Identifying the Arbitrageurs on Mt. Gox: First Insights from the Leaked Dataset

Pietro Saggese[1] Rainer Böhme[2]

Work in progress – please do not cite our results as final.

Abstract

We mine the leaked history of trades on Mt. Gox, the dominant Bitcoin exchange from 2011 to early 2014, with the aim of identifying investors who performed two-point arbitrage between Mt. Gox and three other cryptocurrency exchanges (BTC-e, Bitstamp, Bitfinex). Most importantly, the availability of user identifiers per trade allows us to reconstruct the sequence of actions executed by each investor. We match these sequences to 'ideal' sequences of arbitrage trades, considering the price differences between exchanges and a user-specific estimate of transaction costs. The latter involves a fee model that is inspired by the posted fee schedules and fitted to empirical data. The subset of users whose actual series matches the ideal series best are potential arbitrageurs. Out of about 125,000 users, we identify 2,631 potential arbitrageurs with all three counterpart exchanges, and 14,291 potential arbitrageurs with at least one counterpart exchange. We consider these numbers as upper bounds and argue that post-filtering techniques are required to refine and later validate the results. A preliminary comparison of aggregate statistics between potential arbitrageurs and non-arbitrageurs is given and discussed.

1 Introduction

Arbitrage, the simultaneous purchase and sale of the same asset in two different markets for a risk-free profit, is a key concept in economics and finance. The concept is so important because the *absence* of arbitrage opportunities is a necessary condition for market equilibrium [1]. Intuitively, whenever an arbitrage opportunity emerges, some arbitrageur will exploit it until the mechanism of supply and demand has eliminated the price difference. This 'law of one price' makes the no-arbitrage principle a powerful solution concept in financial theory. It is a common foundation of the Capital Asset Pricing Model (CAPM) [2, 3, 4], the theory of option pricing [5], the efficient market hypothesis [6], and many other theories.

In practice, arbitrage is never risk-free. Since purchase and sale are not executed in an atomic[3] transaction across markets, the arbitrageur bears the risk of incomplete execution or concurrent price changes. Moreover, the asset traded in both markets may not be exactly the same, and there may be political risk premia if the markets operate in different jurisdictions [7]. These

[1]IMT School for Advanced Studies Lucca, Italy. pietro.saggese@imtlucca.it

[2]Department of Computer Science, Universität Innsbruck, Austria. rainer.boehme@uibk.ac.at

[3]Incidentally, this may change with decentralized exchanges on programmable cryptocurrency platforms.

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risks, in addition to other certain transaction costs, impose a lower bound on the price difference needed for profitable arbitrage. The orthodox economic response, in line with the efficient market hypothesis, is to imagine that many small arbitrageurs each take an infinitesimally small portion of the risk (and hence profit). However, Shleifer and Vishny [8] challenge exactly this view in their landmark work on practical arbitrage in financial markets:

> "[A]rbitrage is conducted by relatively few professional, highly specialized investors who combine their knowledge with resources of outside investors to take large positions." (p. 36)

The authors support this claim by referring to the bounded rationality of many investors, "millions of little traders are typically not the ones who have the knowledge and information to engage in arbitrage." (p. 36) While this is plausible and likely cross-checked by expert market participants, the evidence remains anecdotal. Most surprising to us is the fact that 20 years after these statements were published, we still could not find any academic paper that provides an empirical answer to the question 'Who are the arbitrageurs?'.[4] The answer to this question may not only reconcile economic theory with the reality on financial markets, but also refine the assumptions about arbitrageurs in theoretical studies that derive optimal trading strategies in the presence of arbitrageurs [9, and the works cited therein].

In this work we seek to provide a partial answer from a very singular market, namely the exchange market between convertible currency and cryptocurrency in the early years of Bitcoin. The choice of market and time is opportunistic. We mine a leaked dataset of individual and identified trades from Mt. Gox, a meanwhile defunct exchange that enjoyed a dominant market position before its collapse in early 2014.

This manuscript mainly documents a data science approach to the problem. In the next section, we provide some context and describe our approach. On a high level, our pipeline consists of three stages: pre-processing, identification of potential arbitrageurs, and elimination of false positives. Each stage poses specific challenges, which we document and propose solutions for. We intentionally do not report results because we are not confident enough in the output of our analysis, yet. Given the relevance of the question we tackle, we see the need for better validation and we describe our ideas to this end. For these reasons, we appreciate feedback on all stages, but prefer if the work in its current state is not cited for its "results".

2 Background and Approach

2.1 Arbitrage in Cryptocurrency Markets

Bitcoin is a decentralized cryptocurrency system which records transfers between parties denominated in bitcoin (units of cryptocurrency) in a public ledger. By contrast, exchanges are centralized entities in the Bitcoin ecosystem that provide interfaces to conventional payment systems by allowing its users to trade units of cryptocurrency against fiat money [10]. Typical exchanges manage and match orders in a private limit order book, and update their customers' account balances in cryptocurrency or fiat money as trades are executed. As a result, exchanges are the place where price formation occurs. Trades on exchanges are kept in a private ledger and have no effect on the public ledger unless users withdraw cryptocurrency from the exchange to a

[4] Of course, we simply may have missed the relevant source and are grateful for pointers from our readers and the workshop audience.

wallet under their own control. Most exchanges publish aggregate information about prices and volume, but concerns about data quality exist since few cryptocurrency exchanges are regulated and audited by the standards of conventional financial markets [11].

Intuitively, an arbitrageur observes a price difference between two exchanges, buys a bitcoin at the cheaper place, then transfers it to the more expensive place, and then sells it there for a profit. The transfer of bitcoins between exchanges would be observable in the public ledger and could be associated in time with differences in published prices, thereby generating evidence for arbitrage. However, bitcoin transactions are too slow and (at times) too costly for this strategy. Instead, arbitrageurs must maintain a stock of both bitcoins and fiat money in accounts at each exchange in order to react quickly to price differences. The funds can be balanced at a lower frequency and not necessarily correlated with observable price differences. Therefore, we do not gain any advantage from the public ledger in identifying arbitrage transactions. This fact was already mentioned in the first attempt to study arbitrage in Bitcoin [12]. Interestingly, this term paper documents that market participants actively explored arbitrage opportunities in spring 2013[5] by pointing to two websites that track suitable price differences (see Fig. 15 in Appendix D), and one open source software trading bot that exploits arbitrage opportunities.

Besides this form of two-point arbitrage, which requires accounts at two exchanges, one can also consider triangular arbitrage by exploiting relative price differences between three assets (fiat or cryptocurrency) traded against each other in the same exchange. While arbitrage opportunities are measurable from published data, there is no way to identify arbitrage transactions or arbitrageurs from the public ledger. In this regard, cryptocurrency exchanges do not offer researchers any advantage over foreign exchange markets.

2.2 Relation to Prior Work

The body of prior works applying financial econometrics to time series data from cryptocurrency exchanges is vast and not easy to navigate [e. g., 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. We restrict our review to focused studies of arbitrage opportunities and the exploration of market imperfections. The latter are relevant for our method because they inform us about transaction costs, which constrain the exploitability of apparent arbitrage opportunities.

When interpreting prior work, it is important to keep in mind that most studies use rather short and often non-overlapping samples. The maturing market for cryptocurrencies has exhibited extraordinary volatility as it transitioned through several epochs. Consequently, the time-series contain several structural breaks, which make it hard, if not impossible, to draw conclusions that generalize to the cryptocurrency as a whole. To illustrate this, we depict in Figure 1 the sample periods of the works discussed in the following along with the bitcoin price in USD on a logarithmic scale. This accounts best for the order of magnitude differences between epochs.

The early years of Bitcoin trading (which are relevant for our work) are covered by Dong and Dong [24] as well as Badev and Chen [25]. The former team compares official and implied exchange rates between fiat currencies on cryptocurrency exchanges. The authors interpret this measure of triangular arbitrage opportunity as a proxy for the liquidity of Bitcoin. The latter source investigate two-point arbitrage between exchanges, which is close to our interest. But the most remarkable price differences reported fall into the period when the Mt. Gox exchange collapsed, and thus are beyond our sample. The best explanation for these differences is the counter-party risk of the tumbling exchange, rather than unexploited arbitrage opportunities.

[5] `https://bitcointalk.org/index.php?topic=137675.0`,
`https://bitcoin.stackexchange.com/questions/12670/why-dont-people-buy-at-one-exchange-and-sell-at-another` (all accessed on 7 May 2019)

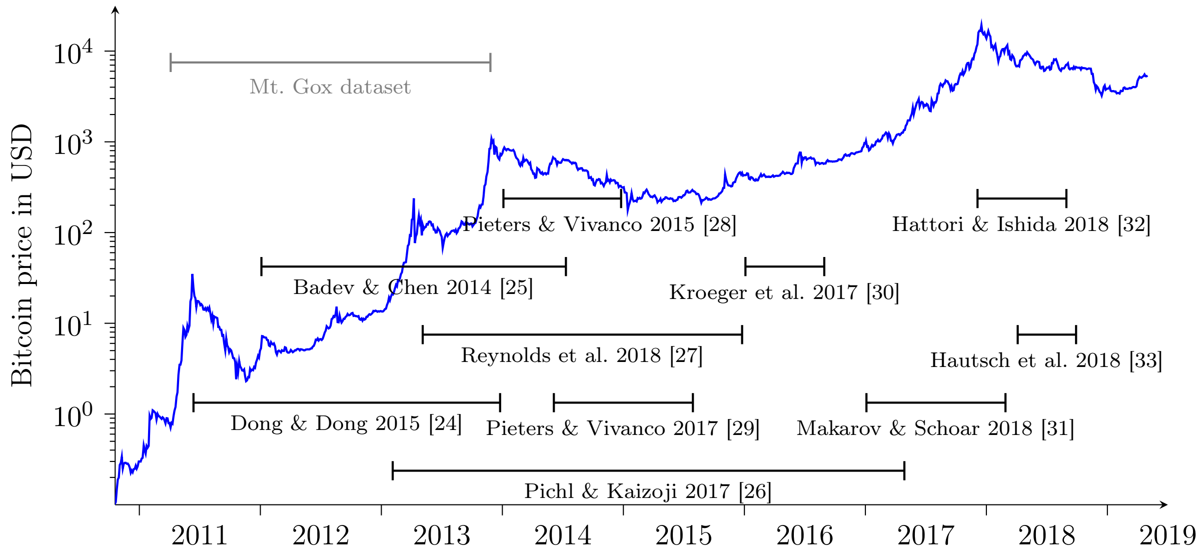


Figure 1: Related work in temporal and market context (Bitcoin price in USD on log scale)

The draft by Kroeger et al. [30], albeit incomplete at the time of writing, is probably closest to our analysis. The authors show that even after accounting for both explicit (fees) and implicit (illiquidity and volatility) transaction costs, arbitrage opportunities pertain between exchanges. In particular, bitcoins are traded at a consistently lower price on one exchange, BTC-e. Reynolds et al. [27] focus on the triangular arbitrage and report persistent mis-pricing when Bitcoin is used as a vehicle currency. Pieters and Vivanco [28] observe that, from January to December 2014, there are arbitrage opportunities between different exchanges. As a noteworthy detail, they remark that the exchange rate for the Argentinian pesos (ARS) on Local Bitcoins, a peer-to-peer exchange, is closer to the ARS black market exchange rate than to the official one. In follow-up work [29], the authors expand this line of thought and try to explain international inconsistencies in the price of Bitcoin with the regulatory environment towards cryptocurrencies. Markets with tighter regulation, approximated by the level of customer identification exchanges require, seem to charge a political risk premium. Related to that, Makarov and Schoar [31] report unexploited arbitrage opportunities for a later time window and suggest that capital controls played an essential role in causing market frictions in 2017. Pichl and Kaizoji [26] study triangular arbitrage without considering transaction costs. They find unexploited opportunities in particular in the Chinese market.

Hattori and Ishida [32] study the nascent futures market for Bitcoin and, somewhat surprisingly, find that it is efficient from March 2018 onwards. The disagreement with the tenor of most other literature can be attributed to the time window and the fact that the futures market operates in a single geographical area. Finally, Hautsch et al. [33] focus on a short time window from April to September 2018. Their main contribution is to derive theoretical bounds to arbitrage, which could arise as a consequence of the settlement time needed for arbitrage strategies that require bitcoin transfers across exchanges.

In summary, based on heterogeneous methods and studying different periods in time with data of different frequency, the literature pretty consistently reports unexploited arbitrage opportunities in cryptocurrency markets. This does not imply that arbitrage does not happen, but might rather indicate that the costs and risks of arbitrageurs are under-estimated. Anecdotal evidence from forums, the existence of web-based arbitrage tools, and code repositories for trading bots indicate that arbitrage does happen [12].

2.3 Data Source

All the reviewed studies have in common that they analyze aggregated price (and sometimes volume) time series. Our approach differs in that we use individual data from the internal ledger of a major exchange, Mt. Gox.

Mt. Gox played a prominent role during the early years of Bitcoin: established in 2010, it was the first exchange and dominated market with more than 90% of total trading volume until early 2012. The competitors Bitstamp and BTC-e entered the market in mid-2011, BTC China at the end of 2011. When Bitfinex started its activity in spring 2013, the competitors of Mt. Gox gained market share pushing Mt. Gox to just under 60% in the summer of 2013. (See Figure 1 in Appendix A of the supplemental material to [34] for market share over time.) Customers of Mt. Gox often experienced delays when withdrawing fiat money. The exchange stopped withdraws altogether on 7 February 2014, and filed for bankruptcy two weeks later. The former CEO was arrested after criminal charges of fraud and embezzlement in 2015, and found guilty of falsifying data in 2019. Exchange closure is a common phenomenon in the cryptocurrency space, and a source of concern for investors, as witnessed by the survival analysis of 80 exchanges in [35].

Our main dataset was leaked to the public in 2014 as a series of CSV files. They contain information on almost 7 million trades in between 1 April 2011 and 29 November 2013. The vast majority (87.7%) of trades are in USD, followed by EUR (7.7%). Figure 2 visualizes selected indicators on how Mt. Gox's user base evolved over time, reaching a total of more than 125,000.

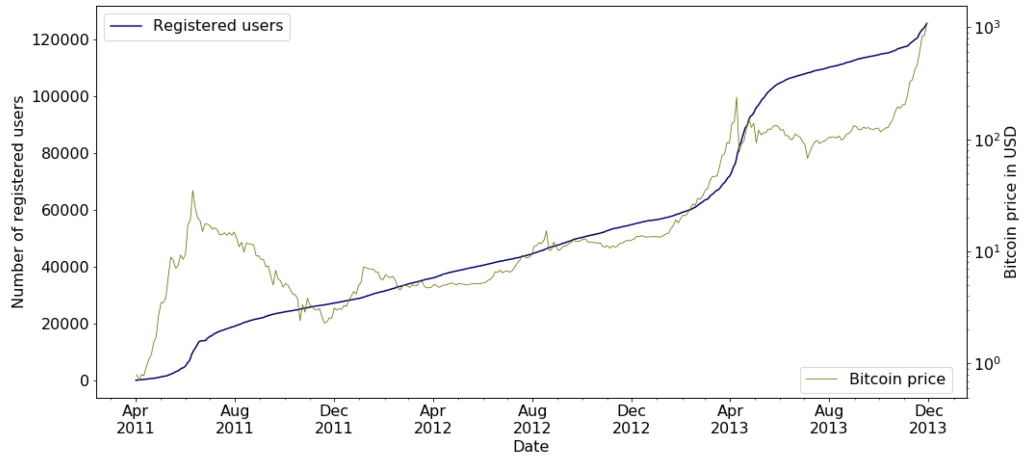
While there is no way to fully verify the correctness of this dataset, our own comparisons to aggregate information support its authenticity. Moreover, according to The Guardian,[6] several members of the bitcoin community claimed to have found their own transactions in the dataset. Finally, certain facts established in the court case against the former CEO of Mt. Gox seem to plausibly explain patterns in the dataset.[7]

Several researchers have analyzed the dataset for other research questions than arbitrage:

- Gandal et al. [34] analyze the actions of two users, *Markus* and *Willy*, who engaged in suspicious trading activity. The authors relate the growth of the 2013 bitcoin price bubble to these users' trades, supporting the claim of potential market manipulation.
- Feder et al. [36] study the impact of distributed denial-of-service (DDoS) attacks on the Mt. Gox exchange. They find that the number of large trades decreases significantly in the days following an attack.
- Scaillet et al. [37] analyze price jumps in the BTC/USD exchange rate. They use the leaked dataset as an approximation for tick-level information and calculate indicators of aggressiveness and market participation from its user identifiers. This kind of information is rarely available in aggregated financial series.
- Chen et al. [38] study trades through the lens of network science. They report that a limited number of users had a significant impact on the bitcoin price, and suspect that some unusual patterns (self-loops, uni- and bi-directional trades, triangular structures) might be indications of market manipulation.

[6] `https://www.theguardian.com/technology/2014/mar/10/mtgox-bitcoin-database-leaked-online-as-hackers-crowdsource-clues` (accessed on 4 May 2019)

[7] This statement is based on personal communication. The authors have not read the Japanese files.



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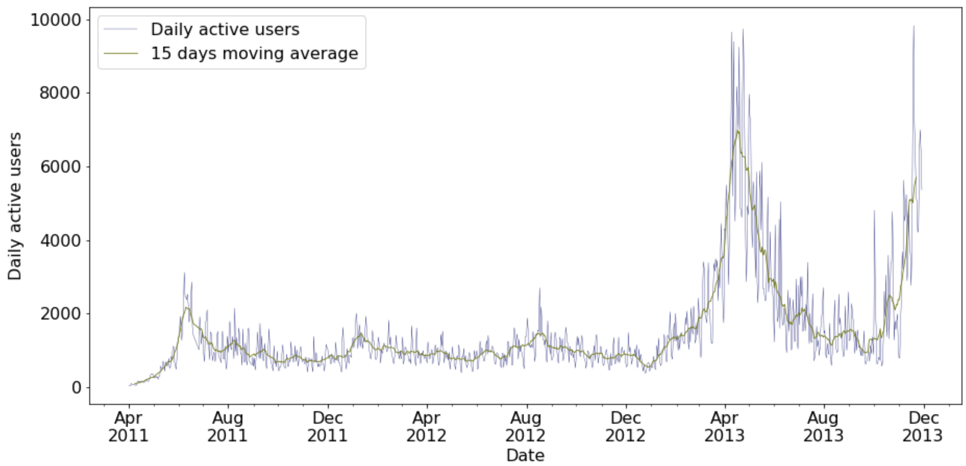

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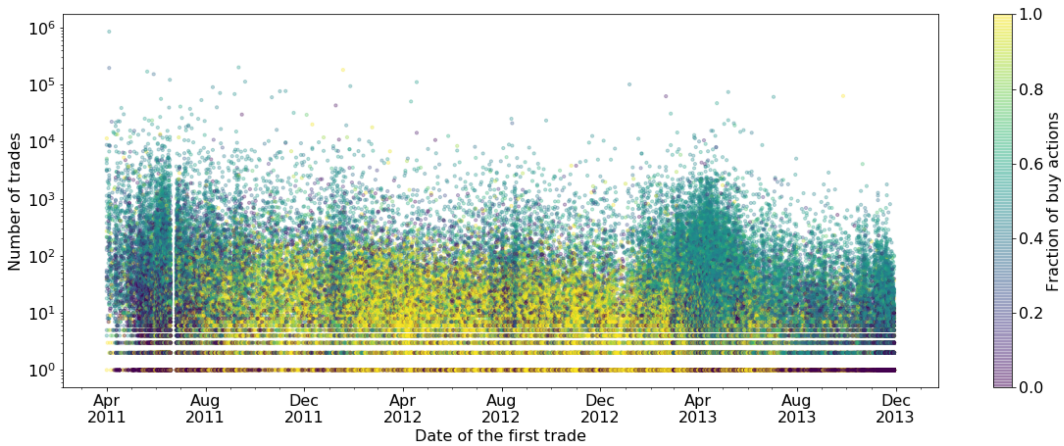


(c)

Figure 2: Descriptive statistics of Mt. Gox users. Panel (a) shows the growth of registered users in relation to the bitcoin price (the latter is reported on a logarithmic scale). Panel (b) shows the number of daily active users. Panel (c) shows a scatterplot where each dot represents a user positions on the x-axis by the first day of activity on Mt. Gox and on the y-axis by the total number of trades; color indicates the fraction of buy actions.

In summary, the dataset is meanwhile well researched. Our own exploration revealed some additional details, which we document in the form of descriptive statistics in Appendix B. We are not aware of any other work looking at arbitrage activity, our main interest in this study.

Finally, a comment on research ethics and data privacy stands to reason. The internal ledger of Mt. Gox contains data that, in principle, can be linked to natural persons by matching it with other records. Moreover, the users appearing in this dataset had no expectation that their individual trades will become public. We therefore take utmost care that none of our analyses singles out users that have not been singled out in other work (which we always document with a proper citation). Specifically, we map all user identifiers in the dataset to a consecutive sequence of integers, preserving the order but not the numbers. Therefore, user identifiers in our figures should not be directly related to identifiers in the data source. Moreover, we do not possess additional data which would allow linking records to natural persons, nor are we aware of a source where this data could be gathered. Therefore, we believe that the harm caused by our study is minimal while there are clear benefits in shedding light into a fundamental question in finance. Readers seeking to replicate the general methods described here are advised to make similar considerations before working with the data.

2.4 Analytical Approach

The internal ledger of one exchange does not allow us to identify two-point arbitrage on the level of individual (pairs of) trades. However, under the assumption that arbitrageurs are users who perform arbitrage consistently over a longer period of time, we can identify the users in the Mt. Gox dataset whose sequence of actions can be plausibly explained with arbitrage exploiting the differences in published prices between Mt. Gox and any counterpart exchange. The high-level idea of our approach is to generate an 'ideal' series of arbitrage actions for each counterpart of Mt. Gox from aggregate information. This ideal series is then matched against the actual trades of each user in the Mt. Gox dataset. The set of users with the highest similarity between ideal and actual trades are likely arbitrageurs.

Figure 3 shows the analytical pipeline of our approach in more detail. It is based on two data sources, the Mt. Gox leaked files and high frequency time series of price and volume information obtained from the Bitcoincharts.com Market API[8] for the three other relevant exchanges in the time period considered: BTC-e (USD and EUR), Bitstamp (USD), and Bitfinex (USD). The pre-processing stage involves data cleaning of the Mt. Gox files. We follow the procedure established in related work and deviate for reasons detailed in Appendix A. Moreover, the high-frequency time series are aggregated to hourly open/high/low/close/volume (OHLCV) series in unified GMT time.

The most interesting parts of our work so far concern the matching stage, where we focus on two aspects: first, the calculation of the ideal series requires a good approximation of the transaction costs. We devise and fit a fee model to estimate the transaction costs as a function of each user's trading history (Section 3.1). Second, we discuss our similarity metrics and report preliminary results on the set of potential arbitrageurs for the three counterpart exchanges (Section 3.2).

As we find that the method is still prone to false positives, we propose a post-filtering approach to narrow down the set of arbitrageurs to users where arbitrage activity can be confirmed from tick data. This part of the work was not completed by the WEIS deadline and will benefit from the feedback received from the workshop participants.

[8] `http://api.bitcoincharts.com/v1/csv/` (accessed on 6 May 2019)

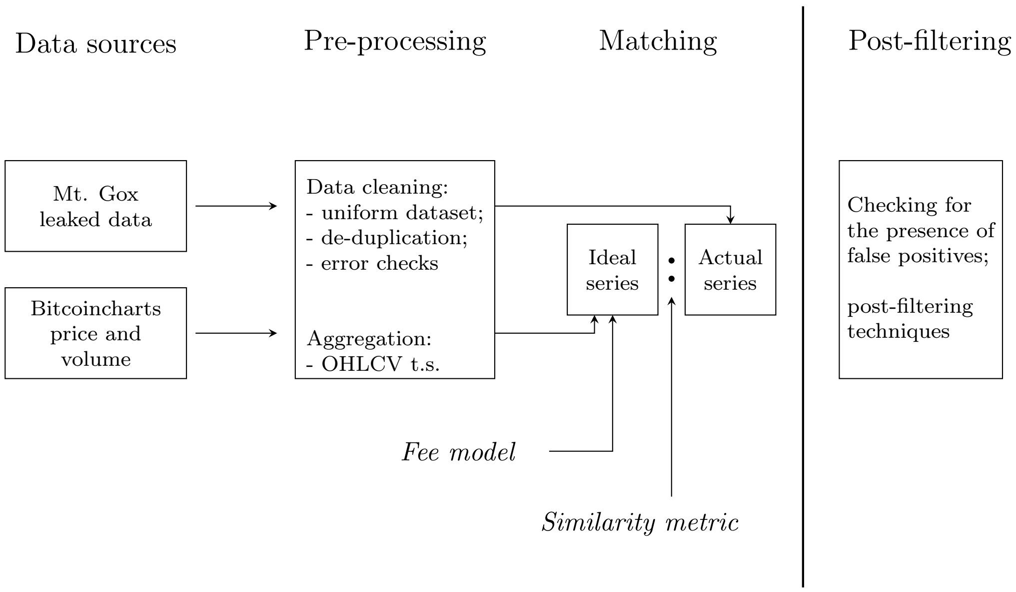


Figure 3: Overview of our analytical approach

3 Method and Results

We describe the method and present the estimated fee model in Section 3.1 before we describe the matching along with our results for the identified potential arbitrageurs in Section 3.2.

3.1 Fee Model

Table 1 shows the fee schedule published by Mt. Gox; we could confirm that the scheme was valid at least from 16 October 2011 until mid-February 2013[9]. Discounts were based on the volume of bitcoins traded by the user over the last 720 hours[10] (that is, 30 days).

We first compare the posted and the real fee schedules. In the Mt. Gox dataset, transaction costs are reported in two entries: bitcoin buyers were charged with fees in bitcoin, while sellers with fees in fiat money[11]. Thus, for each leg of every trade, we compute the actual fees paid as

$$\text{Fee} = \frac{\text{BitcoinFee}}{\text{Bitcoins}} + \frac{\text{MoneyFee}}{\text{Money}}, \tag{1}$$

where 'BitcoinFee' represents the fees paid on the amount of bitcoins traded, while 'MoneyFee' represents the fees paid on the amount of fiat money traded.

Figure 4 shows a sample ($N = 100,000$) of the empirical fees, focusing on the relationship between the actual transaction costs and the past volume traded; each dot represents the fees paid on a leg of a trade. By comparing the posted and the actual schedules, we note that most of the data points fall into the expected volume bands, although deviations exist: first, from 0.40%

[9]http://web.archive.org/web/20111016114851/https://mtgox.com/fee-schedule
http://web.archive.org/web/20140210044241/https://www.mtgox.com/fee-schedule
https://bitcointalk.org/index.php?topic=144558.0 (all accessed on 6 May 2019)
[10]https://en.bitcoin.it/wiki/Mt._Gox (accessed on 6 May 2019)
[11]https://bitcoin.stackexchange.com/questions/10016/does-mtgox-charge-commission-in-usd-or-in-btc (accessed on 6 May 2019)

Table 1: Mt. Gox posted fee schedule. Discounts are based on the user's trading volume over the last 720 hours

Volume (BTCs)	Fees	Volume (BTCs)	Fees
0 to < 100	0.60%	10000 to < 25000	0.30%
100 to < 200	0.55%	25000 to < 50000	0.29%
200 to < 500	0.53%	50000 to < 100000	0.28%
500 to < 1000	0.50%	100000 to < 250000	0.27%
1000 to < 2000	0.46%	250000 to < 500000	0.26%
2000 to < 5000	0.43%	> 500000	0.25%
5000 to < 10000	0.40%	-	-

to 0.20%, many points follow a pattern that cannot be explained by the posted schedule; second, a non-negligible number of dots falls below the threshold of 0.20%, suggesting the existence of privileged users, and a subset of legs is completely exempted from any kind of fee. These legs are further investigated in Figure 5.

The heuristics that we introduce to detect arbitrageurs in Section 3.2 crucially depend on the fees paid by the users. To take them into account, instead of reverse-engineering the posted schedule, we take an empirical approach and fit a simple model that predicts the fees a user would have to pay given his trading history. The fee model is specified as:

$$\begin{aligned}\mathrm{Fee}_i = \beta_0 &+ \beta_1 \cdot \mathrm{LogVol}_i + \beta_2 \cdot \mathrm{VolSmall}_i + \beta_3 \cdot \mathrm{VolBig}_i \\ &+ \beta_4 \cdot \mathrm{LogVol}_i \cdot \mathrm{VolSmall}_i + \beta_5 \cdot \mathrm{LogVol}_i \cdot \mathrm{VolBig}_i \\ &+ \beta_6 \cdot \mathrm{T}_{0i} + \beta_7 \cdot \mathrm{T}_{1i} + \beta_8 \cdot \mathrm{T}_{\mathrm{holid}i} + \epsilon_i.\end{aligned} \tag{2}$$

The independent variables have the following meaning:

- LogVol, the natural logarithm of the volume traded in the last 720 hours by the user who submitted the leg associated to the fee;
- VolSmall, a dummy variable equal to 1 if the volume traded in the last 720 hours is between 100 and 10,000 bitcoins;
- VolBig, a dummy variable equal to 1 if the volume traded in the last 720 hours exceeds 10,000 bitcoins. As it can be seen from Figure 4, discount factors follow a linear trend with different slopes below and above the 10,000 bitcoins threshold. This is the reason why we introduced this dummy variable and the previous one, as well as their interaction terms with the LogVol variable;
- T_0, a dummy variable for trades executed between 1 April 2011 and 23 June 2011;
- T_1, a dummy variable for trades executed between 24 June 2011 and 18 August 2011;
- $\mathrm{T}_{\mathrm{holid}}$, a dummy variable for trades executed on 'special days': from 26 December 2011 to 1 January 2012, from 2 to 7 April 2012; on 9 and 10 November 2012[12].

[12]respectively, Christmas holidays in 2011, Easter holidays in 2012 and first Bitcoin Friday Sale day. `https://bitcoinmagazine.com/articles/bitcoin-friday-sale-happening-today-1352497394/` (accessed on 6 May 2019)

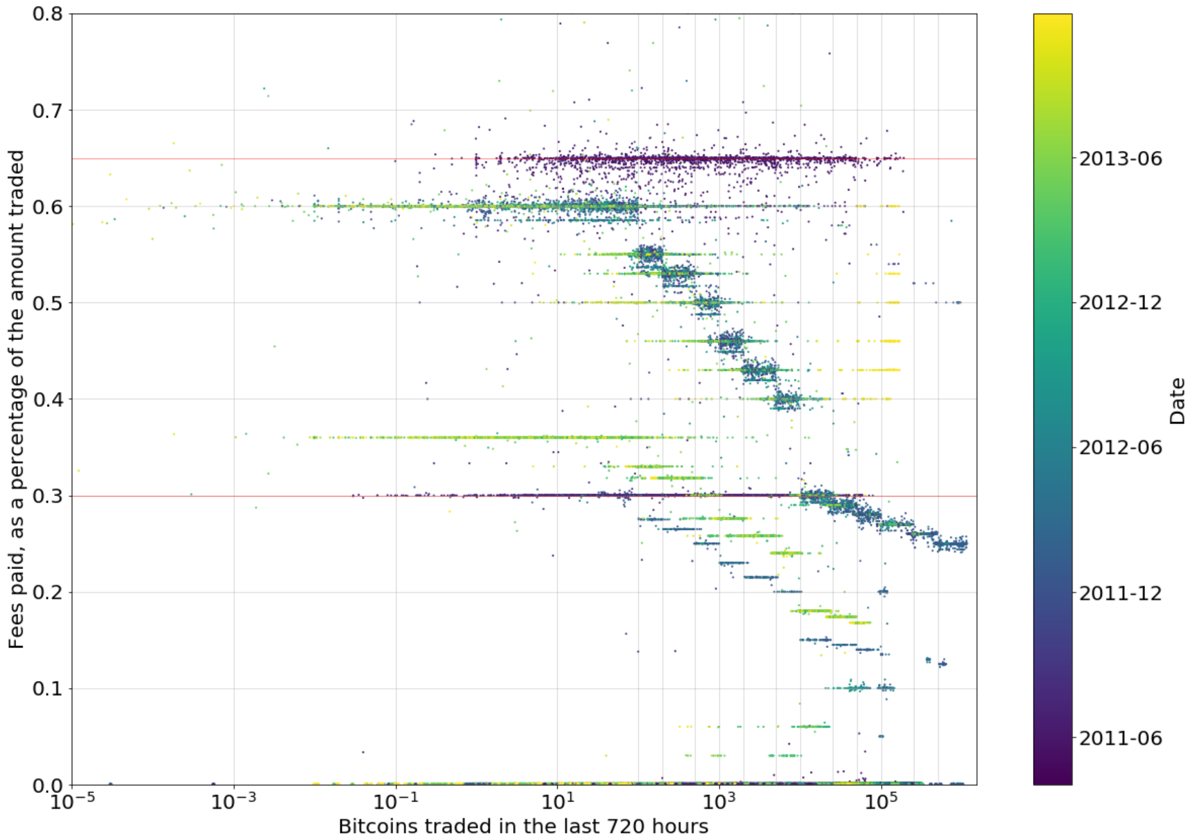
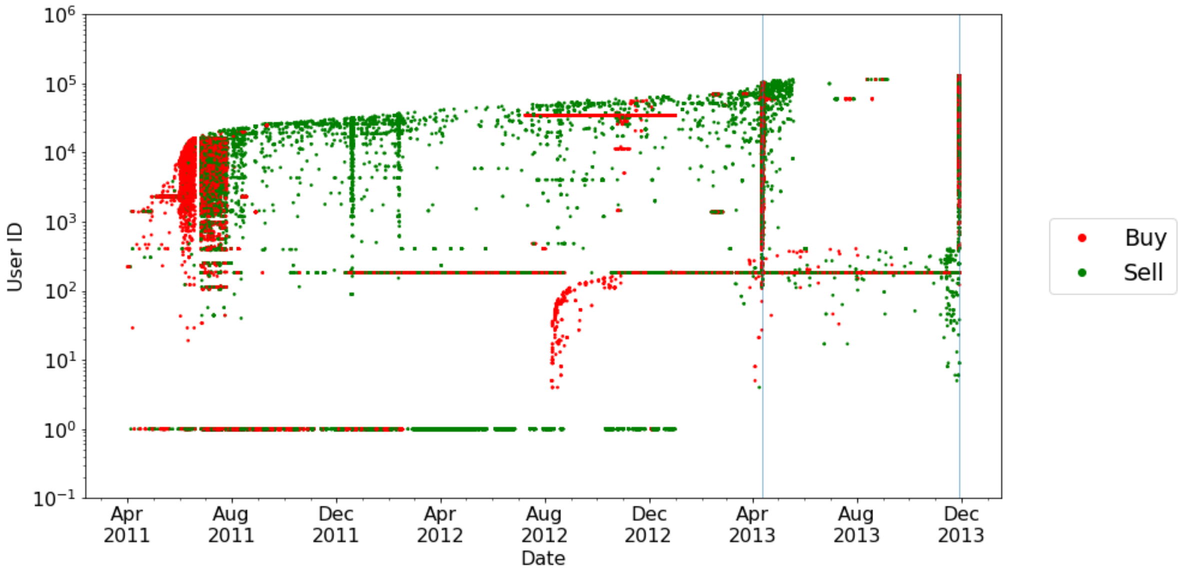


Figure 4: Empirical fee schedule. Fees paid per leg expressed as the percentage of the value of the trade (y-axis). According to Mt. Gox's schedule, fees depend on the amount of bitcoins traded by the user in the last 720 hours (on the x-axis). Color indicates the time of the trade. The lines of the grid help to graphically delimit the volume bands corresponding to different discount brackets. Two horizontal red lines help to identify the two time windows when fees were fixed: 0.65% from 1 April 2011 to 23 June 2011, and 0.3% from 24 June 2011 to 18 August 2011. Many legs pay no fees, and a limited number of legs pays fees as low as 0.1 percent or less. In addition, in some particular circumstances (e.g., Easter and Christmas holidays in 2011 and 2012), fees were halved, thus explaining some of the values that do not correspond to the official fee schedule. The plot shows a random sample of $N = 100,000$ legs.



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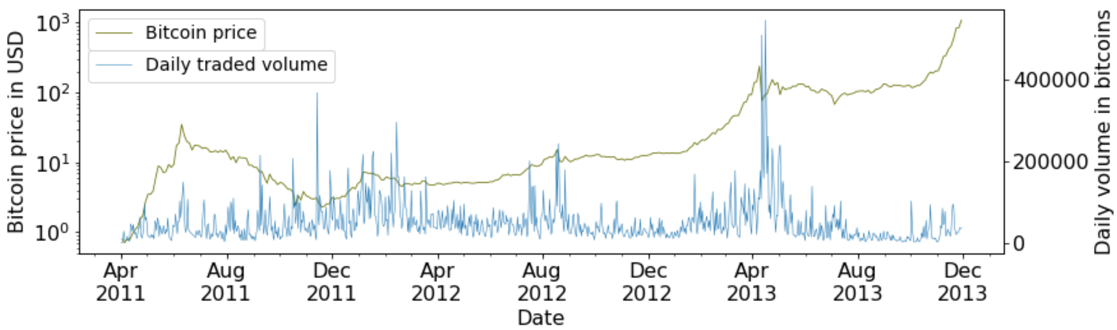


(b)

Figure 5: Patterns of waived fees. Panel (a) reports the distribution of the legs whose associated fees are zero, as a function of time and user ID (the latter on a logarithmic scale, to focus on low IDs). Buy (red) and sell (green) orders differ by color. Many interesting patterns emerge: first, two users with low ID did not pay fees over extended time periods; moreover, they account for $\sim 1,000,000$ zero-fee legs over the total of $\sim 1,700,000$ zero-fee legs. Second, during some days (especially on 19, 20, and 21 December 2011; 12, 13, and 14 April 2013; 28 and 29 November 2013) there were anomalous increases in trade legs with zero fees. Possible explanations include special events, such as a temporary downtime of Mt. Gox on 11 April 2013, and the exchange rate surpassing 1,000 \$/BTC for the first time on 27 November 2013. In both cases, the number of zero-fee trade legs increases shortly afterwards. Panel (b) depicts the daily bitcoin volume traded and the exchange rate in USD (the latter on a logarithmic scale). Observe from both panels that many users with low IDs seem to have *coordinately* executed buy orders from August to around November 2012, and then exclusively sell orders in the the days preceding the Bitcoin price peak. The plot shows a random sample of $N = 200,000$ legs.

Table 2: Fee model for non-zero fees, coefficients fitted with OLS

Dependent variable:	Fee as a percentage of the amount traded				
	Specification				
	(1)	(2)	(3)	(4)	(5)
Intercept	0.636	0.638	0.556	0.560	0.561
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.000070)
LogVol	-0.030	-0.030		-0.001	-0.001
	(0.0000)	(0.0000)		(0.0000)	(0.0000)
VolSmall			-0.106	0.151	0.153
			(0.0001)	(0.0002)	(0.0002)
VolBig			-0.283	-0.206	-0.218
			(0.0001)	(0.0005)	(0.0004)
LogVol $*$ VolSmall				-0.037	-0.037
				(0.0000)	(0.0000)
LogVol $*$ VolBig				-0.006	-0.005
				(0.0001)	(0.0001)
T_0		0.163			0.156
		(0.0002)			(0.0001)
T_1		-0.159			-0.168
		(0.0002)			(0.00011)
T_{holid}		-0.187			-0.191
		(0.0002)			(0.0002)
R^2	0.510	0.605	0.527	0.590	0.688
Obs.	12243707	12243707	12243707	12243707	12243707

Note: All variables are significant at the 0.1% level. This is due to the high number of observations; however, we emphasize that these results are intended not so much to find significant effects as to predict fees. Observations consist of legs whose fees are positive and their value is below 1%.

Table 2 reports the estimated coefficients and goodness-of-fit indicators. In each specification, the constant term approximates the non-discounted official fee of 0.6%, and the response variable is negatively correlated with an increase of the volume traded in the past 720 hours; as expected, when included in the model, T_0 and T_1 respectively increase and decrease the constant term by a factor of around 0.15%, while T_{holid}has an even stronger negative effect (around -0.20%). Finally, again in accordance with our expectations, both β_4 and β_5 are negative; the first one is bigger in absolute terms, thus indicating a steeper variation of discounts given the same variation in volume (as can be seen in Figure 4). We chose a logarithmic model because the break points for the fee schedule in Table 1 follow a logarithmic trend, and thus it was straightforward to consider the estimation on logarithmic volumes. However, to strengthen the results, we explored an alternative model, which is reported in Appendix C, using linear volumes. In that section we also specify a logit model to estimate the probability that a leg pay zero fees given user-specific and time-related variables.

Finally, we need to account for the fact that two-point arbitrage requires *two* trades, one on Mt. Gox and a second one on another exchange. Thus, we also need to include an estimate of the fees that the investor would pay on the counterpart exchange. Relying on each counterpart's posted fees at the time of the analysis[13], we add a transaction cost of 0.2% of the amount traded when the counterpart exchange is BTC-e, and 0.1% for Bitfinex. Since Bitstamp applied a fee schedule similar to Mt. Gox, based on the volume traded in the last 30 days, but ranging from 0.5% to 0.2% and having different bands, we add a term corresponding to the fee paid in Mt. Gox, but rescaled it proportionally to Bitstamp's fee range.

3.2 Matching and Similarity Metric

The fee model gives an approximation of the expected transaction costs, which vary across users and time. This assumption is crucial, since individual transaction costs affect how the same arbitrage opportunity is perceived by different users: the same mis-pricing can be an unexploited arbitrage opportunity for an investor who pays low enough fees, and a costly operation for a trader that pays higher fees. Then, the second step of the analysis consists in defining a heuristic approach to identify the users who are likely to be arbitrageurs: we *match* the sequence of actions executed by each real user, together with the associated fees paid, against an ideal sequence of optimal actions made by a 'perfect arbitrageur'; thus, we introduce a *similarity metric* to classify investors in potential arbitrageurs and non-arbitrageurs.

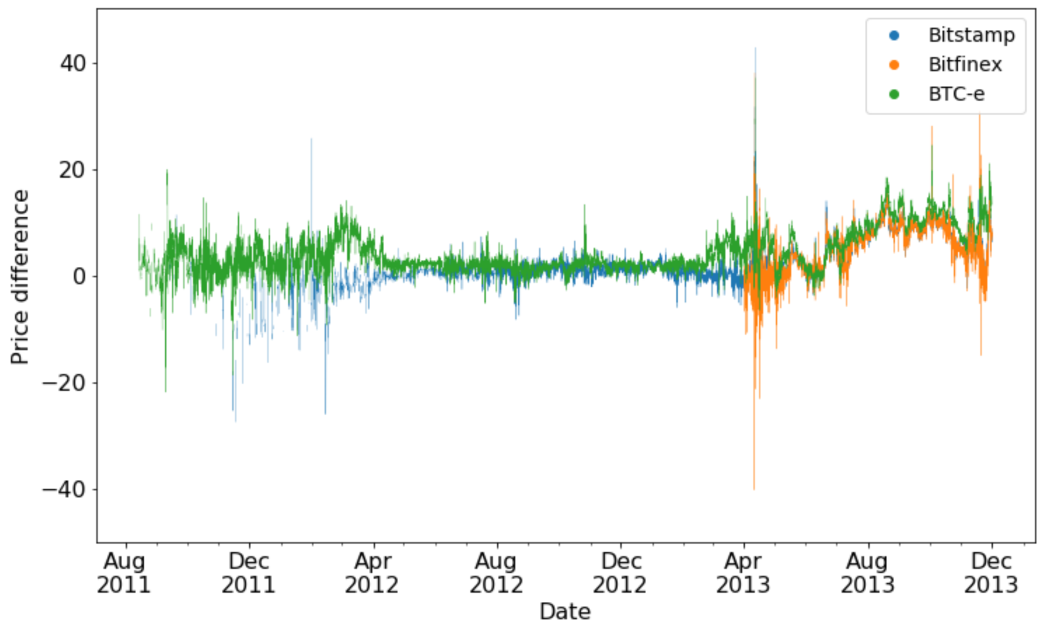


Figure 6: Price differences between Mt. Gox and Bitstamp, Bitfinex, BTC-e, as a percentage of the Mt. Gox price, in USD.

Our work is based on hourly frequency, therefore we first aggregate the price time series for the three counterpart exchanges: Figure 6 shows the bitcoin USD price difference between Mt. Gox and the three counterpart exchanges, as a percentage of the Mt. Gox price, at the hourly level.

[13] Bitfinex: `https://web.archive.org/web/20130513094437/https://bitfinex.com/pages/fees`;
BTC-e: `https://web.archive.org/web/20120805040835/https://btc-e.com/page/2`;
Bitstamp: `https://www.bitstamp.net/article/press-release-bitstamp-introduces-volume-discount-/`
(all accessed on 6 May 2019)

We focus on a counterpart exchange E. For each time interval, the optimal response of an ideal arbitrageur active on Mt. Gox is drawn from the set of actions A = {Buy (B), Sell (S), Hold (H)}: without considering transaction costs, an arbitrageur would always Sell on Mt. Gox if prices are higher on Mt. Gox, and vice versa, he would Buy if they are lower. If there is no mis-pricing, the best action is to Hold. We construct then a sequence of hourly ideal response actions to price changes between Mt. Gox and E, together with the associated maximum transaction costs that would entail zero profits: the information contained in this sequence is common to each user and publicly available.

Second, we aggregate the user actions at the hourly level. When a user submits more than one trade in the same hour, we define the *hourly prevalent action* as Buy if the difference between the amount of bitcoins bought and sold is positive, and as Sell if negative; we exclude hours in which this difference is equal to zero. We made this choice to preserve the original set of users' actions. Fees are aggregated by computing the mean value for each hour. Matching takes place only on the time windows in which the user is active; currently, we do not make any deduction on 'arbitrageur-like' behavior of a user during times of inactivity. This issue is something to be revisited in follow-up work. Moreover, the matching algorithm only compares time-periods in which both a user's action and the price difference between Mt. Gox and the counterpart exchange are available (e.g., Bitfinex data are available only from April 2013 onwards). From the sequence of the users' hourly actions, we construct a first indicator: we count the *changes of state* as the number of alternations of the hourly Buy and Sell actions for each user U.

Then, each user U observes the publicly available information on mis-pricings and maximum transaction costs, and derives from it the *individual* ideal sequence of hourly events, *given the user-specific transaction costs*: while the former is common to each user, the latter depends on each user's trading history and thus is private. The matching algorithm compares this sequence to the individual hourly sequence of actions: an action is considered as 'arbitrage-like' if it corresponds to the one that an ideal arbitrageur would perform in the same time window *and* if the associated transaction costs do not exceed the size of the mis-pricing. Figure 7 shows an illustrative example of the matching procedure for a demonstrative user U.

As a result of the matching algorithm, only a fraction of each user's 'hourly actions' correspond to two-point arbitrage actions. For each user we construct then a second indicator: we compute the fraction of 'arbitrage-like' actions that correspond to two-point arbitrage on the counterpart exchange E. We rank the users based on this parameter in descending order.

We repeat these steps for each of the three counterpart exchanges, considering the price time series denominated in USD. Figure 8 shows three lines, one per exchange; users are ranked in descending order based on the share of 'arbitrage-like' actions. Note that, for each counterpart exchange, actions are matched independently: a user ranked n^{th} in a counterpart exchange might be ranked m^{th} in another. For each counterpart exchange, we take into account only the users for which the matching occurs on more than 10 hours. Consider the illustrative case of Bitfinex: since the price time series is shorter (the data on price differences go from April 2013 to November 2013), a smaller number of users is included in the ranking. Users whose share of 'arbitrage-like' actions is above 0.5 are classified as potential arbitrageurs. All the lines follow a sigmoid trend; in the case of Bitstamp, the right tail follows a smoother decline; this is probably due to the differences in the fee schedules (higher in Bitstamp). In Bitstamp and Bitfinex, with respect to BTC-e, it is possible to notice a steeper decline of the users whose fraction of arbitrage-like actions is high; most importantly, the set of users detected as potential arbitrageurs in BTC-e is larger if compared to the size of the sets obtained on the other counterpart exchanges (12,511 against 4,990 in Bitstamp and 5,045 in Bitfinex). We attribute the dominance of BTC-e to the persistence of larger price differences throughout most of 2013, with respect to the mis-pricings

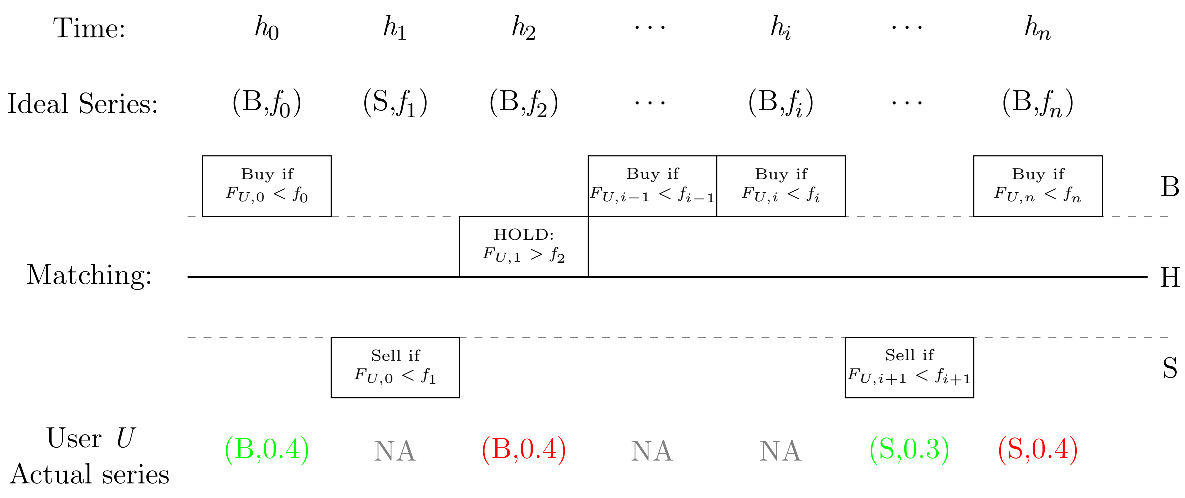


Figure 7: Illustrative example of the matching heuristic. At each point in time we know the buy/sell action that an ideal arbitrageur would execute given the mis-pricing, and a value f for the absolute value of the size of the arbitrage opportunity. $F_{U,i}$ represents the fee paid and varies across time and users. The last row shows the tuples (Prevalent Action, Fee) for a representative user U. Now, for this illustrative example, suppose that all the values f_i are bigger than 0.5%, except $f_2 = 0.3\%$. Once individual fees are included, some of the f values might turn out to be too small for the user to be exploited as arbitrage opportunities, because the fees exceed the size of the mis-pricing; in this case, even though the ideal and actual action at time h_2 correspond, the fee exceeds the price differences: an ideal arbitrageur would then 'Hold' (H); so, this action is counted as 'non-arbitrageur like' for user U. NA stands for Not Active, meaning that in this time window the user did not make any trade; green tuples are classified as 'arbitrageurs-like', while red tuples as 'non arbitrageurs-like'.

in the other counterpart exchanges.

We identify a set of 2,631 users as potential arbitrageurs in all of the three counterpart exchanges. Figure 10 shows to what extent the results overlap. On a total of 125,730 users, 111,439 are always classified as non-arbitrageurs, while 14,291 investors are detected as potential arbitrageurs on at least one exchange. Figure 9 shows the subset of users detected as potential arbitrageurs in each exchange (orange), compared with all the other users (green). While independent of the fraction of *changes of state*, the set is mainly composed of investors who made more Sell actions than Buy actions. This goes against our expectations to find arbitrageur candidates in the upper central portion of the constructed triangle. We hypothesize that the subset of potential arbitrageurs identified on all exchanges might be affected by the structural price increase that occurred in Mt. Gox from Spring 2013. However, further investigation is required.

Table 3 reports the summary statistics on the sets of the identified potential arbitrageurs. The purpose of column (c) is to introduce a benchmark for the comparison of the values in columns (a) and (b), while column (d) is reported for illustrative purposes. Coherently with the interpretation given for Figures 8, 9, and 10, we note that the users in the set of column (a) made more Sell trades and less Buy trades with respect to groups (b) and (c), and the mean user ID in column (a) is higher (84,597 against 57,465 and 49,513 respectively for columns (b) and (c)); furthermore, they moved more fiat currency per trade ($415.70 for column (a), $300.40 and $275.85 for columns (b) and (c)), but moved less bitcoins; thus, they were active in a time window in which prices were high.

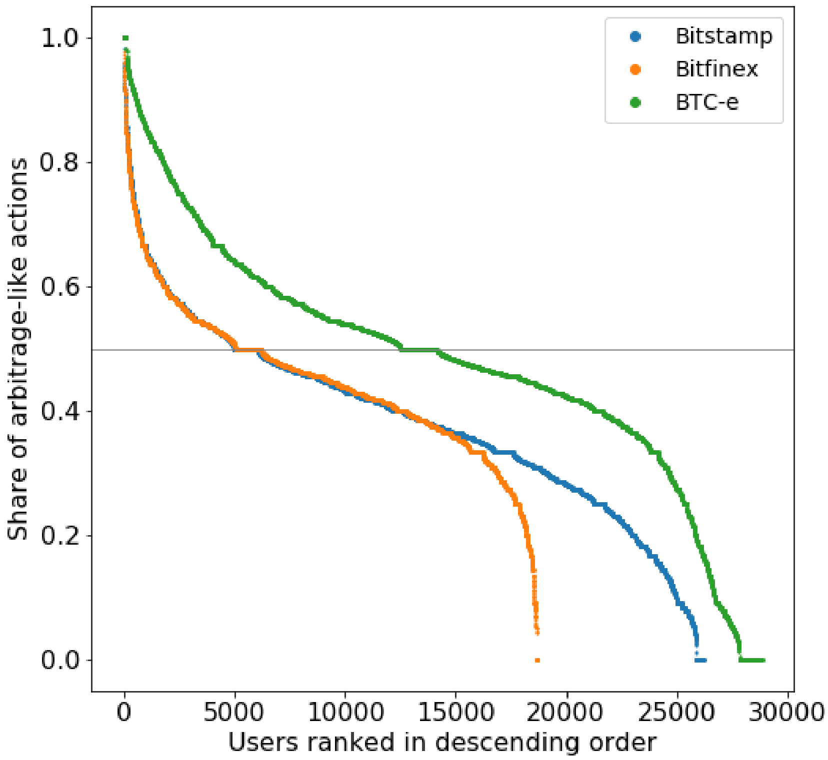


Figure 8: Share of trades that correspond to two-point arbitrage actions with another exchange, per user, evaluated independently for the counterpart exchanges Bitstamp, Bitfinex, BTC-e; users above 0.5 are treated as potential arbitrageurs. Users with less than 10 active hours are excluded from this analysis.

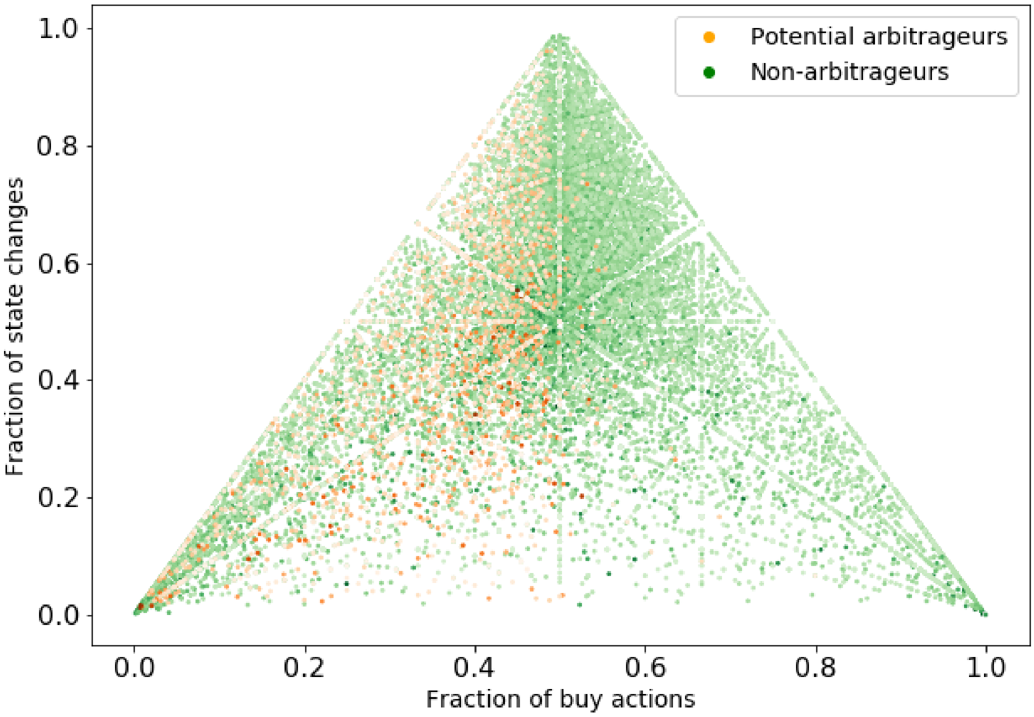


Figure 9: Scatterplot of the users detected as potential arbitrageurs with all three counterpart exchanges (orange), in contrast to all other users (green), as a function of the fraction of buy actions and of the fraction of state changes; the opacity is proportional to the number of actions made by the user.

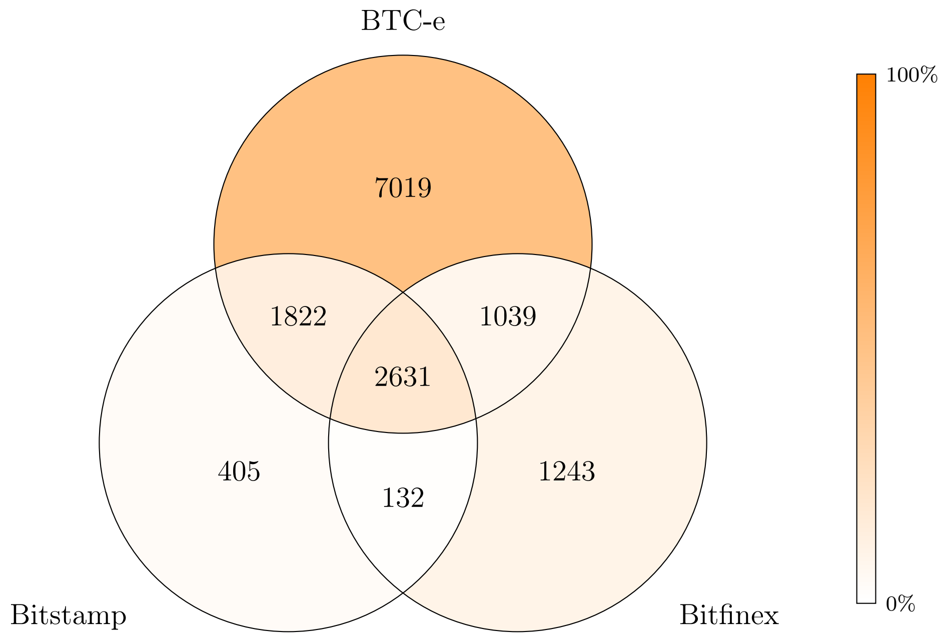


Figure 10: Venn diagram on the number of potential arbitrageurs between Mt. Gox and the three counterpart exchanges considered. The shading is proportional to the fraction of arbitrageurs in each segment.

In conclusion, we consider both the intersection and the union of the sets represented in Figure 10 to be an over-estimation of the actual set of arbitrageurs we try to identify. Thus, we argue that we need to implement post-filtering techniques to remove the false positives and to rule out the alternative hypotheses.

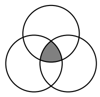
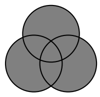
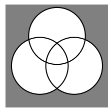
Figures 16 and 17 show the results of the same analysis on the EUR market; since we have data only for the BTC-e exchange platform, we cannot make comparisons among exchanges and the analysis is limited. We report the Figures in Appendix D.

4 Discussion

The aim of this work is to mine Mt. Gox's internal ledger in order to identify users who likely carried out two-point arbitrage between Mt. Gox and one of the three other Bitcoin exchanges. To do so, we pre-process the data sources following related work. We then construct a fee model to estimate user-specific transaction costs, and we define a similarity metric to compare users' actual trades to the actions of an 'ideal arbitrageur', given the individual transaction costs and the price differences between Mt. Gox and the three counterpart exchanges, Bitstamp, BTC-e, and Bitfinex.

At this stage, the paper mainly contributes to the literature by shedding some light on the internal dynamics of the Mt. Gox exchange platform, and in particular on the actual fee scheme paid by the users in Mt. Gox from April 2011 to November 2013. Up to now the contribution on the financial side is limited. A filtering technique is needed to rule out false positives, thus the results for the arbitrageur identification are very preliminary. Moreover, our heuristic is based on longitudinal similarity metric biased against users with few transactions: this restricts the

Table 3: Comparison of potential arbitrageurs to non-arbitrageurs

		Users with $\geq$ 10 hours of activity			All users
		Potential arbitrageurs		Non-arbitrageurs	
		with all 3 exchanges	with at least one exchange		
					
		($n = 2631$)	($n = 14291$)	($n = 18435$)	($n = 125730$)
		(a)	(b)	(c)	(d)
Total trades	mean	376.1	438.1	335.9	111.3
	median	64	72	61	10
Buy trades	mean	137.4	186.8	187.4	55.6
	median	24	27	37	5
Sell trades	mean	238.7	251.3	148.5	55.6
	median	37	41	23	3
Changes of State	mean	67.1	85.2	57.2	20.4
	median	16	18	15	1
Bitcoins bought	mean	478.2	1310.9	1457.4	395.5
	median	27.9	52.4	122.9	9.5
Bitcoins sold	mean	971.6	1738.5	1192.4	395.5
	median	39.5	90.0	57.1	3.0
'Equivalent \$' sent	mean	53944.4	54723.1	67442.1	17932.1
	median	4822.3	3610.1	4328.3	428.0
'Equivalent \$' received	mean	79250.1	66122.4	58718.0	17932.3
	median	7399.1	5560.0	2759.0	241.1
'Equivalent \$' per trade	mean	415.7	300.4	275.9	235.5
	median	194.2	122.6	117.4	91.5
Active hours	mean	69.1	83.1	58.3	22.7
	median	28	32	24	5
Active days	mean	29.3	37.2	26.0	11.3
	median	18	21	16	4
Trades per hour	mean	3.0	3.0	3.5	2.7
	median	2.0	2.0	2.3	1.8
Trades per day	mean	7.5	6.5	7.2	4.1
	median	3.4	3.3	3.7	2.2
User ID	mean	84597.2	57465.6	49513.3	62864.5
	median	95235	54688	44070	62864

A note on the meaning of 'equivalent $'. Columns (c) and (d) include users who made transactions in fiat currency other than USD. Thus, to make results comparable, we converted in USD the value of the trades denominated in different fiat currencies.

ability to comprehensively answer the question if the arbitrageurs are a few big players or many small investors.

Several other open problems remain to be solved. A general limitation of our approach and dataset is that we cannot observe individual behavior on the counterparts exchanges. Only this (or good proxies thereof) would strengthen the evidence that what we are observing is actually two-point arbitrage. The counterpart time series are strongly correlated, mainly from April 2013 on, and we hypothesized in Section 3.2 that this might bias the group of users detected as arbitrageurs on all the counterpart exchanges. The next logical step is to split the dataset into shorter and more homogenous epochs. The matching heuristic has some flaws: we do not account for time-periods without trades, and we consider hourly aggregations instead of an HFT-based approach (the work by Makarov and Schoar [31] contains useful suggestions in this regard). More conceptually, it is not trivial to interpret arbitrage with more than one counterpart exchange involved for the same user because in principle the user could engage in two-point arbitrage between pairs of exchanges not including Mt. Gox. These trades are not included in our dataset. A possible way forward is to compose ideal series with actions involving two-point arbitrage between all pairs of exchanges and consider actions that do not involve Mt. Gox as censored data points.

Even when all of these open problems are addressed, the external validity remains a concern. Can we learn anything substantial about arbitrageurs in conventional financial markets from a nascent niche market that requires technical sophistication and willingness to accept unusual risks? This question must be asked for the early years of Bitcoin as well as the emerging markets in the crypto-token ecosystem. For example, a very recent related work studying automated arbitrage on Ethereum's decentralized token exchanges reports sizable opportunities, which are routinely exploited by a range of competing trading bots [39]. However, the external validity is further compromised by an observer effect: apparently, the race was triggered by a blog post and the release of proof-of-concept code for a trading bot by members of that research team!

Disclosure: the authors have never traded on Mt. Gox.

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A Data Cleaning

Members of the Mt. Gox users community were among the first to explore the leaked dataset. Supposedly they wanted to prove misbehavior of the exchange in the events that lead to its bankruptcy on 28 February 2014. The volunteers analyzed the structure of the dataset, tried to identify potential malicious users, and pointed out key issues to keep in mind. Prior to carrying out our own analysis, we followed their example, and replicated the steps adopted by Gandal et al. [34] and Feder et al. [36], in order to remove duplicates from the dataset. However, we slightly deviate from their method in a way explained and justified in the following.

Previous works use two related methods to detect duplicates: the first one (method *Conservative*) detects rows as duplicates if the following entries are equal: *user id, timestamp, buy/sell action, amount in BTC, amount in Yen.* The other method detects rows as duplicates if the following entries are equal: *user id, timestamp, buy/sell action, amount in BTC.* The latter is more aggressive because it removes a higher number of rows, hence previous works refer to it as *Aggressive.*

However, this approach also treats as duplicates unwanted legs. Consider the case in which a user performs two exactly equivalent trades at the same moment, with the only difference that the complementary leg is executed by different trading partners: both deduplication methods mentioned above remove one of the two exactly equivalent legs. Thus, these methods reduce the dataset more than desirable.

To prevent this behavior, we slightly changed the deduplication method, by modifying the *Aggressive* one and adding the *trade id* value to the set of variables used to detect duplicates (method *TradeId*). As a result, rows are detected as duplicates if the following entries are equal: *trade id, user id, timestamp, buy/sell action, amount in BTC.*

We also implemented another deduplication technique (method *Pairs*), based on the *Aggressive* method, but the legs of a trade are not treated independently: rows are considered as duplicates *only if* both legs are duplicates. We chose this deduplication method for the analyses.

The results appear to be consistent with those obtained by previous works: the rows are slightly less than 14 million, thus the total number of trades is just below 7 million. We were able to find the 'users' Markus and Willy, who traded an amount of BTC consistent with the results reported by Gandal et al. in [34] (335,898 and 268,133 bitcoins, respectively).

To clarify the differences among the different methods, a series of example trades and the resulting deduplications are shown in the following.

Original sample. Table 4 shows the original table. It also corresponds to the deduplication results of method *TradeId*, meaning that in that specific case the *TradeId* method does not remove any duplicate. Consider the following example: rows 937 and 939 show two equal legs, having the same values for *user id, timestamp, buy/sell action, amount in BTC, amount in Yen*; however, since the *trade id* marks them as distinct, they are not treated as duplicates.

Table 4: Original sample and result of *TradeId* deduplication method

	Trade_Id	Date	User_Id	Type	Bitcoins	Money_JPY
930	35837	11-04-04 14:23	2824	buy	10.0	586.89
931	35837	11-04-04 14:23	388	sell	10.0	586.89
932	35838	11-04-04 14:23	3111	buy	10.0	578.42
933	35838	11-04-04 14:23	388	sell	10.0	578.42
934	35839	11-04-04 14:23	2824	buy	10.0	570.20
935	35839	11-04-04 14:23	388	sell	10.0	570.20
936	35840	11-04-04 14:23	3111	buy	10.0	570.00
937	35840	11-04-04 14:23	388	sell	10.0	570.00
938	35841	11-04-04 14:23	1000	buy	10.0	570.00
939	35841	11-04-04 14:23	388	sell	10.0	570.00

Table 5: Result of *Conservative* deduplication technique

	Trade_Id	Date	User_Id	Type	Bitcoins	Money_JPY
930	35837	11-04-04 14:23	2824	buy	10.0	586.89
931	35837	11-04-04 14:23	388	sell	10.0	586.89
932	35838	11-04-04 14:23	3111	buy	10.0	578.42
933	35838	11-04-04 14:23	388	sell	10.0	578.42
934	35839	11-04-04 14:23	2824	buy	10.0	570.20
935	35839	11-04-04 14:23	388	sell	10.0	570.20
936	35840	11-04-04 14:23	3111	buy	10.0	570.00
937	35840	11-04-04 14:23	388	sell	10.0	570.00

Table 6: Result of *Aggressive* deduplication method

	Trade_Id	Date	User_Id	Type	Bitcoins	Money_JPY
930	35837	11-04-04 14:23	2824	buy	10.0	586.89
931	35837	11-04-04 14:23	388	sell	10.0	586.89

Conservative. Here, instead, row 939 is considered a duplicate of row 937. To maintain the dataset coherent, both rows 939 *and* 938 are removed. Row 935 is not a duplicate of row 933 because of the difference in the value of 'Money_JPY'. Table 5 shows the result.

Aggressive. In every trade User 388 is seller at the same date and quantity. Thus, independently on the partner, all are considered as duplicates of the trade at rows 930, 931. Results are shown in Table 6.

Pairs. Here, instead, we remove only trades where *both* legs of a trade are duplicates according to the criterion *user id, timestamp, buy/sell action, amount in BTC*. Note that Trade Id is *not* considered to detect duplicates. As depicted in Table 7, pairs 934, 935 and 936, 937 are removed, while pair 938, 939 is kept, given the presence of a different user w.r.t. previous trades in the *buy* side.

Table 7: Result of *Pairs* deduplication method

	Trade_Id	Date	User_Id	Type	Bitcoins	Money_JPY
930	35837	11-04-04 14:23	2824	buy	10.0	586.89
931	35837	11-04-04 14:23	388	sell	10.0	586.89
932	35838	11-04-04 14:23	3111	buy	10.0	578.42
933	35838	11-04-04 14:23	388	sell	10.0	578.42
938	35841	11-04-04 14:23	1000	buy	10.0	570.00
939	35841	11-04-04 14:23	388	sell	10.0	570.00

Furthermore, Gandal et al. [34] claim that two users executed fraudulent activity and counterfeited trades. They were given a nickname by the Mt. Gox community:

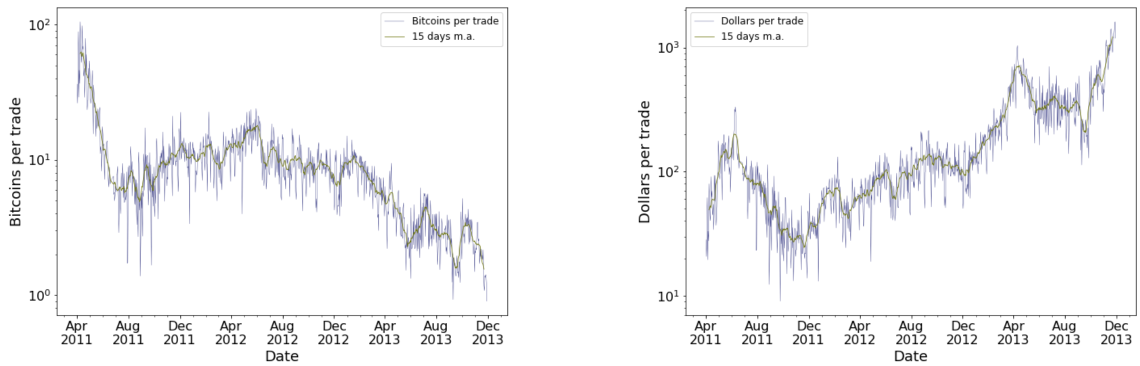
- *Markus.* While the Sell actions he performed seem to be licit trades, those in which he appears as a buyer were manipulated: the amount of fiat currency traded was copied and pasted by previous trades, and the implied exchange rates are seemingly random.
- *Willy.* This nickname has been given to 49 different users that were likely bots handled by the same person. The users that traded with Willy appear to be real, conscious users.

Since our goal is to focus on licit trades, we remove the trades that might affect the regular users' behavior. Thus, we excluded from the dataset the trades where Markus appeared as a buyer, while we kept all his Sell legs and all the Willy trades.

Finally, following the previous work by Scaillet et al. [37] and Chen et al. [38], as a last step we removed the last day from the datset and we corrected the errors caused by the multi-currency trades and by misreported data.

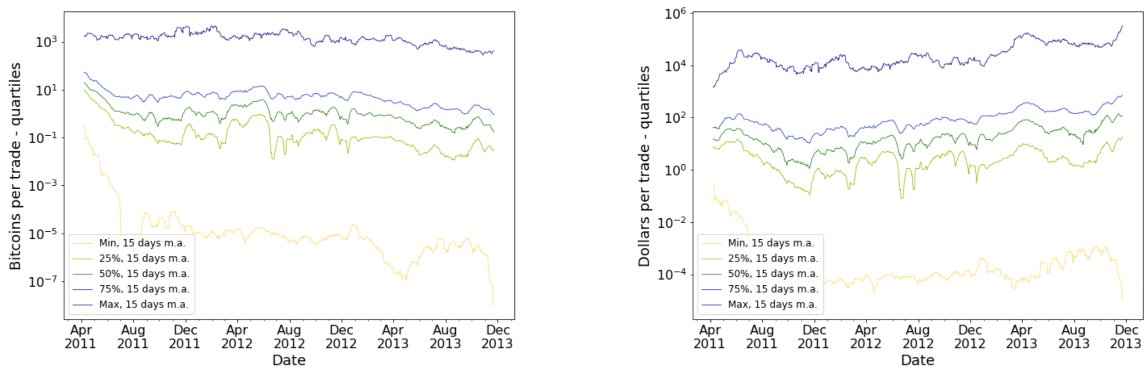
B The Mt. Gox Ecosystem

In this section we provide additional results on the evolution of the Mt. Gox ecosystem in time. Figure 11 shows how the trades evolved in time. In Panel (a) we plot the daily mean amount of bitcoins per trade and in Panel (b) the daily mean amount of dollars exchanged per trade. The first decreases over time, as one would intuitively expect given the huge positive bitcoin price variation. It is less straightforward, instead, to predict a priori the dynamics of the latter; Panel (b) shows that the mean amount of dollars exchanged per trade increased constantly in time. Figure 12 adds further information by plotting their respectives quartiles.



(a) (b)

Figure 11: Panel (a) shows the evolution of the mean amount of bitcoins per trade, while Panel (b) shows the mean amount of Dollars per trade (considering only trades denominated in USD)

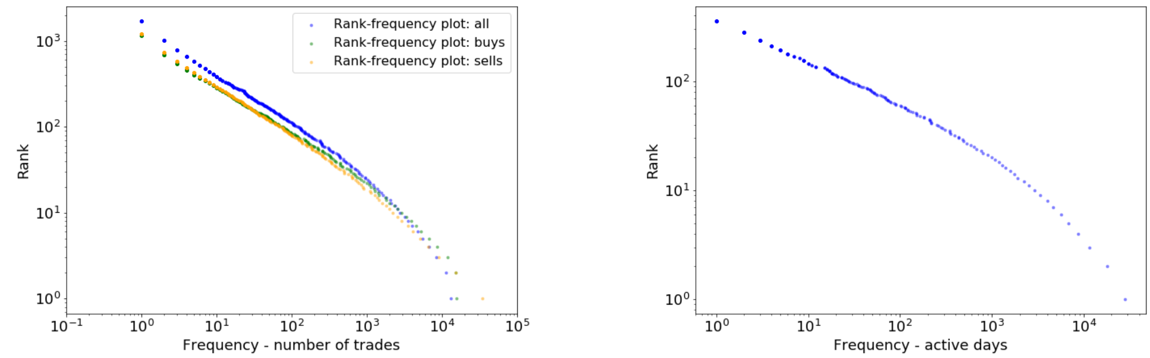


(a) (b)

Figure 12: Quartiles for (a) the amount of bitcoins per trade, and (b) the amount of Dollars per trade (considering only trades denominated in USD)

Figure 13 gives some insight on the users' heterogeneity; following the procedure described by Newman in [40], we constructed in Panel (a) the rank/frequency plot for the trades executed by each user, and in Panel (b) for the number of active days of each user. We do not report the full analysis, but similar results hold for the distributions of Bitcoins bought and sold by each user, as well as for the fiat money that they bought and sold.

Figure 14 represents the 'daily activity line' for the first 1.000 users who traded inside the Mt. Gox platform: for each row representing a user U, column values (that represent days) are white



(a) (b)

Figure 13: Heterogeneous behaviors and fat-tailed distributions: rank frequency plot for (a) the number of trades (Buy, Sell, Total) and (b) the days of activity

if U was active during that day, and black if not active. The choice of the sample is illustrative and arbitrary.

C Fees Scheme

In Section 3.1 we defined an OLS model to predict the user-specific expected fees based on the logarithm of the volume traded by each user. Here we specify an OLS model to predict fees with *linear* volumes; we report the results in Table 8.

The comparison of the two models shows that the overall pattern is similar, but in the logarithmic models the coefficients can be interpreted with less decimal digits, and the parameters' variations of signs are less frequent; in addition to that, in linear models (1) to (3) the intercept is smaller than expected (0.45%) and the interaction terms β_4 and β_5 have positive sign, in contrast with what we would expect. Regarding the explanatory power, the logarithmic model (1) outperforms the same model with linear volume ($R^2 = 0.510$ versus $R^2 = 0.097$), as well as the estimations (2) and (3). Model (4) and (5) have comparable explanatory power, but also in that case the logarithmic model has higher R^2.

Even though not directly comparable, in Kim's [41] work on the cost advantage of Bitcoin over cross-border ATM transactions the models for Bitcoin cross-border transaction costs achieve R-squared values in the order of 0.5 for the time period 2014-2015. Thus, it seems that cryptocurrency fees can be explained with linear regressions at this order of magnitude.

Finally, we propose a logit model to estimate the probability that a leg pay zero fees given user-specific and time-related variables. Using Figure 5 as a reference point, we defined a model where the binary dependent variable is 1 if the trader paid a fee, and 0 otherwise. The model is specified as follows:

$$\begin{aligned} log\left(\frac{\text{Fee}_i}{1-\text{Fee}_i}\right) &= \beta_0 + \beta_1 \cdot \text{LogVol}_i + \beta_2 \cdot \text{Bitcoins}_i + \beta_3 \cdot \text{Date}_i + \\ &+\beta_4 \cdot \text{AnomalousDays}_i + \beta_5 \cdot \text{EarlyAdopters}_i + \beta_6 \cdot \text{AnomalousUsers}_i + \\ &+\beta_7 \cdot \text{Matchers}_i + \beta_8 \cdot \text{Markus}_i + \beta_9 \cdot \text{Willy}_i + \epsilon_i \end{aligned} \tag{3}$$

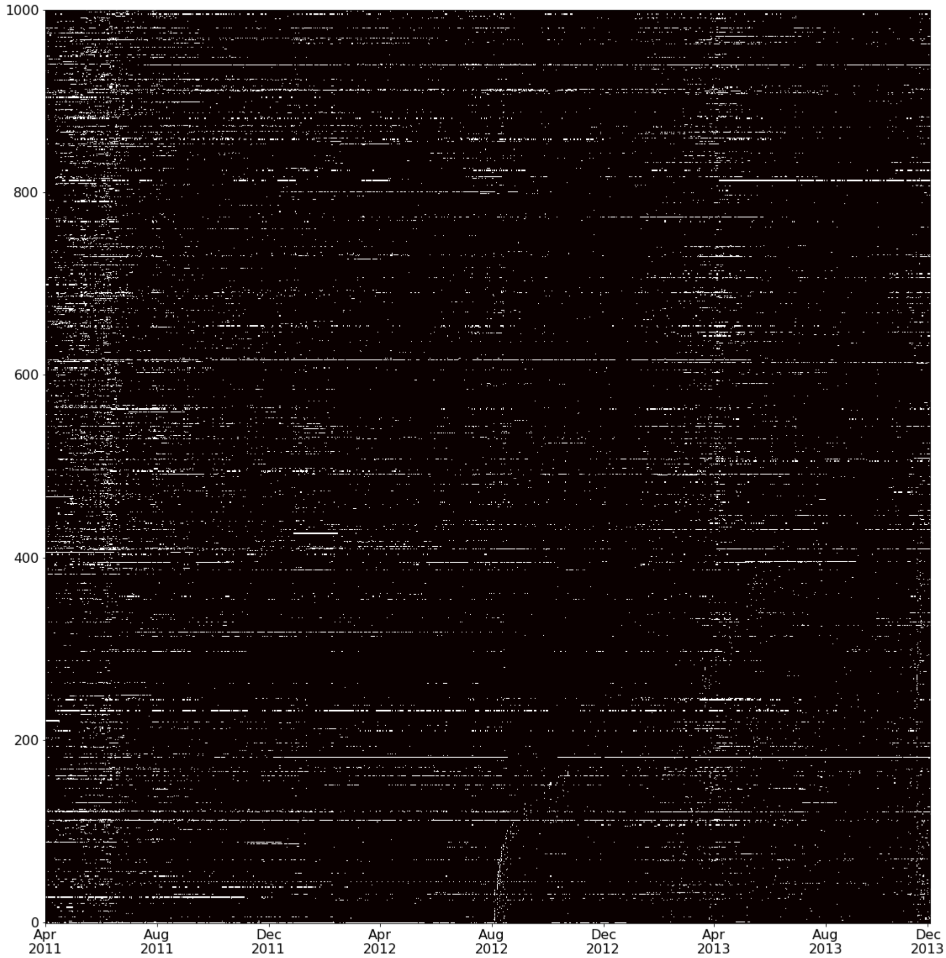


Figure 14: Daily activity map for the first 1000 users - white: active, black: not active

Table 8: Fee model for non-zero fees, coefficients fitted with OLS (linear alternative, not used)

Dependent variable:	Fee as a percentage of the amount traded				
	Specification				
	(1)	(2)	(3)	(4)	(5)
Intercept	0.454	0.456	0.468	0.559	0.561
	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0001)
LinVol	-0.000	-0.000		-0.000	-0.000
	(0.0000)	(0.0000)		(0.0000)	(0.0000)
VolSmall				-0.071	-0.070
				(0.0001)	(0.0001)
VolBig				-0.281	-0.281
				(0.0001)	(0.0001)
LinVol $*$ VolSmall			-0.000	0.000	0.000
			(0.0000)	(0.0000)	(0.0000)
LinVol $*$ VolBig			-0.000	0.000	0.000
			(0.0000)	(0.0000)	(0.0000)
T_0		0.169			0.157
		(0.0002)			(0.0001)
T_1		-0.147			-0.169
		(0.0002)			(0.0001)
T_{holid}		-0.203			-0.192
		(0.0003)			(0.0002)
R^2	0.097	0.192	0.137	0.579	0.677
Obs.	12243707	12243707	12243707	12243707	12243707

Note: All variables are significant at 0.1% level. This is due to the high number of observations; however, we emphasize that these results are intended not so much to find significant effects as to predict fees. Observations consist of legs whose fees are positive and their value is below 1%.

Table 9: Fee model for zero fees, coefficients fitted with the logistic regression

Dependent variable:	Dichotomous variable accounting for the presence of fees				
	Specification				
	(1)	(2)	(3)	(4)	(5)
Intercept	4.940	4.313	1.724	3.177	3.257
	(0.0032)	(0.0045)	(0.0019)	(0.0035)	(0.0069)
LogVol	-0.3647	-0.301			-0.025
	(0.0003)	(0.0004)			(0.0007)
Bitcoins		0.0108			0.004
		(0.0001)			(0.0001)
Date		-0.001	0.001		0.003
		(0.0000)	(0.0000)		(0.0000)
AnomalousDays			-3.283		-6.099
			(0.0034)		(0.0056)
EarlyAdopters				-0.783	-1.236
				(0.0037)	(0.0051)
AnomalousUsers				-5.530	-6.971
				(0.0045)	(0.0065)
Matchers				0.683	0.464
				(0.0048)	(0.0058)
Markus				-1.862	-2.120
				(0.0140)	(0.0146)
Willy				-0.395	-0.609
				(0.0193)	(0.0302)
$pseudo - R^2$	0.152	0.224	0.101	0.469	0.664
Obs.	13990738	13990738	13990738	13990738	13990738

Note: All variables are significant at 0.1% level. This is due to the high number of observations; however, we emphasize that these results are intended not so much to find significant effects as to predict fees.

and the independent variables have the following meaning:

- Bitcoins: the amount of bitcoins traded;
- Date: date in which the trade was executed;
- EarlyAdopters: dummy variable equal to 1 for the first $16,000$ user IDs in sequential order;
- AnomalousDays: dummy variable equal to 1 for the days with an anomalous presence of zero fees trades;
- AnomalousUsers: dummy variable equal to 1 for the Users responsible for the majority of the zero fees, as described in Section 3.1;
- Matchers: dummy variable equal to 1 for the legs whose complementary leg was executed by one of the aforementioned Anomalous Users;
- Markus: dummy variable for the User ID associated to Markus;
- Willy: dummy variable for the User ID associated to Willy;

Models (1) to (3) have very low pseudo-R^2; the largest part of the explanatory power of the model is associated with the variables that contain information on the Users IDs. Coefficients β_1, β_2, and β_3 are very small, suggesting that their effect is significant but limited. As expected, instead, the variables AnomalousDays and AnomalousUsers are associated with negative and high coefficients.The probability that fees are paid decreases for the users defined as early adopters. Unexpectedly, the variable Matchers is associated to higher probability that the order might pay a fee.

D Supplemental Figures and Tables

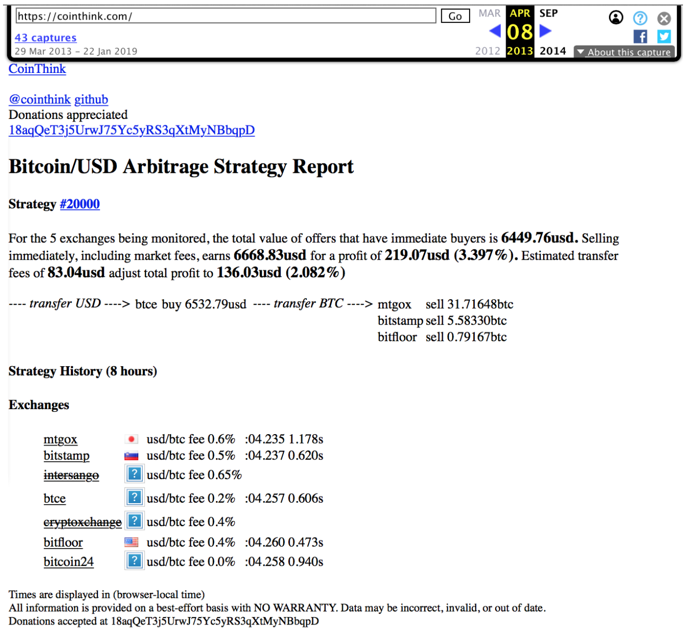


Figure 15: Example of an online arbitrage tool (captured by the Internet Archive on 8 April 2013; last access: 4 May 2019)

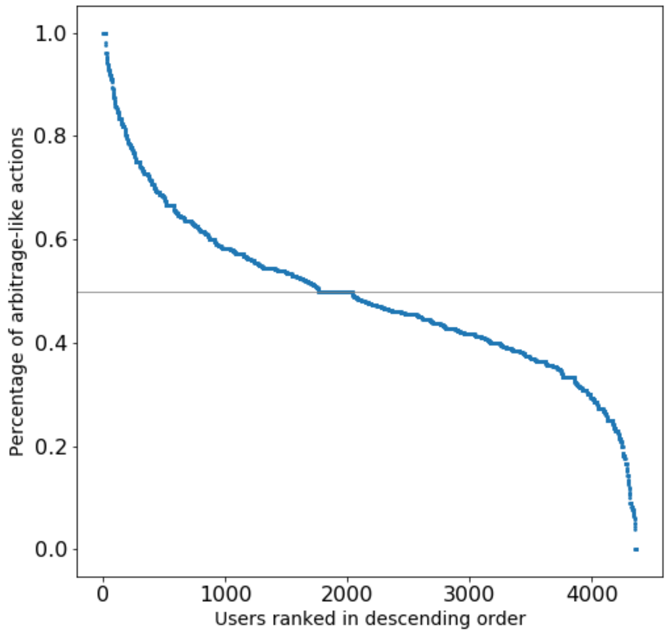


Figure 16: Share of trades that correspond to two-point arbitrage actions with the BTC-e exchange for the EUR market, per user; users above 0.5 are treated as potential arbitrageurs. Users with less than 10 active hours are excluded from this analysis.

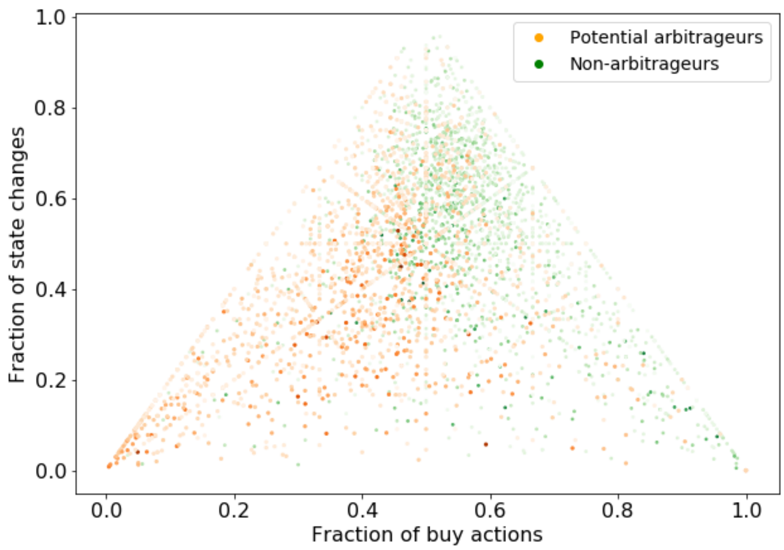


Figure 17: Scatterplot of the users detected as potential arbitrageurs in the BTC-e EUR market (orange), in contrast to the non-arbitrageur users (green), as a function of the fraction of buy actions and of the fraction of state changes; the opacity is proportional to the number of actions made by the user.