi^{q_{CQ}}_{\mathcal{C}_{RP-1}} + \Phi^{q_{CQ}}_{\mathcal{C}_{RP-2}} - \\ \Phi^{q_{CQ}}_{\mathcal{C}_{RP-1}} \Phi^{q_{CQ}}_{\mathcal{C}_{RP-2}}\end{matrix}\right)^{\frac{1}{q_{CQ}}} . e^{i2\Pi.\left(\begin{matrix}\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-1}}} + \Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-2}}} - \\ \Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-1}}} \Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-2}}}\end{matrix}\right)^{\frac{1}{q_{CQ}}}}, \\ \left(\xi_{\mathcal{C}_{RP-1}} \xi_{\mathcal{C}_{RP-2}}\right) . e^{i2\Pi.\left(\Psi_{\xi_{\mathcal{C}_{IP-1}}} \Psi_{\xi_{\mathcal{C}_{IP-2}}}\right)}\end{matrix}\right)$

5. $$\mathcal{C}_{CQ-1} \otimes \mathcal{C}_{CQ-2} = \left(\begin{array}{c} \left(\Phi_{\mathcal{C}_{RP-1}} \Phi_{\mathcal{C}_{RP-2}}\right) . e^{i2\Pi.\left(\Psi_{\Phi_{\mathcal{C}_{IP-1}}} \Psi_{\Phi_{\mathcal{C}_{IP-2}}}\right)}, \\ \left(\begin{array}{c} \xi^{q_{CQ}}_{\mathcal{C}_{RP-1}} + \xi^{q_{CQ}}_{\mathcal{C}_{RP-2}} - \\ \xi^{q_{CQ}}_{\mathcal{C}_{RP-1}} \xi^{q_{CQ}}_{\mathcal{C}_{RP-2}} \end{array} \right)^{\frac{1}{q_{CQ}}} . e^{i2\Pi. \left(\begin{array}{c} \Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-1}}} + \Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-2}}} - \\ \Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-1}}} \Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{RP-2}}} \end{array} \right)^{\frac{1}{q_{CQ}}}} \end{array} \right)$$

6. $$s_{CQ}\mathcal{C}_{CQ-1} = \left(\begin{array}{c} \left(1 - \left(1 - \Phi^{q_{CQ}}_{\mathcal{C}_{RP-1}}\right)^{s_{CQ}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi.\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-1}}}\right)^{s_{CQ}}\right)^{\frac{1}{q_{CQ}}}}, \\ \xi^{s_{CQ}}_{\mathcal{C}_{RP-1}} e^{i2\Pi.\Psi^{s_{CQ}}_{\xi_{\mathcal{C}_{IP-1}}}} \end{array} \right)$$

7. $$\mathcal{C}^{s_{CQ}}_{CQ-1} = \left(\begin{array}{c} \Phi^{s_{CQ}}_{\mathcal{C}_{RP-1}} e^{i2\Pi.\Psi^{s_{CQ}}_{\Phi_{\mathcal{C}_{IP-1}}}}, \\ \left(1 - \left(1 - \xi^{q_{CQ}}_{\mathcal{C}_{RP-1}}\right)^{s_{CQ}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi.\left(1-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{RP-1}}}\right)^{s_{CQ}}\right)^{\frac{1}{q_{CQ}}}} \end{array} \right).$$

Definition 5: [32] For any non-negative numbers $\mathcal{C}_j, j = 1, 2, 3, .., m$, we define the BM operators by

$$BM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_1, \mathcal{C}_2, .., \mathcal{C}_m\right) = \left(\frac{1}{m(m-1)} \sum_{\substack{j,k=1 \\ j \neq k}}^{m} \mathcal{C}_j^{s_{CQ}} \mathcal{C}_k^{t_{CQ}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}} \tag{5}$$

Definition 6: [32] For any nonnegative numbers $\mathcal{C}_j, j = 1, 2, 3, .., m$, we define the GBM operators by

$$GBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_1, \mathcal{C}_2, .., \mathcal{C}_m\right) = \frac{1}{s_{CQ} + t_{CQ}} \left(\prod_{\substack{j,k=1 \\ j \neq k}}^{m} \left(s_{CQ}\mathcal{C}_j + t_{CQ}\mathcal{C}_k\right)\right)^{\frac{1}{m(m-1)}} \tag{6}$$

3. BM OPERATORS BASED ON CQROFSs

The purpose of this section is to explore the notions of BM, WBM, geometric BM, and weighted geometric BM operators based on CQROFSs. Further, the special cases of the established operators are also discussed by some remarks.

Definition 7: For any CQROFN $\mathcal{C}_{CQ-j}, j = 1, 2, 3, .., m$, we define the CQROFBM operator by

$$CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-m}\right) = \left(\frac{1}{m(m-1)} \oplus_{\substack{j,k=1 \\ j \neq k}}^{m} \left(\mathcal{C}^{s_{CQ}}_{CQ-j} \otimes \mathcal{C}^{t_{CQ}}_{CQ-k}\right)\right)^{\frac{1}{s_{CQ}+t_{CQ}}} \tag{7}$$

Based on the operational laws in Definition 4 for CQROFBMs, we explore the following results.

Theorem 1: *The aggregation result from Definition 7 is still a CQROFN such that*

$$CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right)$$

$$= \left\{ \begin{array}{l} \left(\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Phi^{s_{CQ}}_{\mathcal{C}_{RP-j}}\Phi^{t_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}} \times e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Psi^{s_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\Psi^{t_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}}, \\ \left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(\begin{array}{c} 2-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}- \\ \left(1-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}} \times \\ e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(\begin{array}{c} 2-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}} \\ -\left(1-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}} \end{array} \right\}.$$

Proof: For any two CQROFNs

$$\mathcal{C}_{CQ-j}=\left(\Phi_{\mathcal{C}_{RP-j}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-j}}}},\xi_{\mathcal{C}_{RP-j}}e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right) \text{ and } \mathcal{C}_{CQ-k}=\left(\Phi_{\mathcal{C}_{RP-k}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-k}}}},\xi_{\mathcal{C}_{RP-k}}e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-k}}}}\right),$$

based on Definition 4, we get

$$\mathcal{C}^{s_{CQ}}_{CQ-j}=\left(\Phi^{s_{CQ}}_{\mathcal{C}_{RP-j}}e^{i2\Pi\Psi^{s_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}},\left(1-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}\right)^{\frac{1}{q_{CQ}}}e^{i2\Pi\left(1-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}\right)^{\frac{1}{q_{CQ}}}}\right),$$

$$\text{and } \mathcal{C}^{t_{CQ}}_{CQ-k}=\left(\Phi^{t_{CQ}}_{\mathcal{C}_{RP-k}}e^{i2\Pi\Psi^{t_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}},\left(1-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}\right)^{\frac{1}{q_{CQ}}}e^{i2\Pi\left(1-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}\right)^{\frac{1}{q_{CQ}}}}\right).$$

Then we have

$$\mathcal{C}_{CQ-j}^{s_{CQ}} \otimes \mathcal{C}_{CQ-k}^{t_{CQ}} = \left(\Phi_{\mathcal{C}_{RP-j}}^{s_{CQ}} \Phi_{\mathcal{C}_{RP-k}}^{t_{CQ}} e^{i2\Pi \Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{s_{CQ}} \Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{t_{CQ}}}, \left(\begin{array}{c} 2 - \left(1 - \xi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{s_{CQ}} - \left(1 - \xi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{t_{CQ}} \\ - \left(\begin{array}{c} \left(1 - \left(1 - \xi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{s_{CQ}}\right) \\ \left(1 - \left(1 - \xi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{t_{CQ}}\right) \end{array} \right) \end{array} \right)^{\frac{1}{q_{CQ}}} e^{i2\Pi \left(\begin{array}{c} 2 - \left(1 - \Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{s_{CQ}} - \left(1 - \Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{t_{CQ}} \\ - \left(\begin{array}{c} \left(1 - \left(1 - \Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{s_{CQ}}\right) \\ \left(1 - \left(1 - \Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{t_{CQ}}\right) \end{array} \right) \end{array} \right)^{\frac{1}{q_{CQ}}}} \right)$$

and

$$\sum_{\substack{j,k=1 \\ j \neq k}}^{m} \mathcal{C}_{CQ-j}^{s_{CQ}} \otimes \mathcal{C}_{CQ-k}^{t_{CQ}} = \left(\begin{array}{c} \left(1 - \prod_{\substack{j,k=1 \\ j \neq k}}^{m} \left(1 - \left(\Phi_{\mathcal{C}_{RP-j}}^{s_{CQ}} \Phi_{\mathcal{C}_{RP-k}}^{t_{CQ}}\right)^{q_{CQ}}\right) \right)^{\frac{1}{q_{CQ}}} e^{i2\Pi \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{m} \left(1 - \left(\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{s_{CQ}} \Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{t_{CQ}}\right)^{q_{CQ}}\right) \right)^{\frac{1}{q_{CQ}}}}, \\ \prod_{\substack{j,k=1 \\ j \neq k}}^{m} \left(\begin{array}{c} 2 - \left(1 - \xi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{s_{CQ}} - \left(1 - \xi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{t_{CQ}} \\ - \left(\begin{array}{c} \left(1 - \left(1 - \xi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{s_{CQ}}\right) \\ \left(1 - \left(1 - \xi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{t_{CQ}}\right) \end{array} \right) \end{array} \right)^{\frac{1}{q_{CQ}}} e^{i2\Pi \prod_{\substack{j,k=1 \\ j \neq k}}^{m} \left(\begin{array}{c} 2 - \left(1 - \Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{s_{CQ}} - \left(1 - \Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{t_{CQ}} \\ - \left(\begin{array}{c} \left(1 - \left(1 - \Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{s_{CQ}}\right) \\ \left(1 - \left(1 - \Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{t_{CQ}}\right) \end{array} \right) \end{array} \right)^{\frac{1}{q_{CQ}}}} \end{array} \right)$$

Further,

$$\frac{1}{m(m-1)}\sum_{\substack{j,k=1\\ j\neq k}}^{m} \mathcal{C}_{CQ-j}^{s_{CQ}} \otimes \mathcal{C}_{CQ-k}^{t_{CQ}}$$

$$= \left(\begin{array}{l} \left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Phi_{\mathcal{C}_{RP-j}}^{s_{CQ}}\Phi_{\mathcal{C}_{RP-k}}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{s_{CQ}}\Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}}}}, \\ \left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(\begin{array}{c} 2-\left(1-\xi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{s_{CQ}}-\left(1-\xi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{t_{CQ}} \\ -\left(\begin{array}{c}\left(1-\left(1-\xi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{s_{CQ}}\right) \\ \left(1-\left(1-\xi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{t_{CQ}}\right)\end{array}\right)\end{array}\right)^{\frac{1}{q_{CQ}}}\right)^{\frac{1}{m(m-1)}} e^{i2\Pi\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(\begin{array}{c} 2-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{s_{CQ}}-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{t_{CQ}} \\ -\left(\begin{array}{c}\left(1-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{s_{CQ}}\right) \\ \left(1-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{t_{CQ}}\right)\end{array}\right)\end{array}\right)^{\frac{1}{q_{CQ}}}\right)^{\frac{1}{m(m-1)}}} \end{array} \right)$$

and

$$\left(\frac{1}{m(m-1)}\oplus_{\substack{j,k=1\\ j\neq k}}^{m}\left(\mathcal{C}_{CQ-j}^{s_{CQ}}\otimes \mathcal{C}_{CQ-k}^{t_{CQ}}\right)\right)^{\frac{1}{s_{CQ}+t_{CQ}}}$$

$$= \left(\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Phi_{\mathcal{C}_{RP-j}}^{s_{CQ}}\Phi_{\mathcal{C}_{RP-k}}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}} \times e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{s_{CQ}}\Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}}, \right.$$

$$\left(\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(2-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}-\left(\begin{array}{c}\left(1-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}\right)\\ \left(1-\left(1-\xi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}\times\right.$$

$$\left. e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(2-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}-\left(\begin{array}{c}\left(1-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}\right)\\ \left(1-\left(1-\Psi^{q_{CQ}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}\right)\end{array}\right)\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}\right).$$

The proof of the above theorem has been completed.

Further, we explore some properties of $CQROFBM^{s_{CQ},t_{CQ}}$ operator, including idempotency, monotonicity, and boundedness.

Theorem 2: *For any CQROFN $\mathcal{C}_{CQ-j}, j=1,2,3,..,m$, then*

$$CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right)=\mathcal{C}_{CQ}$$

Proof: Suppose $CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right)=(u,v)$. We will first prove the membership function, such that $\Phi'_{\mathfrak{C}_{CQ}}=\Phi_{\mathfrak{C}_{RP}}e^{i2\Pi\Psi_{\Phi_{\mathfrak{C}_{IP}}}}$

Let $\Phi_{\mathcal{C}_{RP}}=\Phi_{\mathcal{C}_{RP-j}},\Psi_{\Phi_{\mathcal{C}_{IP}}}=\Psi_{\Phi_{\mathcal{C}_{IP-j}}}$ and $\Phi_{\mathcal{C}_{RP}}=\Phi_{\mathcal{C}_{RP-k}},\Psi_{\Phi_{\mathcal{C}_{IP}}}=\Psi_{\Phi_{\mathcal{C}_{IP-k}}}$ implies that $\Phi_{\mathcal{C}_{RP}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP}}}}=\Phi_{\mathcal{C}_{RP-j}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-j}}}}$ and $\Phi_{\mathcal{C}_{RP}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP}}}}=\Phi_{\mathcal{C}_{RP-k}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-k}}}}$, then

$$u=\left(\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Phi^{s_{CQ}}_{\mathcal{C}_{RP-j}}\Phi^{t_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}\times e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Psi^{s_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\Psi^{t_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}}\right)$$

$$=\left(\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Phi^{s_{CQ}}_{\mathcal{C}_{RP-j}}\Phi^{t_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}\times e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Psi^{s_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\Psi^{t_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}}\right)$$

$$= \left(\left(1 - \left(\prod_{\substack{j,k=1 \\ j \neq k}}^{m} \left(1 - \left(\Phi_{\mathcal{C}_{RP}} \right)^{q_{CQ}(s_{CQ}+t_{CQ})} \right) \right)^{\frac{1}{m(m-1)}} \right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}} \times e^{i2\Pi \left(1 - \left(\prod_{j,k=1, j\neq k}^{m} \left(1 - \left(\Psi_{\Phi_{\mathcal{C}_{IP}}} \right)^{q_{CQ}(s_{CQ}+t_{CQ})} \right) \right)^{\frac{1}{m(m-1)}} \right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}} \right)$$

$$= \left(\left(1 - \left(\left(1 - \Phi_{\mathcal{C}_{RP}}^{q_{CQ}(s_{CQ}+t_{CQ})} \right)^{m(m-1)} \right)^{\frac{1}{m(m-1)}} \right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}} \times e^{i2\Pi \left(1 - \left(\left(1 - \Psi_{\Phi_{\mathcal{C}_{IP}}}^{q_{CQ}(s_{CQ}+t_{CQ})} \right)^{m(m-1)} \right)^{\frac{1}{m(m-1)}} \right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}} \right)$$

$$= \left(\left(1 - 1 + \Phi_{\mathcal{C}_{RP}}^{q_{CQ}(s_{CQ}+t_{CQ})} \right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}} e^{i2\Pi \left(1 - 1 + \Psi_{\Phi_{\mathcal{C}_{IP}}}^{q_{CQ}(s_{CQ}+t_{CQ})} \right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}} \right) = \Phi_{\mathcal{C}_{RP}} e^{i2\Pi \Psi_{\Phi_{\mathcal{C}_{IP}}}}$$

Based on above approach for truth grade, we also prove falsity grade such that

$\xi'_{\mathcal{C}_{CQ}} = \xi_{\mathcal{C}_{RP}} e^{i2\Pi \Psi_{\xi_{\mathcal{C}_{IP}}}}$. Hence

$$\text{CQROFBM}^{s_{CQ}, t_{CQ}} \left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-m} \right) = \mathcal{C}_{CQ}$$

Theorem 3: *For any two CQROFNs* $\mathcal{C}_{CQ-j} = \left(\Phi_{\mathcal{C}_{RP-j}} e^{i2\Pi \Psi_{\Phi_{\mathcal{C}_{IP-j}}}}, \xi_{\mathcal{C}_{RP-j}} e^{i2\Pi \Psi_{\xi_{\mathcal{C}_{IP-j}}}} \right)$ *and* $\mathcal{C}_{CQ-*k} = \left(\Phi_{\mathcal{C}_{RP-*k}} e^{i2\Pi \Psi_{\Phi_{\mathcal{C}_{IP-*k}}}}, \xi_{\mathcal{C}_{RP-*k}} e^{i2\Pi \Psi_{\xi_{\mathcal{C}_{IP-*k}}}} \right)$, $(j, k = 1, 2, .., m)$, *with conditions* $\Phi_{RP-j} \geq \Phi_{RP-*k}, \Psi_{\Phi_{IP-j}} \geq \Psi_{\Phi_{IP-*k}}, \xi_{RP-j} \leq \xi_{RP-*k}$ *and* $\Psi_{\xi_{IP-j}} \leq \Psi_{\xi_{IP-*k}}$, *then*

$$CQROFBM^{s_{CQ}, t_{CQ}} \left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-m} \right) \geq CQROFBM^{s_{CQ}, t_{CQ}} \left(\mathcal{C}_{CQ-*1}, \mathcal{C}_{CQ-*2}, .., \mathcal{C}_{CQ-*m} \right)$$

Proof: Let $\text{CQROFBM}^{s_{CQ}, t_{CQ}} \left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-m} \right) = (u, v)$ and $\text{CQROFBM}^{s_{CQ}, t_{CQ}} \left(\mathcal{C}_{CQ-*1}, \mathcal{C}_{CQ-*2}, .., \mathcal{C}_{CQ-*m} \right) = \left(u', v' \right)$. The proof of the truth grade, whose real part is as follows: $u' \leq u$. If $\Phi_{*RP-j} \geq \Phi_{RP-*k}, \Psi_{*\Phi_{IP-j}} \geq \Psi_{\Phi_{IP-*k}}, \xi_{*RP-j} \leq \xi_{RP-*k}$ and $\Psi_{*\xi_{IP-j}} \leq \Psi_{\xi_{IP-*k}}$, then we have

$$\Phi_{*RP-j}^{s_{CQ}} \Phi_{*RP-*k}^{t_{CQ}} e^{i2\Pi \left(\Psi_{*\xi_{IP-j}}^{s_{CQ}} \Psi_{*\xi_{IP-*k}}^{t_{CQ}} \right)} \leq \Phi_{RP-j}^{s_{CQ}} \Phi_{RP-k}^{t_{CQ}} e^{i2\Pi \left(\Psi_{\xi_{IP-j}}^{s_{CQ}} \Psi_{\xi_{IP-k}}^{t_{CQ}} \right)},$$

$$\left(1 - \left(\Phi_{*RP-j}^{s_{CQ}} \Phi_{*RP-*k}^{t_{CQ}} \right)^{q_{CQ}} \right) e^{i2\Pi \left(1 - \left(\Psi_{*\xi_{IP-j}}^{s_{CQ}} \Psi_{*\xi_{IP-*k}}^{t_{CQ}} \right)^{q_{CQ}} \right)}$$
$$\geq \left(1 - \left(\Phi_{RP-j}^{s_{CQ}} \Phi_{RP-k}^{t_{CQ}} \right)^{q_{CQ}} \right) e^{i2\Pi \left(1 - \left(\Psi_{\xi_{IP-j}}^{s_{CQ}} \Psi_{\xi_{IP-k}}^{t_{CQ}} \right)^{q_{CQ}} \right)},$$

$$\left(\prod_{\substack{j,k=1 \\ j \neq k}}^{m} \left(1 - \left(\Phi_{*RP-j}^{s_{CQ}} \Phi_{*RP-*k}^{t_{CQ}} \right)^{q_{CQ}} \right) \right)^{\frac{1}{m(m-1)}} e^{i2\Pi \left(\prod_{j,k=1, j\neq k}^{m} \left(1 - \left(\Psi_{*\xi_{IP-j}}^{s_{CQ}} \Psi_{*\xi_{IP-*k}}^{t_{CQ}} \right)^{q_{CQ}} \right) \right)^{\frac{1}{m(m-1)}}}$$

$$\geq \left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Phi_{RP-j}^{s_{CQ}}\Phi_{RP-k}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}} e^{i2\Pi\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Psi_{\xi_{IP-j}}^{s_{CQ}}\Psi_{\xi_{IP-k}}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}}$$

$$\left(\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Phi_{*RP-j}^{s_{CQ}}\Phi_{*RP-*k}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}} \times e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Psi_{*\xi_{IP-j}}^{s_{CQ}}\Psi_{*\xi_{IP-*k}}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}}\right)$$

$$\leq \left(\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Phi_{RP-j}^{s_{CQ}}\Phi_{RP-k}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}} \times e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{m}\left(1-\left(\Psi_{\xi_{IP-j}}^{s_{CQ}}\Psi_{\xi_{IP-k}}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{m(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}}\right)$$

Hence $u' \leq u$. Similarly, $v' \geq v$, for falsity grade. Thus, the final result is shown as

$$\text{CQROFBM}^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right) \geq \text{CQROFBM}^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-*1},\mathcal{C}_{CQ-*2},..,\mathcal{C}_{CQ-*m}\right).$$

Theorem 4: *For any two CQROFNs* $\mathcal{C}_{CQ-j}^{+} = \left(\max_{J}\Phi_{\mathcal{C}_{RP-j}} e^{i2\Pi \max_{j}\Psi_{\Phi_{\mathcal{C}_{IP-j}}}}, \min_{j}\xi_{\mathcal{C}_{RP-j}} e^{i2\Pi \min_{j}\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$ *and* $\mathcal{C}_{CQ-j}^{-} = \left(\min_{J}\Phi_{\mathcal{C}_{RP-j}} e^{i2\Pi \min_{J}\Psi_{\Phi_{\mathcal{C}_{IP-j}}}}, \max_{j}\xi_{\mathcal{C}_{RP-j}} e^{i2\Pi \max_{j}\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$, $(j = 1, 2, .., m)$, *then*

$$\mathcal{C}_{CQ-j}^{-} \leq CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right) \leq \mathcal{C}_{CQ-j}^{+}$$

Proof: Based on monotonicity, we get

$$CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1}^{-},\mathcal{C}_{CQ-2}^{-},..,\mathcal{C}_{CQ-m}^{-}\right) \leq CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right)$$
$$\leq CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1}^{+},\mathcal{C}_{CQ-2}^{+},..,\mathcal{C}_{CQ-m}^{+}\right)$$

By idempotency, we get

$CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1}^{-},\mathcal{C}_{CQ-2}^{-},..,\mathcal{C}_{CQ-m}^{-}\right) = \mathcal{C}_{CQ-j}^{-}$ and $CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1}^{+},\mathcal{C}_{CQ-2}^{+},..,\mathcal{C}_{CQ-m}^{+}\right) = \mathcal{C}_{CQ-j}^{+}$

Then

$$\mathcal{C}_{CQ-j}^{-} \leq CQROFBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right) \leq \mathcal{C}_{CQ-j}^{+}$$

The proof of the above theorem has been completed.

Further, the special cases of the $CQROFBM^{s_{CQ},t_{CQ}}$ operator are shown as

Remark 1: When $t_{CQ} = 0$ in Definition 7, then

$$CQROFBM^{s_{CQ},0}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)$$
$$=\left(\begin{array}{c}\left(\left(1-\left(\prod_{j=1}^{\mathbf{m}}\left(1-\left(\Phi_{\mathcal{C}RP-j}^{s_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}s_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{j=1}^{\mathbf{m}}\left(1-\left(\Psi_{\Phi_{\mathcal{C}IP-j}}^{s_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}s_{CQ}}}}\right),\\ \left(\left(1-\left(1-\left(\prod_{j=1}^{\mathbf{m}}\left(1-\left(1-\xi_{\mathcal{C}RP-j}^{q_{CQ}}\right)^{s_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}}}\right)^{\frac{1}{q_{CQ}}} \times e^{i2\Pi\left(1-\left(1-\left(\prod_{j=1}^{\mathbf{m}}\left(1-\left(1-\Psi_{\xi_{\mathcal{C}IP-1}}^{q_{CQ}}\right)^{s_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}\right)\end{array}\right)$$

Remark 2: When $s_{CQ} = 1, t_{CQ} = 0$ in Definition 7, then

$$CQROFBM^{1,0}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)$$
$$=\left(\begin{array}{c}\left(\left(1-\left(\prod_{j=1}^{\mathbf{m}}\left(1-\left(\Phi_{\mathcal{C}RP-j}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{j=1}^{\mathbf{m}}\left(1-\left(\Psi_{\Phi_{\mathcal{C}IP-j}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}}}}\right),\\ \left(\prod_{j=1}^{\mathbf{m}}\left(\xi_{\mathcal{C}RP-j}^{q_{CQ}}\right)\right)^{\frac{1}{q_{CQ}\mathbf{m(m-1)}}} e^{i2\Pi\left(\prod_{j=1}^{\mathbf{m}}\left(\Psi_{\xi_{\mathcal{C}IP-j}}^{q_{CQ}}\right)\right)^{\frac{1}{q_{CQ}\mathbf{m(m-1)}}}}\end{array}\right)$$

Remark 3: When $s_{CQ} = 0$ in Definition 7, then

$$CQROFBM^{0,t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)$$
$$=\left(\begin{array}{c}\left(\left(1-\left(\prod_{k=1}^{\mathbf{m}}\left(1-\left(\Phi_{\mathcal{C}RP-k}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}t_{CQ}}} \times e^{i2\Pi\left(1-\left(\prod_{k=1}^{\mathbf{m}}\left(1-\left(\Psi_{\Phi_{\mathcal{C}IP-k}}^{t_{CQ}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}t_{CQ}}}}\right),\\ \left(\left(1-\left(1-\left(\prod_{k=1}^{\mathbf{m}}\left(1-\left(1-\xi_{\mathcal{C}RP-k}^{q_{CQ}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{t_{CQ}}}\right)^{\frac{1}{q_{CQ}}} \times e^{i2\Pi\left(1-\left(1-\left(\prod_{k=1}^{\mathbf{m}}\left(1-\left(1-\Psi_{\xi_{\mathcal{C}RP-k}}^{q_{CQ}}\right)^{s_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}\right)\end{array}\right)$$

Remark 4: When $s_{CQ} = 0, t_{CQ} = 1$ in Definition 7, then

$$CQROFBM^{0,1}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) = \left(\begin{array}{c} \left(1-\left(\prod_{k=1}^{\mathbf{m}}\left(1-(\Phi_{\mathcal{C}RP-k})^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{k=1}^{\mathbf{m}}\left(1-\left(\Psi_{\Phi_{\mathcal{C}IP-k}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{q_{CQ}}}}, \\ \left(\prod_{k=1}^{\mathbf{m}}\left(\xi_{\mathcal{C}RP-k}^{q_{CQ}}\right)\right)^{\frac{1}{q_{CQ}\mathbf{m}(\mathbf{m}-1)}} e^{i2\Pi\left(\prod_{k=1}^{\mathbf{m}}\left(\Psi_{\xi_{\mathcal{C}IP-k}}^{q_{CQ}}\right)\right)^{\frac{1}{q_{CQ}\mathbf{m}(\mathbf{m}-1)}}} \end{array}\right)$$

Remark 5: When $s_{CQ} = t_{CQ} = 1$ in Definition 7, then

$$CQROFBM^{1,1}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) = \left(\begin{array}{c} \left(\left(1-\left(\prod_{j\neq k=1}^{\mathbf{m}}\left(1-(\Phi_{\mathcal{C}RP-j}\Phi_{\mathcal{C}RP-k})^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{2q_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{j\neq k=1}^{\mathbf{m}}\left(1-\left(\Psi_{\Phi_{\mathcal{C}IP-j}}\Psi_{\Phi_{\mathcal{C}IP-k}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{2q_{CQ}}}}\right), \\ \left(\begin{array}{c} \left(1-\left(1-\left(\prod_{j\neq k=1}^{\mathbf{m}}\left(\xi_{\mathcal{C}RP-j}^{q_{CQ}}+\xi_{\mathcal{C}RP-k}^{q_{CQ}}-\xi_{\mathcal{C}RP-j}^{q_{CQ}}\xi_{\mathcal{C}RP-k}^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{CQ}}} \times \\ e^{i2\Pi\left(1-\left(1-\left(\prod_{j\neq k=1}^{\mathbf{m}}\left(\Psi_{\xi_{\mathcal{C}IP-j}}^{q_{CQ}}+\Psi_{\xi_{\mathcal{C}IP-k}}^{q_{CQ}}-\Psi_{\xi_{\mathcal{C}IP-j}}^{q_{CQ}}\Psi_{\xi_{\mathcal{C}IP-k}}^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{CQ}}}} \end{array}\right) \end{array}\right)$$

Further, we define the CQROFWBM operator. Suppose weight vector is $\mho_w = (\mho_{w-1}, \mho_{w-2}, .., \mho_{w-\mathbf{m}})^T$, meets $\sum_{j=1}^{\mathbf{m}} \mho_{w-j} = 1$ and $\mho_{w-j} \in [0,1], (j = 1, 2, .., \mathbf{m})$.

Definition 8: For any CQROFN $\mathcal{C}_{CQ-j}, (\mathbf{j} = \mathbf{1}, \mathbf{2}, \mathbf{3}, .., \mathbf{m})$, we define the CQROFWBM operator by

$$CQROFWBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) = \left(\frac{1}{\mathbf{m}(\mathbf{m}-1)} \oplus_{\substack{j,k=1 \\ j\neq k}}^{\mathbf{m}} \left(\left(\mho_{w-j}\mathcal{C}_{CQ-j}\right)^{s_{CQ}} \otimes \left(\mho_{w-k}\mathcal{C}_{CQ-k}\right)^{t_{CQ}}\right)\right)^{\frac{1}{s_{CQ}+t_{CQ}}} \tag{8}$$

Based on the operational laws in Definition 4, we give the following results.

Theorem 5: *The aggregation result from Definition 8 is still a CQROFN such that*

$$CQROFWBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right)=$$

$$\left(\begin{array}{c}\left(\left(1-\left(\prod_{\substack{j,k=1\\j\neq k}}^{\mathbf{m}}\left(1-\left(1-\left(1-\Phi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{\omega_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\Phi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{\omega_{w-k}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}\left(s_{CQ}+t_{CQ}\right)}}\times\right.\\ \left.e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\j\neq k}}^{\mathbf{m}}\left(1-\left(1-\left(1-\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{\omega_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{\omega_{w-k}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}\left(s_{CQ}+t_{CQ}\right)}}}\right),\\ \left(\left(1-\left(1-\left(\prod_{\substack{j,k=1\\j\neq k}}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\xi_{\mathcal{C}_{RP-j}}^{q_{CQ}\omega_{w-j}}\right)^{s_{CQ}}-\left(1-\xi_{\mathcal{C}_{RP-k}}^{q_{CQ}\omega_{w-k}}\right)^{t_{CQ}}-\\ \left(1-\left(1-\xi_{\mathcal{C}_{RP-j}}^{q_{CQ}\omega_{w-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\xi_{\mathcal{C}_{RP-k}}^{q_{CQ}\omega_{w-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}\times\right.\\ \left.e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{j,k=1\\j\neq k}}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}\omega_{w-j}}\right)^{s_{CQ}}-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}\omega_{w-k}}\right)^{t_{CQ}}-\\ \left(1-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}\omega_{w-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}\omega_{w-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}\right)\end{array}\right).$$

Proof: For any two CQROFNs, it is clear that

$$\omega_{w-j}\mathcal{C}_{CQ-j}=\left(\begin{array}{c}\left(1-\left(1-\Phi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{\omega_{w-j}}\right)^{\frac{1}{q_{CQ}}}e^{i2\Pi\left(1-\left(1-\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{\omega_{w-j}}\right)^{\frac{1}{q_{CQ}}}},\\ \xi_{\mathcal{C}_{RP-j}}^{\omega_{w-j}}e^{i2\Pi\left(\Psi_{\xi_{\mathcal{C}_{IP-j}}}^{\omega_{w-j}}\right)}\end{array}\right)\text{, and}$$

$$\omega_{w-k}\mathcal{C}_{CQ-k}=\left(\begin{array}{c}\left(1-\left(1-\Phi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{\omega_{w-k}}\right)^{\frac{1}{q_{CQ}}}e^{i2\Pi\left(1-\left(1-\Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{\omega_{w-k}}\right)^{\frac{1}{q_{CQ}}}},\\ \xi_{\mathcal{C}_{RP-k}}^{\omega_{w-k}}e^{i2\Pi\left(\Psi_{\xi_{\mathcal{C}_{IP-k}}}^{\omega_{w-k}}\right)}\end{array}\right),$$

then $\left(\varpi_{w-j}\mathcal{C}_{CQ-j}\right)^{s_{CQ}} =$

$$\left(\left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{\varpi_{w-j}}\right)^{\frac{s_{CQ}}{q_{CQ}}} e^{i2\Pi\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\right)^{\varpi_{w-j}}\right)^{\frac{s_{CQ}}{q_{CQ}}}}, \left(1-\left(1-\xi^{q_{CQ}\varpi_{w-j}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(1-\Psi^{q_{CQ}\varpi_{w-j}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}\right)^{\frac{1}{q_{CQ}}}}\right)$$

$$\text{and } \left(\varpi_{w-k}\mathcal{C}_{CQ-k}\right)^{t_{CQ}} = \begin{pmatrix} \left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{\varpi_{w-k}}\right)^{\frac{t_{CQ}}{q_{CQ}}} e^{i2\Pi\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}\right)^{\varpi_{w-k}}\right)^{\frac{t_{CQ}}{q_{CQ}}}}, \\ \left(1-\left(1-\xi^{q_{CQ}\varpi_{w-k}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(1-\Psi^{q_{CQ}\varpi_{w-k}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}\right)^{\frac{1}{q_{CQ}}}} \end{pmatrix}.$$

Based on Definition 4, we have

$$\begin{aligned} &\left(\varpi_{w-j}\mathcal{C}_{CQ-j}\right)^{s_{CQ}} \otimes \left(\varpi_{w-k}\mathcal{C}_{CQ-k}\right)^{t_{CQ}} \\ &= \begin{pmatrix} \left(\left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{\varpi_{w-j}}\right)^{\frac{s_{CQ}}{q_{CQ}}} \left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{\varpi_{w-k}}\right)^{\frac{t_{CQ}}{q_{CQ}}}\right) e^{i2\Pi\left(\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\right)^{\varpi_{w-j}}\right)^{\frac{s_{CQ}}{q_{CQ}}} \left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}\right)^{\varpi_{w-k}}\right)^{\frac{t_{CQ}}{q_{CQ}}}\right)}, \\ \left(2-\left(1-\xi^{q_{CQ}\varpi_{w-j}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}-\left(1-\xi^{q_{CQ}\varpi_{w-k}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}-\left(\left(1-\left(1-\xi^{q_{CQ}\varpi_{w-j}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\xi^{q_{CQ}\varpi_{w-k}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}\right)\right)\right)^{\frac{1}{q_{CQ}}} \\ e^{i2\Pi\left(2-\left(1-\Psi^{q_{CQ}\varpi_{w-j}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}-\left(1-\Psi^{q_{CQ}\varpi_{w-k}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}-\left(\left(1-\left(1-\Psi^{q_{CQ}\varpi_{w-j}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Psi^{q_{CQ}\varpi_{w-k}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}\right)\right)\right)^{\frac{1}{q_{CQ}}}}, \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} &\sum_{\substack{j,k=1 \\ j\neq k}}^{m} \left(\varpi_{w-j}\mathcal{C}_{CQ-j}\right)^{s_{CQ}} \otimes \left(\varpi_{w-k}\mathcal{C}_{CQ-k}\right)^{t_{CQ}} \\ &= \begin{pmatrix} \left(\prod_{j,k=1}^{m}\begin{pmatrix} 1-\left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{\varpi_{w-j}}\right)^{s_{CQ}} \\ \left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{\varpi_{w-k}}\right)^{t_{CQ}} \end{pmatrix}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(\prod_{j,k=1}^{m}\left(1-\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\right)^{\varpi_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}\right)^{\varpi_{w-k}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{q_{CQ}}}}, \\ \left(\prod_{j,k=1}^{m}\begin{pmatrix} 2-\left(1-\xi^{q_{CQ}\varpi_{w-j}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}-\left(1-\xi^{q_{CQ}\varpi_{w-k}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}} \\ -\begin{pmatrix} \left(1-\left(1-\xi^{q_{CQ}\varpi_{w-j}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}\right) \\ \left(1-\left(1-\xi^{q_{CQ}\varpi_{w-k}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}\right) \end{pmatrix} \end{pmatrix}\right) e^{i2\Pi\left(\prod_{j,k=1}^{m}\begin{pmatrix} 2-\left(1-\Psi^{q_{CQ}\varpi_{w-j}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}-\left(1-\Psi^{q_{CQ}\varpi_{w-k}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}} \\ -\left(\left(1-\left(1-\Psi^{q_{CQ}\varpi_{w-j}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Psi^{q_{CQ}\varpi_{w-k}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}\right)\right) \end{pmatrix}\right)} \end{pmatrix}.\end{aligned}$$

Further,

$$\frac{1}{\mathbf{m}(\mathbf{m}-1)}\sum_{\substack{\mathrm{j,k}=1\\ \mathrm{j}\neq\mathrm{k}}}^{\mathbf{m}}\left(\varpi_{w-j}\mathcal{C}_{CQ-j}\right)^{s_{CQ}}\otimes\left(\varpi_{w-k}\mathcal{C}_{CQ-k}\right)^{t_{CQ}}$$

$$=\left(\begin{array}{c}\left(\begin{array}{c}\left(1-\left(\prod_{j,k=1}^{\mathbf{m}}\left(1-\left(1-\Phi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{\varpi_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\Phi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{\varpi_{w-k}}\right)^{t_{CQ}}\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{q_{CQ}}}\times\\ e^{i2\Pi\left(1-\left(\prod_{j,k=1}^{\mathbf{m}}\left(1-\left(1-\Psi_{\Phi_{e_{IP-j}}}^{q_{CQ}}\right)^{\varpi_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\Psi_{\Phi_{e_{IP-k}}}^{q_{CQ}}\right)^{\varpi_{w-k}}\right)^{t_{CQ}}\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{q_{CQ}}}}\end{array}\right),\\ \left(\prod_{j,k=1}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\xi_{\mathcal{C}_{RP-j}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}-\left(1-\xi_{\mathcal{C}_{RP-k}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}\\ -\left(\begin{array}{c}\left(1-\left(1-\xi_{\mathcal{C}_{RP-j}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}\right)\\ \left(1-\left(1-\xi_{\mathcal{C}_{RP-k}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}\right)\end{array}\right)\end{array}\right)\right)^{\frac{1}{m(m+1)}} e^{i2\Pi\left(\prod_{j,k=1}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\Psi_{\xi_{e_{IP-j}}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}-\left(1-\Psi_{\xi_{e_{IP-k}}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}-\\ \left(\begin{array}{c}\left(1-\left(1-\Psi_{\xi_{e_{IP-j}}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}\right)\\ \left(1-\left(1-\Psi_{\xi_{e_{IP-k}}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}\right)\end{array}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}}\end{array}\right)$$

$$\left(\frac{1}{\mathbf{m}(\mathbf{m}-1)}\sum_{\substack{\mathrm{j,k}=1\\ \mathrm{j}\neq\mathrm{k}}}^{\mathbf{m}}\left(\varpi_{w-j}\mathcal{C}_{CQ-j}\right)^{s_{CQ}}\otimes\left(\varpi_{w-k}\mathcal{C}_{CQ-k}\right)^{t_{CQ}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}$$

$$=\left(\begin{array}{c}\left(\begin{array}{c}\left(1-\left(\prod_{\substack{\mathrm{j,k}=1\\ \mathrm{j}\neq\mathrm{k}}}^{\mathbf{m}}\left(1-\left(1-\left(1-\Phi_{\mathcal{C}_{RP-\mathrm{j}}}^{q_{CQ}}\right)^{\varpi_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\Phi_{\mathcal{C}_{RP-\mathrm{k}}}^{q_{CQ}}\right)^{\varpi_{w-k}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{q_{CQ}\left(s_{CQ}+t_{CQ}\right)}}\times\\ e^{i2\Pi\left(1-\left(\prod_{\substack{\mathrm{j,k}=1\\ \mathrm{j}\neq\mathrm{k}}}^{\mathbf{m}}\left(1-\left(1-\left(1-\Psi_{\Phi_{e_{IP-\mathrm{j}}}}^{q_{CQ}}\right)^{\varpi_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\Psi_{\Phi_{e_{IP-\mathrm{k}}}}^{q_{CQ}}\right)^{\varpi_{w-k}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{q_{CQ}\left(s_{CQ}+t_{CQ}\right)}}}\end{array}\right),\\ \left(\begin{array}{c}\left(1-\left(1-\left(\prod_{\substack{\mathrm{j,k}=1\\ \mathrm{j}\neq\mathrm{k}}}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\xi_{\mathcal{C}_{RP-\mathrm{j}}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}-\left(1-\xi_{\mathcal{C}_{RP-\mathrm{k}}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}-\\ \left(1-\left(1-\xi_{\mathcal{C}_{RP-\mathrm{j}}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\xi_{\mathcal{C}_{RP-\mathrm{k}}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}\times\\ e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{\mathrm{j,k}=1\\ \mathrm{j}\neq\mathrm{k}}}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\Psi_{\xi_{e_{IP-\mathrm{j}}}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}-\left(1-\Psi_{\xi_{e_{IP-\mathrm{k}}}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}-\\ \left(1-\left(1-\Psi_{\xi_{e_{IP-\mathrm{j}}}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Psi_{\xi_{e_{IP-\mathrm{k}}}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}\end{array}\right)\end{array}\right)$$

The proof of the above theorem has been completed.

Further, we explore some properties of the *CQROFWBM* operator including idempotency, monotonicity, and boundedness.

Theorem 6: *For any CQROFN* $\mathcal{C}_{CQ-j}, \boldsymbol{j = 1, 2, 3, .., m}$, *then*

$$CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) = \mathcal{C}_{CQ}$$

Proof: Straightforward.

Theorem 7: *For any two CQROFNs* $\mathcal{C}_{CQ-j} = \left(\Phi_{\mathcal{C}_{RP-j}} e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-j}}}}, \xi_{\mathcal{C}_{RP-j}} e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$ *and* $\mathcal{C}_{CQ-*k} = \left(\Phi_{\mathcal{C}_{RP-*k}} e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-*k}}}}, \xi_{\mathcal{C}_{RP-*k}} e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-*k}}}}\right), (j, k = 1, 2, .., m)$, *with conditions* $\Phi_{RP-j} \geq \Phi_{RP-*k}, \Psi_{\Phi_{IP-j}} \geq \Psi_{\Phi_{IP-*k}}, \xi_{RP-j} \leq \xi_{RP-*k}$ *and* $\Psi_{\xi_{IP-j}} \leq \Psi_{\xi_{IP-*k}}$, *then*

$$CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) \geq CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}_{CQ-*1}, \mathcal{C}_{CQ-*2}, .., \mathcal{C}_{CQ-*\mathbf{m}}\right)$$

Proof: Straightforward.

Theorem 8: *For any two CQROFNs* $\mathcal{C}^{+}_{CQ-j} = \left(\max_{j} \Phi_{\mathcal{C}_{RP-j}} e^{i2\Pi \max_{j} \Psi_{\Phi_{\mathcal{C}_{IP-j}}}}, \min_{j} \xi_{\mathcal{C}_{RP-j}} e^{i2\Pi \min_{j} \Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$ *and* $\mathcal{C}^{-}_{CQ-j} = \left(\min_{j} \Phi_{\mathcal{C}_{RP-j}} e^{i2\Pi \min_{j} \Psi_{\Phi_{\mathcal{C}_{IP-j}}}}, \max_{j} \xi_{\mathcal{C}_{RP-j}} e^{i2\Pi \max_{j} \Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right), (j = 1, 2, .., m)$, *then*

$$\mathcal{C}^{-}_{CQ-j} \leq CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) \leq \mathcal{C}^{+}_{CQ-j}$$

Proof: According to monotonicity, we get

$$\begin{aligned} & CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}^{-}_{CQ-1}, \mathcal{C}^{-}_{CQ-2}, .., \mathcal{C}^{-}_{CQ-\mathbf{m}}\right) \\ \leq\ & CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) \\ \leq\ & CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}^{+}_{CQ-1}, \mathcal{C}^{+}_{CQ-2}, .., \mathcal{C}^{+}_{CQ-\mathbf{m}}\right) \end{aligned}$$

By idempotency, we get

$CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}^{-}_{CQ-1}, \mathcal{C}^{-}_{CQ-2}, .., \mathcal{C}^{-}_{CQ-\mathbf{m}}\right) = \mathcal{C}^{-}_{CQ-j}$ and $CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}^{+}_{CQ-1}, \mathcal{C}^{+}_{CQ-2}, .., \mathcal{C}^{+}_{CQ-\mathbf{m}}\right) = \mathcal{C}^{+}_{CQ-j}$

Then

$$\mathcal{C}^{-}_{CQ-j} \leq CQROFWBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) \leq \mathcal{C}^{+}_{CQ-j}.$$

The proof of the above theorem has been completed.

Definition 9: For any CQROFN $\mathcal{C}_{CQ-j}, \mathbf{j = 1, 2, 3, .., m}$, we define the CQROFGBM operator by

$$\begin{aligned} & CQROFGBM^{s_{CQ}, t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) \\ & = \left(\frac{1}{s_{CQ} + t_{CQ}} \otimes_{\substack{j,k=1 \\ j \neq k}}^{\mathbf{m}} \left(s_{CQ}\mathcal{C}_{CQ-j} \oplus t_{CQ}\mathcal{C}_{CQ-k}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}} \end{aligned} \tag{9}$$

Based on the operational laws in Definition 4, we give the following result.

Theorem 9: *The aggregation result of the $CQROFGBM^{s_{CQ},t_{CQ}}$ operator is still a CQROFN such that*

$$CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right)$$

$$=\left(\begin{array}{l}\left(\left(1-\left(1-\left(\prod_{\substack{j,k=1\\j\neq k}}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}-\\ \left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}\times\right.\\ \left. e^{i2\Pi\left(1-\left(1-\left(\prod_{j,k=1,\,j\neq k}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}-\\ \left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-k}}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}\right),\\ \left(\left(1-\left(\prod_{\substack{j,k=1\\j\neq k}}^{\mathbf{m}}\left(1-\left(\xi^{s_{CQ}}_{\mathcal{C}_{RP-j}}\xi^{t_{CQ}}_{\mathcal{C}_{RP-k}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}} e^{i2\Pi\left(1-\left(\prod_{j,k=1,\,j\neq k}^{\mathbf{m}}\left(1-\left(\Psi^{s_{CQ}}_{\xi_{\mathcal{C}_{IP-j}}}\Psi^{t_{CQ}}_{\xi_{\mathcal{C}_{IP-k}}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(m-1)}}\right)^{\frac{1}{q_{CQ}(s_{CQ}+t_{CQ})}}}\right)\end{array}\right)$$

Proof: Straightforward.

Further, we explore some properties of the $CQROFGBM^{s_{CQ},t_{CQ}}$ *operator*, such as idempotency, monotonicity, and boundedness.

Theorem 10: *For any CQROFN $\mathcal{C}_{CQ-j}$, $\boldsymbol{j=1,2,3,..,}\mathbf{m}$, then*

$$CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)=\mathcal{C}_{CQ}.$$

Proof: Straightforward.

Theorem 11: *For any two CQROFNs* $\mathcal{C}_{CQ-j}=\left(\Phi_{\mathcal{C}_{RP-j}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-j}}}},\xi_{\mathcal{C}_{RP-j}}e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$ *and* $\mathcal{C}_{CQ-*k}=\left(\Phi_{\mathcal{C}_{RP-*k}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-*k}}}},\xi_{\mathcal{C}_{RP-*k}}e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-*k}}}}\right)$, $(j,k=1,2,..,m)$, *with conditions* $\Phi_{RP-j}\geq\Phi_{RP-*k}$, $\Psi_{\Phi_{IP-j}}\geq\Psi_{\Phi_{IP-*k}}$, $\xi_{RP-j}\leq\xi_{RP-*k}$ *and* $\Psi_{\xi_{IP-j}}\leq\Psi_{\xi_{IP-*k}}$, *then*

$$CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)\geq CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-*1},\mathcal{C}_{CQ-*2},..,\mathcal{C}_{CQ-*\mathbf{m}}\right)$$

Proof: Straightforward.

Theorem 12: *For any two CQROFNs* $\mathcal{C}^{+}_{CQ-j}=\left(\max_j\Phi_{\mathcal{C}_{RP-j}}e^{i2\Pi\max_j\Psi_{\Phi_{\mathcal{C}_{IP-j}}}},\min_j\xi_{\mathcal{C}_{RP-j}}e^{i2\Pi\min_j\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$ *and* $\mathcal{C}^{-}_{CQ-j}=\left(\min_j\Phi_{\mathcal{C}_{RP-j}}e^{i2\Pi\min_j\Psi_{\Phi_{\mathcal{C}_{IP-j}}}},\max_j\xi_{\mathcal{C}_{RP-j}}e^{i2\Pi\max_j\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$, $(j=1,2,..,m)$, *then*

$$\mathcal{C}^{-}_{CQ-j}\leq CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)\leq\mathcal{C}^{+}_{CQ-j}.$$

Proof: According to monotonicity, we get

$$CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{-}_{CQ-1},\mathcal{C}^{-}_{CQ-2},..,\mathcal{C}^{-}_{CQ-\mathbf{m}}\right)$$
$$\leq CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)$$
$$\leq CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{+}_{CQ-1},\mathcal{C}^{+}_{CQ-2},..,\mathcal{C}^{+}_{CQ-\mathbf{m}}\right)$$

By idempotency, we get

$CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{-}_{CQ-1},\mathcal{C}^{-}_{CQ-2},..,\mathcal{C}^{-}_{CQ-\mathbf{m}}\right)=\mathcal{C}^{-}_{CQ-j}$ and $CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{+}_{CQ-1},\mathcal{C}^{+}_{CQ-2},..,\mathcal{C}^{+}_{CQ-\mathbf{m}}\right)=\mathcal{C}^{+}_{CQ-j}$

Then

$$\mathcal{C}^{-}_{CQ-j} \le CQROFGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right) \le \mathcal{C}^{+}_{CQ-j}.$$

The proof of the above theorem has been completed.

Further, the special cases of the $CQROFGBM^{s_{CQ},t_{CQ}}$ *operator* are shown as

Remark 6: When $t_{CQ} = 0$ in Definition 9, then

$$CQROFWGBM^{s_{CQ},0}(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}})$$

$$= \left(\left(\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{s_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}}}\right)^{\frac{1}{q_{CQ}}} \times e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\right)^{s_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}, \left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\xi^{s_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}s_{CQ}}} \times e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\Psi^{s_{CQ}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}s_{CQ}}}}\right).$$

Remark 7: When $s_{CQ} = 1, t_{CQ} = 0$ in Definition 9, then

$$CQROFWGBM^{1,0}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right)$$

$$= \left(\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\Phi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{1}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{1}\right)^{\frac{1}{q_{CQ}}}}, \left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\xi^{1}_{\mathcal{C}_{RP-j}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\Psi^{1}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}}}}\right).$$

Remark 8: When $s_{CQ} = 0$ in Definition 9, then

$$CQROFGBM^{0,t_{CQ}}\left(\mathcal{C}_{CQ-1}, \mathcal{C}_{CQ-2}, .., \mathcal{C}_{CQ-\mathbf{m}}\right)$$

$$= \left(\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(1-\Phi^{q_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m\,(m-1)}}}\right)^{\frac{1}{t_{CQ}}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(1-\Psi^{q_{CQ}}_{\Phi_{\mathcal{C}_{IP-j}}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m\,(m-1)}}}\right)^{\frac{1}{t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}, \left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\xi^{t_{CQ}}_{\mathcal{C}_{RP-j}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m\,(m-1)}}}\right)^{\frac{1}{q_{CQ}t_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\Psi^{t_{CQ}}_{\xi_{\mathcal{C}_{IP-j}}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m\,(m-1)}}}\right)^{\frac{1}{q_{CQ}t_{CQ}}}}\right).$$

Remark 9: When $s_{CQ} = 0, t_{CQ} = 1$ in Definition 9, then

$$CQROFGBM^{0,1}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right) = \left(\begin{array}{c}\left(\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\Phi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{1}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{1}\right)^{\frac{1}{q_{CQ}}}}\right),\\ \left(\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\xi_{\mathcal{C}_{RP-j}}^{1}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{q_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\Psi_{\xi_{\mathcal{C}_{IP-j}}}^{1}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{q_{CQ}}}}\right)\end{array}\right).$$

Remark 10: When $s_{CQ} = t_{CQ} = 1$ in Definition 9, then

$$CQROFGBM^{1,1}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right) = \left(\begin{array}{c}\left(\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(\Phi_{\mathcal{C}_{RP-j}}^{q_{CQ}}+\Phi_{\mathcal{C}_{RP-k}}^{q_{CQ}}-\Phi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\Phi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{CQ}}} \times e^{i2\Pi\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}+\Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}-\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{2}}\right)^{\frac{1}{q_{CQ}}}}\right),\\ \left(\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\xi_{\mathcal{C}_{RP-j}}\xi_{\mathcal{C}_{RP-k}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{2q_{CQ}}} e^{i2\Pi\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(\Psi_{\xi_{\mathcal{C}_{IP-j}}}\Psi_{\xi_{\mathcal{C}_{IP-k}}}\right)^{q_{CQ}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}}\right)^{\frac{1}{2q_{CQ}}}}\right)\end{array}\right).$$

Further, we explore the CQROFWGBM operator. Suppose the weight vector is stated by $\omega_w = \left(\omega_{w-1},\omega_{w-2},..,\omega_{w-\mathbf{m}}\right)^T, \sum_{j=1}^{\mathbf{m}}\omega_{w-j} = 1$ and $\omega_{w-j}\in[0,1], \left(j=1,2,..,\mathbf{m}\right)$.

Definition 10: For any CQROFN $\mathcal{C}_{\mathbf{CQ-j}}, \mathbf{j}=\mathbf{1},\mathbf{2},\mathbf{3},..,\mathbf{m}$, we define the CQROFWGBM operator by

$$CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right) = \left(\frac{1}{s_{CQ}+t_{CQ}}\otimes_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(s_{CQ}\mathcal{C}_{CQ-j}^{\omega_{w-j}}\oplus t_{CQ}\mathcal{C}_{CQ-k}^{\omega_{w-k}}\right)\right)^{\frac{1}{\mathbf{m}(\mathbf{m}-1)}} \tag{10}$$

Based on the operational laws in Definition 4, we give the following result.

Theorem 13: *The aggregation result of $CQROFWGBM^{s_{CQ},t_{CQ}}$ operator is still a CQROFN such that*

$$
\begin{aligned}
&CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-m}\right)\\
&=\left(\begin{array}{l}
\left(\begin{array}{l}
\left(1-\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\Phi_{\mathcal{C}_{RP-j}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}-\left(1-\Phi_{\mathcal{C}_{RP-k}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}-\\ \left(1-\left(1-\Phi_{\mathcal{C}_{RP-j}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Phi_{\mathcal{C}_{RP-k}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}\times\\
e^{i2\Pi\left(1-\left(1-\left(\prod_{j,k=1,\,j\neq k}^{\mathbf{m}}\left(\begin{array}{c}2-\left(1-\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}-\left(1-\Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}-\\ \left(1-\left(1-\Psi_{\Phi_{\mathcal{C}_{IP-j}}}^{q_{CQ}\varpi_{w-j}}\right)^{s_{CQ}}\right)\left(1-\left(1-\Psi_{\Phi_{\mathcal{C}_{IP-k}}}^{q_{CQ}\varpi_{w-k}}\right)^{t_{CQ}}\right)\end{array}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{s_{CQ}+t_{CQ}}}\right)^{\frac{1}{q_{CQ}}}}
\end{array}\right),\\
\left(\begin{array}{l}
\left(1-\left(\prod_{\substack{j,k=1\\ j\neq k}}^{\mathbf{m}}\left(1-\left(1-\left(1-\xi_{\mathcal{C}_{RP-j}}^{q_{CQ}}\right)^{\varpi_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\xi_{\mathcal{C}_{RP-k}}^{q_{CQ}}\right)^{\varpi_{w-k}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}\left(s_{CQ}+t_{CQ}\right)}}\times\\
e^{i2\Pi\left(1-\left(\prod_{j,k=1,\,j\neq k}^{\mathbf{m}}\left(1-\left(1-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-j}}}^{q_{CQ}}\right)^{\varpi_{w-j}}\right)^{s_{CQ}}\left(1-\left(1-\Psi_{\xi_{\mathcal{C}_{IP-k}}}^{q_{CQ}}\right)^{\varpi_{w-k}}\right)^{t_{CQ}}\right)\right)^{\frac{1}{\mathbf{m(m-1)}}}\right)^{\frac{1}{q_{CQ}\left(s_{CQ}+t_{CQ}\right)}}}
\end{array}\right)
\end{array}\right).
\end{aligned}
$$

Proof: Straightforward.

Further, we explore some properties of $CQROFWGBM^{s_{CQ},t_{CQ}}$ operator, such as idempotency, monotonicity, and boundedness.

Theorem 14: *For any CQROFN $\mathcal{C}_{CQ-j}, \boldsymbol{j=1,2,3,..,\mathrm{m}}$, then*

$$CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)=\mathcal{C}_{CQ}.$$

Proof: Straightforward.

Theorem 15: *For any two CQROFNs* $\mathcal{C}_{CQ-j}=\left(\Phi_{\mathcal{C}_{RP-j}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-j}}}},\xi_{\mathcal{C}_{RP-j}}e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$ *and* $\mathcal{C}_{CQ-*k}=\left(\Phi_{\mathcal{C}_{RP-*k}}e^{i2\Pi\Psi_{\Phi_{\mathcal{C}_{IP-*k}}}},\xi_{\mathcal{C}_{RP-*k}}e^{i2\Pi\Psi_{\xi_{\mathcal{C}_{IP-*k}}}}\right),\left(j,k=1,2,..,\mathbf{m}\right)$, *with conditions* $\Phi_{RP-j}\geq\Phi_{RP-*k},\Psi_{\Phi_{IP-j}}\geq\Psi_{\Phi_{IP-*k}},\xi_{RP-j}\leq\xi_{RP-*k}$ *and* $\Psi_{\xi_{IP-j}}\leq\Psi_{\xi_{IP-*k}}$, *then*

$$
\begin{aligned}
&CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)\\
&\geq CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-*1},\mathcal{C}_{CQ-*2},..,\mathcal{C}_{CQ-*\mathbf{m}}\right).
\end{aligned}
$$

Proof: Straightforward.

Theorem 16: *For any two CQROFNs* $\mathcal{C}^{+}_{CQ-j}=\left(\max_J\Phi_{\mathcal{C}_{RP-j}}e^{i2\Pi\max_j\Psi_{\Phi_{\mathcal{C}_{IP-j}}}},\min_j\xi_{\mathcal{C}_{RP-j}}e^{i2\Pi\min_j\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right)$ *and* $\mathcal{C}^{-}_{CQ-j}=\left(\min_J\Phi_{\mathcal{C}_{RP-j}}e^{i2\Pi\min_j\Psi_{\Phi_{\mathcal{C}_{IP-j}}}},\max_j\xi_{\mathcal{C}_{RP-j}}e^{i2\Pi\max_j\Psi_{\xi_{\mathcal{C}_{IP-j}}}}\right),\left(j=1,2,..,\mathbf{m}\right)$, *then*

$$\mathcal{C}^{-}_{CQ-j}\leq CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)\leq\mathcal{C}^{+}_{CQ-j}$$

Proof: Based on monotonicity, we get

$$CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{-}_{CQ-1},\mathcal{C}^{-}_{CQ-2},..,\mathcal{C}^{-}_{CQ-\mathbf{m}}\right)$$
$$\leq CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)$$
$$\leq CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{+}_{CQ-1},\mathcal{C}^{+}_{CQ-2},..,\mathcal{C}^{+}_{CQ-\mathbf{m}}\right)$$

By idempotency, we get

$CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{-}_{CQ-1},\mathcal{C}^{-}_{CQ-2},..,\mathcal{C}^{-}_{CQ-\mathbf{m}}\right)=\mathcal{C}^{-}_{CQ-j}$ and $CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{+}_{CQ-1},\mathcal{C}^{+}_{CQ-2},..,\mathcal{C}^{+}_{CQ-\mathbf{m}}\right)=\mathcal{C}^{+}_{CQ-j}$

Then

$$\mathcal{C}^{-}_{CQ-j}\leq CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}_{CQ-1},\mathcal{C}_{CQ-2},..,\mathcal{C}_{CQ-\mathbf{m}}\right)\leq\mathcal{C}^{+}_{CQ-j}.$$

The proof of the above theorem has been completed.

4. MULTI-ATTRIBUTE GROUP DECISION MAGDM METHOD BASED ON ESTABLISHED OPERATORS

The purpose of this section is to utilize the established operators to solve the MAGDM problems.

4.1. Description of MAGDM Problems

The purpose of the MAGDM Problems is to select the best one from the family of alternatives. Suppose $\mathfrak{D}=\{\mathfrak{D}_1,\mathfrak{D}_2,..,\mathfrak{D}_t\}$, $\mathrm{Ų}=\left\{\mathrm{Ų}_1,\mathrm{Ų}_2,..,\mathrm{Ų}_{\mathbf{m}}\right\}$ and $\mathcal{A}=\{\mathcal{A}_1,\mathcal{A}_2,..,\mathcal{A}_n\}$ respectively represent the families of DMs, alternatives and their attributes. Moreover, we use the CQROFN $\mathcal{C}^{p}_{CQ-jk}=\left(\Phi^{p}_{\mathcal{C}_{RP-jk}}e^{i2\Pi\Psi^{p}_{\Phi_{\mathcal{C}_{IP-jk}}}},\xi^{p}_{\mathcal{C}_{RP-jk}}e^{i2\Pi\Psi^{p}_{\xi_{\mathcal{C}_{IP-jk}}}}\right)$ to express the evaluation value of the alternative $\mathrm{Ų}_j$ under the attribute $\mathcal{A}^p$ given by the DM $\mathfrak{D}_k$, then get the matrices $\mathcal{A}^p=\left[\mathcal{C}^{p}_{jk}\right]_{\mathbf{m}\times n}$. The weight vector of experts is $\mho_{w-j}=\left(\mho_{w-1},\mho_{w-2},..,\mho_{w-t}\right)^T,\sum_{j=1}^{\mathbf{t}}\mho_{w-j}=1$ and $\mho_{w-i}\in[0,1],\left(j=1,2,..,\mathbf{t}\right)$ and the weight vector of attributes is $\omega_{w-j}=\left(\omega_{w-1},\omega_{w-2},..,\omega_{w-\mathbf{n}}\right)^T,\sum_{j=1}^{\mathbf{n}}\omega_{w-j}=1$ and $\omega_{w-j}\in[0,1],\left(j=1,2,..,\mathbf{n}\right)$. Based on the above data, the steps of the algorithm are stated by

4.2. Procedure of the Algorithm

1. Based on Subsection 4.1, we give the decision matrix.

$$r^{p}_{jk}=\left(\mathcal{C}^{p}_{CQ-jk},\mathcal{C}'^{\,p}_{CQ-jk}\right)\quad(11)$$
$$=\begin{cases}\left(\Phi^{p}_{\mathcal{C}_{RP-jk}}e^{i2\Pi\Psi^{p}_{\Phi_{\mathcal{C}_{IP-jk}}}},\xi^{p}_{\mathcal{C}_{RP-jk}}e^{i2\Pi\Psi^{p}_{\xi_{\mathcal{C}_{IP-jk}}}}\right) & \textit{for benefit}\\ \left(\xi^{p}_{\mathcal{C}_{RP-jk}}e^{i2\Pi\Psi^{p}_{\xi_{\mathcal{C}_{IP-jk}}}},\Phi^{p}_{\mathcal{C}_{RP-jk}}e^{i2\Pi\Psi^{p}_{\Phi_{\mathcal{C}_{IP-jk}}}}\right) & \textit{for cost}\end{cases}$$

2. Based on Eq. (12), we can obtain the comprehensive value of each alternative from each DM

$$r^{p}_{j}=\left(\mathcal{C}^{p}_{jk},\mathcal{C}'^{\,p}_{jk}\right)=CQROFWBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{p}_{CQ-j1},\mathcal{C}^{p}_{CQ-j2},..,\mathcal{C}^{p}_{CQ-jn}\right)\quad(12)$$

3. Based on Eq. (13), we can get the comprehensive value of each alternative.

$$r^{p}_{j}=\left(\mathcal{C}^{p}_{j,k},\mathcal{C}'^{\,p}_{j,k}\right)=CQROFWGBM^{s_{CQ},t_{CQ}}\left(\mathcal{C}^{p}_{CQ-j1},\mathcal{C}^{p}_{CQ-j2},..,\mathcal{C}^{p}_{CQ-jn}\right)\quad(13)$$

4. Based on score function, we calculate the score functions of above aggregated values.

5. Rank the score values and examine the best one.
6. The end.

For more clarity, we make flowchart for the above algorithm which is shown in Figure 3.

4.3. Illustrated Numerical Examples

The purpose of this section is to show the reliability and proficiency of the proposed method by some numerical examples.

Example 1: To examine the feasibility and validity of the explored method in this manuscript, we use an investment problem to explain it. In order to select one suitable investment alternative from five companies $Ų = \left\{Ų_1, Ų_2, .., Ų_5\right\}$ which are explained as follows:

$Ų_1$ is a car company

$Ų_2$ is a laptop company

$Ų_3$ is a mobile company

$Ų_4$ is a food company

$Ų_5$ is a furniture company

Further, these companies are evaluated by four attributes $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, .., \mathcal{A}_4\}$, which are explained in Table 1, and three experts $\mathfrak{D} = \{\mathfrak{D}_1, \mathfrak{D}_2, \mathfrak{D}_3\}$ give the evaluation information stated in Tables 2–4. Moreover, the weight vector of experts is $Ʊ_{w-3} = (0.5, 0.35, 0.15)^T$ and the weight vector of attributes is $Ꞷ_{w-4} = (0.35, 0.22, 0.29, 0.14)^T$. The goal is to give a best choice for investment.

For solving this kind of decision problems, the presented approach is better than existing approaches based on the structure of the CQROFS. The CQROFS meets a condition that the sum of q-powers of the real parts (also for imaginary parts) of the truth and falsity grades is not exceeded form unit interval, and it is more general than QROFS, PFS, CPFS, IFS, CIFS, and etc. Because the BM operators are more generalized than various existing operators like weighted averaging, weighted geometric based on some existing notion like QROFS, PFS, CPFS, IFS, CIFS, and etc. Keeping the advantages of the BM operator based on CQROFS, we solve this problem to check the reliability and effectiveness of the explored method.

The decision procedure is shown as follows:

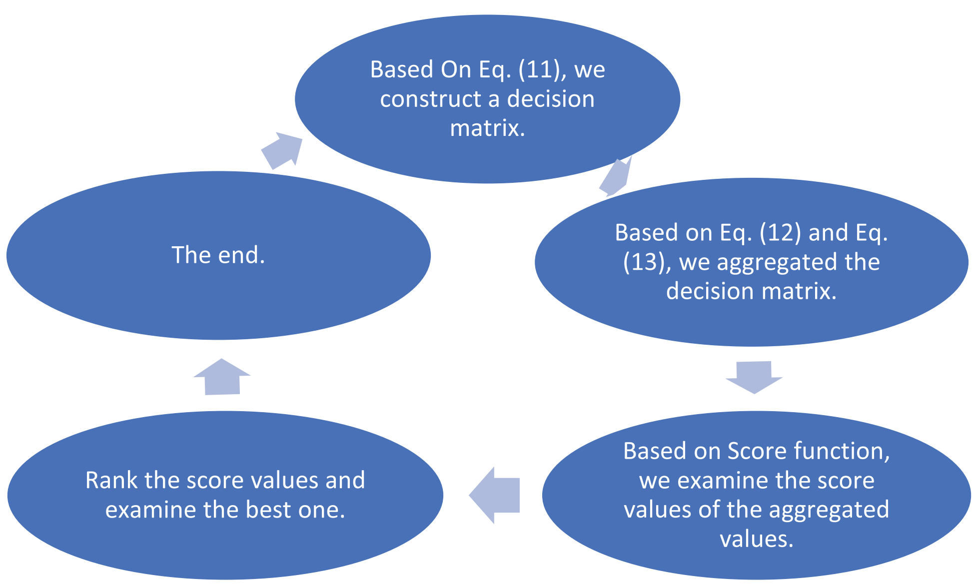

Figure 3 | Graphical interpretation for the procedure of the algorithm of 4.2.

Table 1 | Information about attributes and their representations.

$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$
Risk analysis	Growth analysis	Social-political impact analysis	Environmental impact analysis

Table 2 Complex q-rung orthopair fuzzy decision matrix Z^1 given by $\mathfrak{D}_1$.

Data Representation	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$
Ų$_1$	$\left((0.6)e^{i2\Pi(0.6)}, (0.5)e^{i2\Pi(0.54)}\right)$	$\left((0.67)e^{i2\Pi(0.76)}, (0.78)e^{i2\Pi(0.8)}\right)$	$\left((0.76)e^{i2\Pi(0.87)}, (0.83)e^{i2\Pi(0.74)}\right)$	$\left((0.58)e^{i2\Pi(0.89)}, (0.87)e^{i2\Pi(0.77)}\right)$
Ų$_2$	$\left((0.8)e^{i2\Pi(0.67)}, (0.77)e^{i2\Pi(0.85)}\right)$	$\left((0.7)e^{i2\Pi(0.67)}, (0.81)e^{i2\Pi(0.85)}\right)$	$\left((0.67)e^{i2\Pi(0.55)}, (0.8)e^{i2\Pi(0.9)}\right)$	$\left((0.67)e^{i2\Pi(0.56)}, (0.8)e^{i2\Pi(0.87)}\right)$
Ų$_3$	$\left((0.86)e^{i2\Pi(0.78)}, (0.7)e^{i2\Pi(0.65)}\right)$	$\left((0.73)e^{i2\Pi(0.68)}, (0.82)e^{i2\Pi(0.86)}\right)$	$\left((0.68)e^{i2\Pi(0.66)}, (0.78)e^{i2\Pi(0.89)}\right)$	$\left((0.56)e^{i2\Pi(0.45)}, (0.88)e^{i2\Pi(0.9)}\right)$
Ų$_4$	$\left((0.82)e^{i2\Pi(0.76)}, (0.72)e^{i2\Pi(0.86)}\right)$	$\left((0.6)e^{i2\Pi(0.69)}, (0.9)e^{i2\Pi(0.87)}\right)$	$\left((0.57)e^{i2\Pi(0.78)}, (0.8)e^{i2\Pi(0.79)}\right)$	$\left((0.77)e^{i2\Pi(0.55)}, (0.89)e^{i2\Pi(0.91)}\right)$
Ų$_5$	$\left((0.86)e^{i2\Pi(0.72)}, (0.73)e^{i2\Pi(0.8)}\right)$	$\left((0.67)e^{i2\Pi(0.7)}, (0.84)e^{i2\Pi(0.88)}\right)$	$\left((0.59)e^{i2\Pi(0.54)}, (0.88)e^{i2\Pi(0.87)}\right)$	$\left((0.6)e^{i2\Pi(0.56)}, (0.93)e^{i2\Pi(0.92)}\right)$

Table 3 Complex q-rung orthopair fuzzy decision matrix Z^2 given by D_2.

Data Representation	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$
Ų$_1$	$\left((0.78)e^{i2\Pi(0.76)}, (0.67)e^{i2\Pi(0.54)}\right)$	$\left((0.76)e^{i2\Pi(0.87)}, (0.83)e^{i2\Pi(0.74)}\right)$	$\left((0.76)e^{i2\Pi(0.89)}, (0.87)e^{i2\Pi(0.77)}\right)$	$\left((0.87)e^{i2\Pi(0.83)}, (0.74)e^{i2\Pi(0.76)}\right)$
Ų$_2$	$\left((0.81)e^{i2\Pi(0.67)}, (0.7)e^{i2\Pi(0.85)}\right)$	$\left((0.67)e^{i2\Pi(0.55)}, (0.8)e^{i2\Pi(0.9)}\right)$	$\left((0.67)e^{i2\Pi(0.56)}, (0.8)e^{i2\Pi(0.87)}\right)$	$\left((0.55)e^{i2\Pi(0.8)}, (0.9)e^{i2\Pi(0.67)}\right)$
Ų$_3$	$\left((0.82)e^{i2\Pi(0.68)}, (0.73)e^{i2\Pi(0.65)}\right)$	$\left((0.68)e^{i2\Pi(0.66)}, (0.78)e^{i2\Pi(0.89)}\right)$	$\left((0.68)e^{i2\Pi(0.45)}, (0.88)e^{i2\Pi(0.9)}\right)$	$\left((0.66)e^{i2\Pi(0.78)}, (0.89)e^{i2\Pi(0.68)}\right)$
Ų$_4$	$\left((0.9)e^{i2\Pi(0.69)}, (0.6)e^{i2\Pi(0.86)}\right)$	$\left((0.57)e^{i2\Pi(0.78)}, (0.8)e^{i2\Pi(0.79)}\right)$	$\left((0.57)e^{i2\Pi(0.55)}, (0.89)e^{i2\Pi(0.91)}\right)$	$\left((0.78)e^{i2\Pi(0.8)}, (0.79)e^{i2\Pi(0.57)}\right)$
Ų$_5$	$\left((0.84)e^{i2\Pi(0.7)}, (0.67)e^{i2\Pi(0.8)}\right)$	$\left((0.59)e^{i2\Pi(0.54)}, (0.88)e^{i2\Pi(0.87)}\right)$	$\left((0.59)e^{i2\Pi(0.56)}, (0.93)e^{i2\Pi(0.92)}\right)$	$\left((0.54)e^{i2\Pi(0.88)}, (0.87)e^{i2\Pi(0.59)}\right)$

Table 4 Complex q-rung orthopair fuzzy decision matrix Z^3 given by D_3.

Data Representation	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$
Ų$_1$	$\left((0.87)e^{i2\Pi(0.76)}, (0.54)e^{i2\Pi(0.83)}\right)$	$\left((0.5)e^{i2\Pi(0.6)}, (0.6)e^{i2\Pi(0.77)}\right)$	$\left((0.87)e^{i2\Pi(0.83)}, (0.8)e^{i2\Pi(0.76)}\right)$	$\left((0.87)e^{i2\Pi(0.78)}, (0.74)e^{i2\Pi(0.87)}\right)$
Ų$_2$	$\left((0.85)e^{i2\Pi(0.67)}, (0.55)e^{i2\Pi(0.8)}\right)$	$\left((0.8)e^{i2\Pi(0.67)}, (0.77)e^{i2\Pi(0.87)}\right)$	$\left((0.55)e^{i2\Pi(0.8)}, (0.85)e^{i2\Pi(0.67)}\right)$	$\left((0.55)e^{i2\Pi(0.8)}, (0.9)e^{i2\Pi(0.8)}\right)$
Ų$_3$	$\left((0.65)e^{i2\Pi(0.68)}, (0.64)e^{i2\Pi(0.78)}\right)$	$\left((0.7)e^{i2\Pi(0.78)}, (0.86)e^{i2\Pi(0.67)}\right)$	$\left((0.66)e^{i2\Pi(0.78)}, (0.86)e^{i2\Pi(0.68)}\right)$	$\left((0.66)e^{i2\Pi(0.78)}, (0.89)e^{i2\Pi(0.88)}\right)$
Ų$_4$	$\left((0.86)e^{i2\Pi(0.57)}, (0.78)e^{i2\Pi(0.8)}\right)$	$\left((0.72)e^{i2\Pi(0.76)}, (0.82)e^{i2\Pi(0.89)}\right)$	$\left((0.78)e^{i2\Pi(0.8)}, (0.87)e^{i2\Pi(0.57)}\right)$	$\left((0.78)e^{i2\Pi(0.7)}, (0.79)e^{i2\Pi(0.89)}\right)$
Ų$_5$	$\left((0.8)e^{i2\Pi(0.59)}, (0.54)e^{i2\Pi(0.88)}\right)$	$\left((0.73)e^{i2\Pi(0.72)}, (0.86)e^{i2\Pi(0.92)}\right)$	$\left((0.54)e^{i2\Pi(0.88)}, (0.88)e^{i2\Pi(0.59)}\right)$	$\left((0.54)e^{i2\Pi(0.56)}, (0.87)e^{i2\Pi(0.89)}\right)$

1. Based on Eq. (11), we get the normalized decision matrix. The measured information is same, which is not necessary to require the normalization.
2. Based on Eq. (12), we obtain the comprehensive value of each alternative from each DM (*suppose* $q_{CQ} = 4, s_{CQ} = t_{CQ} = 1$)

   $r_1^1 = \left((0.05)\, e^{i2\Pi(0.12)}, (0.85)\, e^{i2\Pi(0.76)}\right)$. $r_2^1 = \left((0.07)\, e^{i2\Pi(0.04)}, (0.88)\, e^{i2\Pi(0.87)}\right)$.

   $r_3^1 = \left((0.08)\, e^{i2\Pi(0.06)}, (0.88)\, e^{i2\Pi(0.86)}\right)$. $r_4^1 = \left((0.07)\, e^{i2\Pi(0.08)}, (0.90)\, e^{i2\Pi(0.87)}\right)$.

   $r_5^1 = \left((0.07)\, e^{i2\Pi(0.05)}, (0.91)\, e^{i2\Pi(0.88)}\right)$. $r_1^2 = \left((0.11)\, e^{i2\Pi(0.16)}, (0.88)\, e^{i2\Pi(0.75)}\right)$.

   $r_2^2 = \left((0.07)\, e^{i2\Pi(0.05)}, (0.88)\, e^{i2\Pi(0.80)}\right)$. $r_3^2 = \left((0.08)\, e^{i2\Pi(0.05)}, (0.90)\, e^{i2\Pi(0.79)}\right)$.

   $r_4^2 = \left((0.08)\, e^{i2\Pi(0.07)}, (0.87)\, e^{i2\Pi(0.78)}\right)$. $r_5^2 = \left((0.06)\, e^{i2\Pi(0.06)}, (0.92)\, e^{i2\Pi(0.79)}\right)$.

   $r_1^3 = \left((0.14)\, e^{i2\Pi(0.09)}, (0.80)\, e^{i2\Pi(0.84)}\right)$. $r_2^3 = \left((0.08)\, e^{i2\Pi(0.08)}, (0.87)\, e^{i2\Pi(0.81)}\right)$.

   $r_3^3 = \left((0.05)\, e^{i2\Pi(0.09)}, (0.90)\, e^{i2\Pi(0.84)}\right)$. $r_4^3 = \left((0.12)\, e^{i2\Pi(0.07)}, (0.90)\, e^{i2\Pi(0.84)}\right)$.

   $r_5^3 = \left((0.06)\, e^{i2\Pi(0.08)}, (0.89)\, e^{i2\Pi(0.86)}\right)$.
3. Based on Eq. (13), we get the comprehensive value of each alternative ($q_{CQ} = 4, s_{CQ} = t_{CQ} = 1$).

   $r_1 = \left((0.14)\, e^{i2\Pi(0.15)}, (0.21)\, e^{i2\Pi(0.13)}\right)$. $r_2 = \left((0.09)\, e^{i2\Pi(0.08)}, (0.25)\, e^{i2\Pi(0.18)}\right)$.

   $r_3 = \left((0.08)\, e^{i2\Pi(0.08)}, (0.26)\, e^{i2\Pi(0.17)}\right)$. $r_4 = \left((0.12)\, e^{i2\Pi(0.09)}, (0.26)\, e^{i2\Pi(0.19)}\right)$.

   $r_5 = \left((0.08)\, e^{i2\Pi(0.08)}, (0.30)\, e^{i2\Pi(0.20)}\right)$.
4. Based on score function, we calculate the score functions of above aggregated values.

   $S(r_1) = -0.00064, S(r_2) = -0.002333, S(r_3) = -0.00287, S(r_4) = -0.00293, S(r_5) = -0.00465$.
5. Rank the score values and examine the best one company for investment.

   $Ụ_1 \geq Ụ_2 \geq Ụ_3 \geq Ụ_4 \geq Ụ_5$
6. Consequently, $Ụ_1$ is the best one in the above five companies, which is car company.
7. End.

Now we can compare the established method with existing methods in expressing the different fuzzy information, and the results are shown in Table 5.

4.4. Influence on Decision Results for the Different Parameters

The parameters **in the developed operators** play a key role in the final ranking results. In order to show their influence on decision results, the ranking results for the different parameters are shown in the Tables 6–8.

From Tables 6 and 7, we can know these ranking results are changed for the different values of parameters. However, the best one is still $Ụ_1$.

Table 5 Comparison method between the proposed and existing methods.

Methods	Score Function	Ranking	Best Alternatives
Garg and Rani [33]	*Cannot be calculated*	*Cannot be calculated*	*No*
Rani and Garg [34]	*Cannot be calculated*	*Cannot be calculated*	*No*
CPYFS for $q_{CQ} = 2$ in this article	*Cannot be calculated*	*Cannot be calculated*	*No*
Cq-ROFS proposed in this article	$S(r_1) = -0.00064, S(r_2) = -0.002333, S(r_3) = -0.00287, S(r_4) = -0.00293, S(r_5) = -0.00465.$	$Ụ_1 \geq Ụ_2 \geq Ụ_3 \geq Ụ_4 \geq Ụ_5$	$Ụ_1$

From Table 8, it is shown the developed operators based on CQROFS is more general then existing notions due to its constraint, i.e., the sum of q-powers of the real part (also for imaginary part) of the truth and the falsity grades is not exceed from unit interval.

4.5. Comparison of the Established Operators with Some Existing Operators

The explored operators based on CQROFS in this paper is more general than some existing operators due to its constraint, i.e., the sum of q-powers of the real part (also for imaginary part) of the truth and the falsity grades is not exceed from unit interval. Based on comparison between the established method with existing ones, we examine the advantages and superiority of the explored work which is shown in Table 9.

From Table 9, it is clear that the existing operators in [32] are not able to evaluate our considered kinds of information in the form of two-dimension in a single set, and the established operators in this paper are more valuable than existing operators.

To moreover examine the superiority of the explored approach in the MADM environment, we solve a numerical example based on established operator and also for existing operators to show the effectiveness of the explored work. The existing methods were established by Garg and Rani [33], Rani and Garg [34], and Liu *et al.* [30,31] with different kinds of aggregation operators established for CIFSs and CQROFSs.

Table 6 Ranking values for constant parameter $t = 1$ and variable parameter s.

Parameters	Score Values	Ranking
$s_{CQ} = t_{CQ} = 1$	$S(r_1) = 0.177, S(r_2) = 0.114, S(r_3) = 0.110, S(r_4) = 0.126, S(r_5) = 0.095.$	$Ų_1 \geq Ų_2 \geq Ų_4 \geq Ų_3 \geq Ų_5$
$s_{CQ} = 2$, $t_{CQ} = 1$	$S(r_1) = 0.223, S(r_2) = 0.171, S(r_3) = 0.174, S(r_4) = 0.186, S(r_5) = 0.163.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_2 \geq Ų_5$
$s_{CQ} = 5$, $t_{CQ} = 1$	$S(r_1) = 0.186, S(r_2) = 0.147, S(r_3) = 0.152, S(r_4) = 0.161, S(r_5) = 0.145.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_1 \geq Ų_5$
$s_{CQ} = 10$, $t_{CQ} = 1$	$S(r_1) = 0.143, S(r_2) = 0.117, S(r_3) = 0.123, S(r_4) = 0.128, S(r_5) = 0.116.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_2 \geq Ų_5$
$s_{CQ} = 15$, $t_{CQ} = 1$	$S(r_1) = 0.118, S(r_2) = 0.099, S(r_3) = 0.105, S(r_4) = 0.109, S(r_5) = 0.098.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_2 \geq Ų_5$
$s_{CQ} = 20$, $t_{CQ} = 1$	$S(r_1) = 0.100, S(r_2) = 0.088, S(r_3) = 0.093, S(r_4) = 0.095, S(r_5) = 0.087.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_2 \geq Ų_5$

Table 7 Ranking values for constant parameter $s = 1$ and variable parameter t.

Parameters	Score Values	Ranking
$s_{CQ} = t_{CQ} = 1$	$S(r_1) = 0.177, S(r_2) = 0.114, S(r_3) = 0.110, S(r_4) = 0.126, S(r_5) = 0.095.$	$Ų_1 \geq Ų_2 \geq Ų_4 \geq Ų_3 \geq Ų_5$
$s_{CQ} = 1$, $t_{CQ} = 2$	$S(r_1) = 0.226, S(r_2) = 0.173, S(r_3) = 0.176, S(r_4) = 0.189, S(r_5) = 0.167.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_2 \geq Ų_5$
$s_{CQ} = 1$, $t_{CQ} = 5$	$S(r_1) = 0.196, S(r_2) = 0.147, S(r_3) = 0.154, S(r_4) = 0.164, S(r_5) = 0.145.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_1 \geq Ų_5$
$s_{CQ} = 1$, $t_{CQ} = 10$	$S(r_1) = 0.166, S(r_2) = 0.126, S(r_3) = 0.133, S(r_4) = 0.140, S(r_5) = 0.124.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_2 \geq Ų_5$
$s_{CQ} = 1$, $t_{CQ} = 15$	$S(r_1) = 0.146, S(r_2) = 0.113, S(r_3) = 0.121, S(r_4) = 0.126, S(r_5) = 0.112.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_2 \geq Ų_5$
$s_{CQ} = 1$, $t_{CQ} = 20$	$S(r_1) = 0.133, S(r_2) = 0.104, S(r_3) = 0.112, S(r_4) = 0.117, S(r_5) = 0.104.$	$Ų_1 \geq Ų_4 \geq Ų_3 \geq Ų_2 \geq Ų_5$

Table 8 | Ranking values for parameter q.

Parameters	Score Values	Ranking
$q_{CQ} = 3$	$S(r_1) = -0.014, S(r_2) = -0.019, S(r_3) = -0.022, S(r_4) = -0.021, S(r_5) = -0.026.$	$Ṷ_1 \geq Ṷ_2 \geq Ṷ_4 \geq Ṷ_3 \geq Ṷ_5$
$q_{CQ} = 5$	$S(r_1) = -0.0046, S(r_2) = -0.0073, S(r_3) = -0.0089, S(r_4) = -0.0088, S(r_5) = -0.012.$	$Ṷ_1 \geq Ṷ_2 \geq Ṷ_4 \geq Ṷ_3 \geq Ṷ_5$
$q_{CQ} = 8$	$S(r_1) = -0.0013, S(r_2) = -0.0023, S(r_3) = -0.00302, S(r_4) = -0.003, S(r_5) = -0.004.$	$Ṷ_1 \geq Ṷ_2 \geq Ṷ_4 \geq Ṷ_3 \geq Ṷ_5$
$q_{CQ} = 10$	$S(r_1) = -0.0005, S(r_2) = -0.0012, S(r_3) = -0.0016, S(r_4) = -0.0016, S(r_5) = -0.025.$	$Ṷ_1 \geq Ṷ_2 \geq Ṷ_4 \geq Ṷ_3 \geq Ṷ_5$
$q_{CQ} = 15$	$S(r_1) = -0.0000000098, S(r_2) = -0.00026, S(r_3) = -0.00039, S(r_4) = -0.00039, S(r_5) = -0.00072.$	$Ṷ_1 \geq Ṷ_2 \geq Ṷ_4 \geq Ṷ_3 \geq Ṷ_5$

Table 9 | Characteristic comparison between the proposed method and existing methods.

Aggregation Operators	Operator Capture the Interrelation between the Cq-ROFNs	A Parameter Vector Exists to Manipulate the Ranking Results	Contain Two-Dimension Information
q-ROFWA [35]	*No*	*No*	*No*
q-ROFWG [35]	*No*	*No*	*No*
q-ROFHM [36]	*Yes*	*Yes*	*No*
q-ROFWHM [36]	*Yes*	*Yes*	*No*
q-ROFBM [32]	*Yes*	*Yes*	*No*
q-ROFWBM [32]	*Yes*	*Yes*	*No*
q-ROFGBM [32]	*Yes*	*Yes*	*No*
q-ROFWGBM [32]	*Yes*	*Yes*	*No*
Cq-ROFBM	*Yes*	*Yes*	*Yes*
Cq-ROFWBM	*Yes*	*Yes*	*Yes*
Cq-ROFGBM	*Yes*	*Yes*	*Yes*
Cq-ROFWGBM	*Yes*	*Yes*	*Yes*

Note: q-ROFWA, q-rung orthopair weighted averaging; q-ROFWG, q-rung orthopair fuzzy weighted geometric; q-ROFHM, q-rung orthopair fuzzy Heronian mean; q-ROFWHM, q-rung orthopair fuzzy weighted Heronian mean; q-ROFBM, q-rung orthopair fuzzy Bonferroni mean; q-ROFWBM, q-rung orthopair fuzzy weighted Bonferroni mean; q-ROFGBM, q-rung orthopair fuzzy geometric Bonferroni mean; q-ROFWGBM, q-rung orthopair fuzzy weighted geometric Bonferroni mean; Cq-ROFBM, complex q-rung orthopair fuzzy Bonferroni mean; Cq-ROFWBM, complex q-rung orthopair fuzzy weighted Bonferroni mean; Cq-ROFGBM, complex q-rung orthopair fuzzy geometric Bonferroni mean; Cq-ROFWGBM, complex q-rung orthopair fuzzy weighted geometric Bonferroni mean.

Example 2: The information related to this example is given in Example 1. We consider complex pythagorean kinds of information and evaluated the validity and reliability of the established operators in this manuscript, we solve a numerical example whose information is shown in Table 10 and the weight vector of the attributes is $Ꞷ_{w-4} = (0.35, 0.22, 0.29, 0.14)^T$.

The evaluated results are listed in Table 11.

From Table 11, we can see that the proposed method is better than the existing ones in expressing the fuzzy information.

Example 3: The information related to this example is given in Example 1. We consider complex intuitionistic kinds of information and evaluated the validity and reliability of the established operators in this manuscript, we solve a numerical example whose information is shown in Table 12 and the weight vector of the attributes is $Ꞷ_{w-4} = (0.35, 0.22, 0.29, 0.14)^T$.

The evaluated results are listed in Table 13.

From Table 13, it is clear that the all existing operators in [32] are able to evaluate our considered kinds of information for $q_{CQ} = 1$, and they are a special case of the proposed operators.

To give a large space for expressing the fuzzy information and to consider the relationship between attributes, we established some BM operators using CQROFSs. It is clear that the CIFS and CPYFS are a special case of the established CQROFSs. When we set $q_{CQ} = 1$, then the CQROFS is reduced to CIFS, and similarly when we set $q_{CQ} = 2$, then it is reduced to CPYFS. Hence the established operators based on CQROFS are more powerful and more efficient than some existing operators due to its condition and its parameters.

Table 10 | Complex pythagorean fuzzy decision matrix for Example 2.

Data Representation	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$
$Ʉ_1$	$\left((0.6)\,e^{i2\Pi(0.6)},\ (0.5)e^{i2\Pi(0.54)}\right)$	$\left((0.67)\,e^{i2\Pi(0.24)},\ (0.28)e^{i2\Pi(0.22)}\right)$	$\left((0.6)\,e^{i2\Pi(0.5)},\ (0.43)e^{i2\Pi(0.24)}\right)$	$\left((0.58)\,e^{i2\Pi(0.5)},\ (0.47)e^{i2\Pi(0.17)}\right)$
$Ʉ_2$	$\left((0.4)\,e^{i2\Pi(0.67)},\ (0.6)e^{i2\Pi(0.3)}\right)$	$\left((0.7)\,e^{i2\Pi(0.33)},\ (0.31)e^{i2\Pi(0.21)}\right)$	$\left((0.67)\,e^{i2\Pi(0.55)},\ (0.28)e^{i2\Pi(0.25)}\right)$	$\left((0.67)\,e^{i2\Pi(0.5)},\ (0.38)e^{i2\Pi(0.07)}\right)$
$Ʉ_3$	$\left((0.86)\,e^{i2\Pi(0.3)},\ (0.24)e^{i2\Pi(0.4)}\right)$	$\left((0.73)\,e^{i2\Pi(0.23)},\ (0.32)e^{i2\Pi(0.23)}\right)$	$\left((0.68)\,e^{i2\Pi(0.5)},\ (0.28)e^{i2\Pi(0.3)}\right)$	$\left((0.56)\,e^{i2\Pi(0.45)},\ (0.37)e^{i2\Pi(0.19)}\right)$
$Ʉ_4$	$\left((0.8)\,e^{i2\Pi(0.3)},\ (0.22)e^{i2\Pi(0.24)}\right)$	$\left((0.6)\,e^{i2\Pi(0.6)},\ (0.5)e^{i2\Pi(0.11)}\right)$	$\left((0.57)\,e^{i2\Pi(0.5)},\ (0.5)e^{i2\Pi(0.21)}\right)$	$\left((0.77)\,e^{i2\Pi(0.25)},\ (0.29)e^{i2\Pi(0.11)}\right)$
$Ʉ_5$	$\left((0.86)\,e^{i2\Pi(0.22)},\ (0.13)e^{i2\Pi(0.24)}\right)$	$\left((0.67)\,e^{i2\Pi(0.5)},\ (0.34)e^{i2\Pi(0.19)}\right)$	$\left((0.59)\,e^{i2\Pi(0.5)},\ (0.51)e^{i2\Pi(0.2)}\right)$	$\left((0.6)\,e^{i2\Pi(0.5)},\ (0.43)e^{i2\Pi(0.12)}\right)$

Table 11 | Comparison methods between the proposed and existing methods from Example 2.

Methods	Score Function	Ranking
Garg and Rani [33]	*Cannot be calculated*	*Cannot be calculated*
Rani and Garg [34]	*Cannot be calculated*	*Cannot be calculated*
Cq-ROFBM proposed in this article $q_{CQ} = 2$	$S(r_1) = -0.416$, $S(r_2) = -0.375$, $S(r_3) = -0.351$, $S(r_4) = -0.342$, $S(r_5) = -0.337$.	$Ʉ_5 \geq Ʉ_4 \geq Ʉ_3 \geq Ʉ_2 \geq Ʉ_1$
Cq-ROFBM proposed in this article $q_{CQ} = 3$	$S(r_1) = -0.193$, $S(r_2) = -0.158$, $S(r_3) = -0.130$, $S(r_4) = -0.139$, $S(r_5) = -0.130$.	$Ʉ_5 \geq Ʉ_3 \geq Ʉ_4 \geq Ʉ_2 \geq Ʉ_1$

Table 12 | Complex intuitionistic fuzzy decision matrix for Example 3.

Data Representation	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$
$Ʉ_1$	$\left((0.4)\,e^{i2\Pi(0.3)},\ (0.5)e^{i2\Pi(0.54)}\right)$	$\left((0.7)\,e^{i2\Pi(0.24)},\ (0.28)e^{i2\Pi(0.22)}\right)$	$\left((0.36)\,e^{i2\Pi(0.5)},\ (0.43)e^{i2\Pi(0.24)}\right)$	$\left((0.58)\,e^{i2\Pi(0.5)},\ (0.27)e^{i2\Pi(0.17)}\right)$
$Ʉ_2$	$\left((0.4)\,e^{i2\Pi(0.6)},\ (0.6)e^{i2\Pi(0.3)}\right)$	$\left((0.57)\,e^{i2\Pi(0.33)},\ (0.31)e^{i2\Pi(0.21)}\right)$	$\left((0.37)\,e^{i2\Pi(0.55)},\ (0.28)e^{i2\Pi(0.25)}\right)$	$\left((0.67)\,e^{i2\Pi(0.5)},\ (0.18)e^{i2\Pi(0.07)}\right)$
$Ʉ_3$	$\left((0.6)\,e^{i2\Pi(0.3)},\ (0.24)e^{i2\Pi(0.4)}\right)$	$\left((0.53)\,e^{i2\Pi(0.23)},\ (0.32)e^{i2\Pi(0.23)}\right)$	$\left((0.38)\,e^{i2\Pi(0.5)},\ (0.28)e^{i2\Pi(0.3)}\right)$	$\left((0.56)\,e^{i2\Pi(0.45)},\ (0.17)e^{i2\Pi(0.19)}\right)$
$Ʉ_4$	$\left((0.45)\,e^{i2\Pi(0.3)},\ (0.22)e^{i2\Pi(0.24)}\right)$	$\left((0.36)\,e^{i2\Pi(0.6)},\ (0.5)e^{i2\Pi(0.11)}\right)$	$\left((0.37)\,e^{i2\Pi(0.5)},\ (0.5)e^{i2\Pi(0.21)}\right)$	$\left((0.77)\,e^{i2\Pi(0.25)},\ (0.19)e^{i2\Pi(0.11)}\right)$
$Ʉ_5$	$\left((0.56)\,e^{i2\Pi(0.22)},\ (0.13)e^{i2\Pi(0.24)}\right)$	$\left((0.37)\,e^{i2\Pi(0.5)},\ (0.34)e^{i2\Pi(0.19)}\right)$	$\left((0.39)\,e^{i2\Pi(0.5)},\ (0.51)e^{i2\Pi(0.2)}\right)$	$\left((0.6)\,e^{i2\Pi(0.5)},\ (0.23)e^{i2\Pi(0.12)}\right)$

Table 13 | Comparison methods between the proposed and existing methods from Example 3.

Methods	Score Function	Ranking
Garg and Rani [33]	$S(r_1) = -0.570$, $S(r_2) = -0.557$, $S(r_3) = -0.534$, $S(r_4) = -0.547$, $S(r_5) = -0.533$.	$Ʉ_5 \geq Ʉ_4 \geq Ʉ_3 \geq Ʉ_2 \geq Ʉ_1$
Rani and Garg [34]	$S(r_1) = -0.704$, $S(r_2) = -0.672$, $S(r_3) = -0.667$, $S(r_4) = -0.658$, $S(r_5) = -0.651$.	$Ʉ_5 \geq Ʉ_4 \geq Ʉ_3 \geq Ʉ_2 \geq Ʉ_1$
Cq-ROFBM proposed in this article $\mathbf{q}_{CQ} = 2$	$S(r_1) = -0.397$, $S(r_2) = -0.350$, $S(r_3) = -0.328$, $S(r_4) = -0.328$, $S(r_5) = -0.315$.	$Ʉ_5 \geq Ʉ_4 \geq Ʉ_3 \geq Ʉ_2 \geq Ʉ_1$
Cq-ROFBM proposed in this article $\mathbf{q}_{CQ} = 3$	$S(r_1) = -0.171$, $S(r_2) = -0.133$, $S(r_3) = -0.111$, $S(r_4) = -0.125$, $S(r_5) = -0.109$.	$Ʉ_5 \geq Ʉ_3 \geq Ʉ_4 \geq Ʉ_2 \geq Ʉ_1$

5. CONCLUSION

Recently, Liu *et al.* [30,31] explored the novel approach of CQROFS, which is the mixture of the two notions like QROFS and CFS. The CIFS and CPFS are a good tool to the express the fuzzy information. However, CQROFS is more general, to cope with awkward and complicated information due to its outstanding feature that the sum of q-powers of the real part (also for imaginary part) of the truth and real part (also for imaginary part) of the falsity grades is limited to the unit interval. BM operator is an important and meaningful concept to examine the interrelationships between the different attributes. The aims of this manuscript explored the CQROFBM operator, CQROFWBM operator, CQROFGBM operator, and CQROFWGBM operator, and proposed the decision-making method based on the developed operators. Finally, we have used the practical cases to illustrate the feasibility and superiority of the proposed method by comparative analysis with the other existing methods.

In the future, we will extend the proposed approach to the different environment and then apply to the fields of the similarity measures, aggregation operators [39–46].

DATA AVAILABILITY

The data used to support the findings of this study are included within the article.

CONFLICT OF INTEREST

The authors declare that there are no conflict of interest regarding the publication of this article.

AUTHORS' CONTRIBUTIONS

All authors contributed equally.

ACKNOWLEDGMENTS

This paper is supported by the National Natural Science Foundation of China (Nos. 71771140 and 71471172), 文化名家暨 "四个一批" 人才项目 (Project of cultural masters and "the four kinds of a batch" talents) and the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045).

REFERENCES

[1] K.T. Atanassov, Intuitionistic Fuzzy Sets, Physica, Heidelberg, Germany, 1999, pp. 1–137.
[2] L.A. Zadeh, Fuzzy sets, Inf. Control. 8 (1965), 338–353.
[3] H. Garg, K. Kumar, A novel exponential distance and its based TOPSIS method for interval-valued intuitionistic fuzzy sets using connection number of SPA theory, Artif. Intell. Rev. 53 (2020), 595–624.
[4] H. Garg, G. Kaur, Extended TOPSIS method for multi-criteria group decision-making problems under cubic intuitionistic fuzzy environment, Sci. Iran. 27 (2020), 396–410.
[5] R. Joshi, A new multi-criteria decision-making method based on intuitionistic fuzzy information and its application to fault detection in a machine, J. Amb. Intell. Hum. Comput. 11 (2020), 739–753.
[6] P.S. Kumar, Intuitionistic fuzzy zero point method for solving type-2 intuitionistic fuzzy transportation problem, Int. J. Oper. Res. 37 (2020), 418–451.
[7] J.C.R. Alcantud, A.Z. Khameneh, A. Kilicman, Aggregation of infinite chains of intuitionistic fuzzy sets and their application to choices with temporal intuitionistic fuzzy information, Inf. Sci. 514 (2020), 106–117.
[8] P.S. Kumar, Algorithms for solving the optimization problems using fuzzy and intuitionistic fuzzy set, Int. J. Syst. Assur. Eng. Manag. 11 (2020), 189–222.
[9] C. Yue, An intuitionistic fuzzy projection-based approach and application to software quality evaluation, Soft Comput. 24 (2020), 429–443.
[10] R.R. Yager, Pythagorean fuzzy subsets, in 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), IEEE, Edmonton, Canada, 2013, pp. 57–61.
[11] L. Fei, Y. Deng, Multi-criteria decision making in Pythagorean fuzzy environment, Appl. Intell. 50 (2020), 537–561.
[12] M. Akram, F. Ilyas, H. Garg, Multi-criteria group decision making based on ELECTRE I method in Pythagorean fuzzy information, Soft Comput. 24 (2020), 3425–3453.
[13] Q. Zhou, H. Mo, Y. Deng, A new divergence measure of Pythagorean fuzzy sets based on belief function and its application in medical diagnosis, Mathematics. 8 (2020), 142.
[14] B. Oztaysi, S. Cevik Onar, C. Kahraman, Social open innovation platform design for science teaching by using Pythagorean fuzzy analytic hierarchy process, J. Intell. Fuzzy Syst. 38 (2020), 809–819.

[15] P. Song, L. Li, D. Huang, Q. Wei, X. Chen, Loan risk assessment based on Pythagorean fuzzy analytic hierarchy process, in Journal of Physics: Conference Series, IOP Publishing, Dalian, Zhangjiajie, China, 2020, vol. 1437, p. 012101.
[16] A. Guleria, R.K. Bajaj, Pythagorean fuzzy (R, S)-norm discriminant measure in various decision making processes, J. Intell. Fuzzy Syst. 38 (2020), 761–777.
[17] R.R. Yager, Generalized orthopair fuzzy sets, IEEE Trans. Fuzzy Syst. 25 (2016), 1222–1230.
[18] H. Garg, S.M. Chen, Multiattribute group decision making based on neutrality aggregation operators of q-rung orthopair fuzzy sets, Inf. Sci. 517 (2020), 427–447.
[19] T. Senapati, R.R. Yager, Fermatean fuzzy sets, J. Amb. Intell. Hum. Comput. 11 (2020), 663–674.
[20] A.P. Darko, D. Liang, Some q-rung orthopair fuzzy Hamacher aggregation operators and their application to multiple attribute group decision making with modified EDAS method, Eng. Appl. Artif. Intell. 87 (2020), 103259.
[21] R. Verma, Multiple attribute group decision-making based on order-α divergence and entropy measures under q-rung orthopair fuzzy environment, Int. J. Intell. Syst. 35 (2020), 718–750.
[22] C. Zhang, H. Liao, L. Luo, Z. Xu, Multiplicative consistency analysis for q-rung orthopair fuzzy preference relation, Int. J. Intell. Syst. 35 (2020), 38–71.
[23] A.M.D.J.S. Alkouri, A.R. Salleh, Complex intuitionistic fuzzy sets, in AIP Conference Proceedings, American Institute of Physics, Kuala Lumpur, Malaysia, 2012, vol. 1482, pp. 464–470.
[24] D. Ramot, R. Milo, M. Friedman, A. Kandel, Complex fuzzy sets, IEEE Trans. Fuzzy Syst. 10 (2002), 171–186.
[25] R.T. Ngan, M. Ali, D.E. Tamir, N.D. Rishe, A. Kandel, Representing complex intuitionistic fuzzy set by quaternion numbers and applications to decision making, Appl. Soft Comput. 87 (2020), 105961.
[26] H. Garg, D. Rani, New generalised Bonferroni mean aggregation operators of complex intuitionistic fuzzy information based on Archimedean t-norm and t-conorm, J. Exp. Theor. Artif. Intell. 32 (2020), 81–109.
[27] H. Garg, D. Rani, Robust averaging–geometric aggregation operators for complex intuitionistic fuzzy sets and their applications to MCDM process, Arab. J. Sci. Eng. 45 (2020), 2017–2033.
[28] K. Ullah, T. Mahmood, Z. Ali, N. Jan, On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition, Complex Intell. Syst. 6 (2019), 15–27.
[29] M. Akram, S. Naz, A novel decision-making approach under complex Pythagorean fuzzy environment, Math. Comput. Appl. 24 (2019), 73.
[30] P. Liu, Z. Ali, T. Mahmood, A method to multi-attribute group decision-making problem with complex q-rung orthopair linguistic information based on Heronian mean operators, Int. J. Comput. Intell. Syst. 12 (2019), 1465–1496.
[31] P. Liu, T. Mahmood, Z. Ali, Complex q-rung orthopair fuzzy aggregation operators and their applications in multi-attribute group decision making, Information. 11 (2020), 5.
[32] P. Liu, J. Liu, Some q-rung orthopai fuzzy Bonferroni mean operators and their application to multi-attribute group decision making, Int. J. Intell. Syst. 33 (2018), 315–347.
[33] H. Garg, D. Rani, Some generalized complex intuitionistic fuzzy aggregation operators and their application to multicriteria decision-making process, Arab. J. Sci. Eng. 44 (2019), 2679–2698.
[34] D. Rani, H. Garg, Complex intuitionistic fuzzy power aggregation operators and their applications in multicriteria decision-making, Expert Syst. 35 (2018), e12325.
[35] P. Liu, P. Wang, Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making, Int. J. Intell. Syst. 33 (2018), 259–280.
[36] G. Wei, H. Gao, Y. Wei, Some q-rung orthopair fuzzy Heronian mean operators in multiple attribute decision making, Int. J. Intell. Syst. 33 (2018), 1426–1458.
[37] Z. Xu, R.R. Yager, Intuitionistic fuzzy Bonferroni means, IEEE Trans. Syst. Man Cybern. Part B Cybern. 41 (2010), 568–578.
[38] D. Liang, Y. Zhang, Z. Xu, A.P. Darko, Pythagorean fuzzy Bonferroni mean aggregation operator and its accelerative calculating algorithm with the multithreading, Int. J. Intell. Syst. 33 (2018), 615–633.
[39] B. Yatsalo, A. Korobov, B. Öztayşi, C. Kahraman, L. Martínez, A general approach to fuzzy TOPSIS based on the concept of fuzzy multicriteria acceptability analysis, J. Intell. Fuzzy Syst. 38 (2020), 979–995.
[40] D. Banerjee, B. Dutta, D. Guha, L. Martínez, SMAA-QUALIFLEX methodology to handle multicriteria decision-making problems based on q-rung fuzzy set with hierarchical structure of criteria using bipolar Choquet integral, Int. J. Intell. Syst. 35 (2020), 401–431.
[41] B. Dutta, Á. Labella, R.M. Rodríguez, L. Martínez, Aggregating interrelated attributes in multi-attribute decision-making with ELICIT information based on Bonferroni mean and its variants, Int. J. Comput. Intell. Syst. 12 (2019), 1179–1196.
[42] D.F. Li, T. Mahmood, Z. Ali, Y. Dong, Decision making based on interval-valued complex single-valued neutrosophic hesitant fuzzy generalized hybrid weighted averaging operators, J. Intell. Fuzzy Syst. 38 (2020), 4359–4401.
[43] P. Liu, Z. Ali, T. Mahmood, The distance measures and cross-entropy based on complex fuzzy sets and their application in decision making, J. Intell. Fuzzy Syst. (2020), 1–24.
[44] H. Garg, J. Gwak, T. Mahmood, Z. Ali, Power aggregation operators and VIKOR methods for complex q-rung orthopair fuzzy sets and their applications, Mathematics. 8 (2020), 538.
[45] Z. Ali, T. Mahmood, Complex neutrosophic generalized dice similarity measures and their application to decision making, CAAI Trans. Intell. Technol. 5 (2020), 1–10.
[46] N. Jan, Z. Ali, T. Mahmood, K. Ullah, Some generalized distance and similarity measures for picture hesitant fuzzy sets and their applications in building material recognition and multi-attribute decision making, Punjab Univ. J. Math. 51 (2019), 51–70.