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An Ontology for Formal Models of Kinship

Carmen CHUI [a], Michael GRÜNINGER [a], Janette WONG [b]

[a] *Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada*

[b] *Data & Analytics (DNA), Technology & Operations, Royal Bank of Canada, Toronto, Ontario, Canada*

Abstract. The near ubiquity of family relationship ontologies in the Semantic Web has brought on the question of whether any formal analysis has been done in this domain. This paper examines kinship relationships that are normally overlooked in formal analyses of domain-specific ontologies: how are such ontologies verified and validated? We draw inspiration from existing work done in anthropology, where attempts have been made to formally model kinship as atemporal algebraic models. Based on these algebraic models, we provide an ontology for kinship written in first-order logic and demonstrate how the ontology can be used to validate definitions found in Canadian legal laws and data collection documentation.

Keywords. algebraic model, anthropology, kinship, kin term map, relationships, domain ontology

1. Introduction

Despite the near ubiquity of family relationship ontologies within the Semantic Web community, there has been no *formal* analysis of any ontology for this domain that captures familial relationships in anthropology and in legal texts. Often used as an example to illustrate how to develop an ontology in the Web Ontology Language (OWL)[1], there is the *misconception* that family relationships are easy to model in an ontology. The ubiquity of using family relationships gives the impression that this particular domain is trivial to axiomatize. However, extensive work has been done within anthropology which can serve as the basis for the validation of any kinship ontology. In this paper[2], we propose a kinship ontology written in first-order logic to formalize anthropologist Dwight Read's algebraic models of kinship and the associated intended semantics, and to support reasoning problems and queries across demographic datasets found in anthropology.

[1]For example, see the OWL 2 language guide for the Family History Knowledge Base (FHKB) in [1].

[2]An extended version of this paper containing the proofs for the theorems and verification of the ontology can be found online: `http://stl.mie.utoronto.ca/publications/fois_kinship_extended.pdf`

2. Motivation

The popular Family History Knowledge Base (FHKB) presented in [1] was designed to test the limits of OWL reasoners and to maximize the use of inference since it is mostly taxonomic in structure and contains very few non-subclass axioms. There is no ontological basis in its design: no requirements were proposed, no verification nor validation was done, and there was no analysis of its ontological commitments. Further, there have been no formal axiomatizations of notions of kinship outside of FHKB[3]. Instead, we look at the terminology used to describe *kinship* in anthropology and terminology found within legal documentation in the Canadian context.

We need to use *ontologies* to represent these different systems and legal definitions because we want to do data quality and other kinds of queries with respect to that data. Taxonomies are insufficient to carry out such tasks due to their inabilities to *define* additional concepts and provide *explicit* axioms to describe relationships between concepts.

In anthropology, there are established kinship systems that contain various terminology used to describe relationships between people. Kinship patterns were identified by Lewis Henry Morgan [5] and further categorized by George P. Murdock [6], both anthropologists who studied family and kinship structures across different cultures. These various kinship systems[4] and their relationships are depicted and defined graphically in Figure 1: circles and triangles denote female and male, respectively, and colours denote the various relationships with a label describing the relationship underneath. Different societies describe kinship relationships differently. For a more detailed discussion of these kinship systems and terminologies, we refer the reader to [6], [5], and [7].

Additionally, kinship relationships can be defined using Anthony F.C. Wallace and John Atkins' anthropological definitions in [8], shown in Table 1. Abbreviations for familial relationships are as follows: father (Fa), mother (Mo), brother (Br), sister (Si), son (So), and daughter (Da). These terms are most familiar in most English-speaking parts of the world, and are treated as primitives in the English language. More complex kin relationships are treated as the relative product of two or more primitive terms. For example, the definition of grandfather in the first row can be defined as "the father of father" (FaFa) and "the father of mother" (MoFa), while grandmother is defined as "the mother of father" (FaMo) and "the mother of mother" (MoMo).

Further, we are also interested in the legal applications of kinship[5]. In Canada, several legal laws and acts outline the limitations of marriage and in official data collection agencies; these include concepts found in Statistics Canada ('StatsCan')[6], the Marriage (Prohibited Degrees) Act[7], and the Civil Marriage Act[8]. We are interested in defining these legal concepts using a formal kinship ontology.

Additionally, work done in anthropology shows there is interest in representing the structures of kinship algebraically and as formal models. Figures 2 and 3 illustrate me-

[3]There are various other discussions on kinship and reasoning in [2, 3], as well as an ontology pattern presented in [4], but these discussions all pertain to reasoning with OWL.

[4]We also acknowledge other cultural systems for kinship, primarily the Chinese kinship system, which are more descriptive and involves the notion of ages and social institutions. It would be of interest to further extend the ontology presented in this paper to cover this kinship system.

[5]Since the authors live in Canada, the Canadian legal context for these terms and definitions are of interest.

[6]`https://www.statcan.gc.ca/eng/concepts/index`

[7]`https://laws-lois.justice.gc.ca/eng/acts/M-2.1/page-1.html`

[8]`https://laws-lois.justice.gc.ca/eng/acts/c-31.5/page-1.html`

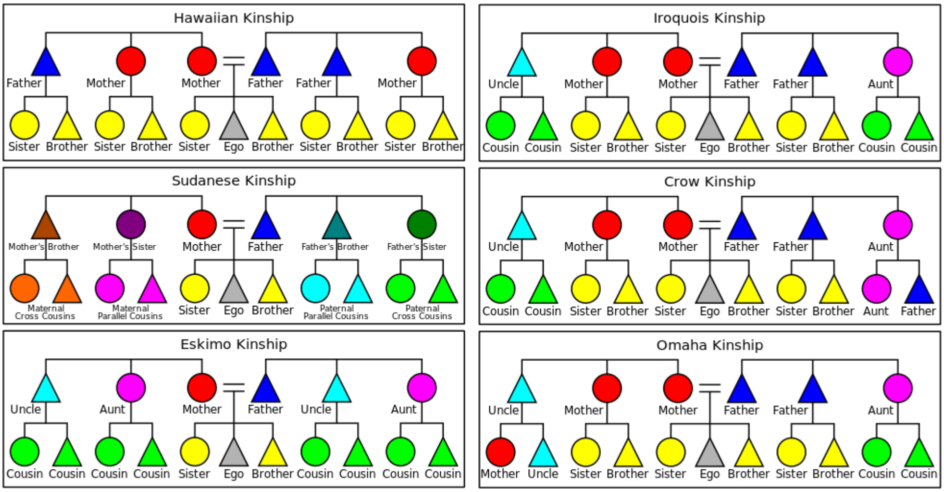


Figure 1. Basic kinship classification systems identified by Murdock in [6]. Triangles and circles denote sex (male/female). Colours denote the different types of relationships in each system systems, with a label for the relationship underneath.

Table 1. Example kinship definitions presented by Wallace and Atkins in [8].

Terminology		**Definition**
grandfather	:	FaFa, MoFa
grandmother	:	FaMo, MoMo
grandson	:	SoSo, DaSo
granddaughter	:	SoDa, DaDa
uncle	:	FaBr, MoBr, FaFaBr, MoFaBr, etc.
aunt	:	FaSi, MoSi, FaFaSi, MoFaSi, etc.
cousin	:	FaBrSo, FaBrDa, MoBrSo, MoBrDa, FaSiSo, FaSiDa, MoSiSo, MoSiDa, FaFaBrSo, FaMoBrSo, MoFaSiDa, etc.
nephew	:	BrSo, SiSo, BrSoSo, SiSoSo, etc.
niece	:	BrDa, SiDa, BrDaDa, SiDaDa, etc.

diation and algebraic structures (kin term map) found in [9–11]. In particular, Read et al. have developed a Kinship Algebraic Expert System (KAES) in [10] which takes kinship terminology and algebraically constructs kin term maps and genealogical diagrams (family trees) of the resulting kin term maps. As we will see later, the algebraic structures produced by Read et al. in [10] and [11] can be used to help formalize kinship relationships as definable relations. Figure 2 outlines *mediation structures* used to describe kinship in [11,12]: these mathematical structures are used to relate two, otherwise unrelated, conceptual categories together using a mediating category[9]. In Figure 2a, a structure for a family with one child is presented: the three categories are shown as the Mother, Father, and Child boxes, each with their own gender attributes. The *spouse* relation links the Mother and Father categories, and are linked to the Child category by the mother

[9]In the context of [12] and [11], the term 'category' refers to *conceptual categories*, which in our ontology are formalized as classes.

and father relations, respectively. Similarly, Figure 2b illustrates a structure for a family with two children and the inclusion of a sibling relation between offspring. As we will see, these mediation structures can be axiomatized with the kinship ontology presented in this paper.

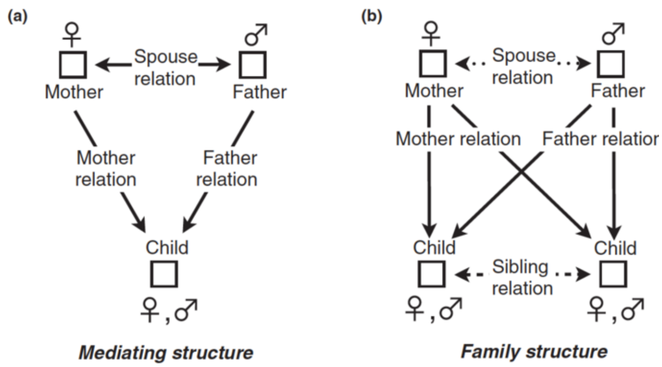


Figure 2. Read's mediation structures for kinship. (a) shows a mediation structure for a family with one child, and (b) shows a mediation structure for a family with two children with the inclusion of a sibling relation. (Figure 2 from [11])

Furthermore, a formal outline of how to generate algebraic kinship structures can be found in [13], where Read asserts that an algebraic structure constructed from kinship terminology is isomorphic to the kin term map structure. He presents a construction methodology[10] that maps the kin terminology with the kin term map. In Section 4, we show how the mediation structures presented in [10, 11] correspond to mathematical graph structures used to verify the kinship ontology presented in this paper.

In the sections that follow, we provide an overview of our axiomatization of Read's algebraic structures and show how our first-order ontology can describe kinship and familial relationships. Regardless of which anthropological kinship system is used to describe relationships, the ontology is sufficient to axiomatize the following: the terms found in each system, the intended semantics of the algebraic structures presented by Read in [11], and definitions in Canadian legal documentation.

3. The Kinship Ontology ($T_{kinship}$)

The idea of representing the *binary* relationships in [11] drives our interest in developing a first-order ontology that sufficiently captures these anthropological concepts of kinship. Herewith we present the kinship ontology, $T_{kinship}$, in first-order logic[11]. The ontology is designed with the mediation structures and kinship term maps from [11] in mind: our focus is on the various kinship relationships presented in anthropology, as these structures have been already established in that field. We emphasize here that we are axiomatizing Read's structures and do not introduce any bias in how these relationships should be axiomatized. Our approach differs from existing work done with ontologies and the

[10]This outlined in detail in Box 13.2 in [13], and in [9].
[11]Available online: `http://colore.oor.net/kinship/`

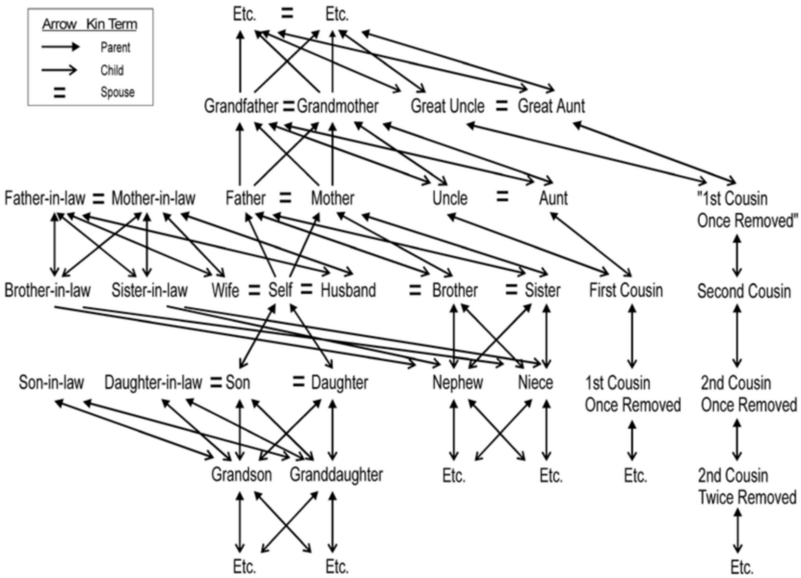


Figure 3. Kin term map that outlines the various relationships. (Figure 5a from [11])

discussion on the purpose and notion of 'roles' (such as role identities discussed in [14] and [15]).

Instead of focusing on which kinship relationship types should be considered as roles, we have taken the existing anthropological kinship terms and have axiomatized them as *binary* relations within the ontology. Further, we present a set of *atemporal* axioms for kinship as the algebraic models presented in [11] are independent of time; while spouses and other relationships may change over time, these changes need to be reflected in a temporal version of the kinship ontology: this is left for a future iteration of the ontology that includes the adoption of relevant time and event ontologies.

The algebra presented in [11] contains one substructure for consanguineal relations (which arise from ancestral lineage) and one substructure for affinal relations (which arise through marriage). The signature of $T_{kinship}$ therefore consists of the two *primitives*: the affinal $hasSpouse(x,y)$ and the consanguineal $ancestorOf(x,y)$ relations, which are read as "x has spouse y" and "x is the ancestor of y," respectively. The axioms of the ontology are organized into the following sets of Common Logic Interchange Format (CLIF) files depicted in Figure 4:

- $T_{ancestor}$ contains axioms pertaining to ancestors (Axioms (1) to (8) in Figure 5).
- T_{spouse} contains axioms pertaining to spouses (Axioms (9) to (12) in Figure 5).
- $T_{kinship}$ imports $T_{ancestor}$ and T_{spouse}, with additional axioms that combine spouses and ancestors.

3.1. Ancestors and Children

Approaches to define kinship relations, such as those presented in [11], begin with the parent/child relation, and then define all other relations through composition. However,

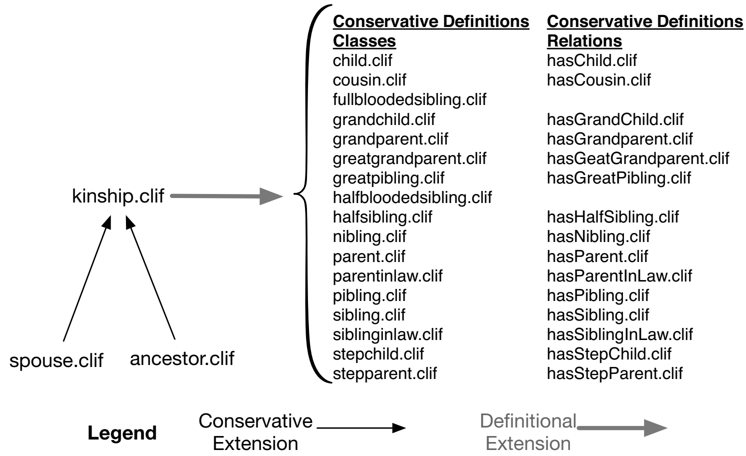


Figure 4. Hierarchy organization in COLORE. Theory names denote the CLIF file names found in the repository. Solid arrows denote conservative extension, dotted arrows denote non-conservative extension, and bolded solid grey arrows denote definitional extension.

the partial ordering over ancestors and descendants is not first-order definable using the $hasChild(x,y)$ relation (in the same way that a discrete linear ordering is not first-order definable using a successor relation). Partial orders are not first-order definable by a theory whose signature consists only of *successor* (see [16]). On the other hand, a successor relation, such as $hasChild(x,y)$, is definable in a discrete partial order using the $ancestorOf(x,y)$ relation. Consequently, $ancestorOf(x,y)$ was selected as a primitive in the ontology.

3.2. Ancestors, Spouses, and Unintended Models

The axioms in $T_{ancestor}$ and T_{spouse} alone are not sufficient. We also need to specify additional constraints between the $ancestorOf(x,y)$ and $hasSpouse(x,y)$ relations, to prevent scenarios such as where grandparents are the spouses of their grandchildren in the models of the ontology. We need axioms that limit how two persons in the domain of the ontology can be related by marriage using $hasSpouse(x,y)$ and by parentage using $ancestorOf(x,y)$. In order to eliminate such unintended models where people with familial relationships would become spouses or parents of each other, we need to introduce an ordering relation to differentiate between ancestors and descendants. Such unintended relationships would be having two siblings becoming spouses, or a grandchild marrying their grandparent. We want to best represent situations that are bound by Canadian laws and thereby adhere to the Criminal Code (R.S.C., 1985, c. C-46)[12], which outlines the conditions for incest when the relationships between two people are by blood. These sorts of relationships are illegal when they are between a person and their parent, child, sibling, grandparent, or grandchild.

Without constraints to limit incestuous relationships in $T_{kinship}$, this would cause models of the ontology to contain circular relationships. For example, this would result in models where a grandparent can be the child of their own child[13], or have grand-

[12] https://laws-lois.justice.gc.ca/eng/acts/C-46/section-155.html
[13] http://colore.oor.net/kinship/output/kinship_greatgrandparents_unintended.model

parents be the spouses of their own grandchild[14]. To eliminate such *unintended* models from the ontology, we use a *discrete partial order* to constrain how the elements of the $hasChild(x,y)$ and $hasSpouse(x,y)$ relations can interact. The $ancestorOf(x,y)$ relation is used to *order* the individuals in the ontology – we utilize the notion of *ordering* found in mathematics and order theory. Axioms (1) to (8) in Figure 5 outlines the axioms used to handle ancestor relationships in the $T_{ancestor}$ module.

We impose the rather strong condition of Axiom 13, (which requires spouses to have no common ancestors) in part because it will be needed to capture the structure of Read's algebra. If the ontology is used for data cleaning, this axiom can be relaxed to allow families with spouses that share a common great-great-grandparent. For example, the British Royal Family consists of third cousins who have married one another: Queen Elizabeth II and Prince Philip are both descendants of Queen Victoria.

$$(\forall x \forall y\, (ancestorOf(x,y) \supset (person(x) \wedge person(y)))). \tag{1}$$

$$(\forall x\, (\neg ancestorOf(x,x))). \tag{2}$$

$$(\forall x \forall y \forall z\, ((ancestorOf(x,y) \wedge ancestorOf(y,z)) \supset ancestorOf(x,z))). \tag{3}$$

$$(\forall x \forall y\, (ancestorOf(x,y) \supset \neg ancestorOf(y,x))). \tag{4}$$

$$\begin{aligned}(\forall x \forall y\, (hasChild(x,y) \equiv & (ancestorOf(x,y) \\ & \wedge \neg(\exists z\, (ancestorOf(x,z) \wedge ancestorOf(z,y)))))).\end{aligned} \tag{5}$$

$$(\forall x \forall y\, (ancestorOf(x,y) \supset (\exists z\, (hasChild(x,z) \wedge (ancestorOf(z,y) \vee (y=z)))))). \tag{6}$$

$$(\forall x \forall y ((ancestorOf(x,y) \supset (\exists z (hasChild(z,y) \wedge (ancestorOf(x,z) \vee (x=z))))))). \tag{7}$$

$$\begin{aligned}(\forall x \forall y \forall z \forall u\, (ancestorOf(u,y) \wedge ancestorOf(z,y) \wedge ancestorOf(x,u) \wedge ancestorOf(x,z) \\ \supset (ancestorOf(u,z) \vee ancestorOf(z,u) \vee (z=u)))).\end{aligned} \tag{8}$$

$$(\forall x \forall y\, (hasSpouse(x,y) \supset (person(x) \wedge person(y)))). \tag{9}$$

$$(\forall x\, (\neg hasSpouse(x,x))). \tag{10}$$

$$(\forall x \forall y\, (hasSpouse(x,y) \supset hasSpouse(y,x))). \tag{11}$$

$$(\forall x \forall y \forall z\, (hasSpouse(x,y) \wedge hasSpouse(x,z) \supset (y=z))). \tag{12}$$

$$(\forall x \forall y \forall z\, ((hasSpouse(x,y) \wedge ancestorOf(z,x)) \supset \neg ancestorOf(z,y))). \tag{13}$$

Figure 5. Axioms for $T_{kinship}$.

3.3. Doesn't Everyone Have A Parent?

We **do not** include this axiom in the ontology:

> A person has a parent who is a person. (For every person, there is another person who is their parent.)

$$(\forall x\, (person(x) \supset (\exists y\, (person(y) \wedge hasParent(x,y) \wedge (x \neq y))))). \tag{14}$$

[14]http://colore.oor.net/kinship/output/kinship_grandparentspouse_unintended.model

Intuitively, this axiom makes sense in real life. However, the knowledge base or ontology of persons should only reflect the elements one wants to examine. We might care about Bob and Alice, but not necessarily Bob's parents or Alice's parents: we do not necessarily need to know the parents of a particular person. It is possible to have an ancestor of a person without them being a parent of that person. Furthermore, this axiom creates *infinite* models when using a model finder like Mace4[15]: for every person in the knowledge base, the program will continually generate more and more elements in the model and will never terminate. Consequently, a model will never be outputted by a model finder.

3.4. What About Gender?

The approach we have taken to axiomatize $T_{kinship}$ is *independent* of gender. Within kinship systems in anthropology, a strict binary gender system of male and female is adopted: this is particularly noticeable in the kinship systems presented in Figure 1. Consequently, we can state that anthropologists have adopted an explicit binary gender ontology in their algebraic representations of relationships.

In contrast, $T_{kinship}$ does not have any inherent bias towards any gender ontology: in the axioms presented in Figure 5, we have made all binary relations as gender-neutral as possible. Due to the gender-neutral nature of the axioms of $T_{kinship}$, we can treat gender as an ontology module that can be imported into $T_{kinship}$ to allow us to make additional distinctions (such as male or female) in order for us to *faithfully interpret* existing work and models done by the anthropology community.

3.5. Kinship Relationships As Defined Relations

With $T_{kinship}$, we can axiomatize the kinship relationships presented in Figures 2 and 3 as *defined relations*. For example, this means that definitions for first cousins once- or twice-removed, and second and third cousins, can be easily axiomatized by extending the ontology with conservative definitions. In the hierarchy organization presented in Figure 4, these definitions are signified as definitional extensions with the bolded grey arrows and are in their own individual CLIF files in the repository. As we will see in Section 5, we can write definitions for classes (such as $grandparent(x)$, $cousin(x)$, $grandchild(x)$) and their corresponding binary relations (such as $hasGrandparent(x,y)$, $hasCousin(x,y)$, $hasGrandchild(x,y)$). For example, the $hasGrandparent(x,y)$ relation has the following definition:

$$(\forall x \forall z\,(hasGrandparent(z,x) \equiv (\exists y\,(hasChild(x,y) \wedge hasChild(y,z))))).$$

From this first-order definition, we can see that the bidirectional equivalence is not definable in OWL. In FHKB, the `grandparent` class is axiomatized as an OWL2 property chain, which allows an ontology user to infer the existence of a property from a chain of properties. This would appear as $hasParent \circ hasParent \sqsubseteq hasGrandParent$. With Read's algebraic approach presented in [10], the grandparent relationship is defined as $P^2 \leftrightarrow$ grandparent in algebraic logic, where P stands for parent. Note that the axioms that arise from the property chain and algebraic approaches correspond to *paths* within

[15] `https://www.cs.unm.edu/~mccune/mace4/`

the kin term maps shown in Figure 3. Consequently, we can state that defined relations in $T_{kinship}$ correspond to paths found in the kinship structures.

We can graphically depict this *model* of the ontology in Figure 6 as a consanguinity graph, where people are nodes and the lines between the nodes represent relationships between the people. Directional arrows indicate a parental relationship between nodes, where the tail-end is the parent node and the arrow-head is the child node (e.g., Lucy is the parent of Alice and Lucy is the child of Alice: $hasChild(Lucy, Alice)$ and $hasParent(Alice, Lucy)$). In order to determine whether two people are related to each other in the graph, all one needs to do is to find a *path*. It is possible for two elements in the graph to not be related at all: for example, Francisco has no relationship with anyone in Figure 6.

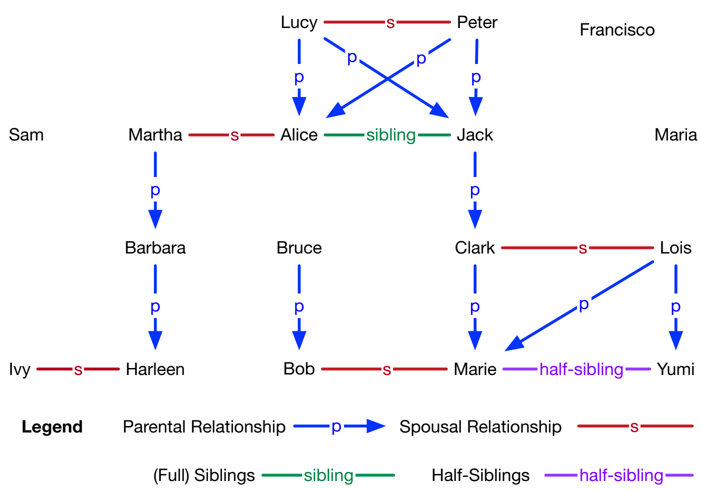


Figure 6. Graphical representation of how defined relations correspond to paths in the underlying consanguinity graph in the model of the ontology. Names of people are nodes of the graph and the lines between notes denote relationships.

To determine how Alice and Bob are related, we would simply have to find a path in the graph between *Bob* and *Alice*. From the example shown in Figure 6, this path would be the path from $hasSpouse(Bob, Marie)$, $hasChild(Clark, Marie)$, $hasChild(Jack, Clark)$, $hasChild(Peter, Jack)$, and $hasChild(Peter, Alice)$. Similarly, to determine if Sam and Jack are related to one another, examining the graph allows us to determine that no path between Sam and Jack can be found, so we can conclude that Sam and Jack are not related to each other.

3.6. Subgraphs as Examples

To show how defined relations can be further generalized, we consider the sibling relationships shown in Figure 7. These are *connected subgraphs* found from the example presented in Figure 6, which show how Alice and Jack are full siblings and that Marie and Yumi are half-siblings. With the ontology, we are able to *define new relations* to demonstrate these relationships. For example, we can define full-blooded siblings as having both parents in common. Conversely, half-siblings have one parent in common.

$$\forall x \forall y\, hasFullBloodedSibling(x,y) \equiv \exists w \exists y \exists z\, hasParent(x,y) \wedge hasParent(x,z) \wedge$$
$$hasParent(w,y) \wedge hasParent(w,z)$$
$$\forall x \forall w\, hasHalfSibling(x,w) \equiv \exists y \exists z\, hasParent(x,y) \wedge hasParent(x,z) \wedge$$
$$hasParent(w,y) \wedge \neg hasParent(w,z)$$

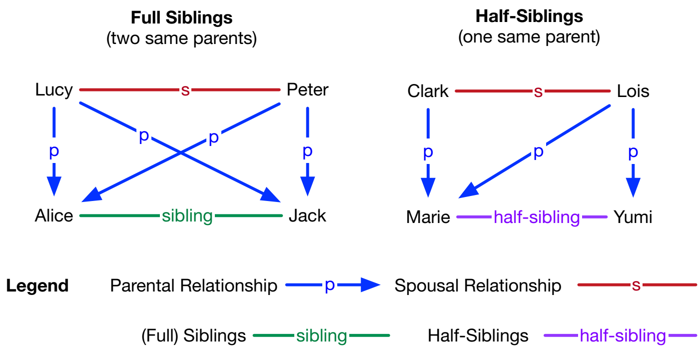


Figure 7. Differences between full- and half-siblings in the models of $T_{kinship}$.

Recall Figure 2 and notice how the models of $T_{kinship}$ resemble the mediation structures presented in [11]. This suggests that, in both cases, the intended structures focus on some underlying classes of graphs, leading to the twin issues of ontology verification and validation. In the sections that follow, we provide the verification of $T_{kinship}$, and the validation of $T_{kinship}$ with respect to the definitions of kinship relationships found in anthropology, the mediation structures developed by the KAES program, and StatsCan documentation.

4. Verification of $T_{kinship}$

Ontology verification is concerned with the relationship between the *intended* models of an ontology and the models of the axiomatization of the ontology. We characterize the models of an ontology up to isomorphism and determine whether these models are equivalent to the intended models of the ontology.

The intended structures for the $ancestorOf(x,y)$ relation is represented by a special class of partial orderings shown in Definition 1. We use the following notation for upper and lower sets from mathematics. For each $\mathbf{x} \in V$, the *upper set* is defined as:

$$U^{\mathbb{P}}[\mathbf{x}] = \{\mathbf{y} : \mathbf{x} \leq \mathbf{y}\}$$

The *lower set* is defined as:

$$L^{\mathbb{P}}[\mathbf{x}] = \{\mathbf{y} : \mathbf{y} \leq \mathbf{x}\}$$

$\mathscr{L}_{\mathbb{P}} = \langle V, E \rangle$ is the lower bound graph for $\mathbb{P}$:

$$(\mathbf{x}, \mathbf{y}) \in E \quad L^{\mathbb{P}}[\mathbf{x}] \cap L^{\mathbb{P}}[\mathbf{y}] \neq \emptyset$$

Definition 1 *A partial ordering $\mathbb{P} = \langle V, \leq \rangle$ is lattice-free iff*

$$\langle L^{\mathbb{P}}[\mathbf{x}], \leq \rangle, \langle U^{\mathbb{P}}[\mathbf{x}], \leq \rangle$$

are semilinear orderings, for each $\mathbf{x} \in V$. $\mathfrak{M}^{lattice\_free}$ denotes the class of discrete lattice-free partial orderings.

Since the *hasSpouse*(x, y) relation is symmetric and irreflexive, it is represented by a special class of simple graphs[16]:

Definition 2 *A scattered edge graph is a simple graph $\mathbb{G} = \langle V, \mathbf{E} \rangle$ such that*

$$\mathbb{G} \cong K_2 \cdot \overline{K_m}$$

$\mathfrak{M}^{scattered\_edge}$ denotes the class of scattered edge graphs.

Models of $T_{kinship}$ are represented by the amalgamation of lattice-free partial orderings and scattered edge graphs:

Definition 3 $\mathbb{P} \oplus \mathbb{G}$ *is a kinship mereograph iff:*

1. $\mathbb{P} = \langle V, \leq \rangle$ *such that* $\mathbb{P} \in \mathfrak{M}^{lattice\_free}$;
2. $\mathbb{G} = \langle V, \mathbf{E} \rangle$ *such that* $\mathbb{G} \in \mathfrak{M}^{scattered\_edge}$;
3. $\mathscr{L}^{\mathbb{P}}([\mathbf{x}]) \cap N^{\mathbb{G}}[\mathbf{x}] = \emptyset$, *for each* $\mathbf{x} \in V$.

$\mathfrak{M}^{kinship\_mereograph}$ denotes the class of kinship mereographs.

Examples of kinship mereographs can be seen in Figures 6 and 7, in which the red edges (spousal relationship) form the scattered edge graph $\mathbb{G}$ and the blue edges (parental relationship) correspond to the Hasse graph of the lattice-free partial ordering $\mathbb{G}$.

Theorem 1 *There exists a bijection $\varphi : Mod(T_{kinship}) \rightarrow \mathfrak{M}^{kinship\_mereograph}$ such that:*

1. $(\mathbf{x}, \mathbf{y}) \in$ **hasSpouse** *iff* $\mathbf{y} \in N^{\mathbb{G}}[\mathbf{x}]$;
2. $(\mathbf{x}, \mathbf{y}) \in$ **ancestorOf** *iff* $\mathbf{x} \in L^{\mathbb{P}}[\mathbf{y}]$.

We can use this characterization of the models of $T_{kinship}$ to exploit the correspondence between the connected substructure of a kinship graph structure and definable relations found in the ontology. If a connected substructure is identified in the graph, a definition for the model that corresponds to the substructure can be written down in first-order logic using the ontology. As a result, the verification of definitional extensions follows from the verification of the primitive kinship theory, $T_{kinship}$.

[16]Notation: K_n is the complete graph with n vertices. $\overline{K_n}$ is the complement of K_n.

5. Validation of $T_{kinship}$

In order to validate $T_{kinship}$, we can use the ontology to axiomatize the relationships found in the aforementioned kinship systems, along with definitions of the kinship relationships found in StatsCan and Canadian legal documents. In particular, we are now able to axiomatize the algebraic models presented by Read: we have taken this independently-derived work from anthropology about kinship and have formalized these intended models that were previously expressed in natural language and in relational algebra. This in contrast to previous approaches where we have formalized the axioms *and* then identified the intended models of the ontology based on *our* interpretations.

For example, we can axiomatize the relationships from the various kinship systems, from Read's algebraic models, and from definitions provided by Wallace and Atkins:

(EX-1) We can generalize the notion of *first cousin* as the child of a parent's sibling. Using the algebraic model from [11] (also in Figure 3), this notion is also captured in the definition for the binary $hasCousin(x,y)$ relation.

$$(\forall x \forall y\, (hasCousin(x,y) \equiv (\exists k \exists w \exists z\, (hasChild(k,z) \wedge hasChild(k,w) \wedge hasChild(z,x) \wedge hasChild(w,y) \wedge (w \neq z))))).$$

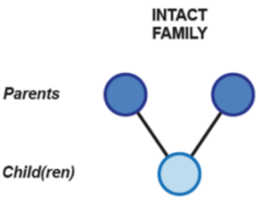
(EX-2) Similarly, we can do the same with concepts like grandchild.

$$(\forall x \forall y\, (hasGrandchild(x,z) \equiv (\exists y \exists z (hasChild(x,y) \wedge hasChild(y,z))))).$$

(EX-3) Additionally, an application of the ontology would be to axiomatize definitions found in StatsCan documentation. For example, StatsCan defines an *intact family* as a family unit where "all children are the biological or adopted children of both married spouses or of both common-law partners [17]." This is also graphically depicted by StatsCan in Figure 8. We can extend $T_{kinship}$ with a new module that contains the $inFamily(x,y)$ relation and the $familygroup(x)$ class to group people together to axiomatize these StatsCan definitions.

$$(\forall x\, (intactfamily(x) \equiv (familygroup(x) \wedge \exists y \exists z\, inFamily(y,x) \wedge inFamily(z,x) \wedge hasSpouse(y,z) \wedge (y \neq z) \wedge (\forall u\, (inFamily(u,x) \wedge (u \neq y) \wedge (u \neq z)) \supset hasChild(y,u))))).$$

(a) Axiom for intact family using $T_{kinship}$.



(b) Figure 3 in [17].

Figure 8. Intact family as defined by StatsCan in [17].

Read proposes a nonassociative algebra $\mathscr{K} = \langle L, \circ, * \rangle$ for kinship relations. The $\circ$ operator represents the composition of consanguineal relations and the $*$ operator represents the composition of spousal (also known as affinal) relations. In [18], Read presents structural equations to construct structural relationships using the American Kinship Terminology (AKT); for example, the sentences in Figure 9 indicate how the algebra can be used to develop the kin term map shown in Figure 3. The $\circ$ operator indicates the composition of kinship terms; for example, 'Self' is the identity term which can be determined using what Read calls a structural equation of 'Parent $\circ$ Child = Self'. A 0 in a structural equation indicates that the terms for 'parent of parent-in-law' (R-4) and 'parent of child-in-law' (R-5) are not valid kin terms in the AKT system.

$$\text{Parent} \circ \text{Child} = \text{Self} \tag{R-1}$$
$$\text{Spouse} \circ \text{Spouse} = \text{Self} \tag{R-2}$$
$$\text{Spouse} \circ \text{Parent} = \text{Parent} \tag{R-3}$$
$$\text{Parent} \circ \text{Parent} \circ \text{Spouse} = 0 \tag{R-4}$$
$$\text{Parent} \circ \text{Spouse} \circ \text{Spouse} = 0 \tag{R-5}$$
$$\text{Spouse} \circ \text{Child} \circ \text{Parent} = \text{Child} \circ \text{Parent} \circ \text{Spouse} \tag{R-6}$$

Figure 9. Example algebraic compositions for relationships in the American Kinship Terminology (AKT) from [9] and [18]. $\circ$ is the composition operator for the structural equation, and 0 indicates that this is not classified as a kin term in AKT.

In order to demonstrate that kinship structures (which are models of $T_{kinship}$) are the *right* class of structures, we show their relationship to Read's nonassociative algebra. The basis for this relationship lies in identifying the graphs that correspond to each class of structures, and then showing how these graphs are related to each other.

The central theorem that shows the relationship between kinship structures and Read's nonassociative algebra $\mathscr{K}$ for kinship relations relies on two classes of graphs that are associated with the respective structures.

Definition 4 *$\mathbb{G} = \langle V, \mathbf{E} \rangle$ is the Hasse graph for a partial ordering $\mathbb{P} = \langle V, \leq \rangle$ if $(\mathbf{x}, \mathbf{y}) \in E$ iff either $\mathbf{x}$ covers $\mathbf{y}$ or $\mathbf{y}$ covers $\mathbf{x}$ in $\mathbb{P}$.*

Definition 5 *Let $\mathbb{M} = \langle V, \circ \rangle$ be a semigroup such that S is a generating set for $\mathbb{M}$.*

A graph $\mathbb{G} = \langle V, \mathbf{E} \rangle$ is the Cayley graph for $\mathbb{M}$ iff $S \subseteq V$ and $(\mathbf{x}, \mathbf{y}) \in E$ iff there exists $\mathbf{z} \in S$ such that $\mathbf{y} = \mathbf{x} \circ \mathbf{z}$.

The idea is that there is a graph homomorphism that maps paths in the Hasse graph of a kinship structure to the kinship relations that are the vertices of the Cayley graph of Read's algebra.

Theorem 2 *Let $\mathbb{K} = \mathbb{P} \oplus \mathbb{G}$ be a kinship structure and let $H(\mathbb{P})$ be the Hasse graph for $\mathbb{P}$.*

$H(\mathbb{P}) \oplus \mathbb{G}$ is homomorphic to the Cayley graph $\Gamma(\mathscr{K})$ for the algebra $\mathscr{K}$.

For example, the path between Bruce and Peter in the kinship structure in Figure 6 is mapped to the following relation in Read's nonassociative algebra:

$$\text{Parent} \circ \text{Spouse} \circ \text{Child} \circ \text{Child} \circ \text{Child}$$

Note that Theorem 2 also means that substructures of kinship structures that are not paths (e.g., the relations depicted in Figure 7) are not mapped to relations in Read's nonassociative algebra.

With the verification and validation of $T_{kinship}$, we have shown that we have represented all the relationships captured in Read's algebraic models in anthropology, the legal context, and statistics collection agencies. Further, the benefit of having a first-order axiomatization of kinship allows us to define relationships that cannot be defined by Read in [9, 11]. The algebraic approach does not include constraints on how spouse and ancestors can be amalgamated. For example, Axiom 13 is a constraint we have included in $T_{kinship}$ based on the current Canadian law for marriage, but such a constraint cannot be represented using Read's approach. While the application of these axioms may depend on the legal context, it is equally important to be able to represent such constraints: algebraically, it is not possible to do so, whereas our first-order axiomatization allows us to further add onto Read's kinship algebra.

6. Lessons Learned & Future Work

We have shown how the kinship ontology can represent definitions developed by anthropologists and our commonsense intuitions of familial relationships. We have extracted these definitions of kinship found in anthropology and have axiomatized the algebraic structures presented within the anthropological community using first-order logic. Further, the ontology is more expressive than the algebraic approach and also supports reasoning. In contrast to existing OWL ontologies for kinship, we have presented an ontology that is not a toy ontology for reasoning in OWL and have been able to validate it with anthropological outside of the ontology community; this is significant since there are additional kinship systems found in anthropology that can be further examined through the use of ontologies.

Future work for this ontology would be to provide a more in-depth ontological analysis of kinship notions independent of anthropology and societal norms. We would like to explore representations of relationships that are weaker than Axiom 13, but stronger than the weakest set of axioms possible with $T_{kinship}$, and how unintended and anomalous models of the ontology interact with one another. As well, we would like to examine how changes in relationships affect the models of the ontology: how do life events, such as marriage and divorce, influence or change the axioms of the ontology? Furthermore, it would be interesting to examine the effects of temporal kinship and how this plays a role with making inferences from data: for example, how have relationships changed over time with census datasets? Using the ontology, can we make additional inferences with datasets from different years to analyze marriage or divorce rates?

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