Who Wants Affordable Housing in their Backyard? An Equilibrium Analysis of Low Income Property Development

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Abstract

We nonparametrically estimate spillovers of properties financed by the Low Income Housing Tax Credit (LIHTC) onto neighborhood residents by developing a new difference-in-differences style estimator. LIHTC development revitalizes low-income neighborhoods, increasing house prices 6.5%, lowering crime rates, and attracting racially and income diverse populations. LIHTC development in higher income areas causes house price declines of 2.5% and attracts lower income households. Linking these price effects to a hedonic model of preferences, LIHTC developments in low-income areas cause aggregate welfare benefits of $116 million. Affordable housing development acts like a place-based policy and can revitalize low-income communities.

1 Introduction

Increasing geographic income segregation and rising housing costs have put the issue of the government's role in promoting affordable housing at the forefront of current policy debates.[1] Subsidized housing policy often focuses on easing low income households' housing costs and providing access to financially out of reach neighborhoods. However, subsidized housing is also a place-based policy. Housing subsidies influence households' choices of neighborhoods and developers' choices of where to build. Subsidy induced changes in the locations of households and housing construction can have important spillovers onto the neighborhood residents.[2] Moreover, these place-based spillovers likely have large economic impacts across the US, as federal, state and local governments spend over $97 billion dollars a year on different forms of housing assistance. A key question is thus how to best allocate affordable housing across neighborhoods.

In this paper, we provide an analysis of the costs and benefits of affordable housing construction to surrounding neighborhood residents and how they vary across demographically different neighborhoods. We study the neighborhood impacts of multifamily housing developments funded through the Low Income Housing Tax Credit (LIHTC). Established in 1986, this program has become an integral component of federal housing policy, funding 21 percent of all multifamily developments over the period 1987-2008. Looking forward, with the construction of publicly run housing projects expected to continue to decline, the LIHTC program is likely to remain one of the main federal government initiatives designed to ensure access to affordable housing by low income households.[3]

We combine data on the location and funding dates for all LIHTC funded projects, housing transaction data from 129 counties, and home buyer race and income data to estimate the effects of LIHTC construction on the surrounding neighborhood. Our estimates show that the impact of affordable housing construction has dramatically different effects on surrounding property values based on whether the affordable housing was built in a relatively richer or poorer neighborhood and whether the neighborhood has a high share of minority residents. LIHTC construction in neighborhoods with a median income below $26,000 in-

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[1] New York City and San Francisco have both announced plans for large expansions of affordable housing units to "ensure diverse and inclusive neighborhoods." Upon entering office, Bill de Blasio, mayor of New York City, unveiled a plan to create and preserve 200,000 units of affordable housing over ten years. In 2014, the mayor of San Francisco, Ed Lee, outlined an initiative to create 30,000 affordable housing units over six years to "ensure San Francisco remains a place where people from every background can call home.".

[2] Previous research suggests households are willing to pay to live near higher income and more educated neighbors (Bayer et al. (2007); Guerrieri et al. (2014); Diamond (2015)). The quality of the housing stock also spills over onto the value of neighboring houses (Rossi-Hansberg et al. (2010), Campbell et al. (2011)).

[3] Section 8 housing vouchers which provide rental subsidies to low income households who rent in the private rental market is the main alternative federally run low-income housing program.

creases local property values by approximately 6.5% within 0.1 miles of the development site In contrast, LIHTC construction in neighborhoods with median incomes above $54,000 leads to housing price declines of approximately 2.5% within 0.1 miles of the development site. These declines, however, are only seen in high income areas with a minority population of below 50%.

To account for these price impacts, we explore how LIHTC development affects other characteristics of the local neighborhood, in particular demographics and local crime rates. We find that the construction of a LIHTC development attracts higher income home buyers in low income areas. Conversely, affordable housing development attracts lower income home buyers in higher income areas with low minority populations. Examining the impact of LIHTC construction on the share of Black home buyers, we find that the introduction of affordable housing leads to decreased segregation in lower income areas. Finally, LIHTC development causes declines in both violent and property crime within low income areas, but does not increase crime in high income areas.

We identify these effects by exploiting the timing of when funding is granted for the development along with the exact geographic location of the affordable housing. Clearly, the neighborhoods targeted by developers to build affordable housing are non-random. However, the timing of the funding is often out of the hands of the developer since there is substantial uncertainty in which year the project will be funded.[4] Further, the exact geographic location of the development site within a broader neighborhood appears to be determined by idiosyncratic characteristics, such as which exact plot of land was for sale at the time.

To harness this identification strategy, we develop a new econometric method for estimating a difference-in-differences style estimator in a non-parametric setting where treatment is a smooth function of distance to the LIHTC site and time since LIHTC funding. We draw on new methods developed in statistics (Charnigo et al. 2011, Charnigo and Srinivasan, 2015) to transform our data on house price levels to data on the derivative of house prices with respect to distance from LIHTC sites. These transformed data allow us to flexibly difference out very local time trends and neighborhood variation in housing prices. Further, by viewing house prices as a smooth function of geographic location, we show how to generalize discrete geographic fixed effects in house prices to a smooth, time-invariant surface of house prices.

Employing a structural, generalized hedonic model of housing choice along the lines of Rosen (1974) and Bajari and Benkard (2005), we translate our estimated price effects into households' preferences for living near LIHTC. The hedonic model allows us to view real estate as a *continuous* choice of quantities of housing and neighborhood characteristics. We

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[4]Developers must apply for LIHTC funds. Acceptance rates vary across states. In California in 2012, only 42.7% of submitted applications received funding.

specifically focus on the continuous choice of distance in miles to a LIHTC development. The key advantage of this approach is that it ensures the existence of an equilibrium price surface, which continuously maps housing and neighborhood characteristics to house prices without needing to specify the determinants of housing supply. As a result of these two key model properties, an agent's optimality condition reduces to a simple equation relating the marginal cost of moving further from a LIHTC site to its marginal benefit, which allows us to recover preferences on an individual basis.

We find that the average household that desires living near LIHTC sites in low income areas is willing to pay approximately 6% of their house price to live 0.1 miles from a LIHTC site. In higher income areas with low minority populations, on the other hand, the average household who chooses to live near LIHTC is willing to pay approximately 1.6% of their total house price to avoid living within 0.1 miles of a LIHTC site.[5] Correlating these preference estimates with information on home buyer demographics, we further find that higher income households are more willing to pay for proximity to or distance from the LIHTC site, consistent with results of Diamond (2016).

We finally use our structural framework, our empirical preference estimates, and census data to calculate the local welfare impact of introducing affordable housing to different types of neighborhoods.[6] We decompose the effect into the welfare impacts of affordable housing on homeowners, renters, and absentee landlords.[7] Our analysis reveals large possible societal gains from building affordable housing in low income areas, with construction of LIHTC in low income, low minority areas increasing total welfare by approximately $116 million. In contrast, building LIHTC in low minority, high income areas leads to losses of approximately $12 million.

Moving LIHTC properties from higher income to lower income neighborhoods may therefore benefit both the residents of the higher and lower income neighborhoods. Of course, these neighborhood benefits must be weighed against the cost and benefits which accrue to the tenants of the affordable housing. Chetty et al. (2016) find that moving young children from high poverty public housing to low poverty areas increases these children's future earnings by a present discounted value of $100 thousand. This effect is not large enough to overcome our estimated benefits to low-income neighborhoods simply because there are many more low income households living in a low-income area than in the affordable housing

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[5]While these households who choose to live near LIHTC sites in high income areas dislike LIHTC proximity, they find it optimal to live there since LIHTC proximity also provides a discount on their home prices. On net, these households prefer to live closer to LIHTC sites in high income areas than to live further away.

[6]A caveat to our local welfare calculation is that we cannot account for possible general equilibrium effects on neighborhoods far away from the LIHTC site.

[7]We do not directly observe rents and renters' location choices, which require us to make some assumptions which allow us to adjust our homeowner results to speak to renter and landlord welfare.

development itself.

Previous work studying the welfare effects of place-based policy subsidizing firm locations has often found it challenging to identify heterogeneous effects of these policies across geographic areas. As stated in Glaeser & Gottlieb (2011) "For these externalities to create a justification for any particular spatial policy, these externalities must be stronger in some places than in others...Economics is still battling over whether such spillovers exist at all, and we are certainly not able to document compelling nonlinear effects." We are able to both document significant spillovers and heterogeneity in these effects across neighborhoods.

A small number of previous studies have examined the impacts of affordable housing on local neighborhoods. Eriksen and Rosenthal (2010) study the crowd-out effects of subsidized affordable housing construction on private rental development and find large crowd-out effects. More closely related, Baum-Snow and Marion (2009) use census data and a regression discontinuity approach to study the effects of LIHTC financed developments in low income neighborhoods on new construction, median incomes, and property values at the census block group level. They also find that housing prices appreciate in low income areas. Schwartz et al. (2006) look at the price impact of affordable housing in New York City and report positive results. Freedman and Owens (2011) study the impact of LIHTC developments on crime at the county level and find mixed results. Our study leverages extensive micro data to study highly local effects of affordable housing in many different parts of the United States. By looking across a wide array of neighborhoods and counties, we show how affordable housing has dramatically different effects on neighborhood residents based on neighborhood income and the minority share of the neighborhood population. Previous studies either focus on a single geographic area (Schwartz et al. 2006, Goujard 2011) or only within low income neighborhoods at a single point in time (Baum-Snow and Marion, 2009). Moreover, none of these studies utilize a structural framework in conjunction with detailed data on buyer characteristics to recover and put structure on individual preferences for proximity to affordable housing.

More broadly, our paper is related to a literature which examines the spillovers to neighborhoods of housing policies. Rossi-Hanbserg et al. (2010) study the impact of urban revitalization programs implemented in the Richmond, Virginia area on local land prices. Campbell et al. (2011) examine the effects of housing foreclosure on housing prices nearby. Ellen et al. (2013) look at how foreclosures impact local crime rates. Autor et al. (2014, 2015) study the impact of ending rent control on nearby real estate prices and crime rates.

Finally, a growing literature has found that higher income individuals are willing to pay more for local neighborhood amenities (Bayer et al (2007), Diamond (2016), Handbury (2013)). A number of recent papers have also argued that higher income or more educated

neighbors endogenously improve local amenities (Card, Mas, and Rothstein (2008), Bayer et al (2007), Guerrieri et al (2014), Diamond (2016)). However, previous work has not had access to micro-level demographic and housing transaction data. Further, LIHTC development provides a quasi-experimental shock to the income mix of ones' neighbors and allows us to identify the distribution of households' preferences for proximity to low-income neighbors.[8]

The paper proceeds as follows. Section 2 provides institutional background detail about the Low Income Housing Tax Credit. Section 3 describes our data sources and Section 4 discusses our estimation procedure for the pricing surface. We present our reduced form results on price and other neighborhood characteristics in Section 5. Section 6 details the hedonic model of housing choice, presents our preference estimates, and conducts welfare analysis. Section 7 concludes.

2 The Low Income Housing Tax Credit

Since its inception in 1986, the Low Income Housing Tax Credit Program has been an integral component in fostering the development of multifamily housing throughout the United States. With an annual tax credit valued at over 8 billion dollars, the program funded 21 percent of all multifamily developments between the years 1987-2008.

Each year, federal tax credits are allocated to the states based on population. These credits are awarded by state authorities to developers of qualified projects. Developers then sell these credits to investors to raise equity capital for their projects and reduce the amount of debt they would otherwise have to borrow. Investors receive a dollar-for-dollar credit against their federal tax liability for a period of 10 years, provided the property continues to comply with all program guidelines.[9]

To qualify for a tax credit under the Low Income Tax Credit Program, federal guidelines require that proposed projects be for construction or rehabilitation of a residential rental property and satisfy either one of two low-income occupancy criteria. At least 20 percent of tenants must earn less than 50 percent of the Area Median Gross Income (AGMI), or alternatively, at least 40 percent of tenants must earn less than 60 percent of AGMI.[10]

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[8]LIHTC development also impacts the overall population density and average age of the neighborhood housing stock. We can't fully separate out preferences for the income level of one's neighbors from preferences for new construction or increased density. However, the differential value of LIHTC development across neighborhoods of different income levels help zoom in on preferences over the income of one's neighbors.

[9]Eriksen (2009) studies the market pricing of these tax credits. He finds that that LIHTC developers in California received on average $0.73 per $1 of tax credit in the years 1999-2005.

[10]Actual income limits depend on household size. The 50 percent of AGMI limit is for a base family size of four members. Income limits are adjusted upward by 4 percentage points for each family member in excess of four. Limits are adjusted downward by 5 percentage points for each family member short of four. These limits are multiplied by 1.2 to get the 60 percent income limits.

If the income of a household in one of the low income units grows to exceed the relevant income limit, then the program requires developers to place a low income tenant into the next unit vacated by a market rate tenant.[11] Additionally, developers must restrict rents, including utility allowance, in low-income units to 30 percent of the relevant income limit, i.e. either 50 percent of 60 percent of AGMI for a minimum affordability period of 30 years.

These criteria are only the minimal requirements as specified by the federal government. In practice, states almost always receive many more project proposals and tax credit allocation requests from developers than they have federal allotments, generally on the order of 2 to 4 times. Each state is therefore required to maintain a "Qualified Application Plan" (QAP) to govern the selection process. These plans usually operate by assigning point scores to various project characteristics and then allocating tax credits based on point totals until funds are exhausted. Such project characteristics include tenant demographics, location, further funding sources, and structural properties of the building. Given this latitude the states enjoy in determining selection criteria, many require developers to go beyond the minimum number of affordable units and the minimum level of affordability.

The funding process in California, in which more than 50 percent of the LIHTC sites in our sample are located, is illustrative of the high degree of competition which exists between developers. Practically all funded projects have the maximum total point score, such that the funding threshold is based on so-called tie-breaker scores. Receiving funding in a given year is by no means guaranteed. According to data provided by the California State Treasurer's Office, only 34.3% of applications were funded in 2007, 38.3% in 2008, 32.2% in 2009, 26.9% in 2010, 57.7% in 2011, and 42.7% in 2012.

The value of tax credits received by selected developers is calculated according to the project's "qualified basis", which essentially reflects the cost of constructing or rehabilitating the low-income units. Once the qualified basis has been determined, the annual tax credit is determined by applying the relevant housing tax credit rate. New construction or substantial rehabilitation projects, which are not otherwise subsidized by the Federal government, receive a 9 percent credit rate, while all other projects receive a 4 percent credit rate. These annual credits are then paid out over a period of 10 years.[12] Section A.1 of the online appendix provides further details on the LIHTC program.

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[11]Many LITHC properties are comprised 100 percent of low-income units. Clearly, this requirement becomes superfluous in such a case.

[12]This calculation is a baseline figure. Congress passed legislation in 1989 affording state allocating agencies the option to increase the qualified basis by up to 30 percent in both "qualified census tracts" (QCTs) and "difficult development areas" (DDAs). Census tracts with 50 percent of households earning below 60 percent of AGMI earn qualified status, subject to a population restriction which is generally non-binding. Metropolitan areas with high ratios of fair market rent to AMGI are designated as DDAs. See Baum-Snow and Marion (2009) for more details.

3 Data

We bring together data from a variety of sources. Our first dataset is from DataQuick, which provides detailed public records on housing characteristics and transactions data collected from county assessor and register of deeds officers. We restrict our analysis to those counties which have transactions history data dating to at least 1996. This leaves us with a sample of approximately 16 million transactions located within 1.5 miles of a LIHTC site in a total of 129 counties in 15 states. Figure 1 provides a map of the counties in our sample. We merge this dataset with data collected by the United States federal government according to the provisions of the Home Mortgage Disclosure Act (HMDA), which provides us with the race and household income of the home buyers.

Information on LIHTC financed projects is provided by the Department of Housing and Urban Development (HUD). This data covers 39,094 projects and almost 2,458,000 low income housing units placed into service between the years 1987 and 2012. Our analysis focuses on the 7098 LIHTC projects located in our sample of 129 counties. See Panel B of Table 1 for summary statistics. Due to DataQuick's coverage of counties, our sample is from more dense, urban areas, relative to the overall distribution of LIHTC sites. However, when comparing the characteristics of LIHTC sites developed in urban areas to our 129 county subsample, our sample looks quite representative.

We finally collect 1990 census data at the tract and block group level. These data provide information on median income levels and minority population shares.

Panel A of Table 1 provides summary statistics. Compared to the United States as a whole, the counties in our sample have a similar black share (11.6% vs 12.1% nationwide), a significantly higher Hispanic share (15.3% vs 8.3% nationwide), a higher renter share (40% ver 34%), and a median income approximately 18% greater. Median income and renter share is higher than the rest of the United States since our sample comprises urban areas and metropolitan areas in relatively high productivity areas such as New England, New York and California. Hispanic share is quite large in our sample since many of our counties are in California and the Southwest, which host a large Hispanic population compared to the rest of the country. Within the census block groups which receive LIHTC developments, the Hispanic and Black share are even higher at 24.0% and 23.6%, respectively. The median incomes are also 33% lower and renter share is 23 percentage points higher in these select block groups than average block groups within our sample of counties.

Panel C of Table 1 also provides summary information about locales within 1 mile, 0.5 miles, 0.2 miles, and 0.1 miles of projects financed through the Low Income Housing Tax Credit program. Average housing prices are about 7% lower and average home buyer

incomes are 5% lower within half a mile of a LIHTC site than those within 1 mile of an LIHTC site. The percent of home buyers which are Black is 11% higher within half a mile of an LIHTC site than within one mile. It is clear LIHTC development is targeted at lower income, higher minority share, and lower housing cost areas. However, housing prices, home buyer incomes, and the Black share of home buyers are quite similar between areas within 0.2 miles of an LIHTC site and those within 0.1 miles. The lack of variation in neighborhood characteristics at these more fine geographic measures help substantiate our identification strategy that precise geographic location of LIHTC development provides quasi-experimental variation. The online appendix reports these summary statistics separately by neighborhood income level and show similar patterns. See Section A.2 of the online appendix for more details on all data sources.

4   Non-Parametric Differences-in-Differences Estimation

Our first goal is to study the reduced-form effect of LIHTC development on local house prices. Our research design to identify the causal effects of LIHTC development on local house prices will use a non-parametric spatial difference in differences strategy. Intuitively, we will compare house prices very close to the LIHTC site before and after LIHTC development versus house price trends slightly further away from the LIHTC site. This allows us to recover the house price impacts of LIHTC developments and how they vary with distance from the LIHTC site and time since development.

Previous research using this style of identification strategy has used a "ring method," where an inner ring is drawn around the treatment location and an outer ring (with a larger radius) is drawn to use as a control group for the inner treated ring. Annual comparisons of house prices in the inner versus outer rings are used to identify the treatment effect. Our method generalizes this concept. Instead of estimating the difference in average house prices between the inner and outer rings around a LIHTC site, we recover a non-parametric function of house prices that illustrates how prices vary with distance from the LIHTC site and the time since development. This gives a more detailed picture of how LIHTC impacts house prices over time and space. It also allows us to recover the gradient of house prices with respect to LIHTC proximity, which will be needed for our hedonic model to recover households' preferences for LIHTC proximity. We find our approach offers a number of benefits over the ring method, which we will discuss in detail. As a robustness test, we will also perform the standard "ring method" analysis.

4.1 The spatial non-parametric differences-in-differences setup

To estimate the non-parametric spatial differences in differences we present a new estimation method using empirical derivatives which we will discuss below. We develop a formal econometric model where we index all LIHTC locations by a unique geographic area l, corresponding to the 1.5 mile radius circle surrounding the LIHTC site. Each location l will have a type $\mathbf{Y} \in \mathcal{L_Y} \subset \mathbb{R}^L$, with $\mathcal{L_Y}$ finite, reflecting the characteristics of the census block group in which the LIHTC site is located; in particular, we focus on median income and minority share. We let individual houses be denoted by j. Within a neighborhood l surrounding a LIHTC site, we associate with each house j its polar coordinates (r_j, θ_j) relative to the LIHTC site in neighborhood l. We denote calendar year by t.

Following a differences-in-differences approach, we assume house prices in neighborhood l are given by:

$$\log p_{jt} = \widetilde{m}_{\mathbf{Y}}\left(r_j, \tau_j\right) + \phi_l\left(r_j, \theta_j\right) + \varphi_l\left(\theta_j, t\right) + \varepsilon_{jt}, \tag{1}$$

where $\widetilde{m}_{\mathbf{Y}}\left(r_j, \tau_j\right)$ is the impact of LIHTC construction on the house transaction price in a neighborhood of type $\mathbf{Y}$. Specifically, r_j measures the distance in miles of house j from the LIHTC site and τ_j measures the difference in years between the transaction year of house j and the LIHTC site announcement date. The non-parametric function $\phi_l\left(r_j, \theta_j\right)$ denotes neighborhood-specific, location "fixed effects."[13] In a standard difference-in-difference framework, treatment is usually assigned to discrete entities (e.g. people, cities, firms), however we generalize this to the continuous case and allow for a smooth surface of housing prices across geographic locations that do not depend on time.[14] The non-parametric function $\varphi_l\left(\theta_j, t\right)$ allows for a distinct time trend for neighborhood l, which also could potentially vary based on θ.[15] The error ε_{jt} recognizes that some housing and neighborhood characteristics will have prices that vary in time in ways that jointly depend on r_j and θ_j beyond this additively separable setup.

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[13]We do not analyze or model possible interaction effects of multiple geographically proximate LIHTC sites. We do not have enough data to study such effects.

[14]One could consider discretizing the geography into something like census tracts or even individual houses and including census tract or house fixed effects in the regression. However, this suffers from a bias/variance trade off where if the geographic units are too large, the regression does not adequately control for neighborhood variation in house prices. However if the geographic units are too small (house fixed effects) a large amount of information is thrown away. In the case of house fixed effects, we would only be able to use repeat sales of the same houses to identify our estimates, even if there were many houses that only transacted once but were located right next to each other. The smooth surface of housing prices over geography attempts to deal with this bias/variance trade off more efficiently.

[15]For example, it could be that in some neighborhoods there is more house price appreciation to the north than to the south for reasons unrelated to LIHTC development. While not controlling for θ is unlikely to bias our estimates, it can help with regard to efficiency.

Crucially, we have assumed that these local price trends do not depend on r. Then, under the identifying assumption that local housing price trends unaccounted for by our location "fixed effects" $\phi_l(r_j, \theta_j)$ and neighborhood time trends $\varphi_l(\theta_j, t)$ are independent of distance to the LIHTC site, r_j, and year since LIHTC funding, τ_j :

$$E\left(\varepsilon_{jt}|\widetilde{m}_{\mathbf{Y}}(r_j, \tau_j), \phi_l(r_j, \theta_j), \varphi_l(\theta_j, t)\right) = 0,$$

we can recover a consistent estimate of the gradient of $\widetilde{m}_{\mathbf{Y}}(r_j, \tau_j)$ with respect to distance. That is, while the overall neighborhood is likely an endogenous choice by developers, due to the unpredictable nature of whether LIHTC funds will be awarded to a given applicant, it is hard for developers to strategically time LIHTC funding with highly local house price trends. Further, due to highly local supply constraints such as the exact location of available lots, the placement of a low-income property is plausibly exogenous with respect to *highly local* price trends. Intuitively, we can obtain a consistent estimate of the non-parametric price gradient by examining price changes close to a LIHTC property vs price changes slightly further away and then using differences-in-differences to "difference out fixed effects". Furthermore, to the extent that the treatment effect ultimately decays towards zero with distance within this area, we can estimate the overall level treatment effect by integrating our estimate of the gradient.

While we set up the estimation in a non-parametric framework, such a strategy has been pursued in previous papers, albeit in a more parametric form. This more parametric approach would compare price changes in an inner circle of certain radius to price changes in an outer ring of certain radius that surrounds the inner ring. The inner ring would be thought to receive the treatment, while the outer ring would act as the control. Examples of this approach include Currie et al. (2013), Autor et al. (2014), Aliprantis and Hartley (2014), and Shoag and Veuger (2015).[16] While this method is quite simple, it has a number of drawbacks. First, it requires the researcher to choose an arbitrary cutoff in distance between the treatment and control rings. Second, it is susceptible to a substantial missing data problem, as it is often the case that at narrow ring choices there could be years in which either the inner or outer ring have no housing transactions. Third, it makes no attempt to control for the time-invariant variation in house prices within each of the two rings, which can lead to substantial variance in the estimator. Fourth, the estimator delivers an average difference in house prices between two rings, but not a derivative at specific distances, which is what is needed for a hedonic model. However, this estimate could be enough if the researcher were only interested in reduced form average effects on house prices. In Section

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[16]Autor et al. (2014) approaches this problem slightly differently using exponential weighting based on distance.

B of the online appendix, we perform simulation studies comparing our setup to the ring method. We find both estimators produce consistent results, but the standard errors on the ring methods are much wider and often cannot reject a zero effect even for modest sized point estimates, which are strongly statistically significant using our non-parametric framework.

4.2   The empirical derivatives estimator

Standard estimation methods for estimating additively separable non-parametric functions, as we have in equation (1) are very computationally challenging to work with when there are many additively separable non-parametric functions.[17] Instead, we build on new methods developed in the statistics literature by Charnigo et al. (2011) and Charnigo and Srinivasan (2015) which allow us to directly estimate the *derivative* of $\widetilde{m}_{\mathbf{Y}}(r_j, \tau_j)$ by "differencing out" the additively separable nonparametric functions. The idea is to estimate gradients of the pricing surface using empirical partial derivatives and then to use kernel regression as a smoothing procedure.   The key advantage of these methods is that they provide a way to difference out the fixed effects and time trends in a spatial, non-parametric setting, just as one would do in a fully parametric differences-in-differences design. The procedure provides substantial computational and efficiency gains over the alternative of estimating the full non-parametric surface in levels and then taking derivatives.   We outline the methodology here.

First, at each housing transaction data point $(r_\iota, \theta_\iota, t_\iota)$, we create "empirical derivatives" of the log house price surface with respect to the distance r to the LIHTC site . Note, ι is simply an indexing variable over all house transaction locations where we will evaluate the empirical derivative. Intuitively, these are just finite differences in house prices with respect to distance from the LIHTC site. To do this, we find housing transactions with similar values of θ and t, but different distances from the LIHTC site. In an ideal world, we would want to use house prices with identical values of θ and t and calculate numerical derivatives of house prices with respect to distance from the LIHTC site. Taking numerical derivatives with respect to r allows us to difference out any time trend in the neighborhood, since the time trend shouldn't vary with distance from the LIHTC site. We then analyze how these numerical derivatives change relative to a base year, allowing us to difference out the time-invariant surface of house prices.

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[17]The standard method is a procedure called back-fitting which is a Gauss-Seidel algorithm where one estimates an individual non-parametric function, given a guess of the other additively separable non-parametric functions. The procedure loops overs each non-parametric function, given the best guess of the others until the method converges. See Hastie and Tibshirani (1990) for more details. This method would be incredibly computationally challenging as we would have to estimate two nonparametric functions for each LIHTC site l along with $\widetilde{m}_{\mathbf{Y}}(r_j, \tau_j)$. This would require iterating over more than 14,000 functions.

To be precise, let $\widetilde{Y}_\iota$ be the empirical derivative at $(r_\iota, \theta_\iota, t_\iota)$ within neighborhood l constructed according to:

$$\widetilde{Y}_{\iota,l} = \sum_{k=1}^{\kappa_n} w_k \frac{\log p_{a(k,\iota,r)} - \log p_{b(k,\iota,r)}}{r_{a(k,\iota,r)} - r_{b(k,\iota,r)}} \tag{2}$$

$$w_k = \frac{k}{\kappa_n (\kappa_n + 1)/2} \tag{3}$$

with the observation subscripts recursively defined by:

$$a(1,\iota,r) = \arg \min_{\{d \in L_{r,\iota} : r_d > r_\iota + l_n\}} r_d, \quad b(1,\iota,r) = \arg \max_{\{d \in L_{r,i} : r_d < r_\iota - l_n\}} r_d \tag{4}$$

$$a(k,\iota,r) = \arg \min_{\{d \in L_{r,\iota} r_d > r_{a(k-1,\iota,r)}\}} r_d, \quad b(k,\iota,r) = \arg \max_{\{d \in L_{r,\iota} : r_d < r_{b(k-1,\iota,r)}\}} r_d \tag{5}$$

where $l_n > 0$.[18]

$$L_{r,\iota} := \left\{ p \in \{1,...,n\} : \frac{(t_p - t_\iota)^2}{(r_p - r_\iota)^2} < \vartheta_n^t, \frac{(\theta_p - \theta_\iota)^2}{(r_p - r_\iota)^2} < \vartheta_n^\theta \right\} \tag{6}$$

Note, again, that ι simply indexes our empirical derivative observations. Equation (2) calculates a numerical derivative with respect to LIHTC distance as the difference in house prices $(\log p_{a(k,\iota,r)} - \log p_{b(k,\iota,r)})$ between a pair of houses $a(k,\iota,r)$ and $b(k,\iota,r)$, divided by the difference in LIHTC distance $(r_{a(k,\iota,r)} - r_{b(k,\iota,r)})$. Equation (2) then calculates these numerical derivatives for κ_n pairs of houses and creates a weighted average of these with weights w_k. This weighted average is our empirical derivative at $(r_\iota, \theta_\iota, t_\iota)$, $\widetilde{Y}_\iota$.

Equations (4), (5), and (6) determine which houses to use for the empirical derivative calculation. Equation (6) first determines the set of "eligible" houses to use in the empirical derivative. Since we are interested in the derivative with respect to LIHTC distance, we ideally would want to compute the empirical derivatives using houses with identical θs and ts. Essentially, we want to hold θ and t fixed and zoom in on house price variation only in the r dimension. Since we cannot choose where to observe house price transactions, we create a tolerance window within which θ and t are "approximately" held fixed. Equation (6) states that houses are in the set eligible for our empirical derivative calculation if both their squared distance in time from t_ι and squared distance in angle from θ_ι are no more than the squared radial distance from r_ι times ϑ_n^t and ϑ_n^θ, respectively. Essentially this means

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[18] l_n ensures we throw away house price transactions extremely close to r_ι. If we have numerical derivatives from transactions which occur at the exact same location as r_i (such as multiple transactions in the same condo building), the denominator in equation (2) will blow up. Thus, we throw out transaction less then $l_n = 0.01$ miles away from r_i.

that houses in the eligible set $L_{r,\iota}$ are similar to $(r_\iota, \theta_\iota, t_\iota)$ in the θ and t dimension, but differ in the r dimension. Visually, this can be thought of house transactions falling within a 3D "bowtie" around $(r_\iota, \theta_\iota, t_\iota)$, as illustrated in Figure 2. ϑ_n is a tuning parameter which determines the width of the bowtie. A large ϑ_n allows more house prices to be eligible for the empirical derivative calculation, but will also add bias since they will have more variation in the θ and t dimension. A smaller ϑ_n allows less data to be used in the empirical derivative calculation, leading to more variance. We search for (at most) κ_n nearby transactions within this "bowtie" of nearby locations.[19]

The house prices $(a(1, \iota, r), ..., a(k, \iota, r))$ are transactions which are further away from the LIHTC site than r_ι and are ordered by radial distance from r_ι. Thus, $a(1, \iota, r)$ is closer to r_ι than $a(2, \iota, r)$. Similarly, the house prices $(b(1, \iota, r), ..., b(k, \iota, r))$ are the transactions closer to the LIHTC site than r_ι and are ordered by radial proximity to r_ι. Thus, $b(1, \iota, r)$ is closer to r_ι than $b(2, \iota, r)$. To construct the empirical derivatives, equations (4) and (5) state that the housing transactions are paired together based on how close they are to r_ι. Thus, the first pair of houses used in the empirical derivative calculation would be $a(1, \iota, r)$ and $b(1, \iota, r)$, where $a(1, \iota, r)$ is the house closest to $(r_\iota, \theta_\iota, t_\iota)$ in the radial (r) dimension that has a distance from the LIHTC site greater than r_ι, and $b(1, \iota, r)$ is the house closest to $(r_\iota, \theta_\iota, t_\iota)$ in the radial dimension which has a distance from the LIHTC site less than r_ι. Note that these houses must also fall within the bowtie tolerance region defined by equation (6). When constructing empirical derivatives in each local area, we use only pre-treatment data in constructing pre-treatment derivatives and only post-treatment data in constructing post-treatment derivatives. This is to ensure that data from the post-treatment period has no effect on the pre-treatment estimates.

Once we have transformed our data on house price levels to data on house price derivatives, we smooth these house price derivatives using a standard kernel estimator. Define the Nadaraya-Watson kernel estimate at (r, t):

$$\widehat{\Phi}_l(r, t) = \frac{n^{-1} \sum_{\iota=1}^{n} K_{\mathbf{H}_n}((r, t) - (r_\iota, t_\iota)) \widetilde{Y}_{\iota, l}}{n^{-1} \sum_{\iota=1}^{n} K_{\mathbf{H}_n}((r, t) - (r_\iota, t_\iota))} \tag{7}$$

where:

$$K_{\mathbf{H}_n}((r, t) - (r_\iota, t_\iota)) = \frac{1}{h_{r,n} h_{t,n}} K\left(\frac{r - r_\iota}{h_{r,n}}, \frac{t - t_\iota}{h_{t,n}}\right) \tag{8}$$

and $K(\cdot, \cdot)$ is the two-dimensional Epichanokov kernel with bandwidths $h_{r,n}, h_{t,n}$. Thus,

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[19]We use κ_n as the maximum number of house price pairs included in the calculation. However, sometimes there are less than κ_n pairs of house prices in the bowtie region. In this case, we use as many pairs as there are available in the data. Formally, the number of available house price pairs is a random variable. We address this in the econometric proofs in the appendix.

$\widehat{\Phi}_l(r,t)$ is constructed around each LIHTC site l.

The following theorem provides a consistent estimate of the gradient treatment effect.

Theorem 1 *Suppose:*

1. $n \to \infty, h_n \to 0, l_n \to 0, \kappa_n \to \infty, \vartheta_n \to 0$

2. $nh_n \to \infty, \kappa_n / (n\vartheta_n^2) \to 0, l_n^2 \kappa_n \to \infty$

Letting T_l denote the treatment year of LIHTC site l of type $\mathbf{Y}$. Then the following:

$$\widehat{\Phi}_l(r, T_l + \tau) - \widehat{\Phi}_l(r, T_l - 1) \tag{9}$$

is a consistent estimate of $\frac{\partial \widetilde{m}_{\mathbf{Y}}(r,\tau)}{\partial r}$, where we assume $\frac{\partial \widetilde{m}_{\mathbf{Y}}(r,-1)}{\partial r} = 0$.

The proof of this result, as well as a general discussion of nonparametric derivative estimation in both the univariate and multivariate setting, are provided in Section D of the online appendix. This result generalizes Charnigo et al. (2011) and Charnigo and Srinivasan (2015) to the case where the data are observed at random locations. Charnigo et. al (2011) analyze a univariate case when the data can be observed at chosen locations. Charnigo and Srinivasan (2015) analyze the multivariate case where the locations of the data are again fixed. Neither of these consider the random design, difference in differences setup. Section C of the online appendix performs Monte Carlo simulations to evaluate the performance of our estimator and coverage rates of our bootstrapped standard errors.

In our empirical work, we define the treatment year T_l as the year in which funds are allocated for the development project, rather than the year the project is placed in service. We do this for two reasons. First, prices are forward looking and thus should reflect anticipated neighborhood effects of low income property development when the project is announced. Second, the construction of the project itself may have direct effects on prices prior to the development being placed into service, but after the funding is announced. We set $\kappa_n = 5$ (the number of house pair used in each empirical derivative), $h_{r,n} = 0.3$ (the bandwidth for smoothing in miles), $h_{t,n} = 5$,(the bandwidth for smoothing in years), $\vartheta_n^t = 1.6$ (the bow-tie width in years), $\vartheta_n^\theta = 0.4$ (the bow-tie width in distance perpendicular to r),and $l_n = .01$ (miles around the empirical derivative location to drop data)[20]. With enough house price

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[20] l_n is required since sometimes there can be transactions at the exact same location (in a condo building) and the distance between the transaction is zero. This is not useful for the empirical derivative calculations since the denominator of the empirical derivatives (distance between the transactions) would be zero. The need to drop data very very close to the site of the empirical derivative shows up in the consistency proofs in the appendix as well.

transactions around a single LIHTC site, we would be able to estimate the price impacts for each site individually, however for power reasons we average our estimates across LIHTC sites of type $\mathbf{Y}$:

$$\frac{\partial \widehat{\widetilde{m}_{\mathbf{Y}}(r,\tau)}}{\partial r} = \frac{1}{N_{\mathbf{Y}}} \sum_{l \in \mathbf{Y}} \left[\widehat{\Phi}_l(r, T_l + \tau) \, dt - \widehat{\Phi}_l(r, T_l - 1) \right],$$

where $N_{\mathbf{Y}}$ is the number of LIHTC sites of type $\mathbf{Y}$ in our data. We use block bootstrapping over LIHTC neighborhoods to obtain standard errors. See Section D of the online appendix for econometric proofs of consistency, Section C for Monte Carlo analysis of our estimator, and Section B for a simulation study comparing our estimator to the simple ring estimator.

5 Reduced Form Results

5.1 Price Effects

We begin by studying the reduced form price effects. For clarity of exposition, we begin by presenting the nonparametric level estimates obtained by integrating the gradient estimates as described in the previous section. Figure 3 illustrates the average impact of LIHTC construction on local house prices across all neighborhoods. First, note that prices leading up to the LIHTC funding are quite flat, validating our identification assumption that absent LIHTC construction, housing prices very close to the LIHTC site would have trended similarly to house prices slightly further away. Nonetheless, it appears from Figure 3 that LIHTC construction has no significant average impact on local house prices. However, this figure masks substantial heterogeneity in the price impact of LIHTC development on local house prices.

To examine such heterogeneity, we re-estimate the price effects for construction in various location types $\mathbf{Y} \in \mathcal{L}_{\mathbf{Y}}$. We begin by dividing the LIHTC sites into four buckets based on the 1990 census median income of the census block group in which the LIHTC site is located. The income quartile cutoffs are \$26,017, \$38,177, and \$54,642 in 2012 dollars. Note that, consistent with the summary statistics evidence provided in Table 1, the cutoff for the top income quartile is still substantially below the average block group median income of \$66,652 for the counties in our sample. Moreover, LIHTC residents must earn no more than 60% of the local area's median gross income, which on average across all our counties is 0.6*\$66,652 =\$39,991. Thus, the bottom quartile of LIHTC sites have residents earning significantly below the average income cutoff, while the top quartile neighborhoods have median incomes

about \$15,000 above the average income cutoff.[21]

Figure 4 illustrates the heterogeneity in price impacts. Panel A shows that LIHTC construction triggers large local price appreciation of approximately 6.5% after 10 years in the bottom income quartile. Panel B shows that LIHTC development has little impact in the second income quartile, beyond maybe small appreciation very close to the development site. We see in panels C and D that construction of affordable housing leads to striking and markedly different effects in both the third and fourth income quartiles, with construction leading to price declines of approximately 3% after 10 years in the third income quartile and declines of 2.5% in the fourth income quartile. The speed of the price decline in the fourth income quartile is dramatic, with practically all losses within 0.1 miles of a LIHTC site over the 10 year period occurring in the first year. However, the price declines in income quartiles 3 and 4 "radiate outwards" as time since LIHTC funding increases. At distances of 0.3 to 0.4 miles away from the LIHTC site, there are modest declines in house prices right away, but they fall over time. It appears the housing market very quickly "prices" the impact of LIHTC very locally, but it takes 5 to 10 years for the house prices 0.3 to 0.4 miles away to fully adjust to the shock. In all cases, we do not see strong evidence for pre-trends in prices, further validating our identification assumption that there are no very local house price trends correlated with LIHTC development. Table 2 summarizes the point estimates shown in these figures, along with standard errors.

We additionally examine the impact of LIHTC development in high minority areas. In particular, we restrict to those LIHTC sites located within a census block group that has a population at least 50% Black or Hispanic based on the 1990 census. We then further classify these sites based on whether they are in low income areas, defined as within the first or second income quartile, or high income areas, defined as within the third or fourth income quartiles.

Figure 5 illustrates the effects of affordable housing construction in high minority areas. Low income, high-minority areas see strong price appreciation of approximately 5% after 10 years resulting from LIHTC development, similar to the overall effect we see in the first and second income quartile. Conversely, prices in high income, high-minority areas remain relatively stable, with no evidence of the house price decline documented above. Thus, the substantial price depreciation seen in high income areas occurs in those neighborhoods with minority populations of below 50%.

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[21]This is an approximate, back of the envelope calculation. AGMI limits vary by county.

5.1.1 Comparison to Ring Method

Our nonparametric estimator gives a lot of detail into the shape and intensity of the local house price responses. However, this does come at a slight cost of computational complexity. Previous methods using this style of identification strategy have employed the "ring method" discussed in the previous section. While this method is simple, it can be quite underpowered. This is due to a severe missing data problem where data is dropped when either the inner or outer ring does not have a transaction in a given year. Furthermore, there can be substantial time-invariant geographic house price variation within the rings (e.g. in the north vs the south) which the ring method does not control for. Finally, our hedonic model requires the actual gradient, which the ring method does not directly provide.

As a robustness check, we have run the ring estimator on our data. We define the inner ring as distances less than 0.2 miles from the LIHTC site, and the control ring as 0.2 to 0.5 miles from the treatment site. The plotted effects are shown in Figure 6. All confidence intervals contain our nonparametric estimated effects. However, only the lowest income neighborhoods show a statistically significant effect due to LIHTC construction. While the point estimates for the higher income neighborhoods are pretty close to those estimated with nonparametrics, the excessive noise in the ring estimator leads us to not be able to reject an effect of zero. We explore the differences between the ring estimator and our nonparametric estimator more directly using simulations in Section B of the online appendix. In those simulations, we also find the standard errors on the ring estimates to be much bigger than those using our nonparametric methods.

5.1.2 Treatment Effects in the Short and Long run

We define the short term effect as the average price gradient impact on LIHTC site l between event years 0 through 5, relative to event years -5 to 0:

$$\frac{\partial \log p^{short}}{\partial r}(r, l) = \frac{1}{5} \int_0^5 \widehat{\Phi}_l(r, T_l + \tau) \, d\tau - \frac{1}{5} \int_{-5}^0 \widehat{\Phi}_l(r, T_l - \tau) \, d\tau.$$

Similarly we define the longer term impact of LIHTC sites as the impact in event years 5 through 10, relative to event years -5 to 0:

$$\frac{\partial \log p^{long}}{\partial r}(r, l) = \frac{1}{5} \int_5^{10} \widehat{\Phi}_l(r, T_l + \tau) \, d\tau - \frac{1}{5} \int_{-5}^0 \widehat{\Phi}_l(r, T_l - \tau) \, d\tau.$$

We decompose these price gradient effects into differential effects based on neighborhood income quartile and minority share. We define $\frac{\partial \widetilde{m}_Y^{short}(r_j)}{\partial r}$ and $\frac{\partial \widetilde{m}_Y^{long}(r_j)}{\partial r}$ as the short term (within 5 years) and long term (6-10 years) gradients of the price effect with respect to

distance (r) of LIHTC development in neighborhoods of type $\mathbf{Y}$:

$$\frac{\partial \log p^{short}(r, l)}{\partial r} = \frac{\partial \widetilde{m}_{\mathbf{Y}}^{short}(r_j)}{\partial r} + \varepsilon_{rl}^{short}, \tag{10}$$

$$\frac{\partial \log p^{long}(r, l)}{\partial r} = \frac{\partial \widetilde{m}_{\mathbf{Y}}^{long}(r_j)}{\partial r} + \varepsilon_{rl}^{long}. \tag{11}$$

We allow the neighborhood price effects to vary by the income quartile of the neighborhood ($\mathbf{Y}_l^{\text{inc}}$) and whether the neighborhood has a high minority share ($\mathbf{Y}_l^{\text{minor}}$) :

$$\frac{\partial \widetilde{m}_{\mathbf{Y}}^{short}(r_j)}{\partial r} = \delta_{\mathbf{Y}_l^{\text{inc}}}^{short}(r) + \beta_{\text{mlow}}^{short}(r) * \mathbf{Y}_l^{\text{minor}} * \mathbf{1}\left[\mathbf{Y}_l^{\text{inc}} \leq 2\right] + \beta_{\text{mhigh}}^{short}(r) * \mathbf{Y}_l^{\text{minor}} * \mathbf{1}\left[\mathbf{Y}_l^{\text{inc}} > 2\right], \tag{12}$$

$$\frac{\partial \widetilde{m}_{\mathbf{Y}}^{long}(r_j)}{\partial r} = \delta_{\mathbf{Y}_l^{\text{inc}}}^{long}(r) + \beta_{\text{mlow}}^{long}(r) * \mathbf{Y}_l^{\text{minor}} * \mathbf{1}\left[\mathbf{Y}_l^{\text{inc}} \leq 2\right] + \beta_{\text{mhigh}}^{long}(r) * \mathbf{Y}_l^{\text{minor}} * \mathbf{1}\left[\mathbf{Y}_l^{\text{inc}} > 2\right]. \tag{13}$$

where $\mathbf{Y}_l^{\text{inc}} \in [1, 2, 3, 4]$ is the income quartile of the neighborhood surrounding LIHTC site l and $\mathbf{Y}_l^{\text{minor}}$ is an indicator variable equal to 1 if LIHTC site is located in a high minority area. $\delta_{\mathbf{Y}_l^{\text{inc}}}^{short}(r)$ and $\delta_{\mathbf{Y}_l^{\text{inc}}}^{long}(r)$ represent the short and long term price gradient impacts of LIHTC development in low minority areas in income quartile $\mathbf{Y}_l^{\text{inc}}$ at distance r. $\beta_{\text{mlow}}^{short}(r)$ and $\beta_{\text{mlow}}^{long}(r)$ measure the differential short and long term impacts in high minority areas within income quartiles 1 and 2. $\beta_{\text{mhigh}}^{short}(r)$ and $\beta_{\text{mhigh}}^{long}(r)$ measure the differential short and long term impacts in high minority areas within income quartiles 3 and 4.

Figures 7 and 8 illustrate our nonparametric estimates in the short-term and long-term of the price *gradient* treatment effects given by equations (12) and (13) as well as 90% confidence intervals. Here, a negative estimate implies prices are increasing as one moves closer to the construction site, while a positive estimate implies prices are decreasing as one moves closer. Note that the results reported in the Q1/Q2 High Minority and Q3/Q4 High Minority plots are differential effects relative to the low minority effects reported in the Q1-Q4 plots. The Q1/Q2 High Minority plot illustrates that there may be slightly larger price appreciation effects in low income areas when the area is also high minority, however the effect is not quite statistically significant. Further, the statistically significant negative impact on the price gradient in the Q3/Q4 High Minority plot demonstrates that high income, high minority areas suffer significantly less price depreciation than high income, low minority areas. This is, of course, consistent with the evidence seen in the second panel of Figure 5. High minority and low income areas receive the most house price appreciation from LIHTC development, while higher income, low minority areas exhibit house price decline.[22]

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[22]Our effects are of a similar magnitude to other nonpriced neighborhood amenities. Gallagher and Greenstone (2008) find that Superfund sponsored clean-ups of hazardous waste sites lead to economically small and statistically indistinguishable from zero changes in local property values. Campbell et al. (2011) report that a foreclosure within a 0.25 mile radius of a given house lowers the predicted log price by 1.7 percent. At extremely small distances the effect is larger, with foreclosures causing price declines of between

5.2   Mechanisms

Taken together, these results seem to imply that LIHTC construction makes low income neighborhoods more desirable regardless of minority share, while making high income, low minority share neighborhoods less desirable. There are a variety of possible explanations for this finding. Most directly, LIHTC renovates or builds new apartment buildings, but also brings in moderately low income tenants to the neighborhood. While we can't unbundle the effects of construction from the impact of the tenants, our results are consistent with the hypothesis that relatively higher (lower) income neighbors are a positive (negative) amenity, as evidenced by LIHTC's heterogeneous effects on local house prices. Clearly in high income neighborhoods LIHTC tenants are likely to be of relatively lower income. In the lowest income areas, though, LIHTC tenants easily could be of higher income than the local average.[23]

Even if LIHTC development is the initial shock that causes these house price changes, there are likely many indirect mechanisms through which LIHTC impacts the desirability of the local neighborhood. We begin by using the merged DataQuick-HMDA data and our non-parametric methods to investigate the impact of LIHTC development on local demographic change. If local residents have preferences over the demographics of their neighbors, the in-migration of LIHTC residents may further attract different types of residents and these new in-migrants could make the neighborhood more or less desirable. Figure 9 reports the average treatment effect in levels (not gradients) from years 0 to 10 on home buyer income. Consistent with our price results, we find that the introduction of affordable housing leads to home buyers with higher incomes of approximately 3%-4% in low income, low minority-share areas. Conversely, such introduction leads to a statistically significant decrease in home buyer income of approximately 1.5% in low minority, top income quartile areas. The effects are muted in high minority areas, with low-income high minority areas not attracting quite as high income home buyers as the low-income non-minority areas. The high income, high minority areas also do not experience declines in home buyer income, unlike the high income, low minority areas.

We next investigate the impact of LIHTC construction on the Black share of home buyers, with the results presented in Figure 10. The average impact on low income, low minority share areas is statistically and economically insignificant. However, low-income,

7.2 and 8.7 percent, depending on specification.

[23]While we do not have direct data on the income distribution of LIHTC tenants across different neighborhood types, Horn and O'Regan (2013) document LITHC tenants are moderately low income. The median LIHTC resident is between 30 and 40 percent AMI. In contrast, over 75 percent of public housing and vouchers tenants are below 30 percent AMI. This suggests that LIHTC residents in the lowest income areas are likely of higher income than their private-market neighbors.

high-minority areas do see a statistically significant decrease in the percentage of Black home buyers. We also see decreases in the Black share of home buyers in the higher income, high minority areas of 3 percentage points. Therefore, it appears that building affordable housing in high minority areas may lead to lower racial segregation.[24]

We explore the impact of affordable housing development on local crime rates in Figure 11. The cities of Chicago, San Francisco, and San Diego provide comprehensive detailed local crime statistics dating from 2001-2014 in Chicago, 2003-2014 in San Diego, and 2007-2014 in San Francisco. These data provide the type of crime, as well as the date and the exact location in the city. Since this is a much smaller sample (we have only have 127 LIHTC sites developed in this time frame in these cities), we cut the data only by high/low income and high/low minority. We find both violent and property crime decline in low income areas, regardless of minority share. However, in higher income areas we do not see any increase in crime, rather property crime may even fall slightly. Lowering crime in low income areas appears to be one of the driving mechanisms through which LIHTC improves low income neighborhoods. In fact, according to survey evidence provided by Buron et al. (2009), increased safety is the third most frequently cited motivation for moving into LIHTC properties, after lower rent and nicer apartments. The same report documents that 80 percent of sampled properties provide some form of security system. The two most popular measures were restricted entry (63 percent) and buzzer systems (56 percent). Over one-third of sampled properties employed a security guard.

In the online appendix, we examine whether the house price impacts of LIHTC vary based on whether the LIHTC is new construction or a rehab of an older apartment building. We find quite similar effects for both rehab and new construction across all types of neighborhoods, suggesting the mechanism is not simply upgrading blighted older buildings (rehabs) or increasing population density (new construction).

6 Hedonic Model of Housing Choice

6.1 Structural Estimation

We now use our reduced form price estimates of the price gradient to structurally recover individual preferences for LIHTC proximity on a household by household basis. We will then

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[24]Horn and O'Regan (2011) find that LIHTC development may directly decrease segregation in certain areas The authors collect data on the racial composition of LIHTC tenants collected from state agencies in Texas, Delaware, and Massachusetts. They find that, in Texas and in low income Delaware census tracts (QCTs), the racial composition of the LIHTC development tends to be less segregated than that of the census tract in which it is located. To the extent that people have preferences for living with individuals of the same race, this may account for some of our estimated effect on the race of new home buyers.

use these preference estimates to quantify the local, long term welfare impact of building affordable housing. To this end, we develop a generalized hedonic model along the lines of Rosen (1974) and Bajari and Benkard (2005). In this framework, a house j is considered a bundle of characteristics $\left(R_j, \mathbf{X}_j, \mathbf{Y}_j, \boldsymbol{\xi}_j\right)$. Here R_j denotes the distance to the nearest LIHTC property. The vector $\mathbf{X}_j$ denotes physical and location characteristics of the house j. These include characteristics which can, to an approximation, be thought of as continuously chosen by agents, such as square-footage, lot size, numbers of beds/baths, and age, as well as discrete choice variables, such as whether the property is a condo or single-family house. As described previously, the variable $\mathbf{Y}_j$ denotes the type of the nearest LIHTC property, that is the median income and minority share of the census block group in which the development is located. This specification allows the agent to view LIHTC properties in different types of neighborhoods as distinct goods with disparate impacts on their utility.[25] Finally, $\boldsymbol{\xi}_j$ is a vector of property and location characteristics of the house which are observable to the home buyer but not to the econometrician. Such variables might include whether there is a finished basement or not. Each household has quasilinear utility over these housing and neighborhood characteristics $R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi}$ and a composite consumption good c whose price is normalized to one.

Under this setup, with minimal assumptions on the utility function and no supply side assumptions, Bajari and Benkard (2005) show that there exists an equilibrium price surface which is Lipschitz continuous with respect to characteristics and that there is a single price for each unique bundle of characteristics. This allow us to write equilibrium house prices as a mapping from characteristics space $p_{jt} = p_t(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi})$. Note that the equilibrium price function can vary with time. This is because in the hedonic framework each different time period is treated as a distinct market, in which market primitives such as consumer preferences or marginal production costs can change.

Home buyers have quasilinear preferences and their optimization problem can be written as:

$$\max_{R, X, Y, \xi, c} U_i(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi}) + c \quad \text{such that} \quad p_t(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi}) + c \leq w_i, \qquad (14)$$

where w_i is the wealth of agent i.[26]

Household i elects his ideal household and neighborhood bundle $(R_i^*, \mathbf{X}_i^*, \mathbf{Y}_i^*, \boldsymbol{\xi}_i^*, c_i^*)$ by

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[25]For example, a given household may find LIHTC proximity desirable when it is built in a low income neighborhood, since it improves neighborhood quality of low income areas. However, LIHTC proximity might be undesirable when it is built in a high income neighborhood, since it could degrade neighborhood quality.

[26]Since quasi-linear utility does not allow for income effects, this model would not be well suited for studying total housing and neighborhood demand. However, we zoom in on a small aspect of this decision: the importance of LIHTC proximity in choosing a place to live. Since this is not the overwhelming driver of house prices, it is plausible to believe income effects are negligible.

maximizing his utility.   Since distance from a LIHTC property is a continuous choice variable, the optimal distance is found by setting the first order condition with respect to R to zero. Doing so relates the slope of the households' utility function with respect to R, i.e. the marginal willingness to pay (MWTP), to the slope of the pricing surface $p_t(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi})$ at the optimal bundle.[27]

$$\frac{\partial U_i\left(R_i^*, \mathbf{X}_i^*, \mathbf{Y}_i^*, \boldsymbol{\xi}_i^*, c_i^*\right)}{\partial R} = \frac{\partial p_t\left(R_i^*, \mathbf{X}_i^*, \mathbf{Y}_i^*, \boldsymbol{\xi}_i^*\right)}{\partial R}. \tag{15}$$

Note that this includes distances very far from any LIHTC where the LIHTC proximity might no longer impact house prices or utility.[28] The households who don't find it worthwhile to live close enough to LIHTC to pay its hedonic price will live at these further distances where proximity is irrelevant for prices and utility. From these very far away households' perspective, it is as if the LIHTC site doesn't exist.

Without any further assumptions on the utility function, we can use our previous non-parametric estimates of the price gradient to recover this local MWTP for each individual household in our data who chooses to live close enough to a LIHTC site such that house prices are influenced.   To do so, we assume the following form for the equilibrium price function:

$$\log p_t(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi}) = \widetilde{m}_{\mathbf{Y}}(R, \tau) + h_t(\mathbf{X}, \boldsymbol{\xi}),$$

where τ is the number of years since LIHTC development and $h_t(\mathbf{X}, \boldsymbol{\xi})$ measures the price impacts of the other physical and neighborhood characteristics. As before, $\widetilde{m}_{\mathbf{Y}}(R, \tau)$ represents the price impact of being R miles away from a LIHTC development built in neighborhood type $\mathbf{Y}$. Implicitly, just as in our empirical specification, we are assuming that the impact of LIHTC distance on house prices does not depend on individual house characteristics.   Under the additional assumption that there is a function from geographic location and time (r, θ, t) to the available characteristics $(\mathbf{X}, \boldsymbol{\xi})$ at that location, and that:

$$h_t(\mathbf{X}, \boldsymbol{\xi}) = \phi_l(r, \theta) + \varphi_l(\theta, t),$$

we then recover our empirical specification (1).   We therefore can estimate an individual

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[27]Households will also optimize over other characteristics variables.   Some of those variables may be categorical. See Bajari and Benkard (2005) for a discussion of how optimization over categorical variables is handled.   Under our assumption of an additively separable utility function, we do not need to examine the optimal choices of households over other housing characteristics for our analysis.

[28]For example, the utility benefit of moving 55.1 miles away from versus 55 miles is likely zero. Similarly, we would expect the hedonic price of LIHTC proximity to also be zero at this distance.

household's MWTP at its optimal bundle by:

$$\frac{\partial U_i\left(R_i^*, \widehat{\mathbf{X}_i^*, \mathbf{Y}_i^*}, \boldsymbol{\xi}_i^*, c_i^*\right)}{\partial R} = \frac{\partial p_t\left(R_i^*, \widehat{\mathbf{X}_i^*, \mathbf{Y}_i^*}, \boldsymbol{\xi}_i^*\right)}{\partial R} = \frac{\partial \widetilde{m}_{\mathbf{Y}}\left(R_i^*, \tau\right)}{\partial r} p_t\left(R_i^*, \mathbf{X}_i^*, \mathbf{Y}_i^*, \boldsymbol{\xi}_i^*\right).$$

That is, the estimated marginal willingness to pay is equal to our non-parametric estimate of the log price gradient at distance R^* multiplied by the observed purchase price.

Note, however, that this procedure only provides information about preferences for each household at that household's optimally chosen bundle. In other words, it does not allow us to recover the full MWTP curve. This is a common issue arising in hedonic work when one does not see many choices by the same household. Since we want to conduct non-local welfare analysis, which essentially requires integrating over the MWTP curve, we follow convention in the literature by specifying a log-linear parametric form for the utility function.

We begin by assuming there is a maximal distance $R_{0,\mathbf{Y}}$ at which proximity to LIHTC of type $\mathbf{Y}$ no longer contributes to agent utility. We identify $R_{0,\mathbf{Y}}$ as the point at which the estimated price gradient for type $\mathbf{Y}$ goes to zero. Furthermore, we see from our heterogeneous estimates of the pricing surface that affordable housing proximity is viewed as an amenity in some areas, e.g. in lower income areas, but a disamenity in others. We assign each type $\mathbf{Y}$ a label $\lambda_{\mathbf{Y}} = G, B$ depending on whether the price gradient is negative or positive respectively. Types with label G are viewed as an amenity and types with label B are viewed as a disamenity. Given these labels, we assume a household's utility increases in R for types with label B and increases in $R_{0,\mathbf{Y}} - R$ for types with label G. In other words, when LIHTC is viewed as an amenity households like proximity, and when LIHTC is viewed as a disamenity households like distance. Give our log-linear specification, we thus have:

$$U_i^G\left(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi}, c\right) = A_G + \gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R\right) \mathbf{1}\left[R \leq R_{0,\mathbf{Y}}\right] + u_i\left(\mathbf{X}, \boldsymbol{\xi}\right) + c \quad (16)$$

$$U_i^B\left(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi}, c\right) = A_B + \gamma_{i,\mathbf{Y}} \log\left(1 + R\right) \mathbf{1}\left[R \leq R_{0,\mathbf{Y}}\right] + u_i\left(\mathbf{X}, \boldsymbol{\xi}\right) + c, \quad (17)$$

where $u_i\left(\mathbf{X}, \boldsymbol{\xi}\right)$ reflects the utility contribution of the house's physical and location characteristics. The parameter $\gamma_{i,\mathbf{Y}}$ is household specific and reflects household i's personal preference for proximity to LIHTC in a neighborhood of type $\mathbf{Y}$. Note that the indicator variables ensures that the contribution of LIHTC proximity to utility disappears beyond distance $R_{0,\mathbf{Y}}$. We finally set the constants A_G, A_B such that the limit of household utility approaches $u_i\left(\mathbf{X}, \boldsymbol{\xi}\right) + c$ as R approaches $R_{0,\mathbf{Y}}$. This leads to $A_G = 0$ and $A_B = \gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}}\right)$.

With these parametric assumptions in place, we can then recover the global MWTP curve for each individual household who optimally chooses $R \leq R_{0,\mathbf{Y}}$. This is, of course,

equivalent to recovering $\gamma_{i,\mathbf{Y}}$ for each household. Given the functional forms, we estimate a household's $\gamma_{i,\mathbf{Y}_i^*}$ by:

$$\frac{\widehat{\gamma_{i,\mathbf{Y}_i^*}}}{1+R_i^*} = \frac{\partial p_t\left(R_i^*, \widehat{\mathbf{X}_i^*}, \mathbf{Y}_i^*, \boldsymbol{\xi}_i^*\right)}{\partial R}, \lambda = B \tag{18}$$

$$\frac{\widehat{\gamma_{i,\mathbf{Y}_i^*}}}{1+R_{0,\mathbf{Y}_i^*}-R_i^*} = \frac{\partial p_t\left(R_i^*, \widehat{\mathbf{X}_i^*}, \mathbf{Y}_i^*, \boldsymbol{\xi}_i^*\right)}{\partial R}, \lambda = G \tag{19}$$

where the form of the price gradient estimate is give above.

6.2   Welfare

Using our recovered parameter estimates, we consider the *long-run, local* welfare impact of introducing unanticipated LIHTC development into a metropolitan area which currently has none. By local welfare, we mean that we will assume the price and amenity value of neighborhoods away from the LIHTC are unchanged due to the LIHTC construction. This assumption would be violated if there are general equilibrium effects, such as if a LIHTC development causes "good neighbors" to move into the LIHTC neighborhoods and leave their old neighborhoods, making these previous neighborhoods decline in amenities. While quantifying the full GE effects of LIHTC would be ideal, we see the heterogeneity in the local welfare impacts as a useful first step in understanding where to target affordable housing.

This is a long-run welfare calculation since we assume moving costs are zero. Moving costs create a wedge between the optimal location of each household and the location the household lived in before LIHTC development. In the long-run, everyone is much more likely to re-optimize their housing choices, although moving costs could still be relevant.

We decompose the welfare effects into the impacts on homeowners, renters, and absentee landlords. We will discuss the equilibrium in both the periods pre and post LIHTC development to highlight the welfare effects on each type of household.

6.2.1   Neighborhood choice before LIHTC development

Prior to construction of a LIHTC development, the local population optimizes over a vector of housing and neighborhood characteristics $(\mathbf{X}, \boldsymbol{\xi})$ according to the following problem:

$$\max u_i\left(\mathbf{X}, \boldsymbol{\xi}\right) + c \quad \text{s.t.} \quad h_0\left(\mathbf{X}, \boldsymbol{\xi}\right) + c \leq y_i \tag{20}$$

where y_i is household income, $u_i\left(\mathbf{X}, \boldsymbol{\xi}\right)$ is the individual-specific utility function over housing and location characteristics defined above, and $h_0\left(\mathbf{X}, \boldsymbol{\xi}\right)$ is the metropolitan equilibrium

hedonic price function over characteristics prior to LIHTC construction.[29] We denote the optimal bundle of characteristics chosen by household i prior to LIHTC construction as $\left(\mathbf{X}_{0,i}^{*}, \boldsymbol{\xi}_{0,i}^{*}\right)$. When LIHTC is built in the following period, we define $R_{i,pre}$ as the distance between household i's chosen location in the pre period and the site of new LIHTC development. Since LIHTC development is unanticipated, $R_{i,pre}$ does not influence his optimization in the period before LIHTC development.

Turning to renters, since we do not observe data directly on rents, we make the assumption that house prices are equal to the present discounted value of rents. Renters therefore face the same optimization as homeowners in the pre-period.

6.2.2 Neighborhood choice after LIHTC development

We assume zero moving costs, such that when affordable housing is built, all households will re-optimize. We view this welfare calculation as the long-run effect on welfare, since reoptimization in reality is sluggish. After LIHTC construction, the possibility to live close to a LIHTC site is within households' choice sets. The utility function is the same as before, but now includes the possibility of receiving utility from LIHTC proximity, as defined in equations (16) and (17):

$$\max_{R,X,\boldsymbol{\xi},c} U_i^{\lambda_{\mathbf{Y}}}\left(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi}, c\right) \quad \text{s.t.} \quad \widetilde{m}_{\mathbf{Y}}\left(R\right) + h_1\left(\mathbf{X}, \boldsymbol{\xi}\right) + c \leq y_i + \widetilde{m}_{\mathbf{Y}}\left(R_{i,pre}\right) + h_0\left(\mathbf{X}, \boldsymbol{\xi}\right).$$

The quantity $\widetilde{m}_{\mathbf{Y}}\left(R_{i,pre}\right) + h_0\left(\mathbf{X}, \boldsymbol{\xi}\right)$ in the budget constraint is the revenue received from the sale of the current home. $\widetilde{m}_{\mathbf{Y}}\left(R_{i,pre}\right)$ represents the house price appreciation for the people whose previous period's location choice happened to be close enough to the new LIHTC site to have their house prices impacted. The expression $\widetilde{m}_{\mathbf{Y}}\left(R\right) + h_1\left(\mathbf{X}, \boldsymbol{\xi}\right)$ is the price of the newly bought home where $h_1\left(\mathbf{X}, \boldsymbol{\xi}\right)$ represents the hedonic prices for all the neighborhood and housing characteristics other than LIHTC proximity in the post period and $\widetilde{m}_{\mathbf{Y}}\left(R\right)$ represents the LIHTC proximity hedonic price.

Our methodology does not allow us to examine the impact of LIHTC construction on outcomes at the broader metropolitan level. That is, we are not able to study how LIHTC construction impacts further away neighborhoods due to equilibrium changes in demographic sorting or the supply of location characteristics across neighborhoods within the metropolitan area. We therefore assume that $h_1\left(\mathbf{X}, \boldsymbol{\xi}\right) = h_0\left(\mathbf{X}, \boldsymbol{\xi}\right)$, i.e. the hedonic price surface for all other house/location characteristics does not change due to the introduction of LIHTC development. This assumes that any housing choice available in the pre-period is still avail-

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[29]We assume that agents do not anticipate LIHTC development when they choose their optimal bundle of characteristics in the pre-period.

able in the post period after LIHTC. We gain significant tractability from this assumption, since it ensure that all households will choose the same bundle of characteristics $(\mathbf{X}_i^*, \boldsymbol{\xi}_i^*)$ pre and post LIHTC construction other than LIHTC proximity. However, it is due to this assumption and its implications that we are truly calculating only a *local* welfare effect.

Renters will reoptimize according to a similar problem:

$$\max_{R,X,\boldsymbol{\xi},c} U_i^{\lambda_{\mathbf{Y}}}(R, \mathbf{X}, \mathbf{Y}, \boldsymbol{\xi}, c) \quad \text{s.t.} \quad \widetilde{m}_{\mathbf{Y}}(R) + h_1(\mathbf{X}, \boldsymbol{\xi}) + c \leq y_i.$$

Note that the key difference is in the budget constraint. Renters do not obtain any income from the sale of a currently owned home.

We can now tabulate the various welfare effects. For homeowners, if their housing choice in the pre period does not end up being close to the new LIHTC site ($R_{pre} \geq R_{0,\mathbf{Y}}$) and their optimal location choice in the post period is also far enough away from the LIHTC such that it does not impact prices or utility ($R_{1,i}^* \geq R_{0,\mathbf{Y}}$), then the welfare impact is $\Delta U_i = 0$. For these households, none of the locations impacted by LIHTC proximity are more desirable than their chosen locations in the pre-period, leading these households to make the same housing choices in the pre and post period. The welfare impact is thus equal to zero.

For homeowners whose pre-period housing choice ends up being close enough to the new LIHTC site such that it impacts housing prices ($R_{pre} < R_{0,\mathbf{Y}}$), but optimally choose to move away from the LIHTC site ($R_{1,i}^* \geq R_{0,\mathbf{Y}}$) in the post period, the welfare impact is given by the change in prices:

$$\Delta U_i = \widetilde{m}_{\mathbf{Y}}(R_{pre}).$$

These households move away from the LIHTC development and choose a new housing-neighborhood bundle that looks identical to the one they chose in the pre-period $(\mathbf{X}_i^*, \boldsymbol{\xi}_i^*)$. They do not gain or lose any utility due to changes in neighborhood or housing characteristics, but do obtain greater or less consumption due to the price appreciation/depreciation $\widetilde{m}_{\mathbf{Y}}(R_{pre})$ of their pre-period home.

For homeowners who optimally choose to live close to the new LIHTC site and pay its hedonic price ($R_{1,i}^* < R_{0,\mathbf{Y}}$), the welfare impact results from both the utility gain/loss due to LIHTC proximity and the change in consumption due to house price appreciation. For LIHTC development in areas where it is viewed as an amenity($\lambda_{\mathbf{Y}} = G$):

$$
\begin{aligned}
\Delta U_i &= \gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R_{1,i}^*\right) + \widetilde{m}_{\mathbf{Y}}(R_{pre}) - \widetilde{m}_{\mathbf{Y}}\left(R_{1,i}^*\right), & \text{if } R_{pre} \leq R_{0,\mathbf{Y}} \\
\Delta U_i &= \gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R_{1,i}^*\right) - \widetilde{m}_{\mathbf{Y}}\left(R_{1,i}^*\right), & \text{if } R_{pre} > R_{0,\mathbf{Y}}.
\end{aligned}
$$

while for LIHTC development in areas where it is viewed as a disamenity($\lambda_{\mathbf{Y}} = B$):

$$\Delta U_i = \gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R_{1,i}^*}{1 + R_{0,\mathbf{Y}}}\right) + \widetilde{m}_{\mathbf{Y}}\left(R_{pre}\right) - \widetilde{m}_{\mathbf{Y}}\left(R_{1,i}^*\right), \quad \text{if } R_{pre} \leq R_{0,\mathbf{Y}}$$

$$\Delta U_i = \gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R_{1,i}^*}{1 + R_{0,\mathbf{Y}}}\right) - \widetilde{m}_{\mathbf{Y}}\left(R_{1,i}^*\right), \qquad\qquad \text{if } R_{pre} > R_{0,\mathbf{Y}}.$$

Turning to renters, the welfare impacts for those who optimally choose to live close to the LIHTC site in the post-period $(R_{1,i}^* < R_{0,\mathbf{Y}})$ are given by:

$$\Delta U_i = \gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R_{1,i}^*\right) - \widetilde{m}_{\mathbf{Y}}\left(R_{1,i}^*\right), \quad \lambda_{\mathbf{Y}} = G$$

$$\Delta U_i = \gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R_{1,i}^*}{1 + R_{0,\mathbf{Y}}}\right) - \widetilde{m}_{\mathbf{Y}}\left(R_{1,i}^*\right), \quad \lambda_{\mathbf{Y}} = B.$$

The welfare impact is zero for renters who optimally choose to live far way from LIHTC $(R_{1,i}^* > R_{0,\mathbf{Y}})$ since they choose the same bundle of house/location characteristics as they did in the pre-period and their rents do not change. Note, that if LIHTC is viewed as an amenity, renters must pay higher rents to locate close to it. If renters do not like living near LIHTC sites, they are compensated for it by the lower rents. On net, renters who chose to live close to LIHTC are made weakly better off than in the pre-period since they could have chosen to live far away from the LIHTC site and receive a utility change of zero relative to the pre-period.

Finally, the welfare impact on absentee landlords whose properties are located at $R_{1,i}^* \leq R_{0,\mathbf{Y}}$ is given by the present value of the change in rents they collect:

$$\Delta U_i = \widetilde{m}_{\mathbf{Y}}\left(R_{1,i}^*\right).$$

The impact is zero for landlords with properties located at distances greater than $R_{0,\mathbf{Y}}$ since rents do not change. Note that landlords are not able to fully capture the amenity value to local renters in raised rents, leading renters to receive some welfare benefits from the increased amenities, even net of increased rental rates. We show this is generically true when renters have heterogeneous preferences in Section E of the online appendix. These impacts are summarized in Tables 3 and 4.

Aggregating all homeowner welfare together when proximity to LIHTC is a disamenity we get:[30]

$$\sum_i \gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R_{1,i}^*}{1 + R_{0,\mathbf{Y}}}\right) * \mathbf{1}(R_{1,i}^* < R_{0,\mathbf{Y}}), \tag{21}$$

---

[30]See Tables 2 and 3 for treatment of both the amenity and disamenity case.

where $\mathbf{1}(R^*_{1,i} < R_{0,\mathbf{Y}})$ is an indicator function for whether household i chooses to live close to the LIHTC site in the post period. The welfare impacts due to house price appreciation do not enter aggregate welfare since it is just a transfer from home sellers to home buyers. A key result of the model is that the *aggregate welfare impacts on homeowners only depend on the direct utility value of LIHTC proximity accruing to the households who choose to live close to the new LIHTC site.* Recall that our estimation methods only allow us to recover the LIHTC preferences for the households who choose to move close LIHTC. The model above shows that these are the *only* households whose preferences need to be estimated.

Aggregate renter welfare when proximity to LIHTC is a disamenity is:

$$\sum_i \left(\gamma_{i,\mathbf{Y}} \log \left(\frac{1 + R^*_{1,i}}{1 + R_{0,\mathbf{Y}}} \right) - \widetilde{m}_{\mathbf{Y}} \left(R^*_{1,i} \right) \right) * \mathbf{1}(R^*_{1,i} < R_{0,\mathbf{Y}}). \tag{22}$$

This is the identical formula to aggregate household welfare, except that it includes the change in rental costs for the renters who choose to pay the hedonic price of LIHTC proximity.

Aggregate landlord welfare is:

$$\sum_i \widetilde{m}_{\mathbf{Y}} \left(R^*_{1,i} \right) * 1(R_{pre} < R_{0,\mathbf{Y}}).$$

Summing the landlord and renter aggregate welfare together will net out the rental cost effects, since this is just a transfer from renters to landlords. This gives:

$$\sum_i \left(\gamma_{i,\mathbf{Y}} \log \left(\frac{1 + R^*_{1,i}}{1 + R_{0,\mathbf{Y}}} \right) \right) * \mathbf{1}(R^*_{1,i} < R_{0,\mathbf{Y}}). \tag{23}$$

Just as in the homeowner case, the aggregate landlord-renter welfare only depends on the direct neighborhood utility value of LIHTC proximity as valued by the renters who optimally choose to live in the vicinity. Summing equations (21) and (23) gives the total welfare value to society.

6.2.3 Measuring the welfare effects

With our welfare derivations in place, we now discuss how we measure the welfare impacts empirically. To this end, let $q^H \left(\gamma_{\mathbf{Y}}, R_1 | \mathbf{Y}, R_1 < R_{0,\mathbf{Y}} \right)$ denote the joint density of preference parameters and distance chosen from the LIHTC site given that the household has chosen to live within $R_{0,\mathbf{Y}}$ miles of a LIHTC site of type $\mathbf{Y}$. This density is directly observed in our data since we recover the γ of each household who chooses to move close to LIHTC and we also observe each household's chosen LIHTC proximity R_1. The aggregate welfare impact of

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a LIHTC site developed in neighborhood of type $\mathbf{Y}$ for homeowners is then simply given by:

$$\Delta U_{agg,\mathbf{Y}}^H = N_H \int \gamma_{\mathbf{Y}} \log \left(1 + R_{0,\mathbf{Y}} - R_{1,i}^*\right) q^H \left(\gamma_{\mathbf{Y}}, R_1, | \mathbf{Y}, R_1 < R_{0,\mathbf{Y}}\right) d\gamma_{\mathbf{Y}} dR_1, \ \lambda_{\mathbf{Y}} = G \quad (24)$$

$$\Delta U_{agg,\mathbf{Y}}^H = N_H \int \gamma_{\mathbf{Y}} \log \left(\frac{1 + R_{1,i}^*}{1 + R_{0,\mathbf{Y}}}\right) q^H \left(\gamma_{\mathbf{Y}}, R_1 | \mathbf{Y}, R_1 < R_{0,\mathbf{Y}}\right) d\gamma_{\mathbf{Y}} dR_1, \ \lambda_{\mathbf{Y}} = B, \quad (25)$$

where N_H is the average number of homeowners who choose to live within $R_{0,\mathbf{Y}}$ miles of a single LIHTC site in neighborhoods of type $\mathbf{Y}$. We only use our estimated preference parameters γ to measure the density of γ at each distance R_1, but not the total quantity of households living at distance R_1. Since our γ estimates come from housing transactions, we don't observe the very-long run total quantity of in-migrants to live close to LIHTC. To get a more accurate measure of the total quantity of households who will choose to move in the long run, we measure the total household count surrounding LIHTC sites in the American Community Survey. We measure N_H from the 5-year pooled ACS block group data on the median number of homeowners per LIHTC site living within $R_{0,\mathbf{Y}}$ miles of the LIHTC site of type $\mathbf{Y}$.[31] These numbers are reported in Table A3.

Since we don't have direct data on renter migration, and therefore cannot directly measure their distribution of γ as we do for homeowners, we need to make some additional assumptions to calculate their welfare. To calculate the impact for renters we assume that the distribution of preference parameters conditional on race and income is the same as that for homeowners, that is:

$$q^R \left(\gamma_{\mathbf{Y}}, R_1 | Race, y, \mathbf{Y}, R_1 < R_{0,\mathbf{Y}}\right) = q^H \left(\gamma_{\mathbf{Y}}, R_1 | Race, y, \mathbf{Y}, R_1 < R_{0,\mathbf{Y}}\right). \quad (26)$$

This allows us to adjust the distribution of home owner preferences according to differences in the distribution of race and income between renters and owners, as measured in the ACS, to create the distribution of preferences among renters. The main caveat to this approach is that even at the same income level and race, renters' preferences may differ from that homeowners for LIHTC proximity. However, since we don't have any information other than home owner purchases and their race and income, we cannot make further adjustments to renter preferences other than using the demographics we observe for homeowners.

We can then calculate the aggregate welfare impact on renters when LIHTC is an amenity $(\lambda_{\mathbf{Y}} = G)$. Let $q^R \left(\gamma_{\mathbf{Y}}, R_1, Race, y | R_1 < R_{0,\mathbf{Y}}\right)$ be the joint density of preferences $(\gamma_{\mathbf{Y}})$, LIHTC proximity (R_1), race $(Race)$, and income (y) for renters who chose to live within $R_{0,\mathbf{Y}}$

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[31] We measure these household counts for the LIHTC sites used in our estimation, not the entire country.

of the LIHTC site. Renter welfare can then be written as:

$$\Delta U_{agg,\mathbf{Y}}^{R} = N_R \sum_{Race} \int \left[\gamma_{\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R_{1,i}^{*}\right) - \widetilde{m}_{\mathbf{Y}}(R_1) \right] q^R \left(\gamma_{\mathbf{Y}}, R_1, Race, y | R_1 < R_{0,\mathbf{Y}}\right) d\gamma_{\mathbf{Y}} dR_1 dy \tag{27}$$

$$= N_R \sum_{Race} \int \left[\gamma_{\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R_{1,i}^{*}\right) - \widetilde{m}_{\mathbf{Y}}(R_1) \right] q^R \left(\gamma_{\mathbf{Y}}, R_1 | Race, y, R_1 < R_{0,\mathbf{Y}}\right) q^R \left(Race, y | R_1 < R_{0,\mathbf{Y}}\right) d\gamma_{\mathbf{Y}} dR_1 dy \tag{28}$$

$$= N_R \sum_{Race} \int \left[\gamma_{\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R_{1,i}^{*}\right) - \widetilde{m}_{\mathbf{Y}}(R_1) \right] q^H \left(\gamma_{\mathbf{Y}}, R_1 | Race, y, R_1 < R_{0,\mathbf{Y}}\right) q^R \left(Race, y | R_1 < R_{0,\mathbf{Y}}\right) d\gamma_{\mathbf{Y}} dR_1 dy \tag{29}$$

$$= N_R \sum_{Race} \int \left[\gamma_{\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R_{1,i}^{*}\right) - \widetilde{m}_{\mathbf{Y}}(R_1) \right] q^H \left(\gamma_{\mathbf{Y}}, R_1, Race, y | R_1 < R_{0,\mathbf{Y}}\right) \frac{q^R\left(Race, y | R_1 < R_{0,\mathbf{Y}}\right)}{q^H\left(Race, y | R_1 < R_{0,\mathbf{Y}}\right)} d\gamma_{\mathbf{Y}} dR_1 dy \tag{30}$$

where going from equation (28) to (29) plugs in equation (26). N_R is the total number of renters in the proximity of the LIHTC site. This number comes from the ACS.

When LIHTC proximity is undesirable $(\lambda_{\mathbf{Y}} = B)$, we have:

$$\Delta U_{agg,\mathbf{Y}}^{R} = N_R \sum_{Race} \int \left[\gamma_{\mathbf{Y}} \log\left(\frac{1 + R_{1,i}^{*}}{1 + R_{0,\mathbf{Y}}}\right) - \widetilde{m}_{\mathbf{Y}}(R_1) \right] q^H \left(\gamma_{\mathbf{Y}}, R_1, Race, y | R_1 < R_{0,\mathbf{Y}}\right) \frac{q^R\left(Race, y | R_1 < R_{0,\mathbf{Y}}\right)}{q^H\left(Race, y | R_1 < R_{0,\mathbf{Y}}\right)} d\gamma_{\mathbf{Y}} dR_1 dy.$$

The joint density of race and income for both renters and homeowners, $q^R\left(Race, y | R_1 < R_{0,\mathbf{Y}}\right)$ and $q^H\left(Race, y | R_1 < R_{0,\mathbf{Y}}\right)$ respectively, are calculated from 5-year pooled American Community Survey micro data. For the price effects due to LIHTC proximity, $\widetilde{m}_{\mathbf{Y}}$, we use our long term price estimates, derived from equation 13. Finally, the aggregate impact on absentee landlords is given by:

$$\Delta U_{agg}^{ll} = N_R \sum_{Race} \int \widetilde{m}_{\mathbf{Y}}(R_1) \, q^H \left(\gamma_{\mathbf{Y}}, R_1, Race, y | R_1 < R_{0,\mathbf{Y}}\right) \frac{q^R\left(Race, y | R_1 < R_{0,\mathbf{Y}}\right)}{q^H\left(Race, y | R_1 < R_{0,\mathbf{Y}}\right)} d\gamma_{\mathbf{Y}} dR_1 dy.$$

6.3   Results

To keep the model parsimonious, we only use the short term and long term price gradient estimates to recover households' preferences for LIHTC proximity. We denote these $\frac{\partial \widetilde{m}_{\mathbf{Y}}^{short}(r_j)}{\partial r}$ and $\frac{\partial \widetilde{m}_{\mathbf{Y}}^{long}(r_j)}{\partial r}$, as in equations (12) and (13).

Figure 7 plots the estimates of the short-term price gradient effects and Figure 8 plots the estimates of the long-term price gradient effects. These are labeled the "non-parametric estimates" in the figures. To use these estimates within our structural model to recover preferences, we set the gradient equal to zero at all distances past the point where the gradient first hits the x-axis and crosses zero or the point at which it comes the closest to zero. These estimates are also shown in Figures 9 and 10.[32] We use the point at which the price effect goes to zero as our estimate of $R_{0,\mathbf{Y}}$, the distance beyond which household utility is no longer impacted by LIHTC proximity.

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[32]Our raw estimates are never statistically different from zero at any distance past the point where gradient first hits the x-axis.

Using the estimated price gradients, we use equations (18) and (19) to estimate each household's preference to live near LIHTC within neighborhood type **Y**. We designate LIHTC proximity to be desirable within neighborhoods where LIHTC development caused price appreciation, while we designate LIHTC proximity as undesirable within areas where it causes price decline.

We now examine how households' preferences for LIHTC proximity vary with race and income of home buyers. We measure how much each household is willing to pay (as a share of their house price) to live 0.1 miles from an LIHTC development. Panel A of Figures 12 and 13 shows that higher income households are willing to pay the most to live close the LIHTC development in Q1 income areas, conditional on wanting to live in a Q1 income area. This is true in both high and low minority areas. We find minority home buyers are willing to pay more to live in high minority, Q1 income areas than non-minority home buyers, while the reverse is true in low minority Q1 income areas. Overall, households choosing to live near LIHTC sites in Q1 income areas are willing to pay about 6% of their house price to live 0.1 miles from an LIHTC site. The preferences in Q2 areas are essentially zero, as reflected in the essentially zero price effects discussed previously. Within low minority Q3 areas and Q4 areas, we find that higher income households are willing to pay slightly more to avoid living 0.1 miles from an LIHTC site. We also find that minority home buyers are less deterred by LIHTC development in Q3 and Q4 low minority areas than non-minority home buyers. Overall, these households are willing to pay about 1.6% of their house price to avoid living within 0.1 miles of LIHTC. Within Q3 and Q4 high minority areas, the effects are economically insignificant, consistent with the economically insignificant price effects discussed previously. The online appendix reports similar effects measured in dollars, instead of in house price percentages, as these will be the numbers more closely linked to those used in the next section for the welfare calculation. [33]

Turning to welfare, Panel A of Table 5 reports the average willingness to pay for LIHTC development per homeowner, renter, and landlord impacted by LIHTC development within neighborhoods of different types. Within Q1 income, low minority areas, the average homeowner would be willing to pay $23,403 for LIHTC development. The average renter would be willing to pay $6502 and the average landlord would be willing to pay $6011.[34] In Q1, high minority areas, the average homeowner would be willing to pay $16,857, the average renter would be willing to pay $6475, and the average landlord would be willing to pay $6099. There are substantial benefits to the community from LIHTC development in Q1 income

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[33]These figures are dominated by the fact that higher income households buy more expensive houses, creating a strong link between home buyer income and willingness to pay for LIHTC proximity.

[34]The sum of the welfare benefits to landlords and renters is less than that of homeowners because renters tend to be lower income, lowering their willingness to pay.

areas. However, the opposite is true in low minority Q3 and Q4 areas. Within Q4 areas, the average homeowner would be willing to pay $3972 to deter LIHTC development, the average renter would be willing to pay $67 for LIHTC development and the average landlord would be willing to pay $2416 to deter LIHTC development. In high minority, Q4 income areas, residents benefit from LIHTC development.

Table 5 scales these numbers to aggregate effects to get total willingness to pay by society from LIHTC development in different areas. The units are reported in thousands of dollars. Since low income areas tend to be quite dense, as evidenced by Table A3, the aggregate benefit to homeowners from LIHTC development in Q1 income low minority areas is $57.9 million. The aggregate benefit to renters is $29.2 million and the benefit to landlords is $29.0 million. In total, society would be improved by $116.2 million from a single LIHTC development in a low income, low minority area. This number is even bigger in low income, high minority areas, ($211 million), simply because there tend to be more people living close to LIHTC in these areas. Conversely, development of LIHTC in a Q4, low minority area leads to an aggregate welfare loss of $12.1 million.

These place-based effects are large and meaningful relative to the typical cost of development, around $300,000 per unit in California and likely lower elsewhere. This suggests the development cost of a typical 82 unit building would be $24.6 million, substantially less than the positive benefits in low income areas, and more than the welfare losses in the high income areas.

6.4   Discussion

It is important to recognize that there are some important caveats to our analysis that prevent us from fully capturing all indirect channels through which LIHTC development could impact welfare. First, we can only capture the local welfare impacts of introducing a tax credit property into neighborhoods of varying type. This analysis does not include indirect general equilibrium effects on the amenity values on house prices of neighborhoods away from the LIHTC sites. For example, an extreme scenario is that LIHTC attracts "good neighbors" to low-income areas, forcing other neighborhoods to lower their stock of "good neighbors." In this zero sum game of good neighborhood allocation, the aggregate welfare effects could be zero, while the local welfare effects to the LIHTC neighborhood would be positive. However, documenting that LIHTC effects are extremely heterogeneous across space is the first step toward gauging the scope for place based policies. For externalities to create a justification for spatial policy, such externalities must be stronger in some places than in others. As noted by Glaeser & Gottlieb (2011), the placed-based policy literature

is often uncertain whether such spillovers exist and rarely documents nonlinear effects. Our results clearly document the heterogeneity in neighborhood impacts, which is a key first step to designing optimal spatial policy.

Second, we are not attempting to analyze the effectiveness or welfare impact of the LIHTC program versus private market development. That is, we are not performing a policy evaluation of the LIHTC program. First, we do not observe the counterfactual. It may, for instance, be the case that LIHTC development is simply crowding out private multifamily properties. Conversely, it seems quite likely that, due to frictions, private developers cannot fully internalize the positive externalities of such construction in low-income areas, leaving scope for government intervention, but answering this question is beyond the scope of the current paper.

With these caveats in place, we believe that our estimates can still be useful in helping federal, state, and local agencies think about where to locate affordable housing developments. If government agencies face constraints in the number of subsidized properties they can finance, our estimates would suggest that development should be strongly targeted to low income areas. We must caveat this claim as well, though. Our welfare estimates view LIHTC development as a purely place based policy. Yet the location of affordable housing also influences the welfare of the tenants living in the affordable housing, which our numbers above do not capture.

Recent work by Chetty et al. (2015) finds that young children strongly benefit from growing up in lower poverty neighborhoods. While the neighborhoods types analyzed in Chetty et al. (2016) do not map directly into our definitions, they report that a child moving out of public housing and into a low poverty area gains $99,000 in presented discounted value of future income over a lifetime. If we use this number to benchmark the potential gains to LIHTC tenants living in a Q4 low minority area versus a Q1 low minority neighborhood, and assume each apartment has two children in it, the average LIHTC development would improve the welfare of these children by $26.7 million.[35] This is an underestimate of the total welfare benefits to these households, as it only values the increased earnings and not other benefits which have been documented, such as better measures of mental health and lower obesity rates (Kling et al, 2007). The benefits to these tenants ($26.7 million) more than offsets the losses to local residents in these high income areas ($12.1 million), which makes development in these higher income areas look desirable.

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[35]We assume the LIHTC remains affordable for 30 years and that each apartment will house a household with two children for 15 years. Thus, two households will leave in each apartment for 15 years, sequentially. We discount the present value of children's income of the 2nd household moving in the apartment 15 years after it was built by 3%, same discount rate used in Chetty et al. (2016). We assume the LIHTC site at 82 apartments.

However, if placing such a property in a high income area means that one less property must be allocated to a low income area, then there is an opportunity cost of $116 million, which is more than 4 times the welfare benefits documented by Chetty et al. (2016). This makes development of affordable housing in low income areas appear to be a very effective policy in improving the economic welfare of many low-income households, even when taking into account the current MTO evidence. A final caveat to these points is that there may likely be diminishing marginal returns to new housing in poor areas if these policies were scaled substantially. Yet, this is likely also true of the benefits estimated by Chetty et al (2016) as well.

7 Conclusion

In this paper, we study multifamily housing developments funded through the Low Income Housing Tax Credit (LIHTC) to quantify the costs and benefits of affordable housing development on surrounding neighborhoods. Leveraging new econometric methods, we find that LIHTC construction has heterogeneous effects on local house prices based on neighborhood characteristics. In lower income areas, house prices appreciate substantially over the long-run in response to the introduction of affordable housing projects. Areas with a high minority share also experience significant price appreciation when a LIHTC development is built. On the other hand, prices in areas with higher median incomes and low minority shares tend to depreciate over the long-run.

We investigate the mechanisms underlying these price effects. Development in low income, low minority share areas leads to a rise in the income of subsequent home buyers, while development in high income, low minority share areas leads to a decrease in home buyer income. In low income areas, LIHTC development leads to a reduction in both violent and property crime. This is consistent with evidence documenting that the overwhelming majority of LIHTC properties have a security system in place, as well as the stated desire of LIHTC residents for secure housing. Finally, we find that LIHTC construction in high minority share areas leads to an increase in the non-Black share of subsequent home buyers. This result is particularly significant since it illustrates that reduced segregation, a stated goal of many affordable housing programs, may be achieved by locating subsidized, rent-controlled properties in high minority share areas, due to the resulting neighborhood and demographic change.

We employ a structural, generalized hedonic model of housing choice to link the estimated price effects to individual preferences for proximity to low income housing. In lower income areas where affordable housing developments are viewed as an amenity, higher income

households are willing to pay more for proximity. Conversely, higher income households are willing to pay more to live further away from affordable housing developments in higher income areas where such properties are viewed as a disamenity. White households are willing to pay more than minority households to locate further away from the tax credit property in high income, high minority share areas and have a lower willingness to pay for proximity in lower income, high minority share areas. Such results suggest that white households may have a preference for neighborhood homogeneity which interacts with how they view the amenities/disamenities provided by LIHTC construction.

Our results show that affordable housing development has large welfare impacts as a place based policy, which more than offset the welfare impacts to tenants living in affordable housing. Given the goals of many affordable housing policies is to decrease income and racial segregation in housing markets, these goals might be achieved by investing in affordable housing in low income and high minority areas, which will then spark in-migration of high income and a more racially diverse set of residents. These housing market spillovers leading to broader neighborhood change could make a larger dent in lowering racial and income segregation in the housing market than policies which try to achieve these goals by targeting higher income or low minority areas with affordable housing developments.

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A Data & Policy Appendix

A.1 The Low Income Housing Tax Credit

In 1986, Congress passed the Tax Reform Act. As part of this legislation, Congress increased the Home Mortgage Interest Deduction and modified the treatment of imputed rent and local property taxes to further incentivize investment in owner-occupied housing. Concerned that such provisions would decrease the supply of affordable rental housing for low-income individuals, Congress introduced the Low Income Housing Tax Credit (LIHTC) as part of the Act to encourage the development of multifamily housing and thus serve as a balancing measure.

Each year, federal tax credits are allocated to the states based on population. In particular, each state receives the inflation-adjusted equivalent of $2.30 per resident, as measured in 2014 dollars. These credits are awarded by state authorities to developers of qualified projects. Developers then sell these credits to investors to raise equity capital for their projects and reduce the amount of debt they would otherwise have to borrow. Investors receive a dollar-for-dollar credit against their federal tax liability for a period of 10 years, provided the property continues to comply with all program guidelines.[36]

To qualify for a tax credit under the Low Income Tax Credit Program, federal guidelines require that proposed projects be for construction or rehabilitation of a residential rental property and satisfy either one of two low-income occupancy criteria. At least 20 percent of tenants must earn less than 50 percent of the Area Median Gross Income (AGMI), or alternatively, at least 40 percent of tenants must earn less than 60 percent of AGMI.[37] The AGMI is based on data from the Internal Revenue Service, the American Housing Survey, and the decennial Census. It is calculated annually by the Department of Housing and Urban Development (HUD) for all metropolitan areas and counties. If the income of a household in one of the low income units grows to exceed the relevant income limit, then the program requires developers to place a low income tenant into the next unit vacated by a market rate tenant.[38] Additionally, developers must restrict rents, including utility allowance, in low-income units to 30 percent of the relevant income limit, i.e. either 50 percent of 60 percent of AGMI for a minimum affordability period of 30 years.

Note that these criteria are only the minimal requirements as specified by the federal government. In practice, states almost always receive many more project proposals and tax credit allocation requests from developers than they have federal allotments, generally on the order of 2 to 4 times. Each state is therefore required to maintain a "Qualified Application Plan" (QAP) to govern the selection process. These plans usually operate by assigning point scores to various project characteristics and then allocating tax credits based on point totals until funds are exhausted. Such project characteristics include tenant

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[36] Eriksen (2009) studies the market pricing of these tax credits. He finds that that LIHTC developers in California received on average $0.73 per $1 of tax credit in the years 1999-2005.

[37] Actual income limits depend on household size. The 50 percent of AGMI limit is for a base family size of four members. Income limits are adjusted upward by 4 percentage points for each family member in excess of four. Limits are adjusted downward by 5 percentage points for each family member short of four. These limits are multiplied by 1.2 to get the 60 percent income limits.

[38] Many LITHC properties are comprised 100 percent of low-income units. Clearly, this requirement becomes superfluous in such a case.

demographics, location, further funding sources, and structural properties of the building. Given this latitude the states enjoy in determining selection criteria, many require developers to go beyond the minimum number of affordable units and the minimum level of affordability.

The funding process in California, in which more than 50 percent of the LIHTC sites in our sample are located, is illustrative of the high degree of competition which exists between developers. Practically all funded projects have the maximum total point score, such that the funding threshold is based on the so-called tie-breaker score, which reflects ratios of committed public funds to total developer cost, as well as eligible basis to total developer cost. Moreover, state guidelines specify certain allotments of funding for various set-asides, such as rural, non-profit, and small developments, as well as well as certain allotments by geographic region. The tie-breaker score threshold thus varies annually and by region and set-aside, depending on the number of applications received in that segment in a given year. Receiving funding in a given year is by no means guaranteed, reflecting the high degree of competition. According to data provided by the California State Treasurer's Office, only 34.3% of applications were funded in 2007, 38.3% in 2008, 32.2% in 2009, 26.9% in 2010, 57.7% in 2011, and 42.7% in 2012.

The value of tax credits received by selected developers is calculated according to the project's "qualified basis", which essentially reflects the cost of constructing or rehabilitating the low-income units. Specifically, the project's "eligible basis" is the value of all depreciable development costs, such as construction, engineering, soil tests, and utility connection fees. It does not include land acquisition costs. The qualified basis is then calculated by multiplying the eligible basis by the "applicable fraction." This is the smaller of two percentages, the fraction of low income units in the development, or the fraction of total square footage allotted to low income units. Once the qualified basis has been determined, the annual tax credit is determined by applying the relevant housing tax credit rate. New construction or substantial rehabilitation projects, which are not otherwise subsidized by the Federal government, receive a 9 percent credit rate, while all other projects receive a 4 percent credit rate. These annual credits are then paid out over a period of 10 years.[39]

Since its inception in 1986, the Low Income Housing Tax Credit Program has been an integral component in fostering the development of multifamily housing throughout the United States. With an annual tax credit valued at over 8 billion dollars, the program funded 21 percent of all multifamily developments between the years 1987-2008.

A.2   Data

A.2.1   DataQuick

Our first dataset is from DataQuick, which provides detailed public records housing characteristics and transactions data collected from county assessor and register of deeds

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[39]This calculation is a baseline figure. Congress passed legislation in 1989 affording state allocating agencies the option to increase the qualified basis by up to 30 percent in both "qualified census tracts" (QCTs) and "difficult development areas" (DDAs). Census tracts with 50 percent of households earning below 60 percent of AGMI earn qualified status, subject to a population restriction which is generally non-binding. Metropolitan areas with high ratios of fair market rent to AMGI are designated as DDAs. See Baum-Snow and Marion (2009) for more details.

officers. This dataset covers over 109 million properties from 1988-2012 in over 2,300 jurisdictions and provides information such as sales price, transaction type, loan amount, number of beds, number of baths, square-footage, lot size, age, etc. The quality of the DataQuick data is not uniform across the country. Certain states, such as Texas and Utah, do not require the prices of housing transactions to be a matter of public record. Thus, DataQuick does not report house prices for those states. Other states, such as Illinois, provide prices in their records but do not collect information regarding the number of bathrooms. Finally, not all of the counties covered by DataQuick have records dating back to 1988. Coverage of a significant number of counties began in 1996.

We restrict our analysis to those counties which have transactions history data dating to at least 1996. From this subset, we then restrict to those counties which have at least an average of 1000 residential arm-length sales per year. This leaves us with a sample of approximately 16 million transactions located within 1.5 miles of a LIHTC site in a total of 129 counties in 15 states, concentrated largely in the major metropolitan areas of New England, Florida, California, Illinois, North Carolina, Tennessee, the Southwest and the Pacific Northwest. Figure 1 provides a map of the counties in our sample.

A.2.2   Home Mortgage Disclosure Act Data

We merge this dataset with data collected by the United States federal government according to the provisions of the Home Mortgage Disclosure Act (HMDA). Passed in 1975 due to concerns over redlining in the mortgage market in urban, minority areas, this legislation requires all lending institutions to report public loan data. Implemented as Regulation C by the Federal Reserve Board, it was amended in 1989 in response to the Financial Institutions Reform, Recovery, and Enforcement Act (FIRREA).[40] These amendments greatly expanded the mortgage lenders covered under HMDA and required reporting of significant demographic information of both loan applicants and borrowers. The government provides public historical archives of this data covering the period 1991-2012. It includes information on loan census tract, loan amount, loan provider, and borrower demographics such as race, income, and sex. Since there is not a unique loan ID on which we can match the DataQuick data to the HMDA data, we perform a fuzzy merge. In particular, we merge the data according to census tract, year, loan amount, and bank name. This results in a match rate of approximately 80 percent. To examine whether our merge procedure linking the housing transaction data to the HMDA data introduces selection biases, Table 1 also reports house prices using both the DataQuick database alone as well as the merged DataQuick-HMDA database. The housing prices of those matched to the HMDA data are about 10% higher than the average housing transaction, however this does not vary with distance to a LIHTC site.

A.2.3   HUD LIHTC Database

Information on LIHTC financed projects is provided by the Department of Housing and Urban Development (HUD). This data covers 39,094 projects and almost 2,458,00 low

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[40]The rule-writing authority of Regulation C was transferred from the Federal Reserve Board to the Consumer Financial Protection Bureau on July 21, 2011.

income housing units placed into service between the years 1987 and 2012. Note that the data therefore reflects only those projects approved by the state allocating agencies and not all project proposals submitted to them. The dataset includes detailed geographic information regarding the project location, the type of construction, the year the project was placed in service, the year funds for the project were allocated, and the number of units designated as low-income. Geocoded information is missing for some of the projects and we exclude from the sample any projects for which the year funds allocated variable is missing. We are left with 7098 LIHTC projects located in our sample of 129 counties. See Panel C of Table 1 for summary statistics. Due to DataQuick's coverage of counties, our sample is from more dense, urban areas, relative to the overall distribution of LIHTC sites.

B   Comparison of Ring Method vs Empirical Derivatives

We will assess the performance of the standard ring difference-in-differences method with our new empirical derivatives method by a simulation study. We will match characteristics of our problem to make the simulation study as close as possible to the scenario we analyze in our data. First, we will simulate data for L affordable housing developments, each subscripted by l. We assume the price of a house j located near LIHTC site l has a price at time t as defined by:

$$
\begin{aligned}
p_{jt} &= 1 + 0.05 * (0.8 - r)^2 * \mathbf{1}\,(t > 0) * \mathbf{1}\,(r < 0.8) + \alpha_l * t + \beta_l^{lat} LAT_j + \beta_l^{lon} LON_j + \varepsilon_{jt}, \\
&\quad \varepsilon_{jt} \sim N\,(0, 0.036)\,, \ \beta_l^{lat} \sim N\,(0, 0.036)\,, \ \beta_l^{lon} \sim N\,(0, 0.036)\,, \ \alpha_l \sim N\,(0, 0.025)\,.
\end{aligned}
$$

The true treatment effect of interest is: $0.05 * (0.8 - r)^2 * \mathbf{1}\,(t > 0) * \mathbf{1}\,(r < 0.8)$, while each neighborhood l as its own time invariant surface of house prices, as represented by the plane: $\beta_l^{lat} LAT_j + \beta_l^{lon} LON_j$ and (LAT_j, LON_j) are the coordinates of house j and β_l^{lat}, β_l^{lon} measure the effect of the location on house prices, independent of time period. These parameters are drawn from a normal distribution for each neighborhood l. $\alpha_l * t$ represents a neighborhood specific time trend for house prices, where α_l is drawn from a normal distribution for each neighborhood. ε_{jt} is a house specific error that is normally distributed.

To make this simulation comparable to the real dataset, we allow there to be $L = 1721$ LIHTC sites used for the simulation (the number in each of our 4 income quartiles in the real data). We allow there to be 58 housing transactions per square mile per year around each LIHTC site, which is what we have in our real LIHTC data.

The standard ring difference in difference method is to compare the change in house prices in an inner ring to the change in house prices in an outer ring. This discretizes the spatial component of the data. Time is also treated as discrete, measured by year. We define our inner ring as the average house price transaction in year T within a 0.25 mile radius 0.25:

$$
R_{lT}^{in} = \frac{\sum_{jt} p_{jt} * \mathbf{1}\,(Yr\,(t) = T) * \mathbf{1}\,(r \le 0.25)}{\sum_{jt} \mathbf{1}\,(Yr\,(t) = T) * \mathbf{1}\,(r \le 0.25)}.
$$

where T indexes years (as opposed to exact dates) and $Yr\,(t)$ is a function that maps

exact dates to the year of the transaction. We define a middle ring in year T at radius $0.25 < r \leq 0.5$:

$$R_{lT}^{mid} = \frac{\sum_{jt} p_{jt} * \mathbf{1}\left(Yr\left(t\right) = T\right) * \mathbf{1}\left(0.25 < r \leq 0.5\right)}{\sum_{jt} \mathbf{1}\left(Yr\left(t\right) = T\right) * \mathbf{1}\left(0.25 < r \leq 0.5\right)},$$

and an outer ring at $0.5 < r \leq 0.75$:

$$R_{lT}^{out} = \frac{\sum_{jt} p_{jt} * \mathbf{1}\left(Yr\left(t\right) = T\right) * \mathbf{1}\left(0.5 < r \leq 0.75\right)}{\sum_{jt} \mathbf{1}\left(Yr\left(t\right) = T\right) * \mathbf{1}\left(0.5 < r \leq 0.75\right)}.$$

The inner treatment effect in year T is defined as the difference in inner versus outer rings in year T, relative to the year before treatment (treatment occurs in year 0):

$$\text{Treat\_inner} = \left(R_{lT}^{in} - R_{lT}^{out}\right) - \left(R_{l,-1}^{in} - R_{l,-1}^{out}\right).$$

The middle ring treatment is defined similarly:

$$\text{Treat\_middle} = \left(R_{lT}^{mid} - R_{lT}^{mid}\right) - \left(R_{l,-1}^{in} - R_{l,-1}^{out}\right).$$

We calculate inner and middle ring treatment effects and plot their path over time, along with bootstrapped standard errors, where the bootstrap sampling is down at the neighborhood l level (the same as we use for our empirical derivative method). Results are plotted in panels C and D of Figure A4. For both the inner and outer ring, we fail to reject a point estimate of zero treatment effect. The ring estimate's confidence interval does contain the true point estimate as we would hope. The estimates are very noisy because the make no attempt to control for the within-ring variation in house prices due to $\beta_l^{lat} LAT_j + \beta_l^{lon} LON_j$.

In comparison, Panels A and B plot the estimated non-parametric effect at 0.1 miles from the LIHTC set and 0.4 miles from the LIHTC site. Despite these being nonparametric estimates, the standard errors are smaller than in the ring method and we reject a point estimate of zero in both cases. The true value is also contained in the 95% confidence intervals. Despite being a nonparametric estimate, the empirical derivative method perform better because it accounts for the time-invariant spatial variation in house prices: $\beta_l^{lat} LAT_j + \beta_l^{lon} LON_j$. It also makes better use of thin data by smoothing nearby data together with house transactions on unavailable in a given year-ring Our method also produces a fully non-parametric plot of house price effects, as seen in Figure A5, while the ring method only delivers a couple discrete numbers.

This is not to say that the empirical derivative methods will always have smaller confidence intervals. If the time-invariant spatial variation in house prices $\left(\beta_l^{lat} LAT_j + \beta_l^{lon} LON_j\right)$ is not very important within the rings used for analysis, we would expect the ring method to produce more powerful estimates than the nonparametrics, since it does not need to recover as much information from the data. Also, as with all nonparametric estimators, there will be some bias in the estimates due to smoothing. We assess this bias-variance trade off in the Monte Carlo simulation in the next appendix.

C   Monte Carlo Simulations

We perform some Monte Carlo simulations to assess the bias and variance in our estimator, as well as the coverage ratio of our standard errors. We will do this under a variety of value of the tuning parameters to valid their impacts on the estimates. We will use the same model as in the previous appendix to simulate our data to feed into our estimator:

$$p_{jt} = 1 + 0.05 * (0.8 - r)^2 * \mathbf{1}\,(t > 0) * \mathbf{1}\,(r < 0.8) + \alpha_l * t + \beta_l^{lat} LAT_j + \beta_l^{lon} LON_j + \varepsilon_{jt},$$
$$\varepsilon_{jt} \tilde{\ } N\,(0, 0.036)\,, \ \beta_l^{lat} \tilde{\ } N\,(0, 0.036)\,, \ \beta_l^{lon} \tilde{\ } N\,(0, 0.036)\,, \ \alpha_l \tilde{\ } N\,(0, 0.025)\,.$$

We will also keep the size of the dataset the same as used in the previous simulation. We allow there to be $L = 1721$ LIHTC sites used for each simulation (the number in each of our 4 income quartiles in the real data). We allow there to be 58 housing transactions per square mile per year around each LIHTC site, which is what we have in our real LIHTC data. We will perform 100 monte carlo simulation for each set of tuning parameters.

There are 2 tuning parameters in our empirical derivative estimator: κ and u. κ determines the number of pairs of house prices to use in empirical derivative calculation, as shown in equation (2). In our main estimates we set $\kappa = 5$, meaning we use 5 pairs of house prices (where one house within each pair is located closer to the LIHTC site and one is located further away from the point of derivative evaluation). Using a large number of house price pairs should lead to less variance in our estimator, since it uses more data, however it can also increase the bias of our estimate, since it will incorporate house prices that are further away from the site of derivative evaluation. We will try values of $\kappa = 5$ and $\kappa = 3$ in our monte carlo simulation.

Panel A of Figure A6 plots the bias in our derivative estimator. We see slightly more bias in our estimator for the higher values of k, as expected. This is also seen in Panel B of Figure A6, which shows the bias in the levels estimate (where we just integrate the derivative estimate). Panels C and D of Figure A6 plot the variance of our estimator across the monte carlo simulations. As expected, the higher values of κ lower the variance in our estimator, highlighting the bias-variance trade off.

The second tuning parameter determines the width of the "bow-tie" ϑ used to select houses eligible for the empirical derivative estimation. In our main estimates, we use a value of $\vartheta = 0.5$, lower values of ϑ allow houses located far away in the time and θ dimension from the point of derivative evaluation to be included in the calculation. Higher values force the estimator to only use house prices with very similar values of θ and t for derivative estimation. For our monte carlo simulation, we evaluate $\vartheta = 0.5$ and $\vartheta = 0.2$. Lower values of ϑ should increase variance in our estimator, as it does a less accurate job of controlling for θ and t. However this should also decrease the bias in our estimator, since it allows more of the data used for derivative estimator to be very close to the point of evaluation in the r direction, regardless of θ and t. Looking at Panel A of Figure A6, this is exactly what we see. There is less bias in the derivative estimator at lower values of ϑ. Panel B of Figure A6 reports the bias in the levels estimate. Again, lower values of ϑ lead to less bias. Panels C and D of Figure A6 plot the variance in our estimator across the monte carlo simulations. As predicted, lower levels of ϑ increase the variance of our estimator. As expected, there is the standard bias-variance trade off.

Regardless of the tuning parameters, Panel A shows that the bias in the derivative estimate is at its worst at a distance of 0.8. This is not surprising, since the true derivative at this point is kinked. The nonparametric estimator smooths over this kink, leading to bias.

To assess whether our bootstrapping procedure produces accurate confidence intervals, Figure A7 plots the coverage rates of our 95% confidence intervals. Note that we adjust our point estimate by the estimated bias from the previous figure to allows us to look at the coverage ratios of our confidence intervals, independent of the bias in our estimator due to the choice of tuning parameters. We find all of our coverage ratios are centered around 95%, which makes us very comfortable with our standard error calculations.

D   Econometric Proofs

We begin by developing general econometric methods for estimating empirical derivatives. We will then apply these results to prove Theorem 1 in the main text. We develop our general econometric results by first focusing on the univariate case. This will ease exposition and provide intuition. Results for the multivariate case follow closely the logic of the univariate case.

D.1   Univariate Case

Suppose we draw an iid sample of size n from the following nonparametric model

$$Y_i = m(x_i) + \varepsilon_i$$

where $E(\varepsilon_i|x_i) = 0$ and $Var(\varepsilon_i|x_i) = \sigma^2$. We assume that $m(x)$ is twice continuously differentiable. The variable x_i is distributed according to the continuous density function:

$$f(x) : [x_{\min}, x_{\max}] \to (0, \infty).$$

We propose the following Nadaraya-Watson kernel estimator for the first derivative $m'(x_i)$:

$$
\begin{aligned}
\widehat{m}'(x) &= \frac{n^{-1} \sum_{i=1}^{n} K_{h_n}(x - x_i) \widetilde{Y}_i}{n^{-1} \sum_{i=1}^{n} K_{h_n}(x - x_i)} \\
\widetilde{Y}_i &= \sum_{j=1}^{k_{n,i}} w_j \frac{Y_{i,+j} - Y_{i,-j}}{x_{i,+j} - x_{i,-j}} \mathbf{1}[k_{n,i} > 0] \\
w_j &= \frac{j}{k_{n,i}(k_{n,i} + 1)/2},
\end{aligned}
\tag{31}
$$

with the observations $\{(x_{i,+j}, Y_{i,+j})\}_{j=1}^{k_{n,i}} \{(x_{i,-j}, Y_{i,-j})\}_{j=1}^{k_{n,i}}$ defined recursively by:

$$
\begin{aligned}
x_{i,+1} &= \arg \min_{x > x_i + l_n} x, & x_{i,-1} &= \arg \max_{x < x_i - l_n} x \\
x_{i,+j} &= \arg \min_{x > x_{i,+j-1}} x, & x_{i,-j} &= \arg \max_{x < x_{i,-j-1}} x
\end{aligned}
$$

for $j = 2, ..., k_{n,i}$ where $l_n > 0$. Note that $Y_{i,+j} = m(x_{i,+j}) + \varepsilon_{i,+j}$. The random variable $k_{n,i}$ is defined as:

$$
\begin{aligned}
k_{n,i} &= \min(|L_{n,i}|, |U_{n,i}|, \kappa_n) \\
L_i &= \{x_p : x_p < x_i - l_n\} \\
U_i &= \{x_p : x_p > x_i + l_n\}
\end{aligned}
$$

for some $\kappa_n > 0$. We define $K_{h_n}(x - x_i)$ as

$$
K_{h_n}(x - x_i) = \frac{1}{h_n} K\left(\frac{x - x_i}{h_n}\right),
$$

where $K(\cdot)$ is a kernel function. We have the following result:

Theorem 2 *Assume the random design model above and suppose:*

1. *$K(u)$ is bounded, compactly supported, and symmetric*

2. *$n \to \infty, h_n \to 0, l_n \to 0, \kappa_n \to \infty$*

3. *$nh_n \to \infty, \kappa_n n^{-1} \to 0, l_n^2 \kappa_n \to \infty$*

   Then $\widehat{m}'(x) \to^p m'(x)$ for all $x \in (x_{\min}, x_{\max})$.

We prove the theorem through a sequence of lemmas. In what follows, we denote the indicator variable $\mathcal{I}_{n,i} = \mathbf{1}[k_{n,i} > 0]$. We also denote $P_i(\cdot) = P(\cdot|x_i)$ and $E_i[\cdot] = E[\cdot|x_i]$.

Lemma 3 *The estimate*

$$
n^{-1} \sum_{i=1}^{n} K_{h_n}(x - x_i)
$$

converges in probability to $f(x)$ for all $x \in (x_{\min}, x_{\max})$.

Proof. This is a standard result. See Hardle (1990). ∎

Lemma 4 *As $n \to \infty$, $P(k_{n,i} < \infty) \to 0$.*

Proof. Trivial. ∎

Lemma 5 *Conditional on $x_i = u$ for any $u \in (x_{\min}, x_{\max})$, as $n \to \infty$ the difference $\left(x_{i,+k_{n,i}} - x_{i,-k_{n,i}}\right)\mathcal{I}_{n,i}$ converges in probability to zero.*

Proof. Fix $\delta > 0$. Since $l_n \to 0$, there exists N_1 such that for all $n > N_1$, $l_n < \delta/4$. Conditional on $x_i = u$, we have:

$$
P_i\left(\left(x_{i,+k_{n,i}} - x_{i,-k_{n,i}}\right)\mathcal{I}_{n,i} > \delta\right) \leq F_B(2\kappa_n; n-1, p_\delta)
$$

where $F_B \left(\cdot ; n - 1, p_\delta \right)$ denotes the binomial distribution and

$$p_\delta = \int_{u-\delta/2}^{u-\delta/4} f \left(s \right) ds + \int_{u+\delta/4}^{u+\delta/2} f \left(s \right) ds > 0.$$

Since $\kappa_n n^{-1} \to 0$ as $n \to \infty$, there exists N_2 such that $\kappa_n n^{-1} < p_\delta/4$ for all $n \geq N_2$. Let $N = \max \left\{ N_1, N_2, -2 \ln \left(\delta \right) / p_\delta^2 \right\}$. Then by Hoeffding's inequality, for all $n > N$:

$$\begin{aligned} P_i \left(\left(x_{i,+k_{n,i}} - x_{i,-k_{n,i}} \right) \mathcal{I}_{n,i} > \delta \right) & \leq & \exp \left(-2 \frac{\left(\left(n-1 \right) p_\delta - 2\kappa_n \right)^2}{n-1} \right) \\ & \leq & \exp \left(-\frac{\left(n-1 \right) p_\delta^2}{2} \right) \\ & \leq & \delta, \end{aligned}$$

which proves the claim. ∎

Lemma 6 *As $n \to \infty$, the bias:*

$$\left| E \left[\frac{1}{n} \sum_{i=1}^{n} K_{h_n} \left(x - x_i \right) \widetilde{Y}_i \right] - m' \left(x \right) f \left(x \right) \right| \to 0.$$

Proof. The idea of the proof is to apply a first-order Taylor expansion to the empirical derivatives. We then must show that the remainder term in the expansion converges to zero as the sample size grows. We can write the bias of the estimator as:

$$\begin{aligned} & \left| E \left[\frac{1}{n} \sum_{i=1}^{n} K_{h_n} \left(x - x_i \right) \widetilde{Y}_i \right] - m' \left(x \right) f \left(x \right) \right| \\ = & \left| E \left[\frac{1}{n} \sum_{i=1}^{n} K_{h_n} \left(x - x_i \right) \sum_{j=1}^{k_{n,i}} w_j \frac{Y_{i,+j} - Y_{i,-j}}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right] - m' \left(x \right) f \left(x \right) \right| \end{aligned}$$

Denote expectation conditional on the observed sample covariates as $E_x[\cdot]$. By applying Taylor's theorem, we have

$$
E_x\left[\sum_{j=1}^{k_{n,i}} \frac{Y_{i,+j} - Y_{i,-j}}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i}\right]
$$

$$
= m'(x_i) \sum_{j=1}^{k_{n,i}} w_j \frac{x_{i,+j} - x_{i,-j}}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i}
$$

$$
+ \frac{1}{2} \sum_{j=1}^{k_{n,i}} w_j \frac{m''(\zeta_{i,i+j})(x_{i,+j} - x_i)^2 - m''(\zeta_{i,i-j})(x_i - x_{i,-j})^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i}
$$

$$
= m'(x_i)\mathcal{I}_{n,i} + \frac{1}{2} \sum_{j=1}^{k_{n,i}} w_j \frac{m''(\zeta_{i,i+j})(x_{i,+j} - x_i)^2 - m''(\zeta_{i,i-j})(x_i - x_{i,-j})^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i}
$$

where $\zeta_{i,i+j} \in (x_i, x_{i,+j})$ and $\zeta_{i,i-j} \in (x_{i,-j}, x_i)$. By the law of iterated expectations and the triangle inequality, the bias is thus bounded above by

$$
\left| E\left[\frac{1}{n} \sum_{i=1}^{n} K_{h_n}(x - x_i) m'(x_i) \mathcal{I}_{n,i}\right] - m'(x) f(x) \right|
$$

$$
+ \left| E\left[\frac{1}{n} \sum_{i=1}^{n} K_{h_n}(x - x_i) \sum_{j=1}^{k_{n,i}} w_j \frac{m''(\zeta_{i,i+j})(x_{i,+j} - x_i)^2 - m''(\zeta_{i,i-j})(x_i - x_{i,-j})^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i}\right] \right|.
$$

We know that that $P(k_{n,i} > 0) \to 1$. The first term thus goes to zero as $n \to \infty$ according to the usual proof for consistency of Nadaraya-Watson estimators. See, for example, Hardle (1990). To show that the second term converges to zero, note first that it is bounded above by:[41]

$$
\sup_x |m''(x)| \int |K_{h_n}(x - x_i)| \left| E_i\left[\sum_{j=1}^{k_{n,i}} w_j \frac{(x_{i,+j} - x_i)^2 + (x_i - x_{i,-j})^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i}\right] \right| f(x_i)\, dx_i
$$

$$
\leq \sup_x |m''(x)| \int |K_{h_n}(x - x_i)| E_i\left[\sum_{j=1}^{k_{n,i}} w_j \frac{(x_{i,+j} - x_i)^2 + (x_i - x_{i,-j})^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i}\right] f(x_i)\, dx_i
$$

$$
\leq \sup_x |m''(x)| \int |K_{h_n}(x - x_i)| E_i\left[\sum_{j=1}^{k_{n,i}} w_j (x_{i,+j} - x_{i,-j}) \mathcal{I}_{n,i}\right] f(x_i)\, dx_i
$$

$$
\leq \sup_x |m''(x)| \int |K_{h_n}(x - x_i)| E_i\left[(x_{i,+k_{n,i}} - x_{i,-k_{n,i}}) \mathcal{I}_{n,i}\right] f(x_i)\, dx_i
$$

---

[41] Recall that $E_i[\cdot] = E[\cdot|x_i = u]$.

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By Hardle (1990), it thus suffices to show that $E_i\left[\left(x_{i,+k_{n,i}} - x_{i,-k_{n,i}}\right)\mathcal{I}_{n,i}\right]$ converges to zero. But we know by Lemma 5, $P_i\left(\left(x_{i,+k_{n,i}} - x_{i,-k_{n,i}}\right)\mathcal{I}_{n,i} > 0\right)$ converges to zero. Since $\left(x_{i,+k_{n,i}} - x_{i,-k_{n,i}}\right)\mathcal{I}_{n,i}$ is almost surely bounded, convergence in probability implies $E_i\left[\left(x_{i,+k_{n,i}} - x_{i,-k_{n,i}}\right)\mathcal{I}_n\right.$ converges to zero as well. This completes the proof. ∎

Lemma 7 *As $n \to \infty$, the variance:*

$$Var\left(n^{-1}\sum_{i=1}^{n} K_{h_n}\left(x - x_i\right)\widetilde{Y}_i\right) \to 0.$$

Proof. The variance can be decomposed as:

$$\frac{1}{n}Var\left[K_{h_n}\left(x - x_i\right)\widetilde{Y}_i\right] + \frac{1}{n^2}\sum_{i\neq j}Cov\left(K_{h_n}\left(x - x_i\right)\widetilde{Y}_i, K_{h_n}\left(x - x_j\right)\widetilde{Y}_j\right) \qquad (32)$$

By the law of total variance:

$$\frac{1}{n}Var\left[K_{h_n}\left(x - x_i\right)\widetilde{Y}_i\right] = \frac{1}{n}\left[E\left[Var_i\left(K_{h_n}\left(x - x_i\right)\widetilde{Y}_i\right)\right] + Var\left(E_i\left[K_{h_n}\left(x - x_i\right)\widetilde{Y}_i\right]\right)\right]$$

$$\to \frac{1}{n}E\left[K_{h_n}^2\left(x - x_i\right)Var_i\left(\widetilde{Y}_i\right)\right] + \frac{1}{n}Var\left(K_{h_n}\left(x - x_i\right)m'\left(x_i\right)\right)$$

The second term approaches zero as $n \to \infty$ by Hardle (1990). Also by Hardle (1990), for the first term it suffices to show that:

$$Var_i\left(\widetilde{Y}_i\right) \to 0.$$

The idea of the proof is to again expand the empirical derivatives using Taylor's theorem. We then use the law of total variance to condition on the observables. We then prove that the variance converges to zero. Applying a Taylor expansion, the variance becomes:

$$Var_i\left(\begin{array}{c}\sum_{j=1}^{k_{n,i}} w_j \frac{\varepsilon_{i,+j} - \varepsilon_{i,-j}}{x_{i,+j} - x_{i,-j}}\mathcal{I}_{n,i} \\ + \sum_{j=1}^{k_{n,i}} w_j \frac{m''\left(\zeta_{i,i+j}\right)\left(x_{i,+j} - x_i\right)^2 - m''\left(\zeta_{i,i-j}\right)\left(x_i - x_{i,-j}\right)^2}{x_{i,+j} - x_{i,-j}}\mathcal{I}_{n,i}\end{array}\right)$$

Using the law of total variance, this can be decomposed as:

$$Var_i\left(\begin{array}{c}E_x\left[\sum_{j=1}^{k_{n,i}} w_j \frac{\varepsilon_{i,+j} - \varepsilon_{i,-j}}{x_{i,+j} - x_{i,-j}}\mathcal{I}_{n,i}\right] \\ + E_x\left[w_j \frac{m''\left(\zeta_{i,i+j}\right)\left(x_{i,+j} - x_i\right)^2 - m''\left(\zeta_{i,i-j}\right)\left(x_i - x_{i,-j}\right)^2}{x_{i,+j} - x_{i,-j}}\mathcal{I}_{n,i}\right]\end{array}\right) \qquad (33)$$

$$+ E_i\left[Var_x\left(\sum_{j=1}^{k_{n,i}} w_j \frac{\varepsilon_{i,+j} - \varepsilon_{i,-j}}{x_{i,+j} - x_{i,-j}}\mathcal{I}_{n,i}\right)\right]$$

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We first note that

$$
E_x \left[\sum_{j=1}^{k_{n,i}} w_j \frac{\varepsilon_{i,+j} - \varepsilon_{i,-j}}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right] = 0,
$$

since $E\left(\varepsilon_{i+j}|x_{i,+j}\right) = E\left(\varepsilon_{i-j}|x_{i,-j}\right) = 0$. Thus the first term in (33) becomes:

$$
Var_i \left(E_x \left[\sum_{j=1}^{k_{n,i}} w_j \frac{m''\left(\zeta_{i,i+j}\right)\left(x_{i,+j} - x_i\right)^2 - m''\left(\zeta_{i,i-j}\right)\left(x_i - x_{i,-j}\right)^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right] \right)
$$

$$
= \quad E_i \left[\sum_{j=1}^{k_{n,i}} w_j \frac{m''\left(\zeta_{i,i+j}\right)\left(x_{i,+j} - x_i\right)^2 - m''\left(\zeta_{i,i-j}\right)\left(x_i - x_{i,-j}\right)^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right]
$$

$$
- E_i \left[\sum_{j=1}^{k_{n,i}} w_j \frac{m''\left(\zeta_{i,i+j}\right)\left(x_{i,+j} - x_i\right)^2 - m''\left(\zeta_{i,i-j}\right)\left(x_i - x_{i,-j}\right)^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right]^2
$$

The final term converges to zero by the preceding lemma. The first term is:

$$
E_i \left[\left(\sum_{j=1}^{k_{n,i}} w_j \frac{m''\left(\zeta_{i,i+j}\right)\left(x_{i,+j} - x_i\right)^2 - m''\left(\zeta_{i,i-j}\right)\left(x_i - x_{i,-j}\right)^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right)^2 \right]
$$

$$
\leq \quad E_i \left[\left(\sum_{j=1}^{k_{n,i}} w_j \left| \frac{m''\left(\zeta_{i,i+j}\right)\left(x_{i,+j} - x_i\right)^2 - m''\left(\zeta_{i,i-j}\right)\left(x_i - x_{i,-j}\right)^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right| \right)^2 \right]
$$

$$
\leq \quad \left[\sup_x m''\left(x\right) \right]^2 E_i \left[\left(\sum_{j=1}^{k_{n,i}} w_j \frac{\left(x_{i,+j} - x_i\right)^2 + \left(x_i - x_{i,-j}\right)^2}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right)^2 \right]
$$

$$
\leq \quad \left[\sup_x m''\left(x\right) \right]^2 E_i \left[\left(x_{i+k_{n,i}} - x_{i-k_{n,i}} \right)^2 \mathcal{I}_{n,i} \right],
$$

which approaches zero as $n \to \infty$ by Lemma 5 and almost sure boundedness. Thus, the first term in (33) converges to zero. Turning to the second term in (33):

$$E_i \left[Var_x \left(\sum_{j=1}^{k_{n,i}} w_j \frac{\varepsilon_{i,+j} - \varepsilon_{i,-j}}{x_{i,+j} - x_{i,-j}} \mathcal{I}_{n,i} \right) \right]$$

$$= E_i \left[2\sigma^2 \sum_{j=1}^{k_{n,i}} \frac{w_j^2}{(x_{i,+j} - x_{i,-j})^2} \mathcal{I}_{n,i} \right]$$

$$\leq \frac{2\sigma^2}{4l_n^2} E_i \left[\sum_{j=1}^{k_{n,i}} \frac{j^2}{k_{n,i}^2 (k_{n,i} + 1)^2 / 4} \mathcal{I}_{n,i} \right]$$

$$\leq \frac{\sigma^2}{3l_n^2} E_i \left[\frac{2k_{n,i} + 1}{k_{n,i} (k_{n,i} + 1)} \mathcal{I}_{n,i} \right]$$

$$\leq \frac{\sigma^2}{3} E_i \left[\frac{1}{l_n^2 k_{n,i}} \mathcal{I}_{n,i} \right]$$

$$\leq \frac{\sigma^2}{3} \left[\frac{1}{l_n^2 \kappa_n} + \frac{1}{l_n^2} P (k_{n,i} < \kappa_n) \right]$$

The first term in brackets approaches zero by assumption. For sufficiently large n, by applying $\kappa_n n^{-1} \to 0$ and Hoeffding's inequality as in the previous lemma, we have for sufficiently large $n > N$:

$$\frac{1}{l_n^2} P (k_{n,i} < \kappa_n) \leq \frac{1}{l_n^2} \exp \left(-\frac{(n-1) p_N^2}{2} \right)$$

where:

$$p_N = \int_{x_{\min}}^{u - l_N} f (s) \, ds + \int_{u + l_N}^{x_{\max}} f (s) \, ds.$$

Since $l_n^2 \kappa_n \to \infty$, $\kappa_n n^{-1} \to 0$, and $n \exp (-n) \to 0$, it follows that:

$$\frac{1}{l_n^2} P (k_{n,i} < \kappa_n) \to 0$$

as desired.

Turning to the covariance in equation (32), we apply the law of total covariance:[42]

$$\frac{1}{n^2} \sum_{i \neq j} Cov\left(K_{h_n}\left(x - x_i\right)\widetilde{Y}_i, K_{h_n}\left(x - x_j\right)\widetilde{Y}_j\right)$$

$$= \frac{n-1}{n} E\left[K_{h_n}\left(x - x_i\right) K_{h_n}\left(x - x_j\right) Cov_{ij}\left(\widetilde{Y}_i, \widetilde{Y}_j\right)\right]$$

$$+ \frac{n-1}{n} Cov\left(K_{h_n}\left(x - x_i\right) E_{ij}\left[\widetilde{Y}_i\right], K_{h_n}\left(x - x_j\right) E_{ij}\left[\widetilde{Y}_j\right]\right)$$

$$\rightarrow \frac{n-1}{n} E\left[K_{h_n}\left(x - x_i\right) K_{h_n}\left(x - x_j\right) Cov_{ij}\left(\widetilde{Y}_i, \widetilde{Y}_j\right)\right]$$

$$+ \frac{n-1}{n} Cov\left(K_{h_n}\left(x - x_i\right) m'\left(x_i\right), K_{h_n}\left(x - x_j\right) m'\left(x_j\right)\right)$$

$$= \frac{n-1}{n} E\left[K_{h_n}\left(x - x_i\right) K_{h_n}\left(x - x_j\right) Cov_{ij}\left(\widetilde{Y}_i, \widetilde{Y}_j\right)\right].$$

By Hardle (1990), we thus need to show $Cov_{ij}\left(\widetilde{Y}_i, \widetilde{Y}_j\right)$ converges to zero as $n \rightarrow \infty$. This will be the case as long as the probability that there is overlapping data used to form the empirical derivatives $\widetilde{Y}_i$ and $\widetilde{Y}_j$ goes to zero as $n \rightarrow \infty$. This can easily be shown by Hoeffding's inequality, using the exact same approach as in Lemma 5. ∎

The previous two lemmas show that the numerator of equation (31) converges in mean-square error to zero, which implies convergence in probability. The theorem thus follows by Slutsky's theorem.

D.2 Multivariate Case

We now extend our estimation procedure to the multidimensional case. Suppose we draw an iid sample of size n from the following nonparametric model

$$Y_i = m\left(\mathbf{X}_i\right) + \varepsilon_i$$

where $E\left(\varepsilon_i | \mathbf{X}_i\right) = 0$ and $Var\left(\varepsilon_i | \mathbf{X}_i\right) = \sigma^2$. We assume that $\mathbf{X}_i = \left(x_{1,i}, ... x_{D,i}\right) \in \mathbb{R}^D$ and m is twice continuously differentiable in all of its arguments. The variables $\mathbf{X}_i$ are distributed according to the continuous joint density function:

$$f\left(\mathbf{X}\right) : \prod_{d=1}^{D} \left[x_{d,\min}, x_{d,\max}\right] \rightarrow \left(0, \infty\right).$$

---

[42]We denote $E_{ij}\left[\cdot\right] = E\left[\cdot | x_i, x_j\right]$ and $Cov_{ij}\left(\cdot\right) = Cov\left(\cdot | x_i, x_j\right).$

We propose the following Nadaraya-Watson kernel estimator for the first derivative $\partial m / \partial x_d$:

$$\frac{\partial \widehat{m}}{\partial x_d} = \frac{n^{-1} \sum_{i=1}^{n} K_{\mathbf{H}_n} (\mathbf{X} - \mathbf{X}_i) \widetilde{Y}_i}{n^{-1} \sum_{i=1}^{n} K_{\mathbf{H}_n} (\mathbf{X} - \mathbf{X}_i)} \qquad (34)$$

$$\widetilde{Y}_i = \sum_{j=1}^{k_{n,i}} w_j \frac{Y_{a(j,i,d)} - Y_{b(j,i,d)}}{x_{d,a(j,i,d)} - x_{d,b(j,i,d)}} \mathbf{1} [k_{n,i} > 0]$$

$$w_j = \frac{j}{k_{n,i} (k_{n,i} + 1) / 2},$$

with the observations $\left\{ \left(\mathbf{X}_{a(j,i,d)}, Y_{a(j,i,d)} \right) \right\}_{j=1}^{k_n} \left\{ \left(\mathbf{X}_{b(j,i,d)}, Y_{b(j,i,d)} \right) \right\}_{j=1}^{k_n}$ defined recursively by:

$$a(1,i,d) = \arg \min_{\left\{ p \in L_{d,i} : x_{d,p} > x_{d,i} + l_n \right\}} x_{d,p}, \quad b(1,i,d) = \arg \max_{\left\{ p \in L_{d,i} : x_{d,p} < x_{d,i} - l_n \right\}} x_{d,p}$$

$$a(j,i,d) = \arg \min_{\left\{ p \in L_{d,i} : x_{d,p} > x_{d,a(j-1,i,d)} \right\}} x_{d,p}, \quad b(j,i,d) = \arg \max_{\left\{ p \in L_{d,i} : x_{d,p} < x_{d,b(j-1,i,d)} \right\}} x_{d,p}$$

for $j = 2, ..., k_{n,i}$ where $l_n > 0$ and:

$$L_{d,i} := \left\{ p \in \{1, ..., n\} : \frac{(x_{q,p} - x_{q,i})^2}{(x_{d,p} - x_{d,i})^2} < \vartheta_n \text{ for all } q \neq d \right\}$$

for some $\vartheta_n > 0$. The random variable $k_{n,i}$ is defined as:

$$k_{n,i} = \min \left(|L_{n,i}|, |U_{n,i}|, \kappa_n \right)$$
$$L_i = \{ p \in L_{d,i} : x_{d,p} < x_{d,i} - l_n \}$$
$$U_i = \{ p \in L_{d,i} : x_{d,p} > x_{d,i} + l_n \}$$

for some $\kappa_n > 0$. Letting $\mathbf{H}_n = (h_{1,n}, ..., h_{D,n})$, note that:

$$K_{\mathbf{H}_n} (\mathbf{X} - \mathbf{X}_i) = \frac{1}{h_{1,n} \cdots h_{D,n}} K \left(\frac{x_1 - x_{1,i}}{h_{1,n}}, ..., \frac{x_D - x_{D,i}}{h_{D,n}} \right),$$

where $K(\cdot)$ is a kernel function. We have the following result:

Theorem 8 *Assume the random design model above and suppose:*

1. *$K(u)$ is bounded, compactly supported, and spherically symmetric.*

2. *$n \to \infty, h_n \to 0, l_n \to 0, \kappa_n \to \infty, \vartheta_n \to 0$*

3. *$nh_n \to \infty, \kappa_n \left(n\vartheta_n^{D-1} \right)^{-1} \to 0, l_n^2 \kappa_n \to \infty$*

Then:

$$\frac{\partial \widehat{m}}{\partial x_d} (\mathbf{X}) \to^p \frac{\partial m}{\partial x_d} (\mathbf{X})$$

for all $\mathbf{X} \in \prod_{d=1}^{D} (x_{d,\min}, x_{d,\max})$.

We again prove the result in a sequence of lemmas.

Lemma 9 *The estimate*

$$n^{-1} \sum_{i=1}^{n} K_{\mathbf{H}_n} (\mathbf{X} - \mathbf{X}_i)$$

converges in probability to $f(\mathbf{X})$ *for all* $\mathbf{X} \in \prod_{d=1}^{D} (x_{d,\min}, x_{d,\max})$.

Proof. This is a standard result. See Hardle (1990). ∎

Lemma 10 *As* $n \to \infty$, $P(k_{n,i} < \infty) \to 0$.

Proof. Unlike the univariate case, we have to do some work to prove this. The issue is that the "bowtie" may collapse to a line too quickly. Since $\kappa_n \to \infty$, without loss of generality it suffices to show that $P(|U_{n,i}| < \infty) \to 0$. We first note that since f is a continuous function from a compact set to $(0, \infty)$ it must achieve a minimum, which we denote as $\Delta > 0$. Choose any $M \geq 0$. It is then straightforward to check via multiple integration that for $n > M + 1$:

$$P(|U_{n,i}| \leq M) \leq F_B(M; n-1, p_\Delta)$$

where:

$$p_\Delta = \frac{\Delta \vartheta_n^{(D-1)/2} (x_{d,\max} - x_{d,i})^D}{D!}$$

By Hoeffding's inequality:

$$
\begin{aligned}
P(|U_{n,i}| \leq M) &\leq \exp\left(-2\frac{((n-1)\,p_\Delta - M)^2}{n-1}\right) \\
&= \exp\left(-2\left[(n-1)\,p_\Delta^2 - 2p_\Delta M + \frac{M^2}{n-1}\right]\right).
\end{aligned}
$$

Since $\kappa_n \left(n\vartheta_n^{D-1}\right)^{-1} \to 0$, it must be that $n\vartheta_n^{D-1} \to \infty$. It therefore follows that $(n-1)\,p_\Delta^2 \to \infty$. Two two final terms in the bracket approach zero. Therefore, $P(|U_{n,i}| \leq M)$ converges to zero. ∎

Lemma 11 *As* $n \to \infty$, *the bias:*

$$\left| E\left[n^{-1} \sum_{i=1}^{n} K_{\mathbf{H}_n} (\mathbf{X} - \mathbf{X}_i) \widetilde{Y}_i\right] - \frac{\partial m}{\partial x_d}(\mathbf{X}) f(\mathbf{X}) \right| \to 0.$$

Proof. Write the bias as:

$$\left| E\left[\frac{1}{n} \sum_{i=1}^{n} K_{\mathbf{H}_n} \left(\mathbf{X} - \mathbf{X}_i \right) \widetilde{Y}_i \right] - \frac{\partial m}{\partial x_d} \left(\mathbf{X} \right) f \left(\mathbf{X} \right) \right|$$

$$= \left| E\left[\sum_{i=1}^{n} K_{\mathbf{H}_n} \left(\mathbf{X} - \mathbf{U} \right) \sum_{j=1}^{k_n} w_j \frac{Y_{a(j,i,d)} - Y_{b(j,i,d)}}{x_{d,a(j,i,d)} - x_{d,b(j,i,d)}} \mathcal{I}_{n,i} \right] - \frac{\partial m}{\partial x_d} \left(\mathbf{X} \right) f \left(\mathbf{X} \right) \right|$$

where $\mathbf{U} = (u_1, ..., u_D)$ and $d\mathbf{U} = du_1 \cdots du_D$. Conditioning on the sample and applying Taylor's theorem as in the univariate case, we have:

$$E_{\mathbf{X}}\left[\sum_{j=1}^{k_n} w_j \frac{Y_{a(j,i,d)} - Y_{b(j,i,d)}}{x_{d,a(j,i,d)} - x_{d,b(j,i,d)}} \mathcal{I}_{n,i} \right]$$

$$= \sum_{j=1}^{k_n} \sum_{q=1}^{D} w_j \frac{\partial m}{\partial x_q} \left(\mathbf{U} \right) \frac{x_{q,a(j,i,d)} - x_{q,b(j,i,d)}}{x_{d,a(j,i,d)} - x_{d,b(j,i,d)}} \mathcal{I}_{n,i}$$

$$+ \sum_{j=1}^{k_n} \sum_{|\alpha|=2} w_j \frac{R_\alpha \left(\mathbf{X}_{a(j,i,d)} \right) \left(\mathbf{X}_{a(j,i,d)} - \mathbf{X}_i \right)^\alpha - R_\alpha \left(\mathbf{X}_{b(j,i,d)} \right) \left(\mathbf{X}_{b(j,i,d)} - \mathbf{X}_i \right)^\alpha}{x_{d,a(j,i,d)} - x_{d,b(j,i,d)}} \mathcal{I}_{n,i}$$

where we have used the multi-index notation with $\alpha \in \mathbb{N}^D$ and where $R_\alpha \left(\cdot \right)$ denotes the remainder of the Taylor expansion. By the triangle inequality, the bias is therefore bounded above by

$$\left| E\left[K_{\mathbf{H}_n} \left(\mathbf{X} - \mathbf{U} \right) \frac{\partial m}{\partial x_d} \left(\mathbf{U} \right) \mathcal{I}_{n,i} \right] - \frac{\partial m}{\partial x_d} \left(\mathbf{X} \right) f \left(\mathbf{X} \right) \right|$$

$$+ \left| \int K_{\mathbf{H}_n} \left(\mathbf{X} - \mathbf{U} \right) \sum_{j=1}^{k_n} \sum_{q \neq d} \frac{\partial m}{\partial x_q} \left(\mathbf{U} \right) \sqrt{\vartheta_n} \mathcal{I}_{n,i} f \left(\mathbf{U} \right) d\mathbf{U} \right|$$

$$+ \left| \int K_{\mathbf{H}_n} \left(\mathbf{X} - \mathbf{U} \right) E_i \left[\sum_{j=1}^{k_n} \sum_{|\alpha|=2} w_j \frac{R_\alpha \left(\mathbf{X}_{a(j,i,d)} \right) \left(\mathbf{X}_{a(j,i,d)} - \mathbf{X}_i \right)^\alpha - R_\alpha \left(\mathbf{X}_{b(j,i,d)} \right) \left(\mathbf{X}_{b(j,i,d)} - \mathbf{X}_i \right)^\alpha}{x_{d,a(j,i,d)} - x_{d,b(j,i,d)}} \mathcal{I}_{n,i} f \left(\mathbf{U} \right) d\mathbf{U} \right] \right|.$$

The first term converges to zero by the usual consistency proof for multivariate Nadaraya-Watson estimates and since $P\left(k_{n,i} > 0 \right) \to 1$. We can bound the remainder according to the Taylor uniform bound. The second term converges to zero since $\vartheta_n \to 0$ as $n \to \infty$. The final term is therefore bounded above by:

$$\sup_{|\alpha|=2} \sup_{\mathbf{X}} \left| D^\alpha m \left(\mathbf{X} \right) \right| \int \left| K_{\mathbf{H}_n} \left(\mathbf{X} - \mathbf{U} \right) \right| E_i \left[\sum_{j=1}^{k_n} \sum_{|\alpha|=2} w_j \frac{\left(\mathbf{X}_{a(j,i,d)} - \mathbf{X}_i \right)^\alpha + \left(\mathbf{X}_{b(j,i,d)} - \mathbf{X}_i \right)^\alpha}{x_{d,a(j,i,d)} - x_{d,b(j,i,d)}} \mathcal{I}_{n,i} \right] f \left(\mathbf{U} \right) d\mathbf{U}$$

$$\leq \sup_{|\alpha|=2} \sup_{\mathbf{X}} \left| D^\alpha m \left(\mathbf{X} \right) \right| \int \left| K_{\mathbf{H}_n} \left(\mathbf{X} - \mathbf{U} \right) \right| E_i \left[\begin{array}{c} \sum_{j=1}^{k_n} w_j \left(1 + (D-1)\sqrt{\vartheta_n} + D(D-1)\vartheta_n \right) \\ \times \left(x_{d,a(j,i,d)} - x_{d,b(j,i,d)} \right) \mathcal{I}_{n,i} \end{array} \right] f \left(\mathbf{U} \right) d\mathbf{U},$$

which converges to zero by Hoeffding's inequality as in the univariate case and since $\vartheta_n \to 0$ as $n \to \infty$. ∎

Lemma 12 *As $n \to \infty$, the variance:*

$$Var\left(n^{-1}\sum_{i=1}^{n}K_{\mathbf{H}_n}\left(\mathbf{X}-\mathbf{X}_i\right)\widetilde{Y}_i\right) \to 0.$$

Proof. The proof follows exactly as in the univariate case, applying the multidimensional Taylor formula in place of the unidimensional Taylor formula. ■

The numerator of equation (34) converges in mean-square error to zero, which implies convergence in probability. The theorem follows by Slutsky's theorem.

D.3   Proof of Theorem 1

Given the previous results, the proof is straightforward. The only complication is that 2-dimensional (only in r and t) smoothing is used in the Nadaraya-Watson estimator, rather than 3-dimensional smoothing. It is straightforward, however, from the work above to see that:[43]

$$\widehat{\Phi}_l\left(r, T_l + \tau\right) \quad \to \quad {}^p\frac{\partial \widetilde{m}_{\mathbf{Y}}\left(R, \tau\right)}{\partial R} + \int_0^{2\pi}\frac{\partial \phi_l\left(r, \theta\right)}{\partial r}f\left(\theta|T_l + \tau, r\right)d\theta$$

$$\widehat{\Phi}_l\left(r, T - 1\tau\right) \quad \to \quad {}^p\int_0^{2\pi}\frac{\partial \phi_l\left(r, \theta\right)}{\partial r}f\left(\theta|T_l - 1, r\right)d\theta$$

By the assumption in footnote 18, $f\left(\theta|T_l + \tau, r\right) = f\left(\theta|T_l - 1, r\right)$, so:

$$\widehat{\Phi}_l\left(r, T_l + \tau\right) - \widehat{\Phi}_l\left(r, T_l - 1\right) \to^p \frac{\partial \widetilde{m}_{\mathbf{Y}}\left(R, \tau\right)}{\partial R},$$

as desired.

E   Renter Surplus

In this appendix, we show that generically renters will earn economic surplus in the hedonic equilibrium. The basic intuition is that given a fixed distribution of rental locations in space, when renters are heterogeneous, market clearing will generally require that certain renters are inframarginal at the location where they live. To fix ideas, we begin with a simple discrete case. We then show that the basic result holds in the continuous hedonic model as well.

E.1   Discrete Case

To start, suppose there are two locations, which we designate as location A and location B. Suppose that there are 3 rental units at location A and 7 rental units at location B. There are two types of renters, which we designate as type x and type y. The number of each type

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[43]Recall that we assume $\frac{\partial \widetilde{m}_{\mathbf{Y}}(R,-1)}{\partial R} = 0$.

as well as their respective valuations of living at each location are provided in the following table:

	# of Renters	Value of A	Value of B
x	5	6	2
y	5	3	0

Each agent also has an outside option, which we call location Z, that yields a total utility value of zero.

First, note that market clearing requires both types of agents to live at location B. Thus, due to the presence of the outside option, it must be that $p(B) = 0$. It is then straightforward to see that $p(A) = 4$. If $p(A) < 4$, then no type x renters would choose location B and market clearing would be violated. Similarly, if $p(A) > 4$, then no renter would choose to live at location A and market clearing would be violated. So equilibrium prices are $p(A) = 4$ and $p(B) = 0$. In equilibrium, all five type y agents live at location B. Two of the type x agents live at location B and three type x agents live at location A.

Note that the type x agents earn economic rents at both locations. Since both types of agents must live at location B due to market clearing, the type x agents will be inframarginal at location B and therefore earn economic surplus. But then to induce type x agents to live at location A, equal economic surplus must be offered to the type x agents at location A as well.

E.2 Continuous Case

We next show that renters continue to earn economic surplus in the continuous case. We consider the canonical linear-quadratic hedonic model as described in Tinbergen (1956), Epple (1987), Ekeland et al. (2004), and others. Suppose that the supply of rental units is normally distributed over the real line with mean μ_Q and variance σ_Q^2. Given a hedonic price function $p(R)$, a renter i chooses her optimal location according to the problem:

$$\gamma_i R + \frac{1}{2} A R^2 - p(R),$$

which yields the first-order condition:

$$\gamma_i + AR - p'(R) = 0.$$

We assume that renters are heterogeneous with γ_i normally distributed with mean μ_γ and variance σ_γ^2. We conjecture that the hedonic price function has a quadratic form:

$$p(R) = \pi_0 + \pi_1 R + \frac{1}{2} \pi_2 R^2.$$

A renter i thus optimally chooses the location:

$$R_i^* = \frac{\gamma_i - \pi_1}{\pi_2 - A}.$$

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The density of demand therefore follows a normal distribution. Market clearing requires that the density of demand equal the density of supply. Since both demand and supply are normally distributed, it suffices that the means and variances equal each other. We therefore have a system of two equations in two unknowns:

$$\frac{\mu_\gamma - \pi_1}{\pi_2 - A} = \mu_Q$$

$$\frac{\sigma_\gamma^2}{(\pi_2 - A)^2} = \sigma_Q^2$$

which yields:

$$\pi_1 = \mu_\gamma - \mu_Q (\pi_2 - A)$$

$$\pi_2 = \frac{\sigma_\gamma}{\sigma_Q} + A.$$

Finally, we assume that agents have an outside option offering a reservation utility of zero. As is well known, this pins down the initial condition at $R = 0$ in the underlying differential equation and sets $\pi_0 = 0$. If $\pi_0 > 0$, renters living at $R = 0$ would earn negative utility and thus choose the outside option instead.

As a representative example, consider the model above with the following parameters:

A	1
μ_γ	3
σ_γ	3
μ_Q	0
σ_Q	5

In Figure A8, we plot the equilibrium utility:

$$\gamma_i R_i^* + \frac{1}{2} A R_i^{*2} - p(R_i^*)$$

for renters with varying γ_i. As is clear from the figure, all agents with the exception those with $\gamma_i = 3$, who optimally chooses $R = 0$, earn economic surplus in equilibrium.

Table 1: Summary Statistics

Panel A: 1990 Census Block Group Data

	Whole US	Counties in Sample	Block Groups with LIHTC Development Sites
1990 Black Share	0.1212	0.1164	0.2394
	[0.2399]	[0.2334]	[0.3122]
1990 Hispanic Share	0.0827	0.1526	0.2394
	[0.1668]	[0.2091]	[0.2511]
1990 Median Income	56482	66652	44890
	[28730]	[32620]	[23278]
1990 Renter Share	0.3357	0.4029	0.6336
	[0.2397]	[0.2778]	[0.2658]

Panel B: LIHTC Developments

	All LIHTC	Non-Rural LIHTC	Estimation Sample
Year Funds Allocated	1997.6	1997.7	2000.8
	[6.70]	[6.76]	[5.55]
# Low Income Units	60.3	68.5	82.2
	[71.5]	[79.1]	[86.3]
% Units Low Income	0.97	0.96	0.96
	[0.13]	[0.14]	[0.13]
New Construction	0.63	0.59	0.58
	[0.48]	[0.49]	[0.49]
Rehab Existing Building	0.37	0.41	0.42
	[0.48]	[0.49]	[0.49]
In Central City	0.46	0.61	0.64
	[0.50]	[0.49]	[0.48]
In Metro,Non-Central City	0.30	0.39	0.36
	[0.46]	[0.49]	[0.48]
In Rural Area	0.24	0	0
	[0.43]	[0]	[0]
Observations	32799	24843	6882

Panel C: DataQuick & HMDA Data

	Transactions<1 mi of LIHTC Site	Transactions<.5 mi of LIHTC Site	Transactions<.2 mi of LIHTC Site	Transactions<.1 mi of LIHTC Site
Housing Transaction Price	309186	273864	258183	259868
	[335132]	[390521]	[404485]	[471315]
Housing Transaction Price	326821	291613	274882	273789
-HMDA Matched	[317237]	[364186]	[376996]	[411524]
% of Home Buyers Black	0.0581	0.0704	0.0761	0.0753
	[0.4621]	[0.4737]	[0.4800]	[0.4802]
Income of Home Buyer	98745	93273	91566	92137
	[51151]	[51099]	[52624]	[54364]
Housing Transactions	12,007,578	3,431,529	807,669	242,004

Panel D: Crime Rates within 1 mi of LIHTC Sites

	Low Minority Income Q1/2	High Minority Income Q1/2	Low Minority Income Q3/4	High Minority Income Q3/4
Annual Violent Crimes	617.1	586.9	383.3	453.0
per Square Mile	[2720.7]	[1557.8]	[2044.2]	[1154.0]
Annual Property Crimes	2523.2	1083.1	1495.1	982.3
per Square Mile	[9701.8]	[2787.6]	[7155.5]	[2670.2]
Observations	678030	2452968	989424	559950

Note: All prices inflated to 2012 dollars. Standard deviations in brackets. Crime data covers San Diego 2003-2014, Chicago 2001-2014, and San Francisco 2007-2014 for 127 LIHTC Sites. An observation for crime data is a 0.025 square mile area.

Table 2: Reduced Form Log Price Impacts

LIHTC Sites in Tracts with Median Inc<$26,017

Distance to LIHTC:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Log Price Impact :	0.0640***	0.0532***	0.0458***	0.0404***	0.0356***	0.0304***	0.0247***	0.0183***	0.0121***	0.0071**	0.0039*	0.0018
	[0.0166]	[0.0134]	[0.0113]	[0.0099]	[0.0086]	[0.0073]	[0.0061]	[0.0051]	[0.0041]	[0.0031]	[0.0022]	[0.0011]

LIHTC Sites in Tracts with $26,017≤ Median Income < $38,177

Distance to LIHTC:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Log Price Impact :	0.0053	0.0018	-0.0016	-0.0029	-0.0026	-0.0021	-0.0017	-0.0012	-0.0008	-0.0007	-0.0004	-0.0000
	[0.0098]	[0.0082]	[0.0070]	[0.0062]	[0.0056]	[0.0050]	[0.0044]	[0.0037]	[0.0031]	[0.0024]	[0.0016]	[0.0008]

LIHTC Sites in Tracts with $38,177≤ Median Income < $54,642

Distance to LIHTC:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Log Price Impact :	-0.0250***	-0.0207***	-0.0149**	-0.0096*	-0.0054	-0.0024	-0.0008	-0.0001	0.0003	0.0001	-0.0002	-0.0002
	[0.0079]	[0.0063]	[0.0053]	[0.0047]	[0.0042]	[0.0038]	[0.0033]	[0.0028]	[0.0023]	[0.0018]	[0.0013]	[0.0007]

LIHTC Sites in Tracts with $54,642≤ Median Income

Distance to LIHTC:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Log Price Impact :	-0.0244***	-0.0205***	-0.0166***	-0.0123***	-0.0088**	-0.0062*	-0.0043	-0.0028	-0.0018	-0.0012	-0.0008	-0.0005
	[0.0080]	[0.0063]	[0.0052]	[0.0044]	[0.0039]	[0.0036]	[0.0033]	[0.0029]	[0.0025]	[0.0019]	[0.0014]	[0.0007]

LIHTC Sites in Tracts with Median Income<$38,177, Minority Share ≥0.50

Distance to LIHTC:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Log Price Impact :	0.0347***	0.0262***	0.0195**	0.0152**	0.0126*	0.0105*	0.0085	0.0066	0.0046	0.0028	0.0017	0.0012
	[0.0111]	[0.0092]	[0.0078]	[0.0068]	[0.0061]	[0.0054]	[0.0048]	[0.0043]	[0.0036]	[0.0029]	[0.0020]	[0.0011]

LIHTC Sites in Tracts with Median Income≥$38,177, Minority Share ≥0.50

Distance to LIHTC:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Log Price Impact :	-0.0019	-0.0037	-0.0020	0.0011	0.0032	0.0051	0.0056	0.0052	0.0044	0.0026	0.0007	0.0000
	[0.0112]	[0.0092]	[0.0078]	[0.0068]	[0.0061]	[0.0055]	[0.0049]	[0.0043]	[0.0035]	[0.0028]	[0.0020]	[0.0011]

Note: All prices inflated to 2012 dollars. Bootstrapped standard errors in brackets. Effects reported are the average house price impact between 0 and 22 years versus 6 to 1 year before treatment. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3: Welfare Impacts of LIHTC on Households (LIHTC Amenity)

ΔU_i	Homeowners	Renters	Landlords
$R_{pre} > R_{0,\mathbf{Y}}, R^*_{1,i} > R_{0,\mathbf{Y}}$	0	0	0
$R_{pre} > R_{0,\mathbf{Y}}, R^*_{1,i} < R_{0,\mathbf{Y}}$	$\gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R^*_{1,i}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$	$\gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R^*_{1,i}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$	0
$R_{pre} < R_{0,\mathbf{Y}}, R^*_{1,i} > R_{0,\mathbf{Y}}$	$\widetilde{m}_{\mathbf{Y}}\left(R_{pre}\right)$	0	$\widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$
$R_{pre} < R_{0,\mathbf{Y}}, R^*_{1,i} < R_{0,\mathbf{Y}}$	$\gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R^*_{1,i}\right) + \widetilde{m}_{\mathbf{Y}}\left(R_{pre}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$	$\gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R^*_{1,i}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$	$\widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$
Total by group:	$\sum_i \gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R^*_{1,i}\right) \cdot 1\left(R^*_{1,i} < R_{0,\mathbf{Y}}\right)$	$\sum_i \left(\gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R^*_{1,i}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)\right) \cdot$ $1\left(R^*_{1,i} < R_{0,\mathbf{Y}}\right)$	$\sum_i \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right) \cdot$ $1\left(R_{pre} < R_{0,\mathbf{Y}}\right)$
Total Overall:	$\sum_i \gamma_{i,\mathbf{Y}} \log\left(1 + R_{0,\mathbf{Y}} - R^*_{1,i}\right) \cdot 1\left(R^*_{1,i} < R_{0,\mathbf{Y}}\right)$		

Note: R_{pre} represents distance of household's chosen location before LIHTC development from new LIHTC development site. $R^*_{1,i}$ represents chosen distance from LIHTC development site when household reoptimizes location post LIHTC development. $R_{0,\mathbf{Y}}$ represents max distance at which LIHTC proximity can impact utility. Each row and column pair represents a different household type. Entry summarizes welfare impact of LIHTC development when agents view LIHTC proximity as an amenity.

Table 4: Welfare Impacts of LIHTC on Households (LIHTC Disamenity)

ΔU_i	Homeowners	Renters	Landlords
$R_{pre} > R_{0,\mathbf{Y}}, R^*_{1,i} > R_{0,\mathbf{Y}}$	0	0	0
$R_{pre} > R_{0,\mathbf{Y}}, R^*_{1,i} < R_{0,\mathbf{Y}}$	$\gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R^*_{1,i}}{1 - R_{0,\mathbf{Y}}}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$	$\gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R^*_{1,i}}{1 - R_{0,\mathbf{Y}}}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$	0
$R_{pre} < R_{0,\mathbf{Y}}, R^*_{1,i} > R_{0,\mathbf{Y}}$	$\widetilde{m}_{\mathbf{Y}}\left(R_{pre}\right)$	0	$\widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$
$R_{pre} < R_{0,\mathbf{Y}}, R^*_{1,i} < R_{0,\mathbf{Y}}$	$\gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R^*_{1,i}}{1 - R_{0,\mathbf{Y}}}\right) + \widetilde{m}_{\mathbf{Y}}\left(R_{pre}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$	$\gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R^*_{1,i}}{1 - R_{0,\mathbf{Y}}}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$	$\widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)$
Total by group:	$\sum_i \gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R^*_{1,i}}{1 - R_{0,\mathbf{Y}}}\right) \cdot 1\left(R^*_{1,i} < R_{0,\mathbf{Y}}\right)$	$\sum_i \left(\gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R^*_{1,i}}{1 - R_{0,\mathbf{Y}}}\right) - \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right)\right) \cdot$ $1\left(R^*_{1,i} < R_{0,\mathbf{Y}}\right)$	$\sum_i \widetilde{m}_{\mathbf{Y}}\left(R^*_{1,i}\right) \cdot$ $1\left(R_{pre} < R_{0,\mathbf{Y}}\right)$
Total Overall:	$\sum_i \gamma_{i,\mathbf{Y}} \log\left(\frac{1 + R^*_{1,i}}{1 - R_{0,\mathbf{Y}}}\right) \cdot 1\left(R^*_{1,i} < R_{0,\mathbf{Y}}\right)$		

Note: R_{pre} represents distance of household's chosen location before LIHTC development from new LIHTC development site. $R^*_{1,i}$ represents chosen distance from LIHTC development site when household reoptimizes location post LIHTC development. $R_{0,\mathbf{Y}}$ represents max distance at which LIHTC proximity can impact utility. Each row and column pair represents a different household type. Entry summarizes welfare impact of LIHTC development when agents view LIHTC proximity as a disamenity.

Table 4: Mean Utility Benefit per Household from LIHTC Construction

Panel A: Low Minority Areas

	Income Q1	Income Q2	Income Q3	Income Q4
Average Benefit per Home Owner	23403*	208	-3636***	-3972**
	[13426]	[1618]	[912]	[1921]
Average Benefit per Renter	6502	67	234	67
	[4061]	[907]	[377]	[115]
Average Benefit per Landlord	6011**	46	-2843***	-2416**
	[3030]	[1730]	[951]	[1027]

Panel B: High Minority Areas

	Income Q1	Income Q2	Income Q3	Income Q4
Average Benefit per Home Owner	16857***	2414	996	3255
	[5440]	[1847]	[3138]	[5419]
Average Benefit per Renter	6475**	190	342	971
	[2634]	[666]	[1010]	[1773]
Average Benefit per Landlord	6099**	1288	375	1090
	[2496]	[1597]	[1684]	[2201]

Note: Mean welfare benefit from LIHTC construction to households who choose to live nearby. Utility is measured in 2012 dollars. To decompose effect between renters and landlords we assume the present discounted value of future rents is equal to house prices.

Table 5: Total Utility Benefit to Households from LIHTC Construction

Panel A: Low Minority Areas

	Income Q1	Income Q2	Income Q3	Income Q4
Aggregate Benefit to Home Owners	57,945*	61.46	-3,789***	-9,008**
	[33243]	[479]	[950]	[4358]
Aggregate Benefit to Renters	29,208	23.87	258	78.48
	[19626]	[563]	[442]	[152]
Aggregate Benefit to Landlords	29,048**	28.42	-3,331***	-3,208**
	[14644]	[1074]	[1114]	[1364]
Aggregate Benefit to Society	116201**	113.7	-6,861***	-12,138**
	[53163]	[1136]	[1780]	[5494]

Panel B: High Minority Areas

	Income Q1	Income Q2	Income Q3	Income Q4
Aggregate Benefit to Home Owners	63,460***	1,446	3,615	14,508
	[20480]	[1113]	[11394]	[24148]
Aggregate Benefit to Renters	73,417**	314.7	1,903	4,861
	[32059]	[1382]	[6087]	[9613]
Aggregate Benefit to Landlords	74,236**	2,672	2,262	5,907
	[30379]	[3314]	[10148]	[11933]
Aggregate Benefit to Society	211,113***	4,433	7,780	25,277
	[62794]	[3829]	[23577]	[40943]

Note: Total welfare benefit from LIHTC construction to households. Utility is measured in thousands of 2012 dollars. To decompose effect between renters and landlords we assume the present discounted value of future rents is equal to house prices.

Figure 1: Counties Used in Analysis
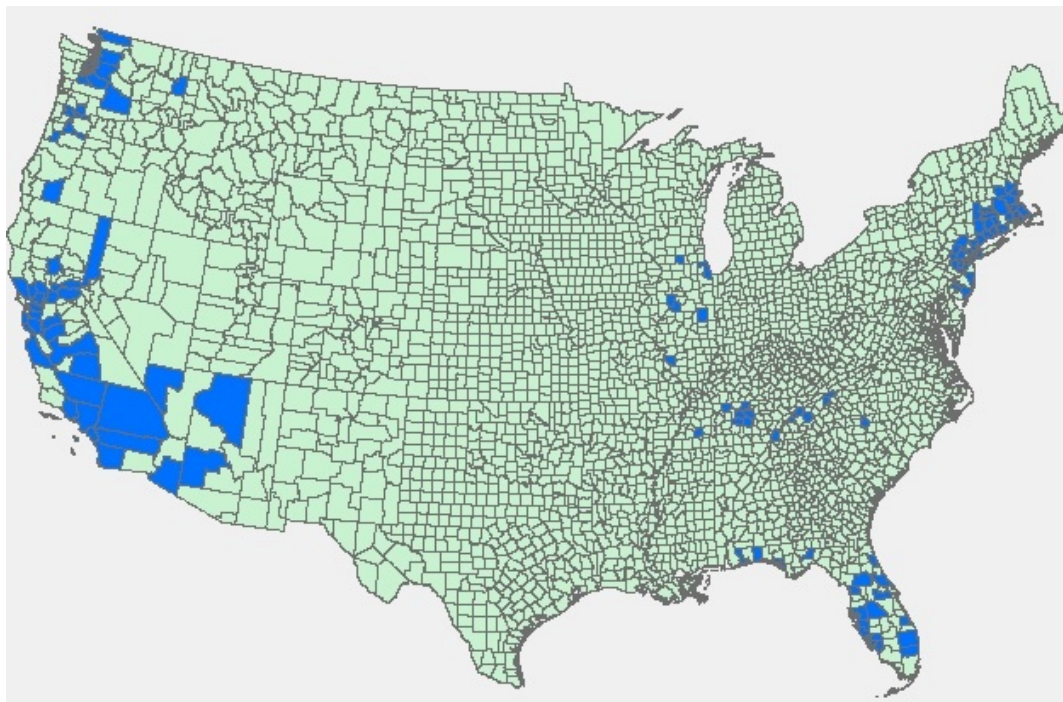
Note: Counties were selected based on whether the history of housing transaction data began in 1996 or earlier and had at least an average of 1000 arm-length transactions per year. This gives 129 counites in 15 states and covers 31% of the US population.

Figure 2: Example of Bowtie Threshold Used in Empirical Derivatives
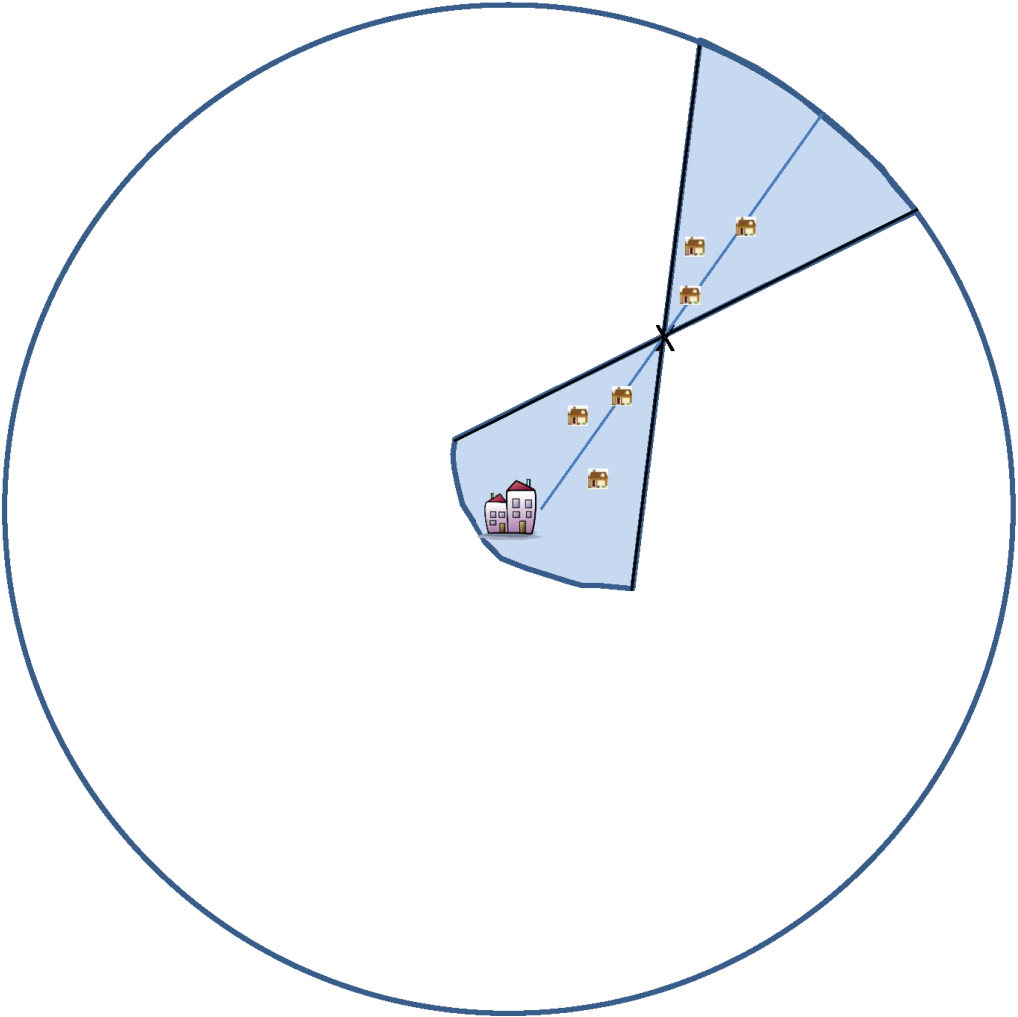
Note: LIHTC site is located in the middle of the ring. The site marked X is where the empirical derivative with respect to LIHTC distance is being measured. Houses inside the shaded region are candidates for the empirical derivative calculation.

Figure 3: Average Price Impact of LIHTC
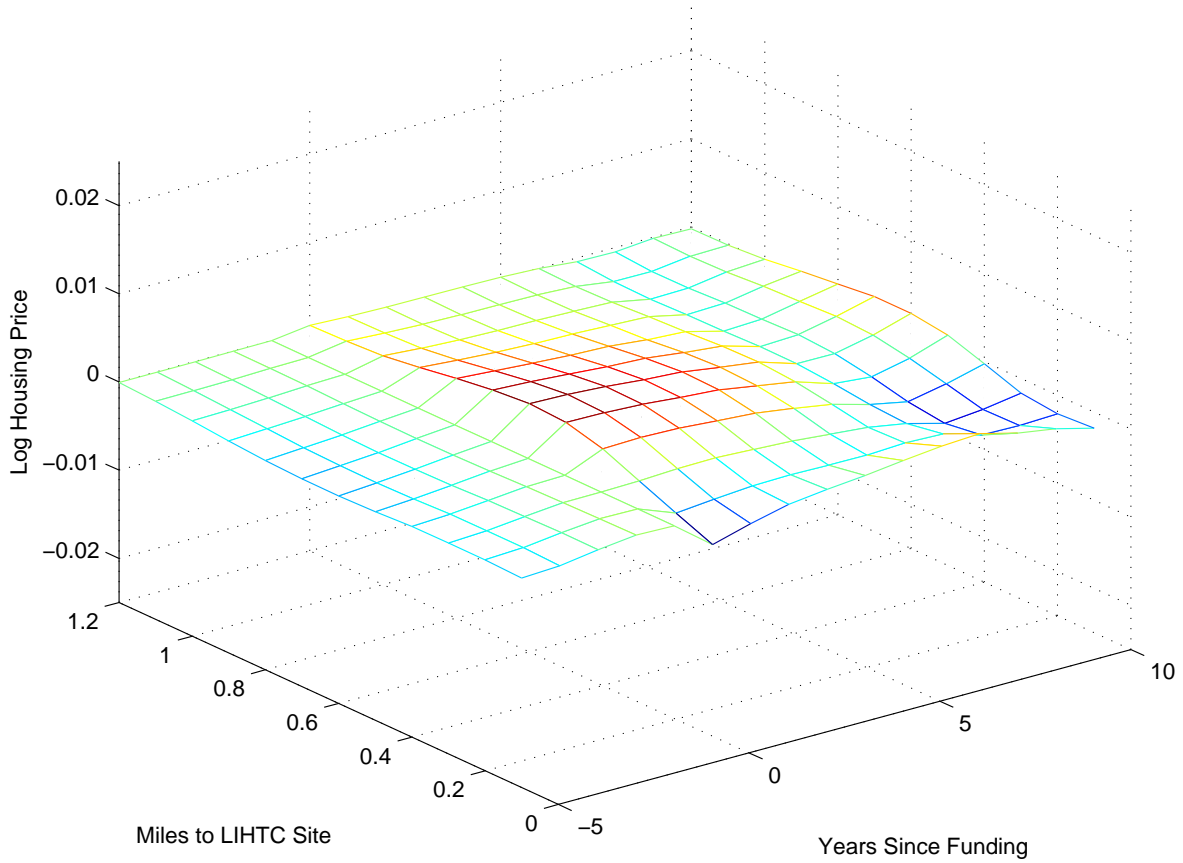
Note: Kernel smoothed estimates of log house prices using Nadaraya-Watson estimator with Epanechnikov kernel. Estimates integrate over the estimated derivatives to measure log price levels at a given distance from LIHTC site, relative to 1.4 miles away.

Figure 4: Price Impact of LIHTC by Neighborhood Median Income

(a) Q1 Income Neighorhoods
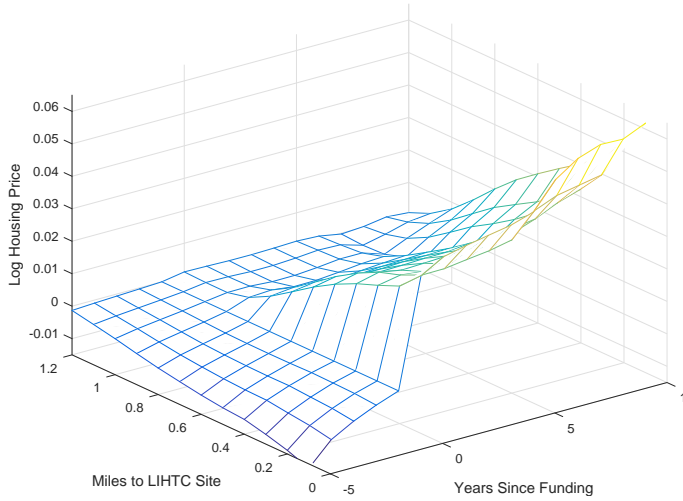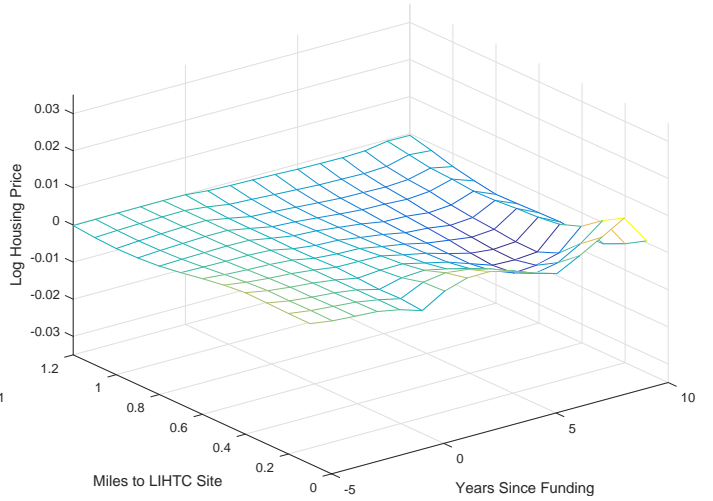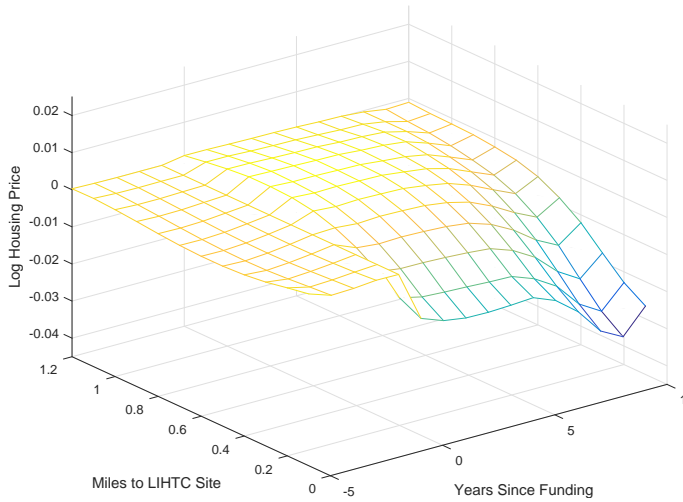
(b) Q2 Income Neighorhoods

(c) Q3 Income Neighorhoods

(d) Q4 Income Neighorhoods

Note: Kernel smoothed estimates of log house prices using Nadaraya-Watson estimator with Epanechnikov kernel. Estimates integrate over the estimated derivatives to measure log price levels at a given distance from LIHTC site, relative to 1.4 miles away. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site.

Figure 5: Price Impact of LIHTC by Median Income within High Minority Neighborhoods

(a) Q1 & Q2 Income, High Minority Neighorhoods
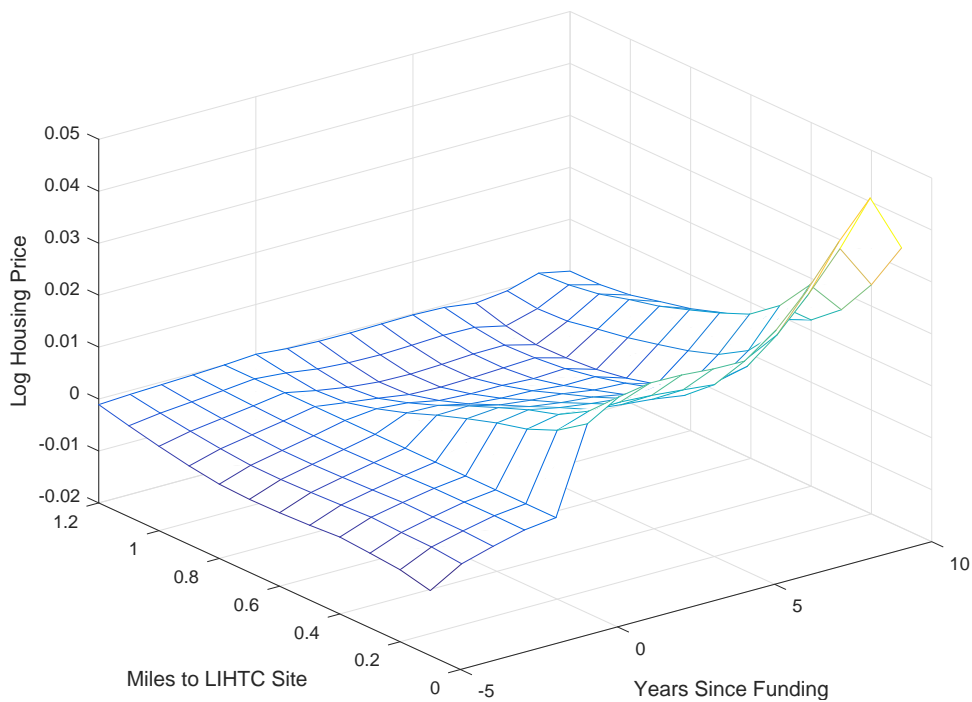
(b) Q3 & Q4 Income, High Minority Neighorhoods
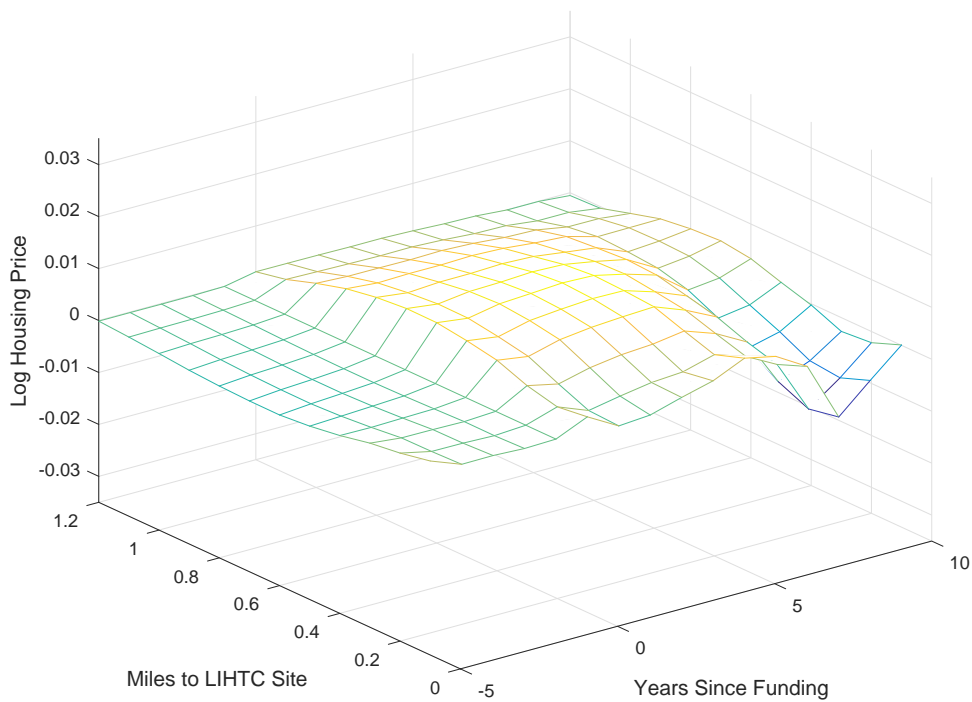
Note: Kernel smoothed estimates of log house prices using Nadaraya-Watson estimator with Epanechnikov kernel. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site.

Figure 6: Ring Method Estimated Treatment Effects by Neighborhood Income

(a) Q1 Income Neighorhoods
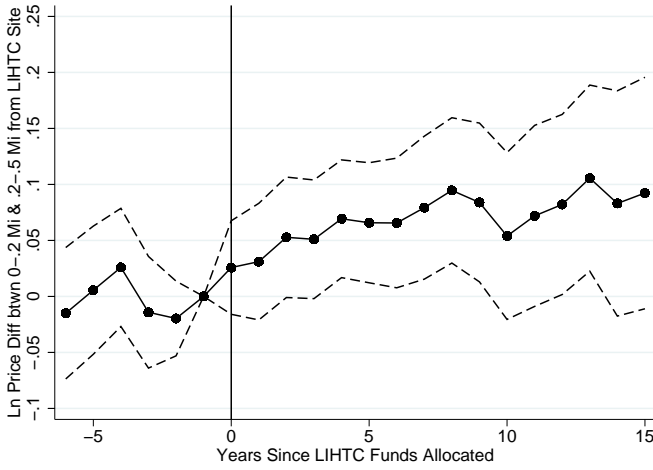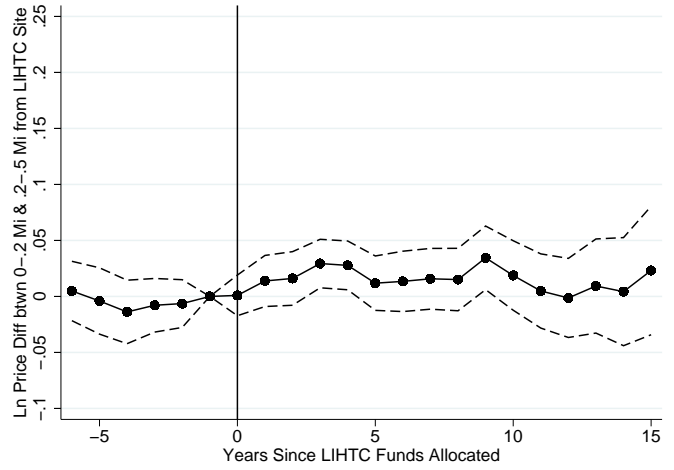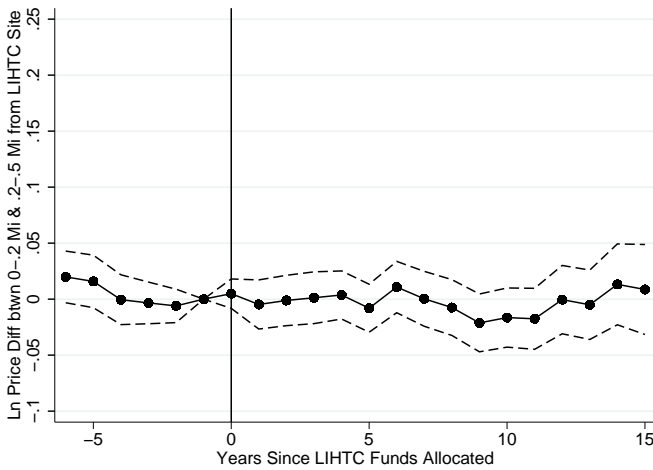
(b) Q2 Income Neighborhoods

(c) Q3 Income Neighborhoods

(d) Q4 Income Neighborhoods
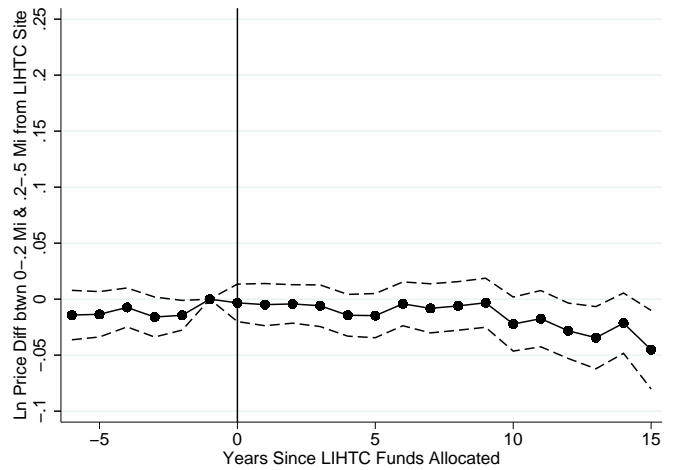
Note: Willingness to pay is measured in 2012 dollars. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LI-HTC site. Reported preferences are for households who choose to live close to LIHTC development. 95% confidence intervals shown.

Figure 7: Impact of LIHTC on Derivative of Log House Prices wrt miles from LIHTC vs miles to LIHTC:

Impacts 0 to 5 years post LIHTC Funding

Note: Kernel smoothed estimates of log house price derivatives with respect to distance to LIHTC using Nadaraya-Watson estimator with Epanechnikov kernel. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site. An LIHTC project is consider high minority share if at least 50% of the census block group where the LIHTC project is located was Black or Hispanic as reported in the 1990 census. Dashed lines are fully non-parametric estimate and confidence intervals. Solid line represents effect truncated to zero for distances beyond which the first time the non-parametric estimate crosses zero or gets closest to zero. These price effects are used in structural model of preference estimation. Standard errors estimated using a block-bootstrap with 500 simimulations where sampling is done over LIHTC sites. Dashed lines measure 90% confidence interval.

Figure 8: Impact of LIHTC on Derivative of Log House Prices wrt miles from LIHTC vs miles to LIHTC:

Impacts 6 to 10 years post LIHTC Funding

Note: Kernel smoothed estimates of log house price derivatives with respect to distance to LIHTC using Nadaraya-Watson estimator with Epanechnikov kernel. Household median income quartile cutoffs are \$26017, \$38177, and \$54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site. An LIHTC project is consider high minority share if at least 50% of the census block group where the LIHTC project is located was Black or Hispanic as reported in the 1990 census. Dashed lines are fully non-parametric estimate and confidence intervals. Solid line represents effect truncated to zero for distances beyond which the first time the non-parametric estimate crosses zero or gets closest to zero. These price effects are used in structural model of preference estimation. Standard errors estimated using a block-bootstrap with 500 simiulations where sampling is done over LIHTC sites. Dashed lines measure 90% confidence interval.

Figure 9: Impact of LIHTC on Homebuyer Income:

Impacts 0 to 10 years post LIHTC Funding
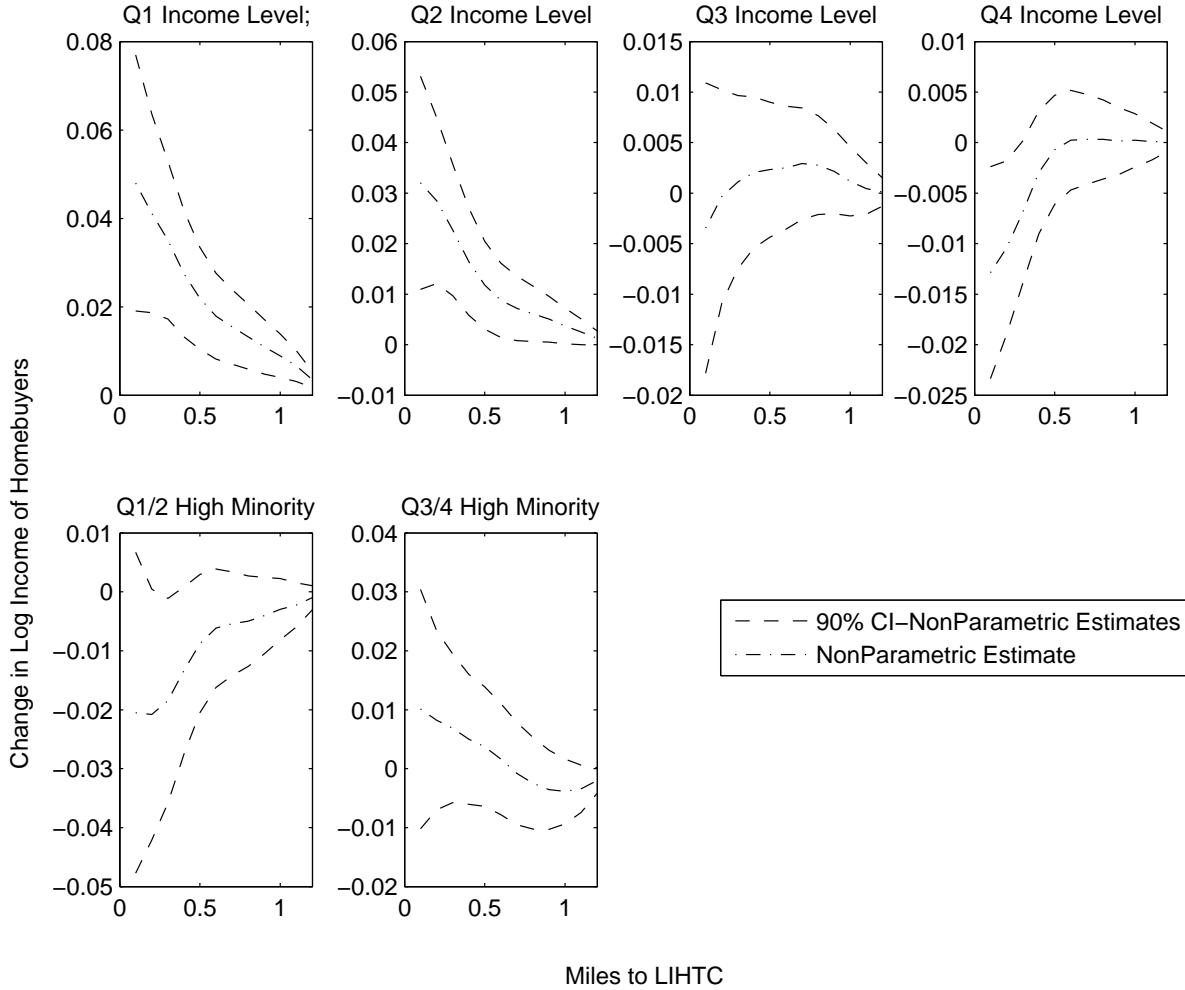
Note: Kernel smoothed estimates of log homebuyer income which are black using Nadaraya-Watson estimator with Epanechnikov kernel. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site. An LIHTC project is consider high minority share if at least 50% of the census block group where the LIHTC project is located was Black or Hispanic as reported in the 1990 census. Dashed lines are fully non-parametric estimate and confidence intervals. Standard errors estimated using a block-bootstrap with 500 simimlations where sampling is done over LIHTC sites. Dashed lines measure 90% confidence interval.

Figure 10: Impact of LIHTC on Black Share of Homebuyers:

Impacts 0 to 10 years post LIHTC Funding
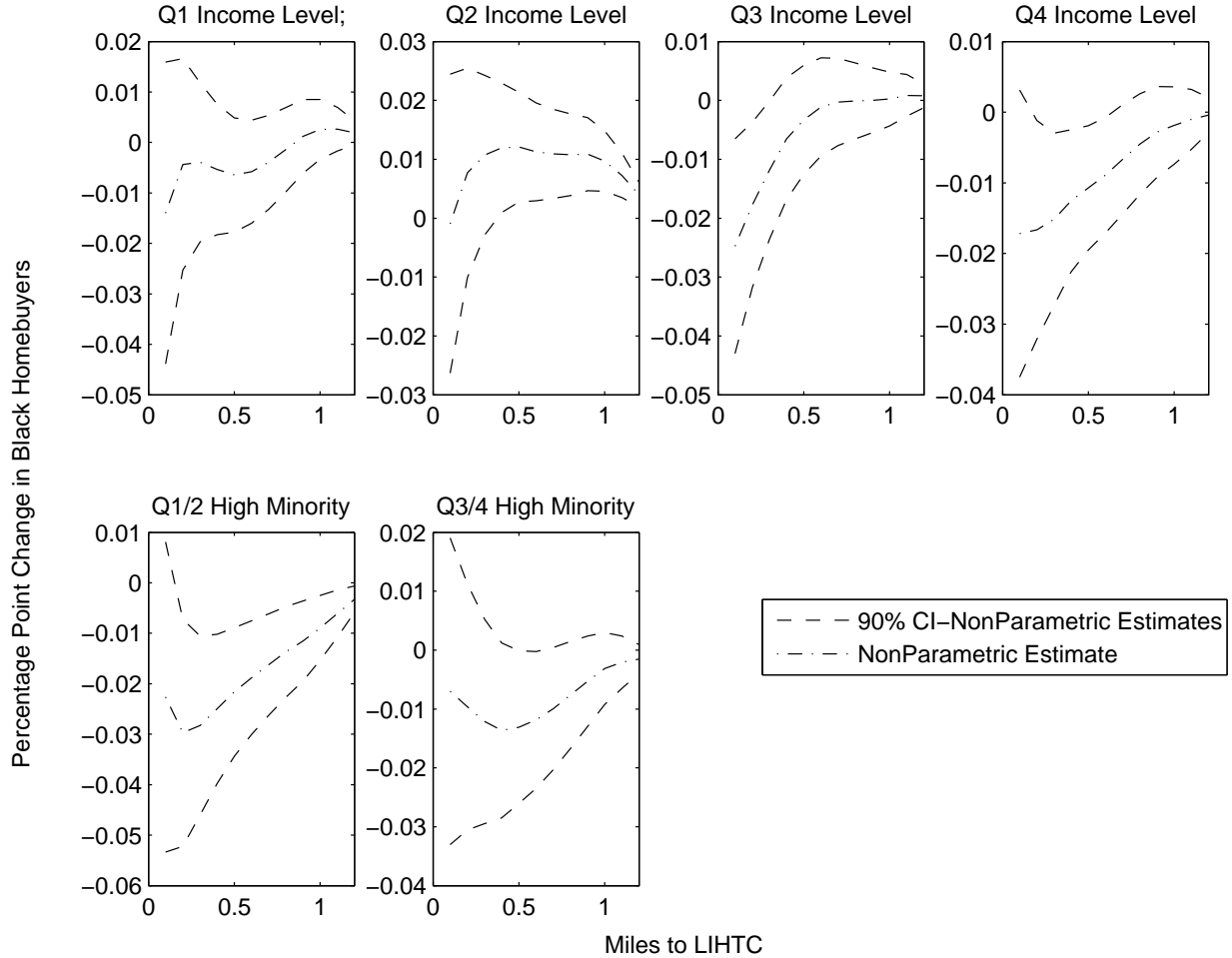
Note: Kernel smoothed estimates of percent of homebuyers which are black using Nadaraya-Watson estimator with Epanechnikov kernel. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site. An LIHTC project is consider high minority share if at least 50% of the census block group where the LIHTC project is located was Black or Hispanic as reported in the 1990 census. Dashed lines are fully non-parametric estimate and confidence intervals. Solid line represents effect truncated to zero for distances beyond which the first time the non-parametric estimate crosses zero or gets closest to zero. These price effects are used in structural model of preference estimation. Standard errors estimated using a block-bootstrap with 500 simiulations where sampling is done over LIHTC sites. Dashed lines measure 90% confidence interval.

Figure 11: Crime Impacts of LIHTC by Neighborhood Median Income

(a) Property Crime in High Minority Neighorhoods
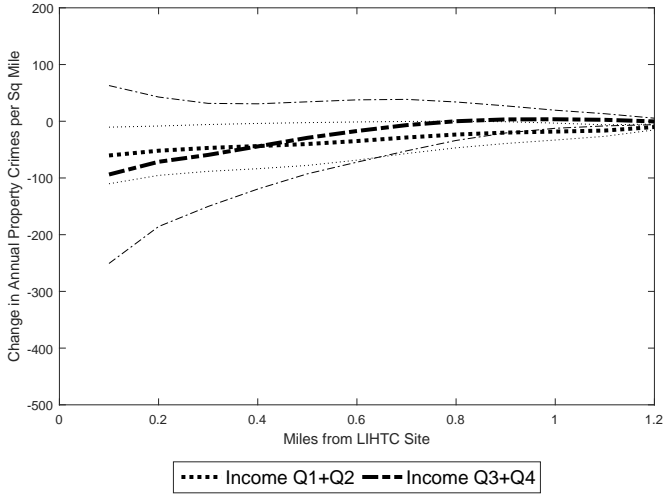
(b) Property Crime in Low Minority Neighorhoods
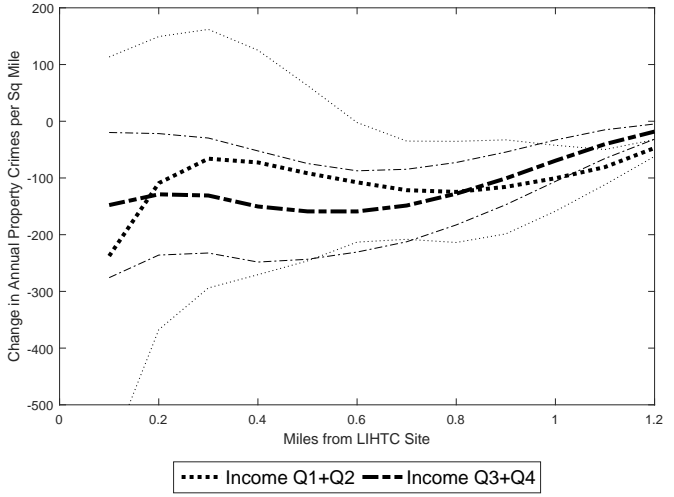
(c) Violent Crime in High Minority Neighorhoods
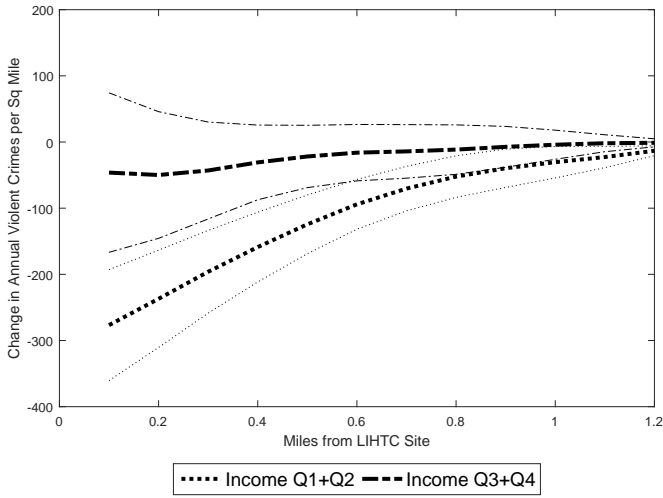
(d) Violent Crime in Low Minority Neighorhoods
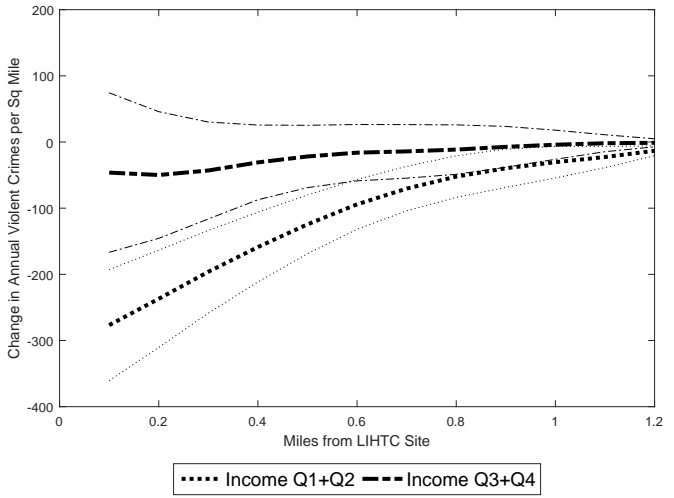
Note: Kernel smoothed estimates of annual crimes per square mile using Nadaraya-Watson estimator with Epanechnikov kernel. Estimates integrate over the estimated derivatives to measure log crimes per square mile at a given distance from LIHTC site, relative to 1.4 miles away. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site.

Figure 12: Mean Willingness to Pay to Live 0.1 miles from LIHTC: Low Minority Areas

(a) Q1 Income Neighorhoods

(b) Q2 Income Neighorhoods
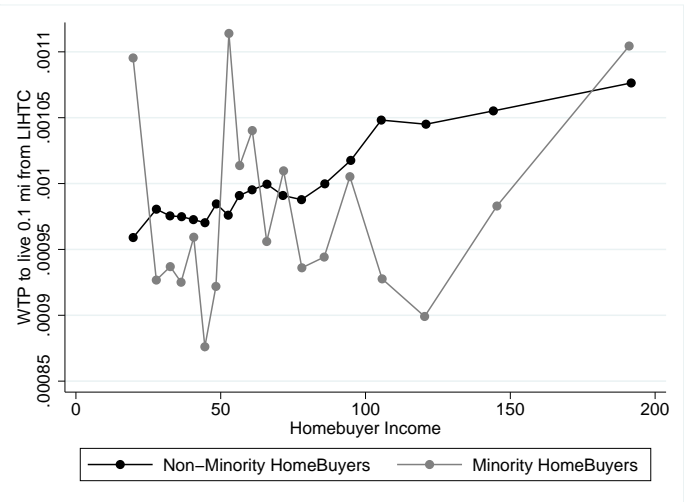
(c) Q3 Income Neighborhoods
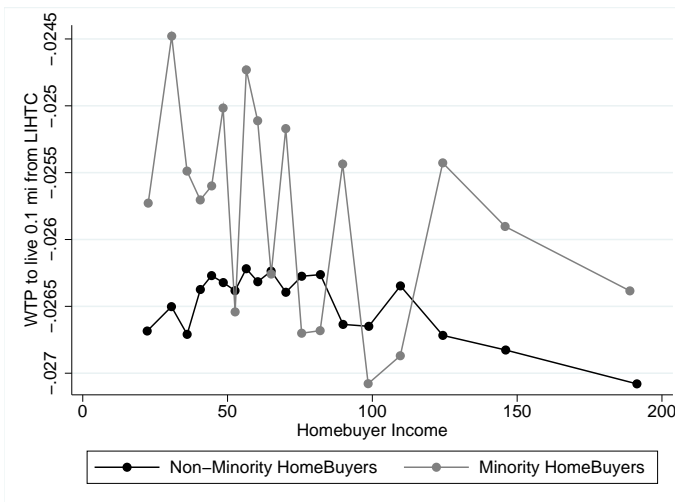
(d) Q4 Income Neighborhoods
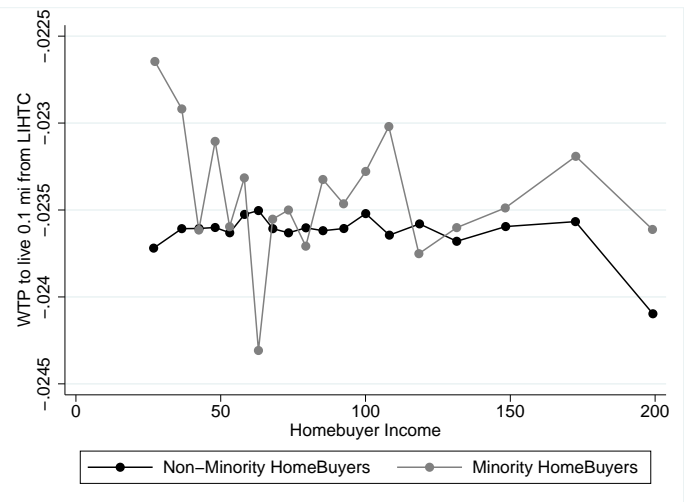
Note: Willingness to pay is measured as a percentage of the hombuyer's houseprice. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site. Reported preferences are for households who choose to live close to LIHTC development.

Figure 13: Mean Willingness to Pay to Live 0.1 miles from LIHTC: High Minority Areas

(a) Q1 Income Neighorhoods

(b) Q2 Income Neighorhoods
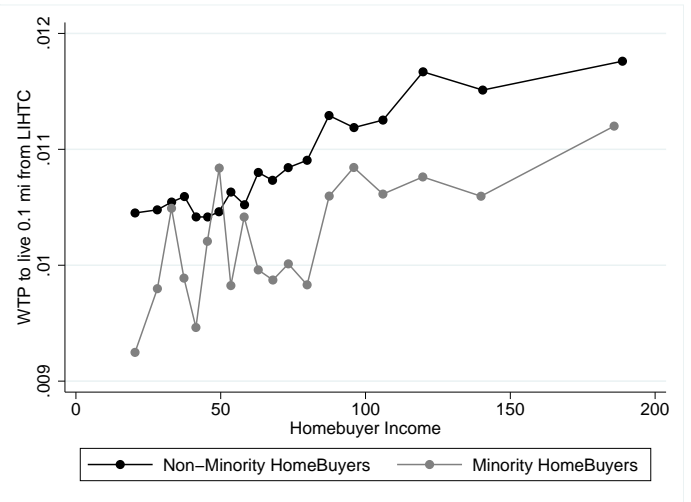
(c) Q3 Income Neighborhoods
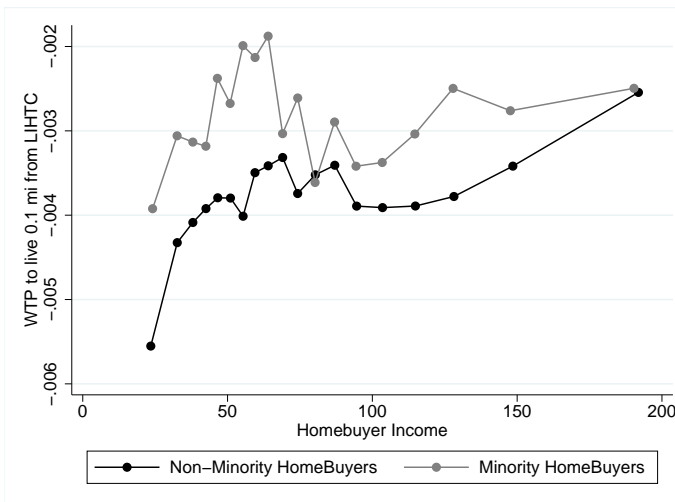
(d) Q4 Income Neighborhoods
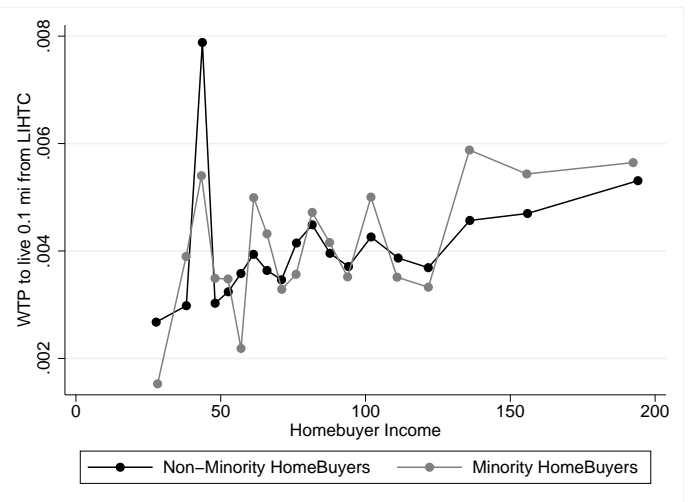
Note: Willingness to pay is measured as a percentage of the hombuyer's houseprice. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site. Reported preferences are for households who choose to live close to LIHTC development.

Table A1: Summary Statistics by Neighborhood Income Quartile

Panel A: LIHTC Sites in Block Groups with Median Income<$26,017

	Transactions<1 mi of LIHTC Site	Transactions<.5 mi of LIHTC Site	Transactions<.2 mi of LIHTC Site	Transactions<.1 mi of LIHTC Site
Housing Transaction Price	257600	231679	224353	229799
	[329316]	[339910]	[381076]	[453349]
Housing Transaction Price -HMDA Matched	277499	252776	242647	244480
	[310175]	[318216]	[351499]	[416517]
% of Home Buyers Black	.09364	0.1310	0.1513	0.1459
	[0.5250]	[0.5489]	[0.5383]	[0.5090]
Income of Home Buyer	93420	90547	90928	92547
	[53672]	[54649]	[56557]	[57717]

Panel B: LIHTC Sites in Block Groups with $26,017 ≤ Median Income< $38,177

	Transactions<1 mi of LIHTC Site	Transactions<.5 mi of LIHTC Site	Transactions<.2 mi of LIHTC Site	Transactions<.1 mi of LIHTC Site
Housing Transaction Price	233492	205286	193702	198689
	[267110]	[296370]	[279210]	[279210]
Housing Transaction Price -HMDA Matched	249363	220514	211160	214544
	[240353]	[258190]	[274176]	[294745]
% of Home Buyers Black	0.0716	0.0882	0.0907	0.0907
	[0.4790]	[0.4984]	[0.5023]	[0.5102]
Income of Home Buyer	86386	81136	81022	82374
	[49050]	[48963]	[49971]	[50617]

Panel C: LIHTC Sites in Block Groups with $38,177 ≤ Median Income< $54,642

	Transactions<1 mi of LIHTC Site	Transactions<.5 mi of LIHTC Site	Transactions<.2 mi of LIHTC Site	Transactions<.1 mi of LIHTC Site
Housing Transaction Price	274424	242592	231821	231749
	[281324]	[279365]	[326449]	[445992]
Housing Transaction Price -HMDA Matched	291458	258873	244756	238676
	[266944]	[260891]	[261478]	[281637]
% of Home Buyers Black	0.0689	0.0757	0.0771	0.0724
	[0.4753]	[0.4787]	[0.4850]	[0.0485]
Income of Home Buyer	93285	87950	86099	85123
	[49194]	[48734]	[50185]	[52580]

Panel D: LIHTC Sites in Block Groups with $54,642 ≤ Median Income

	Transactions<1 mi of LIHTC Site	Transactions<.5 mi of LIHTC Site	Transactions<.2 mi of LIHTC Site	Transactions<.1 mi of LIHTC Site
Housing Transaction Price	369676	334890	314001	316961
	[377497]	[461491]	[488588]	[544460]
Housing Transaction Price -HMDA Matched	385589	350551	327842	327336
	[358489]	[448765]	[465937]	[507898]
% of Home Buyers Black	0.0415	0.0471	0.0517	0.0504
	[0.4274]	[0.4294]	[0.4303]	[0.4775]
Income of Home Buyer	107289	102083	98220	99220
	[51190]	[51151]	[52628]	[54084]

Note: All prices inflated to 2012 dollars. Standard deviations in brackets.

Table A2: Reduced Form Log Price Impacts: New Construction vs Rehab Sites

LIHTC Sites in Tracts with Median Inc<$26,017

Dist to LIHTC:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Rehab Sites												
	0.0791***	0.0643***	0.0560***	0.0538***	0.0517***	0.0474***	0.0399***	0.0300***	0.0197***	0.0110**	0.0052	0.0023
	[0.0208]	[0.0174]	[0.0149]	[0.0130]	[0.0115]	[0.0100]	[0.0085]	[0.0072]	[0.0059]	[0.0045]	[0.0032]	[0.0017]
New Construction Sites												
	0.0490*	0.0441**	0.0387**	0.0300**	0.0228*	0.0159	0.0109	0.0071	0.0042	0.0025	0.0017	0.0010
	[0.0253]	[0.0200]	[0.0169]	[0.0147]	[0.0126]	[0.0105]	[0.0088]	[0.0073]	[0.0060]	[0.0046]	[0.0032]	[0.0017]

LIHTC Sites in Tracts with $26,017≤ Median Income < $38,177

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Rehab Sites												
	0.0211	0.0185	0.0146	0.0115	0.0103	0.0093	0.0081	0.0068	0.0056	0.0043	0.0032	0.0019
	[0.0152]	[0.0126]	[0.0109]	[0.0096]	[0.0086]	[0.0077]	[0.0069]	[0.0061]	[0.0051]	[0.0040]	[0.0028]	[0.0014]
New Construction Sites												
	-0.0089	-0.0118	-0.0139	-0.0132	-0.0115	-0.0099	-0.0084	-0.0069	-0.0055	-0.0046	-0.0031	-0.0015
	[0.0135]	[0.0114]	[0.0099]	[0.0087]	[0.0080]	[0.0071]	[0.0062]	[0.0053]	[0.0043]	[0.0034]	[0.0023]	[0.0012]

LIHTC Sites in Tracts with $38,177≤ Median Income < $54,642

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Rehab Sites												
	-0.0205	-0.0187*	-0.0129	-0.0062	-0.0000	0.0045	0.0069	0.0077	0.0075*	0.0057*	0.0036	0.0019
	[0.0131]	[0.0106]	[0.0089]	[0.0078]	[0.0071]	[0.0066]	[0.0058]	[0.0050]	[0.0041]	[0.0031]	[0.0023]	[0.0012]
New Construction Sites												
	-0.0361***	-0.0298***	-0.0235***	-0.0183***	-0.0145***	-0.0115**	-0.0096**	-0.0083**	-0.0068**	-0.0055**	-0.0041**	-0.0021*
	[0.0108]	[0.0086]	[0.0072]	[0.0062]	[0.0056]	[0.0050]	[0.0045]	[0.0039]	[0.0033]	[0.0027]	[0.0019]	[0.0011]

LIHTC Sites in Tracts with $54,642≤ Median Income

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1.	1.2
Rehab Sites												
	-0.0320**	-0.0243**	-0.0179**	-0.0123*	-0.0078	-0.0045	-0.0024	-0.0010	-0.0002	0.0002	0.0007	0.0008
	[0.0144]	[0.0105]	[0.0085]	[0.0074]	[0.0067]	[0.0061]	[0.0055]	[0.0049]	[0.0041]	[0.0033]	[0.0023]	[0.0012]
New Construction Sites												
	-0.0224**	-0.0201***	-0.0173***	-0.0136**	-0.0104**	-0.0082*	-0.0063	-0.0047	-0.0033	-0.0025	-0.0019	-0.0012
	[0.0093]	[0.0077]	[0.0065]	[0.0057]	[0.0051]	[0.0046]	[0.0041]	[0.0035]	[0.0028]	[0.0021]	[0.0015]	[0.0008]

Note: All prices inflated to 2012 dollars. Bootstrapped standard errors in brackets. Effects reported are the average house price impact between 0 and 22 years versus 6 to 1 year before treatment. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A3: Median # of Households Impacted by LIHTC Development

Panel A: Low Minority Areas

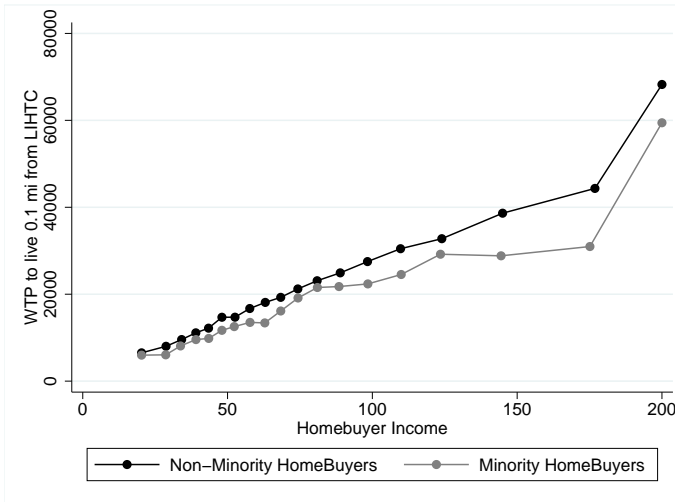|	Income Q1	Income Q2	Income Q3	Income Q4
# Renting HHs Impacted by LIHTC	4832.5	621	1171.5	1328
# Owning HHs Impacted by LIHTC	2476	296	1042	2268
Observations	658	884	1768	2463

Panel B: High Minority Areas

	Income Q1	Income Q2	Income Q3	Income Q4
# Renting HHs Impacted by LIHTC	12171	2075	6028	5422
# Owning HHs Impacted by LIHTC	3764.5	599	3630	4456.5
Observations	2248	1817	1267	340

Note: Median number of renting and home owning households living within the area impacted by LIHTC development, as meared in the 2007-2012 ACS. Standard deviation in brackets below. Each observation is a neighborhood around a LIHTC development.

Figure A1: Mean Willingness to Pay to Live 0.1 miles from LIHTC: Low Minority Areas

(a) Q1 Income Neighorhoods

(b) Q2 Income Neighorhoods

(c) Q3 Income Neighborhoods

(d) Q4 Income Neighborhoods

Note: Willingness to pay is measured in 2012 dollars. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site. Reported preferences are for households who choose to live close to LIHTC development.

Figure A2: Mean Willingness to Pay to Live 0.1 miles from LIHTC: High Minority Areas

(a) Q1 Income Neighborhoods

(b) Q2 Income Neighborhoods
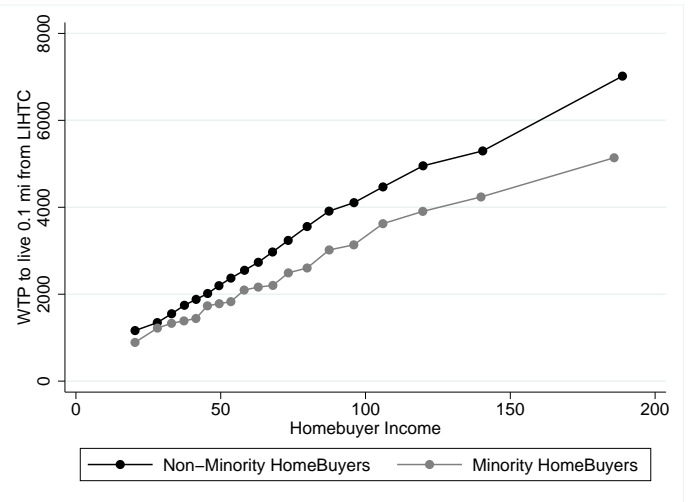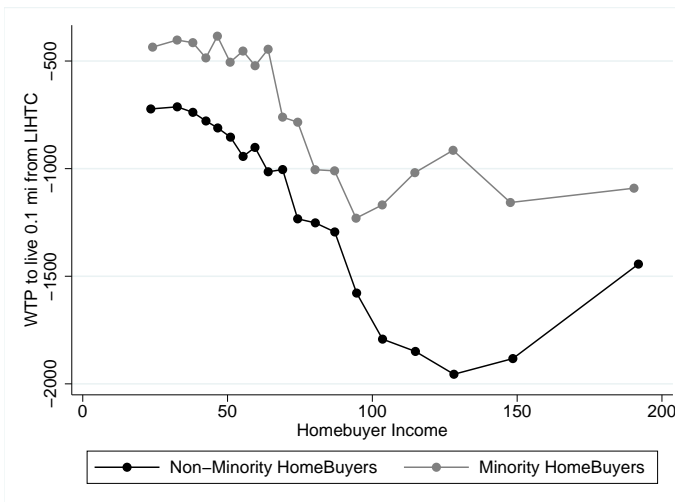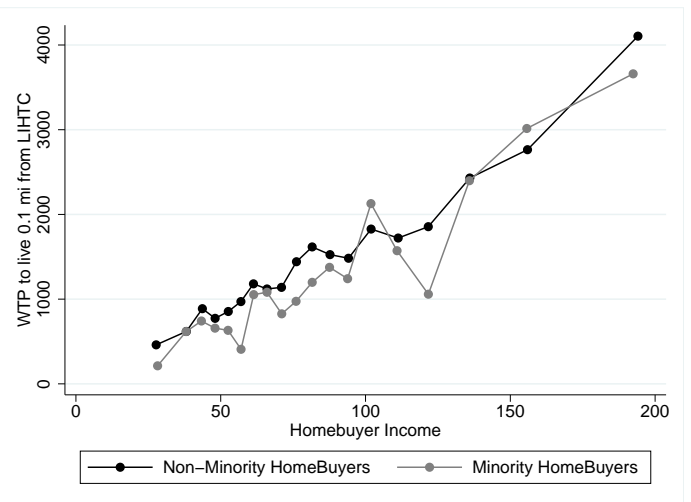
(c) Q3 Income Neighborhoods

(d) Q4 Income Neighborhoods

Note: Willingness to pay is measured in 2012 dollars. Household median income quartile cutoffs are $26017, $38177, and $54642 in 2012 dollars, as reported in the 1990 Census block group of the LIHTC site. Reported preferences are for households who choose to live close to LIHTC development.

Figure A3: Ring Analysis vs Empirical Derivative Simulation Study
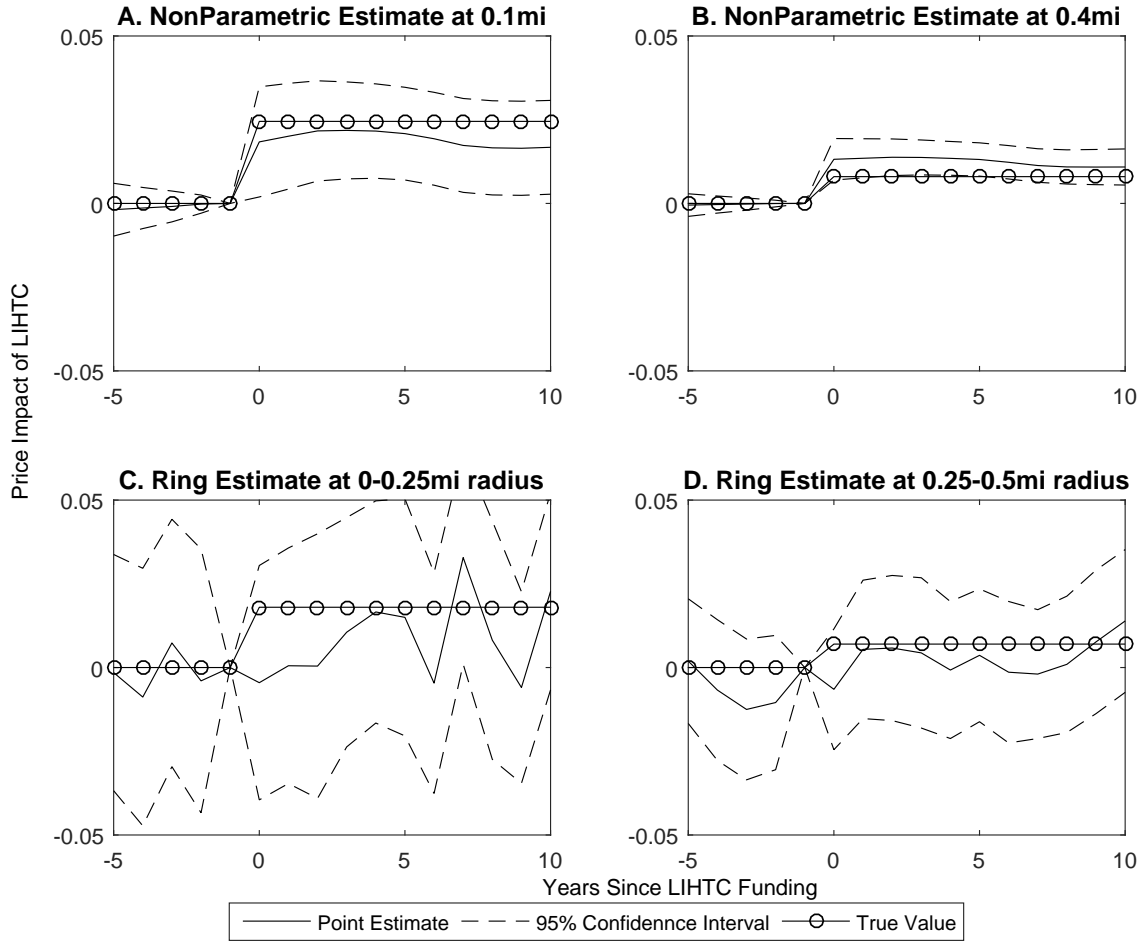
Note: Data is simulated to compare estimation methods. Error bands represnt 95% confidence interval.

Figure A4: Empirical Derivative Simulation Study Surface Estimate

NonParametric Price Surface Estimate
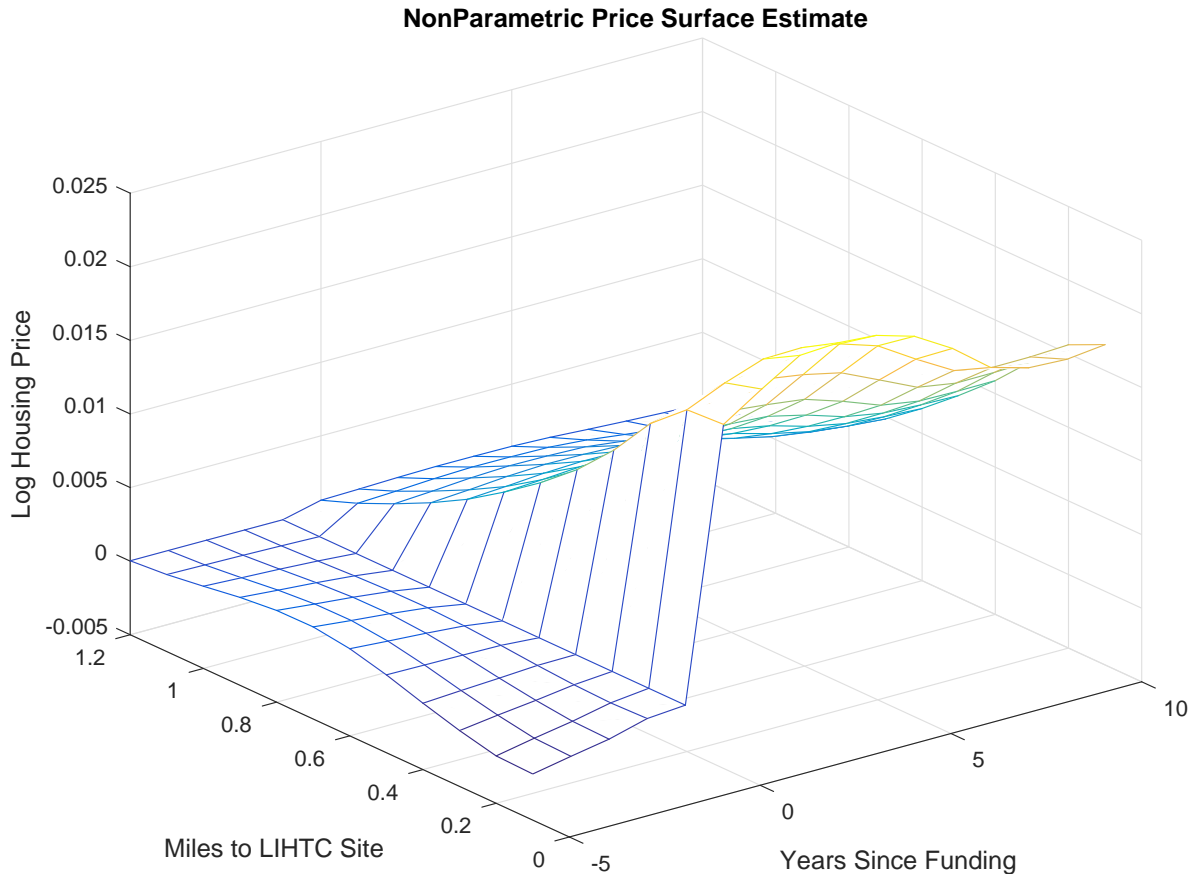
Note: Data is simulated to compare estimation methods.

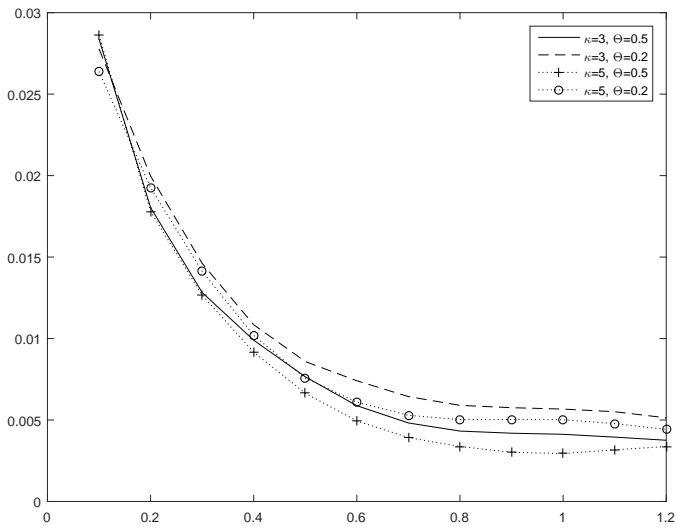Figure A5: Monte Carlos Results of Empircal Derivative Estimator Varying Tuning Paramters
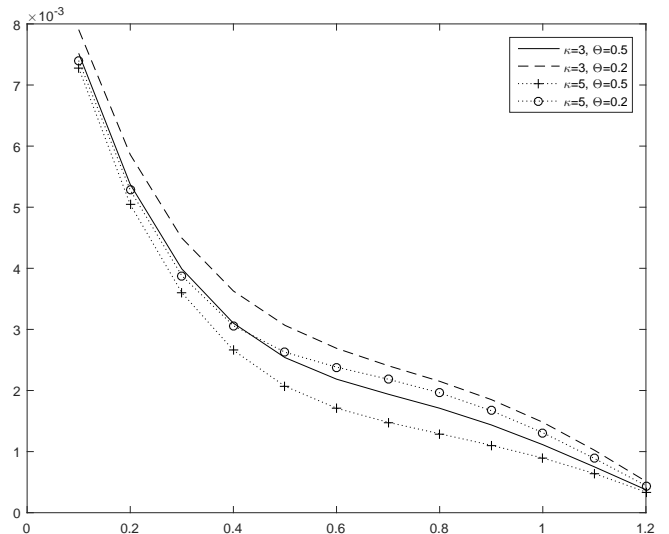
(a) Bias in Derivative Estimator
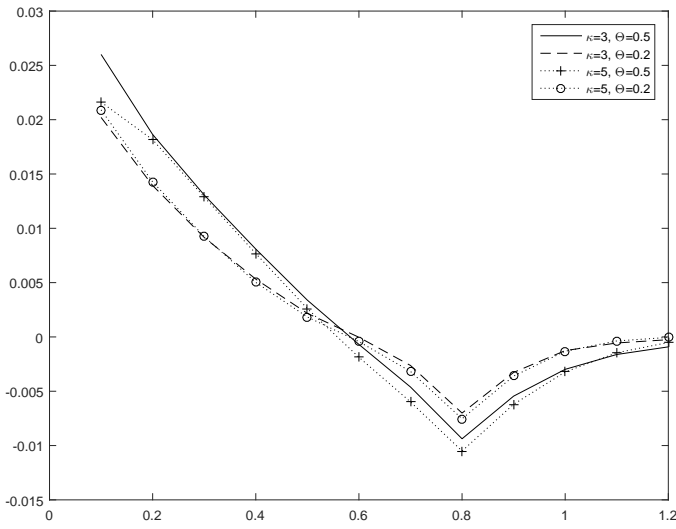
(b) Bias in Level Estimator
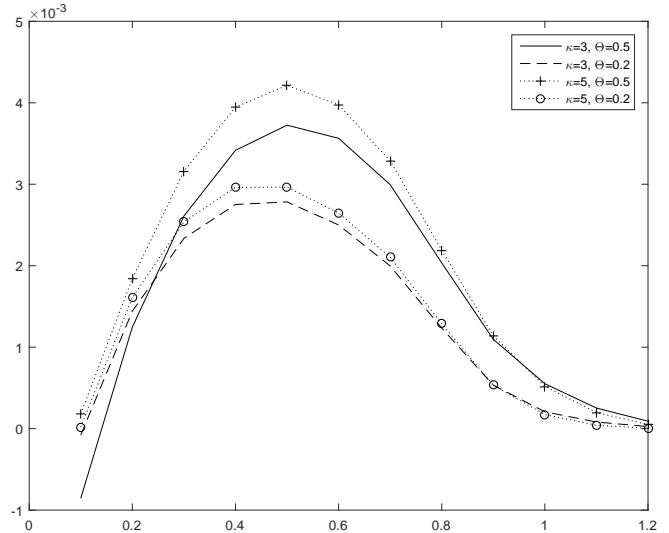
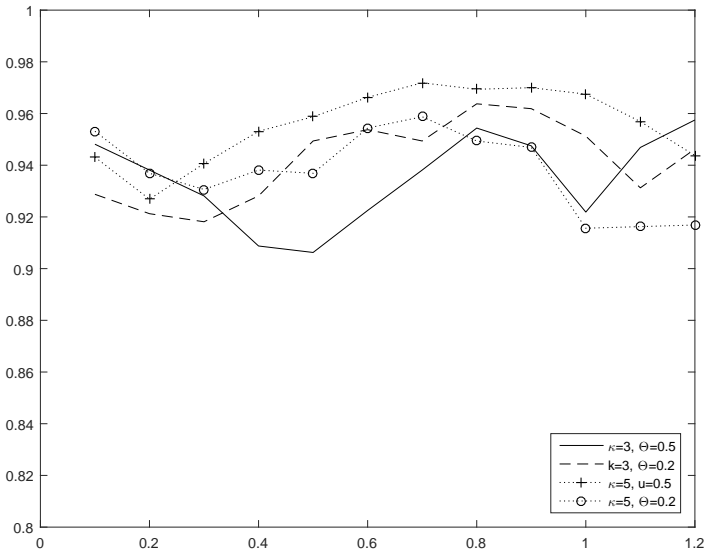(c) Standard Dev in Derivative Estimator
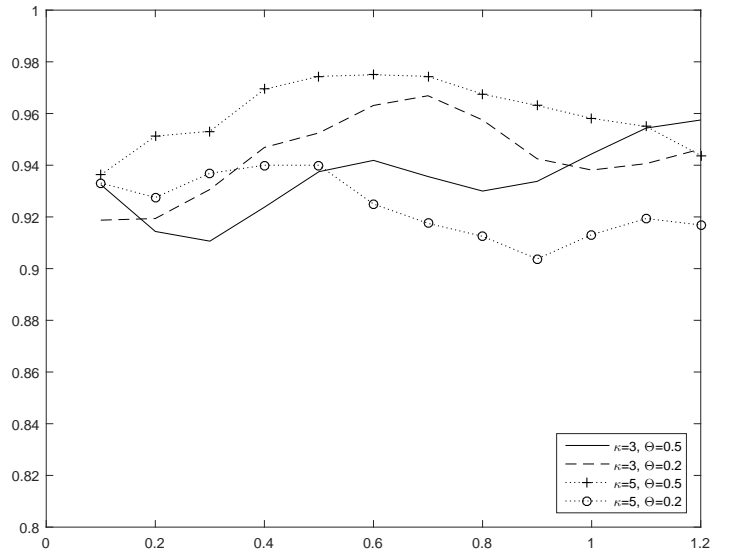
(d) Standard Dev in Level Estimator

Note: Each set of tuning parameters performs 100 monte carlo simulations. See text for further details.

Figure A6: Coverage Ratios from Monte Carlos Results

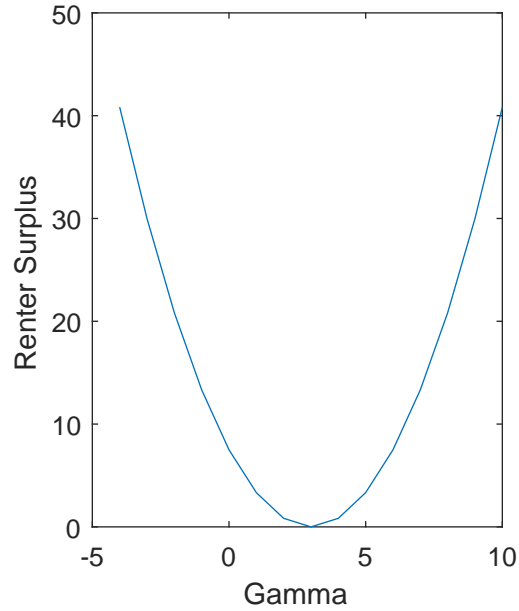(a) 95% CI Coverage Ratios for Derivative Estimator

(b) 95% CI Coverage Ratios for Level Estimator

Note: Each set of tuning parameters performs 100 monte carlo simulations. Coverage ratios are adjusted for estimated bias. Standard errors are block bootstrapped at neighborhood level using 100 samples.

Figure A7: Renter Surplus in Linear-Quadratic Hedonic Model

Note: This figure shows the equilibrium renter surplus of the linear-quadratic hedonic model detailed in Appendix E. Equilibrium renter surplus is a function of the preference parameter γ_i. The other parameters of the model are $A = 1$, $\mu_\gamma = 3$, $\sigma_\gamma = 3$, $\mu_Q = 0$, and $\sigma_Q = 5$. The figure shows that all renters, except the measure zero set with $\gamma_i = 3$, earn positive surplus in equilibrium.