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# Statistical Tools to Evaluate the Performance of Current Control Strategies of Power Converters and Drives

Gustavo Rivas, Jorge Rodas, *Senior Member, IEEE,* and Jesus Doval Gandoy, *Member, IEEE*

***Abstract*—The proposal of new current control techniques for power converters and electric motor drives has been one of the main research topics in the fields of power converters and drives during the past years. Usually, when evaluating and comparing the performance of current controllers, various figures of merit (FMs) are used, e.g. the mean squared error or the absolute error between the reference and the measurement. Here it is shown that such FMs have a random nature. Nevertheless, only one result is reported in many published articles, for each FMs. Also, it is not indicated whether or not more than one trial has been performed to obtain the FM. In that case, opposite conclusions can be reached when two current controllers are compared, depending on the chosen results. In this sense, the number, $n$, of experimental runs required to accurately compare any FM, is proposed in order to address this problem. Likewise, a statistical comparison procedure is introduced to evaluate the relative performance of two controllers using any FM. Also, based on the proposed statistical comparison methodology compared to other criteria, an exhaustive simulation analysis is presented comparing the accuracy of decision-making. Finally, a real data set application based on experimental results is used to illustrate the proposed procedure.**

***Index Terms*—Central limit theorem, current controller performance, hypothesis test, interval estimation, power converters and drives.**

## I. Introduction

Power converters and drives are widely used in many applications including electric vehicles, ship propulsion, wind energy conversion systems, distributed generation, among others [1], [2]. As a consequence, high-performance closed-loop current control strategies are constantly proposed and compared in the literature [3]. According to the existing literature, new current controllers are usually validated experimentally and then compared with a known or traditional control strategy [4], [5].

A common way to compare the performance of current controllers is to use a, so-called, figure of merit (FM). For instance, in [6] the mean square tracking error of the phase currents (MSE) and the total harmonic distortion in the phase currents (THD) are used as FMs. In [7] the performance criteria are related to the MSE of the currents in several sub-spaces such as $d-q$ and $x-y$. References [8]red-[11] also use THD as an FM but [8] includes the average error between the measured current and its reference, [9] uses the harmonic distortion, [10] uses switching frequency and [11] includes the torque ripple.

Typically, when two current controllers are compared using an FM, only one value is reported. If several experiments were to be conducted for each current controller, then random numbers for the FMs will be obtained from an unknown probability distribution function (PDF). Such random variations may be due to several reasons, e.g. temperature, noise, measurements, among others. An exhaustive review of the sources of errors in the measures can be seen in [12]. As will be shown later in this paper, based on experimental results, an FM can vary by up to 56% between one trial and another. According to the best of the authors' knowledge, no criteria has previously been published to treat this issue. A single trial has the potential to give an unfair comparison between controllers. Thus, the results of many published articles that present a comparison of performance between current controllers based on FM may not be reproducible. In this context, the main questions to be answered in this paper are:

1) How to confidently determine which controller has the best performance.
2) How to quantify how much better it is.
3) How many experimental runs are necessary to validate the conclusions.
4) How reliable the comparison based on statistical performance criteria is.

FMs are random variables since they varies from trial to trial. So, a statistical treatment to experimental measurements is allowed [12], [13]. The objective of this paper is to introduce the use of traditional statistical techniques to make robust comparisons between two current control techniques.

Thus, the main contributions of this work are to establish the random nature of FMs and to provide an easy-to-understand methodology to rigorously deal with this randomness. The procedure also gives the minimum experimental runs ($n$) necessary to make robust comparisons between two current control methods based on FMs.

The rest of the paper is organized as follows. Statistical concepts as well as the comparison methodology are explained in Section II. The system, control techniques, experimental

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setup, and FMs used to demonstrate the proposal are explained in Section III. In Section IV, an exhaustive analysis based on experimental results obtained from two predictive current controllers (PCC) applied to a motor drive is presented. Then, the detailed procedure based on a real data set is shown and explained in Section V. Finally, conclusions are summarised in Section VI.

## II. Theoretical environment

This section introduces the statistical concepts that will be used in the sequel. Most of the proposal takes the recommendations for Type A evaluation of standard uncertainty ([12] section 4.2 p. 10) as a starting point. Specifically, the estimation of a confidence interval (CI) will be first introduced. The CI gives a range of values that, with high probability, contain the true value of a PDF parameter. Furthermore, with this estimation, as will be explained next, it is possible to determine the current controller with the best performance as well as how much better (or worse) it is. FMs generally measure the deviation between the current prediction and the theoretical reference. Therefore, the lower these values are, the greater the efficiency of the current controller will be.

The estimation of the CI consists of determining two values, lower limit (LL) and upper limit (UL), denoted by $a$ and $b$, respectively, as shown in Fig. 1. Given the interval $[a, b]$ and fixing a probability $1-\alpha$, also known as confidence level (CL) (see Fig. 1) the following can be verified:

$$\mathcal{P}(a \leq \delta \leq b) = 1 - \alpha, \tag{1}$$

where $\delta$ denotes the parameter to estimate, $\mathcal{P}(\cdot)$ denotes the probability and $\alpha \in (0, 1)$.

Let $X$ and $Y$ be two random variables representing the same FM for two different current control methods A and B, respectively. Suppose that they have an unknown PDF with mean $\mu_x$ and $\mu_y$ and variance $\sigma_x^2$ and $\sigma_y^2$, respectively. Then, to treat the problem of comparison on the efficiency of two current control strategies by the estimation of a CI by the difference in means

$$\delta = \mu_x - \mu_y, \tag{2}$$

and the estimation of a CI by the means $\mu_x$ and $\mu_y$.

For this purpose, the central limit theorem (CLT) is now introduced. By considering $X_1, X_2, ..., X_n$ as a succession of independent and identically distributed random variables of a probability distribution with mean $\mu$ and with a variance $\sigma^2 \neq 0$. Then, if $n \to \infty$, the random variable

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \tag{3}$$

converges to a normal distribution with mean and variance

$$\mu_{\overline{x}} = \mu, \;\; \sigma_{\overline{x}}^2 = \frac{\sigma^2}{n}, \tag{4}$$

respectively.

By CLT, the standardized asymptotic distribution of the sample mean $Z = \frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$ converges in distribution to the standard normal $\mathcal{N}(0, 1)$.

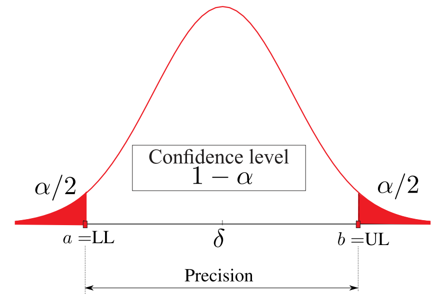


Fig. 1. CL, LL and UL for a generic $\delta$ parameter.

### A. CI estimation

The distribution $Z$ converges to a $\mathcal{N}(0, 1)$ for an infinite (large) number of repetitions of an experiment under the same conditions. In a real application, it is not possible to perform infinite experimental runs. Therefore, based on a sample of $x_i$ observations $1 \leq i \leq n$, a valid estimate of $\mu$ can be obtained. Here the objective is to estimate an interval that contains the true value of the parameter $\mu_x$ (or $\mu_y$) and with an acceptable CL probability and precision.

The CL represents a probability that for a specific sample size $n$, based on the sample distribution of the selected estimator, the true parameter be contained. On the other hand, the precision is related to the length of the CI. For instance, if the CI is $[a, b]$ (see Fig. 1), then the shorter the length, the more accurate the estimate of $\mu_x$ will be.

According to (4), $\mu_{\overline{x}} = \mu$, the expected value of the sample distribution of the mean converges with the value of the population mean. Therefore, after estimating a CI by the sample mean, the population's mean is indirectly being estimated. This purpose is shown below.

By applying the CLT, the bilateral CI with $1-\alpha$ CL for $\mu_{\overline{x}}$ is:

$$\mathrm{CI}_{1-\alpha}(\mu_{\overline{x}}) = \overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \tag{5}$$

being $z_{\alpha/2}$ the percentile $\alpha/2$ of the standard normal distribution and $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$. Note that as $z_{\alpha/2} = -z_{1-\alpha/2}$, for simplicity the first one is used in this paper.

***Remark 1****. In most of the real applications $\sigma^2$ is unknown, so in this situation, if $X \sim \mathcal{N}(\mu_x, \sigma)$ even for small $n$ ($n < 30$) the way of applied* (5) *providing better results is what follows:*

$$\mathrm{CI}_{1-\alpha}(\mu_{\overline{x}}) = \overline{x} \pm t_{\alpha/2, n-1} \frac{\hat{S}_x}{\sqrt{n}} \tag{6}$$

*where $t_{\alpha/2,n-1}$ is the $\alpha/2$ percentile of a $t$-distribution with $n-1$ degrees of freedom and $\hat{S}_x$ is an estimator of $\sigma$, defined by*

$$\hat{S}_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}. \tag{7}$$

Now, the estimation of the CI for $\delta$ is presented. The random variable to consider in this case is the difference in means $\overline{X} - \overline{Y}$. Similar to the previous case for $\mu_x$, now the goal is to estimate $\mathrm{CI}_{1-\alpha}(\mu_{\overline{x}-\overline{y}})$. The variance of the difference of the means in independent samples (assuming $n$ and $\sigma$ equal) is:

$$\sigma^2_{\overline{x}-\overline{y}} = \sigma^2_{\overline{x}} + \sigma^2_{\overline{y}} = \frac{\sigma^2}{n} + \frac{\sigma^2}{n} = \frac{2\sigma^2}{n}. \tag{8}$$

Assuming equal variances, the $\mathrm{CI}_{1-\alpha}(\mu_{\overline{x}-\overline{y}})$ is:

$$\mathrm{CI}1-\alpha(\mu_{\overline{x}-\overline{y}}) = \overline{x} - \overline{y} \pm z_{\alpha/2}\sigma\sqrt{\frac{2}{n}}. \tag{9}$$

***Remark 2****. Assuming equality but unknown variance $\sigma^2$, in relation to* ***Remark 1****,* (9) *must be:*

$$\mathrm{CI}_{1-\alpha}(\mu_{\overline{x}-\overline{y}}) = \overline{x} - \overline{y} \pm t_{\alpha/2,2n-2}\frac{\hat{S}_p}{\sqrt{n}} \tag{10}$$

*where $t_{\alpha/2,2n-2}$ denotes the $\alpha/2$ percentile of a t-distribution with $2n-2$ degrees of freedom and $\hat{S}_p$ is an estimator of a pooled standard deviation (SD), defined by*

$$\hat{S}_p = \sqrt{\hat{S}^2_x + \hat{S}^2_y} \tag{11}$$

*where $\hat{S}_y$ is defined analogously as* (7)*.*
***Remark 3****. Assuming inequality and unknowns variances,* (9) *must be:*

$$\mathrm{CI}_{1-\alpha}(\mu_{\overline{x}-\overline{y}}) = \overline{x} - \overline{y} \pm t_{\alpha/2,\nu}\frac{\hat{S}_p}{\sqrt{n}} \tag{12}$$

*where $t_{\alpha/2,\nu}$ is the $\alpha/2$ percentile of a t-distribution with $\nu$ degrees of freedom, being $\nu$ approximately by Welch-Satterthwaite equation [22] as*

$$\nu \approx (n-1)\frac{\left(\hat{S}^2_x + \hat{S}^2_y\right)^2}{\hat{S}^4_x + \hat{S}^4_y}. \tag{13}$$

### *B. Proposed number of experimental runs ($n$)*

The comparison between controllers, using the statistical method described in this article, starts from a lack of knowledge between the efficiencies of both controllers, known as the null hypothesis $H_o$. On the other hand, there is the hypothesis that the researcher wishes to validate (or discard), which corresponds to the alternative hypothesis $H_1$. In this case, these hypotheses are:

$$H_0 : \delta = 0$$

vs.

$$H_1 : \delta = \tau, \quad \tau \in \{R - 0\}$$

The $H_o$ assumes that both controllers (A and B) are equally efficient based on some FMs. While the $H_1$ establishes the superiority of one of them.

The decision rule will be that if the $0 \in \mathrm{CI}_{1-\alpha}(\mu_{\overline{x}-\overline{y}})$, it is not possible to reject the $H_0$. To accept the $H_1$ it must be fulfilled that $\tau \in \mathrm{CI}_{1-\alpha}(\mu_{\overline{x}-\overline{y}})$ and $0 \notin \mathrm{CI}_{1-\alpha}(\mu_{\overline{x}-\overline{y}})$.

This method of decision is not foolproof. Note that a decision is going to be made based on a sample. So, there is the possibility of making mistakes. Table I summarizes the cases of errors to consider. When a true $H_0$ is rejected, the Type I error is committed, whose probability of occurrence is complementary of the CL, and therefore represented by $\alpha$. While the Type II error, whose probability is represented by $\beta$, occurs when a $H_0$ is not rejected when it is false.

TABLE I
TYPE I AND TYPE II ERRORS

| | | Real situation | |
|---|---|---|---|
| | | $\delta = \tau$ ($H_0$ false) | $\delta = 0$ ($H_0$ true) |
| Decision | $\delta = \tau$ | No error | Type I error $\alpha$ |
| based on sample | $\delta = 0$ | Type II error $\beta$ | No error |

Fig. 2 shows the distribution of the mean difference under $H_0$ (the left distribution centered on $\delta = 0$) and under $H_1$ (the right distribution centered on $\delta = \tau$). When $\tau$ is near to 0 the CIs under $H_0$ and $H_1$ could be intercepted. The probability that this does not occur is known as the power $1-\beta$.

For a bilateral $1-\alpha$ CL, under $H_0$ (see Fig. 2), the UL of the CI is:

$$\mathrm{UL} = z_{1-\alpha/2}\sigma\sqrt{\frac{2}{n}} \tag{14}$$

and for unilateral power $1-\beta$, under $H_1$ (see Fig. 2), the LL of the CI is:

$$\mathrm{LL} = \tau - z_{1-\beta}\sigma\sqrt{\frac{2}{n}}. \tag{15}$$

The required sample size $n$ is the one that allows the overlapping of the statistic distributions under $H_0$ and under $H_1$ to provide the specified $\alpha$ and $\beta$ values. Note that if $n$ increases, the distributions become more pointed and the overlap decreases, therefore, the risks $\alpha$ and $\beta$. In Fig. 2, the vertical solid line marks the value from which one or the other decision will be made. The criterion is that the limits defined in (14) and (15) do not overlap, that is:

$$z_{1-\alpha/2}\sigma\sqrt{\frac{2}{n}} \leq \tau - z_{1-\beta}\sigma\sqrt{\frac{2}{n}}. \tag{16}$$

The difference $\tau$ proposed in $H_1$ can be established (for example) in relative terms to $\mu_x$, given a $\epsilon \in (0,1)$, thus, $\tau = \epsilon\mu_x$ , isolating the $n$ in the previous inequality is:

$$n \geq \frac{2(z_{1-\alpha/2} + z_{1-\beta})^2\sigma^2}{\mu^2_x\epsilon^2}. \tag{17}$$

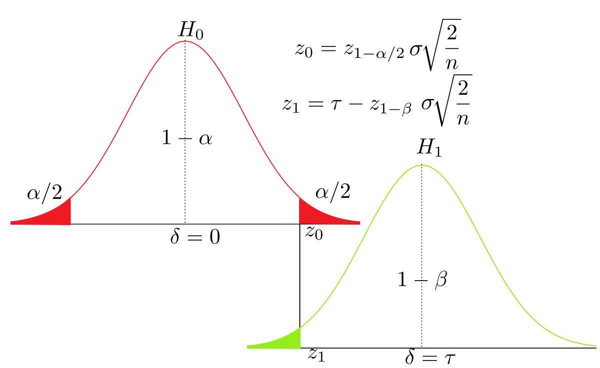


Fig. 2. Distribution of the mean difference under $H_o$ and $H_1$.

Note that in (17) $z_{1-\alpha/2}$, $z_{1-\beta}$ and $\epsilon$ are quality comparisons parameters related to CL, Power and precision, respectively (later will be used in Table V). Moreover, in (17) the value of $\sigma$ and $\mu_x$ are unknown. So, a practical solution to this situation is to conduct a pilot test run based on $n_0$ experimental runs, and estimate $\mu_x$ by following the classical arithmetic mean $\overline{x}$ while $\sigma$ is estimated as the pooled standard sampling error $\hat{S}_p$ defined in (11) and $\hat{S}_x$ and $\hat{S}_y$ as in (7) replacing $n$ by $n_0$. Finally, (17) can be rewritten as:

$$n \geq \frac{(z_{1-\alpha/2} + z_{1-\beta})^2(\hat{S}_x^2 + \hat{S}_y^2)}{\overline{x}^2\epsilon^2}. \tag{18}$$

***Remark 4***. *The way to narrow the CI is when $\epsilon \to 0$. Note that based on* (18) *this implies a sample size increases. But unlike other areas, for the context of this article, getting a high sample number is not a problem, so we can be as precise as we want.*

### *C. Practical considerations*

The application of the described method is subjected to the compared and continuous FMs and meets the following:

- Normality. The FM to be compared follows a normal distribution for each current controller.
- Homoscedasticity. Current controllers to be compared should have the same FM variations. If the FM variations are different, the Welch [22] two-sample t-test should be used.
- Independence. The FMs used to make the comparison must be obtained independently between the current controllers to be compared.

When the conditions mentioned above are fulfilled, the application of the described methods is appropriate.

One of the main objectives of this article is to facilitate the application of the described statistical method. To this end, let $x$ and $y$, denote two vectors containing the $n$ values of some FM, for two different current control methods A and B, respectively. All computations required will be performed by using programs written in the R language [37].

**Step I. Normality test**

The estimation of the CIs in (6), (10) and (12) have in common that when it is working with samples (specially with moderate size) the accuracy of the calculations depends on the FMs to have a normal distribution. The way to check this assumption is through a hypothesis test. The normality hypotheses are established as follows

$$H_o: x \text{ (or } y\text{) has normal distribution}$$

vs.

$$H_1: x \text{ (or } y\text{) has not a normal distribution.}$$

Note that the value of $\alpha$ should be chosen before the test. This also means that the maximum Type I error that can get has a probability equals to $\alpha$. Then, the Shapiro-Wilk statistical test will be used. The decision criterion will be as follows: if the p-value is bigger than $\alpha$, then the $H_o$ is not rejected; otherwise, $H_1$ is accepted and the CIs cannot be computed with (18). The R-code is:

```
shapiro.test(x)$p.value
```

**Step II. Homoscedasticity test**.

The CI estimation by (10) assumes equal variances for the same FM of both current controllers. Then, as in the previous case, this supposition must be checked. The hypothesis is established as follows

$$H_o: x \text{ and } y \text{ have equal variances}$$

vs.

$$H_1: x \text{ and } y \text{ do not have equal variances.}$$

If there is insufficient statistical evidence to reject the hypothesis of normality, the statistical test to be used will be the F-test as follows

```
var.test(x,y)$p.value
```

If the null hypothesis of equality of variances is rejected, (12) should be used instead of (10).

**Step III. CIs estimation**.

Following is shown how to find the CIs using the `t.test()` function of R. In the context of ***Remark 1***, to find the $\text{CI}_{0.95}(\mu_{\overline{x}})$ (or $\text{CI}_{0.95}(\mu_{\overline{y}})$), the code is

```
t.test(x,conf.level = 0.95)$conf.int
```

Likewise, in the context of ***Remark 2***, to find the $\text{CI}_{0.95}(\mu_{\overline{x}-\overline{y}})$, the code is

```
t.test(x,y, paired=FALSE,
var.equal=TRUE,conf.level = 0.95)$conf.int
```

In the same way, according to ***Remark 3***, to find the $\text{CI}_{0.95}(\mu_{\overline{x}-\overline{y}})$, the code is

```
t.test(x,y, paired=FALSE,
var.equal=FALSE,conf.level =
0.95)$conf.int
```

### *D. Scalability of the proposed method*

.

Suppose it is necessary to compare a large number of variables, say $k$ variables from two different controllers. The computation of a large number of variables does not imply any complication other than the iterative repetition of the proposed method as shown below.

Let `xi1`, `xi2`, $1 < i \leq k$ be vectors representing the $i$-th variable of two controllers 1 and 2, respectively. Let `ni` be a $k$-dimensional vector of $n$-size that contains the results of the sample size calculations for a 95% CL, a power equal to 80% and a relative error of 5%. The R-code for each `ni` is:

```
ni=(qnorm(0.975)+qnorm(0.80))^2*(sd(xi1)^2
      +sd(xi2)^2)/(mean(xi1*0.05)^2)2
```

Then, the R-code to decide the sample size to compare the $k$ variables is:

```
max(ni)
```

It should be noted that when selecting the maximum of `ni`, the restriction of (18) is fulfilled, this establishes that the $n$ found is the minimum value that ensures it meets the quality criteria, in the comparisons for specific values of: CL, power and precision.

Similarly, for the comparison of the $k$ variables we proceed as follows: Let `pi` be a $k$-dimensional vector that contains the results of the CI for the difference of means at a 95% CL. The R-code for each `pi` is:

```
pi = t.test (xi1, xi2, paired = FALSE,
var.equal = TRUE, conf.level = 0.95) $
conf.int
```

The following section introduces the context in which a case study will be shown for the practical application of the concepts developed in this section.

## III. Multiphase machine description and control

The system, that will be used as a showcase is a multiphase machine (more than 3 phases). This type of machine is capable of continuous operation even if one or more phases are open or with faults [24]. Moreover, the possibility to split the power into more phases allows the use of reduced components as well as leads, for a more efficient use of the cable cross-sectional area [25]. All these advantages have been the motivation for industrial applications of multiphase machines in propulsion systems [26], wind energy conversion systems [27], among others. However, a higher number of phases leads to control challenges.

Advanced control strategies of multiphase machines have been a main research topic for more than a decade [28]. Most of the control approaches are extensions of well-known techniques for conventional three-phase machines such as direct torque control [29] or field-oriented control [30], and nonlinear controllers like sliding mode control [31] and MPC [32]. The extension of the aforementioned control techniques to the post-fault operation has been also proposed [33]. Fig. 3 shows the field-oriented control technique applied to a multiphase machine which consists of a proportional-integral (PI) outer speed control with an inner current controller. This schematic is considered in this paper with MPC as a current controller.

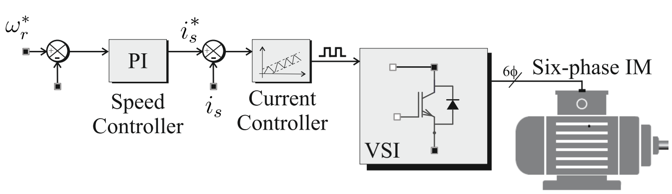


Fig. 3. Schematic diagram of the field-oriented control for a six-phase IM.

The most popular type of multiphase machine, namely asymmetrical six-phase induction machine (IM), is considered in this paper. This machine is driven by a 2-level voltage source inverter (VSI). By using first-order Euler approximation, the discrete form representation of the state variable can be expressed in the form:

$$\begin{bmatrix} i_{s\alpha[k+1]} \\ i_{s\beta[k+1]} \\ i_{sx[k+1]} \\ i_{sy[k+1]} \\ i_{r\alpha[k+1]} \\ i_{r\beta[k+1]} \end{bmatrix} = \mathbb{A}. \begin{bmatrix} i_{s\alpha[k]} \\ i_{s\beta[k]} \\ i_{sx[k]} \\ i_{sy[k]} \\ i_{r\alpha[k]} \\ i_{r\beta[k]} \end{bmatrix} + \mathbb{B}. \begin{bmatrix} v_{s\alpha[k]} \\ v_{s\beta[k]} \\ v_{sx[k]} \\ v_{sy[k]} \end{bmatrix} \tag{19}$$

where $i_{s\alpha}$, $i_{s\beta}$ are the $\alpha-\beta$ stator current, $i_{sx}, i_{sy}$ are the $x-y$ stator current and $i_{r\alpha}$, $i_{r\beta}$ represent the unmeasurable $\alpha-\beta$ rotor currents. The input voltages are denoted by $v_{s\alpha}$, $v_{s\beta}$, $v_{sx}$, $v_{sy}$ and finally, $\mathbb{A}$ and $\mathbb{B}$ are defined in [31].

### A. Conventional MPC-based current control of Six-Phase IM

MPC is a technique based on the model of the system. For power converters, MPC takes advantage of the discrete nature of the model. A cost function shown below in (20) is used to define the desired behavior, such as current tracking. For a six-phase IM, the cost function is typically evaluated 49 times, and then, the voltage vector that minimizes the cost function is selected and applied to the six-phase machine through the VSI during the next sample time.

$$J = |i^*_{s\alpha\beta[k+1]} - \hat{i}_{s\alpha\beta[k+1]}|^2 + \lambda_{xy}.|i^*_{sxy[k+1]} - \hat{i}_{sxy[k+1]}|^2. \tag{20}$$

Readers are referred to [34] for more details regarding MPC applied to power converter and drives, and [32], [35] for MPC applied to multiphase machines.

### B. Modulated Model Predictive Control of Six-Phase IM

The second current controller that will be used in this paper is modulated model predictive control (M2PC) proposed in [36]. The M2PC uses a modulation stage based on a switching pattern to generate a fixed switching frequency. The duty cycles are generated by using two active vectors and a null vector which are applied to the converter using a given switching pattern.

## IV. Comparative and Sensitive Analysis

This section focuses on the numerical and experimental analysis of the results obtained with the proposed method, and also compares the efficiency of other alternatives, in order to compare the performance of current controllers based on FMs, i.e. the average of the values obtained and/or the most favorable value.

### A. Experimental setup

To compare the two current control techniques, the test rig shown in Fig. 4 was used. This system consisted of a six-phase IM fed by a commercial VSI while a constant $V_{\rm dc}$ voltage is used from a direct current (dc) power supply system. The current controllers are implemented in the dSPACE MABXII DS1401 rapid prototyping platform. The results obtained have been captured and processed using MATLAB/Simulink script. The experimental tests have been performed with current sensors LA 55-P s, which had a frequency bandwidth from dc up to 200 kHz. The current measurements have been then converted to digital form using a 16-bit analog to digital converter. The six-phase IM position has been obtained with a 1024-pulses per revolution incremental encoder to estimate the rotor speed. Also, a 5 HP (3.7285 kW) eddy current brake has been used to introduce a variable mechanical load on the system.

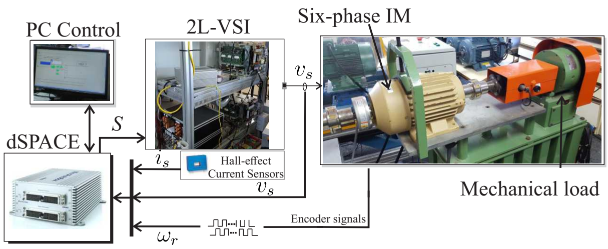


Fig. 4. Six-phase IM experimental setup.

The performance of both controllers were evaluated by using MSE and THD as FMs. The MSE between the reference and the measured stator currents in the ($\alpha$-$\beta$) and ($x$-$y$) sub-spaces is performed by using the following equation

$$\mathrm{MSE}(i_{\sigma s}) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(i_{\sigma s} - i^{*}_{\sigma s})^2} \tag{21}$$

where $n$ is the quantity of studied samples, $i^{*}_{\sigma s}$ the stator current reference, $i_{\sigma s}$ the measured stator current taking into account that $\sigma \in \{\alpha,\ \beta,\ x,\ y\}$. While, the THD is obtained as follows:

$$\mathrm{THD}(i_s) = \sqrt{\frac{1}{i_{s1}^2}\sum_{k=2}^{n}(i_{sk})^2} \tag{22}$$

where $i_{s1}$ corresponds to the fundamental stator current and $i_{sk}$ is the harmonic stator current (multiple of the fundamental stator current).

TABLE II
DESCRIPTIVE STATISTICAL PARAMETERS FOR THE FMS MSE$_\alpha$ AND MSE$_\beta$, BASED ON 200 EXPERIMENTAL TEST FOR EACH MPC AND M2PC CONTROLLERS.

| Descriptive statistical parameters | MSE$_\alpha$ MPC | MSE$_\alpha$ M2PC | MSE$_\beta$ MPC | MSE$_\beta$ M2PC |
|---|---|---|---|---|
| Minimum | 0.0615 | 0.0933 | 0.0616 | 0.0963 |
| Maximum | 0.0678 | 0.1087 | 0.0684 | 0.1120 |
| Variation (%) | 10.244 | 16.506 | 11.039 | 16.303 |
| Mean | 0.0654 | 0.1019 | 0.0649 | 0.1045 |
| SD | 0.0011 | 0.0030 | 0.0014 | 0.0030 |
| CV (%) | 1.7540 | 2.9860 | 2.2170 | 2.8380 |

TABLE III
DESCRIPTIVE STATISTICAL PARAMETERS FOR THE FMS MSE$_x$ AND MSE$_y$, BASED ON 200 EXPERIMENTAL TEST FOR EACH MPC AND M2PC CONTROLLERS.

| Descriptive statistical parameters | MSE$_x$ MPC | MSE$_x$ M2PC | MSE$_y$ MPC | MSE$_y$ M2PC |
|---|---|---|---|---|
| Minimum | 0.4802 | 0.0804 | 0.4767 | 0.0844 |
| Maximum | 0.5241 | 0.0877 | 0.5244 | 0.0922 |
| Variation (%) | 9.1420 | 9.0800 | 10.0060 | 9.2420 |
| Mean | 0.5013 | 0.0844 | 0.5020 | 0.0880 |
| SD | 0.0086 | 0.0013 | 0.0081 | 0.0014 |
| CV (%) | 1.7230 | 1.5760 | 1.6150 | 1.6440 |

TABLE IV
DESCRIPTIVE STATISTICAL PARAMETERS FOR THE FMS THD$_\alpha$ AND THD$\beta$, BASED ON 200 EXPERIMENTAL TEST FOR EACH MPC AND M2PC CONTROLLERS.

| Descriptive statistical parameters | THD$_\alpha$ MPC | THD$_\alpha$ M2PC | THD$_\beta$ MPC | THD$_\beta$ M2PC |
|---|---|---|---|---|
| Minimum | 5.9080 | 7.7730 | 5.9440 | 8.1870 |
| Maximum | 9.2270 | 9.7540 | 8.9810 | 10.4810 |
| Variation (%) | 56.1780 | 25.4860 | 51.0940 | 28.0200 |
| Mean | 7.3760 | 9.0080 | 7.2810 | 9.1980 |
| SD | 0.8082 | 0.3428 | 0.8188 | 0.4142 |
| CV (%) | 10.9570 | 3.8050 | 11.2460 | 4.5030 |

### B. FM's random behavior

The random nature of the FM's is first demonstrated based on a total of 400 experimental tests, obtained for 200 tests for each MPC [32] and M2PC [36] current controllers, under the same condition: 16 kHz sampling time, 1500 rpm rotor speed, $i_q = 1$ A and $i_d = 1$ A. Applying (21)-(22), the following FMs are computed: MSE in $\alpha - \beta$ and $x - y$ sub-space while the THD is computed for the $\alpha - \beta$ sub-space. The measure used to synthesize the dispersion of the results with respect to the mean is the coefficient of variation (CV), defined as the percentage that represents the SD in relation to the mean. The most relevant statistical parameters obtained from the FMs, based on the experimental results, are presented in Tables II-IV.

By analyzing the obtained statistical results in Tables II-IV, the range of variation (between the minimum and the maximum) of the results for the MSE is less than for the THD. For the MSE in both sub-spaces, it is between 9% to 16% while for THD it is between 25% to 56%. These results show the dispersion between different experiments. Then justifies the treatment of FMs as random variables. Another important aspect to highlight is the CV in relation to the mean. In accordance with the range of variation, for the obtained MSE, the CV is between 1% and 3%, while for THD it is between 4% and 11%. The different range of variation observed in MSE and THD is because they are calculated with different equations, see (21) and (22). Also, it will be shown that the results of the MSE are more homogeneous than those of the THD. This dispersion has an impact on the computation of the required sample size ($n$), since the greater the variability, the larger $n$ is needed, keeping the quality parameters constant as will be shown next.

### C. Quality parameters impact

Next, the impact of the quality parameters (CL, power, and $\epsilon$) of comparisons and the variability of each FM on the calculation of the $n$ is presented. In this context, a pilot number of experimental runs $n_0 = 20$ is used in (18). For a very high CL (99%), a high power (90%) and a very small relative error ($\epsilon = 0.01$) the $n$ can be very large, reaching values of 1220 necessary experiments as shown in Table V. Note that in addition to the quality parameters in the comparisons, the variability inherent to FM affects the calculation of the $n$ as shown in (18). In the case of THD, as shown in Table IV, the variability is much greater than for MSE, therefore a larger $n$ is required to satisfy the quality parameters in the imposed comparisons. On this basis, the required $n$ will differ depending on the FM used to compare controllers. As a compromised value between the required $n$ and quality parameters, it is suggested to use CL = 95% ($z_{0.975} = 1.96$), power = 80% ($z_{0.80} = 0.84$) and $\epsilon$ = 0.05, which corresponds to the last line of Table V.

TABLE V
CALCULATION OF THE $n$ FOR DIFFERENT FMS AND QUALITY PARAMETERS.

| Quality parameters | | | Calculation of $n$ | | | |
|---|---|---|---|---|---|---|
| CL | Power | $\epsilon$ | MSE$_\alpha$ | MSE$_x$ | THD$_\alpha$ | Max. |
| 99% | 90% | 0.01 | 246 | 21 | 1220 | 1220 |
| 99% | 80% | 0.01 | 193 | 16 | 958 | 958 |
| 95% | 90% | 0.05 | 7 | 1 | 35 | 35 |
| 95% | 80% | 0.05 | 6 | 1 | 26 | 26 |

### D. Accuracy analysis comparisons

In this subsection, in order to analyze the properties of the proposed method, Type I errors and power for different scenarios are compared to other comparisons criteria.

One of the main issue addressed in this work is the lack of publication of the criteria used to conclude that one controller is better than another based on the measurements of the FMs obtained in experimental trials. Typically, FMs are used to evaluate a current controller performance where the lower value implies better performance. With this premise, four different criteria are presented that are very plausible of application.

Define $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ as two $n$-size vectors of the same FM for two controllers A and B.

To establish the first criterion (denoted C1) let us first suppose that the minimum value of vector $x$ is less than the minimum value of vector $y$. So far, it could be said that controller A is better than B. However, as previously demonstrated, the dispersion in the experimental results can be very large. Therefore, it could be that the maximum value of vector $x$ is greater than the maximum of vector $y$. Based on the previous result, controller B is better than A. Finally, by joining the two situations, the C1 criterion arises where it is not possible to decide which of the two controllers is better, which is equivalent to saying that there is not sufficient evidence that one is better than the other.

The second criterion (denoted C2) is possibly the most common. This is that when the average value of the FM of one controller is less than another, this one is better than the other.

One possible situation is that all the values of the FM of controller A are less than all the values obtained by controller B (or vice versa). In this case, ensuring that controller A is better than controller B (or vice versa) is a very consistent conclusion. This criterion will be denoted as C3.

The last criterion (denoted as C4) is very similar to C3. The difference is that not all the results are minor, but the intersection is partial. If the elements of $x$ outside the intersection are all less than the minimum of $y$, then controller A is better. Whereas, if the elements of $x$ outside the intersection are all greater than the maximum of $y$, then B is the best controller.

Next, the algorithm involved in the definition of each described criterion is presented.

- Criterion C1: If $min(y) > min(x)$ and $max(y) < max(x)$ or $min(x) > min(y)$ and $max(x) < max(y)$, then neither is better than the other, based on FM analyzed; otherwise one is more efficient than the other.
- Criterion C2: If $\bar{x} < \bar{y}$, then A is better than B.
- Criterion C3: If $max(x) < min(y)$, then A is better than B.
- Criterion C4: If $min(x) < min(y)$ and $max(x) < max(y)$, then A is better than B.

To estimate Type I error, 10,000 observations of two random variables $X$ and $Y$ with normal distribution with mean equal to 10 and for different CV=(1%, 3%, 7%, 11%) were generated. In the case of unequal variances, the relationship between them is $V_x/V_y = (1.2, 1.5)$. Based on 1000 Monte Carlo simulations, the Type I error was estimated as the fraction of trials that rejected the null hypothesis of equality of means between the two controllers proposed method the difference did not exist. In all simulations, the value $\alpha = 0.05$ is used for the proposed method. The obtained results are presented in the Tables VI to X for different $n$ values. Tables VI-VIII show the estimation of Type I error for the proposed method and C1 criterion. Note that C1 is the only criterion that considers the possibility of equality in comparisons. The estimated Type I error is very close to the nominal one for all cases and sample sizes for the proposed method. For C1, for the case of equal variances (Table VI), it commits a very high Type I error, around 50%. However, for unequal variances C1 is a little more effective, committing a Type I error for large samples around 40% and 15% according to Tables VII and VIII, respectively. One of the main advantages of the proposed method is that it is possible to control the Type I error since this estimate is around the nominal one. Also, it is possible to control Type II error and its associated measure, known as the power of the test.

Now, the proposed method is compared with C2, C3, and C4 criteria. For the estimation of power, 10000 observations of two random variables X with normal distribution with $\mu$ = 10 and Y with normal distribution with $\mu$ equals to 10.3 and 10.5 and equal variances for different CV=(1%, 3%, 7%, 11%) were generated. For the case of unequal variances, the following relationship $V_x/V_y = 1.2$ was considered. In this sense, the simulations carried out to consider two distributions that differ by 3% and 5% from the mean. The experiment, based on 1 000 Monte Carlo simulations, estimates the power as the fraction

TABLE VI
ESTIMATION OF TYPE I ERROR UNDER EQUAL VARIANCES, FOR PROPOSED METHOD (PM) AND C1.

| CV | $n=5$ | | $n=20$ | | $n=35$ | | $n=65$ | |
|---|---|---|---|---|---|---|---|---|
| | PM | C1 | PM | C1 | PM | C1 | PM | C1 |
| 1% | 5% | 57% | 6% | 51% | 5% | 50% | 5% | 54% |
| 3% | 5% | 57% | 6% | 52% | 4% | 54% | 5% | 54% |
| 7% | 5% | 57% | 5% | 50% | 5% | 50% | 5% | 54% |
| 11% | 4% | 53% | 5% | 50% | 5% | 50% | 5% | 54% |

TABLE VII
ESTIMATION OF TYPE I ERROR UNDER UNEQUAL VARIANCES ($V_x/V_y = 1.2$), FOR PROPOSED METHOD (PM) AND C1.

| CV | $n=5$ | | $n=20$ | | $n=35$ | | $n=65$ | |
|---|---|---|---|---|---|---|---|---|
| | PM | C1 | PM | C1 | PM | C1 | PM | C1 |
| 1% | 5% | 56% | 5% | 43% | 5% | 42% | 5% | 38% |
| 3% | 4% | 53% | 5% | 46% | 5% | 43% | 6% | 40% |
| 7% | 5% | 56% | 5% | 43% | 4% | 42% | 6% | 40% |
| 11% | 5% | 55% | 5% | 44% | 5% | 42% | 6% | 40% |

TABLE VIII
ESTIMATION OF TYPE I ERROR UNDER UNEQUAL VARIANCES ($V_x/V_y = 1.5$), FOR PROPOSED METHOD (PM) AND C1.

| CV | $n=5$ | | $n=20$ | | $n=35$ | | $n=65$ | |
|---|---|---|---|---|---|---|---|---|
| | PM | C1 | PM | C1 | PM | C1 | PM | C1 |
| 1% | 4% | 48% | 5% | 27% | 4% | 21% | 6% | 14% |
| 3% | 5% | 50% | 5% | 28% | 5% | 21% | 5% | 16% |
| 7% | 5% | 50% | 5% | 27% | 4% | 21% | 6% | 14% |
| 11% | 5% | 49% | 5% | 27% | 5% | 19% | 6% | 14% |

of trials that rejected the null hypothesis of equality of means between the two controllers when there is a difference. For the proposed method, the values of $\alpha = 0.05$ and a power = 0.80 ($\beta = 0.2$) were used. The results are shown for a sample size calculated with the following quality parameters in the comparisons: CL = 95%, power = 80%, $\epsilon = (0.03, 0.05)$ based on a pilot sample $n_0 = 20$. In Tables, IX and X it is verified that the estimated power for the proposed method is around the nominal 80%. In the case of equal variances (Table IX), C2 is the most powerful, reaching almost an estimated power of 100%. In other words, C2 rejected the null hypothesis practically every time it had to do so (the means generated are different). C3 criterion has good power properties only for the case where CV = 1%, that is, for very small and homogeneous samples. As the heterogeneity in the sample increases, the C3 criterion is practically incapable of detecting the existing differences. C4 criterion is more powerful in relation to the proposed method for very homogeneous and moderately homogeneous samples. For CV above 7%, the proposed method is more powerful. For the case of unequal variances (Table X), the results are similar to the case of equal variances.

TABLE IX
POWER ESTIMATION UNDER EQUAL VARIANCES, USING THE PROPOSED METHOD (PM), C2, C3 AND C4 CRITERIA.

| $\epsilon$ | CV | $n$ | PM | C2 | C3 | C4 |
|---|---|---|---|---|---|---|
| 3% | 1% | 3 | 77% | 100% | 88% | 100% |
| 3% | 5% | 19 | 85% | 100% | 1% | 85% |
| 3% | 7% | 47 | 86% | 100% | 1% | 70% |
| 3% | 11% | 91 | 80% | 100% | 1% | 64% |
| 5% | 1% | 2 | 74% | 100% | 100% | 100% |
| 5% | 5% | 6 | 74% | 100% | 19% | 91% |
| 5% | 7% | 13 | 70% | 99% | 1% | 83% |
| 5% | 11% | 27 | 78% | 100% | 1% | 68% |

TABLE X
POWER ESTIMATION UNDER UNEQUAL VARIANCES, USING THE PROPOSED METHOD (PM), C2, C3 AND C4 CRITERIA.

| $\epsilon$ | CV | $n$ | PM | C2 | C3 | C4 |
|---|---|---|---|---|---|---|
| 3% | 1% | 4 | 86% | 100% | 75% | 100% |
| 3% | 3% | 15 | 73% | 100% | 0% | 72% |
| 3% | 5% | 50 | 75% | 100% | 0% | 52% |
| 3% | 7% | 95 | 72% | 99% | 0% | 38% |
| 5% | 1% | 3 | 85% | 100% | 99% | 100% |
| 5% | 3% | 9 | 84% | 100% | 3% | 91% |
| 5% | 5% | 18 | 75% | 100% | 0% | 73% |
| 5% | 7% | 45 | 85% | 100% | 9% | 62% |

## V. REAL DATA SET APPLICATION

Now, a step by step procedure will be carried out to explain the proposed comparison procedure between two controllers, in this case, MPC [32] and M2PC [36].

### A. Number of experimental trials

For this step, it is necessary to choose the quality parameters for the comparison. The suggested values in Subsection IV-C will be used. As the $n$ calculation in (18) contains unknown parameters, a pilot test is carried out with $n_0 = 20$. The obtained values for both controllers are shown in Table XI.

TABLE XI
$n$ CALCULATION BY USING (18).

| FM | MPC ($X$) | | M2PC ($Y$) | $n$ calculation |
|---|---|---|---|---|
| | $\hat{S}_x$ | $\overline{x}$ | $\hat{S}_y$ | Eq. (18) |
| $\text{MSE}_\alpha$ | 0.000891 | 0.064825 | 0.002476 | 6 |
| $\text{MSE}_x$ | 0.005677 | 0.000932 | 0.492535 | 1 |
| $\text{THD}_\alpha$ | 0.585782 | 7.127265 | 0.270637 | 26 |

Note that, as it was already demonstrated in the previous section, for each FMs to be compared, different $n$ values are obtained. Then, the highest $n$ obtained must be used to evaluate the FMs of both controllers, being $n = 26$ in this case. This is due to the fact that the $n$ calculation gives us the minimum value for which the quality criteria in the comparisons are satisfied. Therefore, increasing the $n$ value does not affect the quality performance.

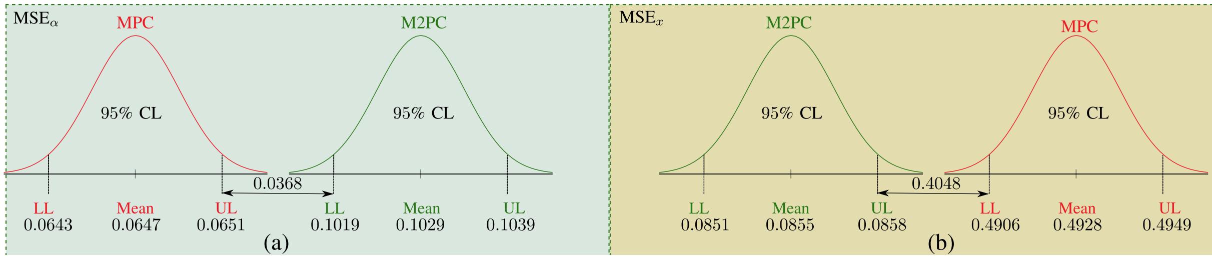


Fig. 5. Normal distribution plots for the FMs mean for MPC and M2PC PCCs (a) MSE$_\alpha$ and (b) MSE$_x$.

### B. Statistical comparisons

Now, it is followed by the same steps described in subsection II-C applying to the case study.

**Step I. Normality test**

It is used $\alpha = 0.05$. The $H_0$ are not rejected due to all p-values are greater than 0.05. The $H_0$ and $H_1$ are established as follows:

$H_0$ : The FM (i.e. the MSE$_\alpha$) has a normal distribution for both controllers (MPC and M2PC).

$H_1$ : The FM (i.e. the MSE$_\alpha$) does not have a normal distribution for both controllers (MPC and M2PC).

Based on the obtained results show in Table XII, THD$_\alpha$ does not meet the assumption of normality. A good alternative in these cases is to raise the quality of the comparison parameters to increase the sample size and re-apply the normality test. If the lack of normality persists, the proposed statistical procedure cannot be applied to compare the THD$_\alpha$. In particular, in this case, if you want to compare the THD$_\alpha$, it is necessary to perform a non-parametric test. This is however beyond the scope of this paper and not discussed further.

TABLE XII
NORMALITY TEST

| FM | Controller | p-value | Normal distribution |
|---|---|---|---|
| MSE$_\alpha$ | MPC | 0.4371 | Yes |
| MSE$_\alpha$ | M2PC | 0.7289 | Yes |
| MSE$_x$ | MPC | 0.9852 | Yes |
| MSE$_x$ | M2PC | 0.5739 | Yes |
| THD$_\alpha$ | MPC | 0.0417 | No |
| THD$_\alpha$ | M2PC | 0.8437 | Yes |

Note that $\alpha$-axis is representative of the $\alpha - \beta$ plane, the results for the $\beta$-axis are virtually the same. A similar remark can be made regarding the $x$-axis, representing the $x - y$ plane. Therefore, the comparing procedure will continue with MSE$_\alpha$ and MSE$_x$.

**Step II. Homoscedasticity test.**

As in the previous step, $\alpha = 0.05$. The hypotheses are:

$H_0$ : The variance of the FM are equal for both controllers.

$H_1$ : The variance of the FM are not equal for both controllers.

The obtained p-values shown in Table XIII are less than 0.05. Therefore, the equal variance hypothesis is rejected. Note that this result is consistent with the results presented in Table II where the variance (SD$^2$) for the MSE$_\alpha$ of the M2PC ($0.0030^2$) is around 7 times greater than the MPC ($0.0011^2$). Table III even shows that the variance for the MSE$_x$ of the MPC ($0.0086^2$) is around 44 times greater than the variance of M2PC ($0.0013^2$). This has an impact on the CI estimation, as shown in (12).

TABLE XIII
HOMOSCEDASTICITY TEST

| FM | p-value | Equal variance |
|---|---|---|
| MSE$_\alpha$ | 0.0001 | No |
| MSE$_x$ | 0.0001 | No |

**Step III. CI estimation.**

As it was introduced in the first paragraph of Section II, the estimation of the CIs, allows to know which current controller is better as well as how much better it is. By following **Step III** of subsection II-C, the CI for the mean of each FM for both controllers is computed, see Table XIV. Now, it is then possible to identify which controller is better (or not) according to the FM. It is important to highlight that a controller will not always be better for all FMs considered since each one captures different characteristics. In our case study, according to Table XIV and Fig. 5(a), by comparing MPC and M2PC current controllers regarding MSE$_\alpha$ as FM, it is possible, with sufficient statistical evidence, to conclude at the 95% CL that the MPC is better than the M2PC. From the same table and from Fig. 5(b), it can also be concluded, at the 95% CL, that the M2PC is better than the MPC, based on the MSE$_x$ as FM. Note that the simultaneous control of $\alpha - \beta$ and $x - y$ planes is still an open issue in the field of MPC applied to multiphase machines. As it can be seen from the case example, MPC gives better $\alpha - \beta$ at the expense of a worse $x - y$ current tracking. While MP2C performance is opposite. Both conclusions derive from the fact that the CIs estimated for the different controllers are not intercepted, as shown in Fig. 5. This indicates that its average efficiency for the FM considered is different. Furthermore, the CI with the lowest values is the best, since what is being sought is to follow a current and this is better when the measured current is closer to the reference current.

According to ***Remark 3***, it is estimated the CI for the difference of the means between both controllers in Table XV to understand how much better the controller is on average concerning the FM considered. Based on MSE$_\alpha$, MPC is on average 57% to 61% more efficient than M2PC at 95% confidence level. On the other hand, the M2PC is on average

TABLE XIV
95% CI FOR THE FM MEAN.

| Controller | FM | Mean ($n = 26$) | CI LL | CI UL |
|---|---|---|---|---|
| MPC | MSE$_\alpha$ | 0.0647 | 0.0643 | 0.0651 |
| M2PC | | 0.1029 | 0.1019 | 0.1038 |
| MPC | MSE$_x$ | 0.4928 | 0.4906 | 0.4949 |
| M2PC | | 0.0855 | 0.0851 | 0.0858 |

4.74% to 4.79% more efficient than MPC regarding the MSE$_x$. Fig. 6 summarizes the normal distribution for the difference of the means.

TABLE XV
95% CI FOR THE FM MEAN DIFFERENCES.

| Controller | FM | Mean difference ($n = 26$) | CI LL | CI UL |
|---|---|---|---|---|
| MPC M2PC | MSE$_\alpha$ | 0.0382 | 0.037 | 0.0392 |
| MPC M2PC | MSE$_x$ | 0.4073 | 0.4051 | 0.4095 |

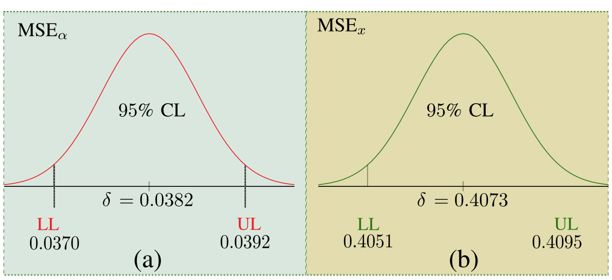


Fig. 6. Normal distribution plots for the difference of the means ($\delta$) for both FMs for MPC and M2PC PCCs (a) MSE$_\alpha$ and (b) MSE$_x$.

## VI. Conclusion and Future Work

As the first contribution of this paper, the random nature of FMs has been proven. Based on obtaining a large number of experimental tests and on the calculation of descriptive statistics, it is shown that FMs can differ both in the central value and in the dispersion and these characteristics are related to the definition adopted in their calculation.

The second contribution is that given the need to adopt scientific criteria that allows establishing a precise comparability procedure between current controllers applied to power converters and electric drives, a well-established and simple statistical method has been proposed. It consists of the estimation of a CI for the mean of the FM considered and also for the difference of means. The theoretical and practical development have been adequate so that it can be clear and understandable for non-statistical researchers working on the control of power electronics and electric motor drives.

Then, the contributions of the article have allowed:

- In case there is a difference between the performance of the current controllers, correctly decide the best controller based on FM. This provides the advantage that although another criterion could be very powerful when two equally efficient controllers are compared, since this criterion always chooses the one with the lower sample mean, it will commit a 100% Type I error. Therefore, this criterion is not admissible since it is not robust in the face of situations that may occur in reality and that are unknown in advance.
- Establish a rank of supremacy according to the FM considered when the difference is detected.
- Regardless of the variability in the observations, the conclusions obtained will continue to be valid for the sample size calculated according to quality parameters. A remarkable disadvantage of all the other study criteria, is that the amount of sample necessary to ensure the achievement of certain quality standards in the comparisons is not known.
- The robustness of the proposed statistical methodology is established by taking into account the estimated Type I error and the estimated power. It allows to conclude that the proposed method is capable of meeting the quality criteria based on the quality parameters.

Note that the proposed procedure can be used to compare sensor-based controllers such as current control, torque control, speed control and so on. Also it can be applied to any power converters (i.e. multilevel converters, matrix converters, etc) and drives (i.e. conventional or multiphase IMs). Moreover, the same procedure can be used for other FMs and different experimental conditions.

A limitation of the proposed method is that it can be applied only to FMs that comply with the assumption of normality. This gives rise to another line of research on non-parametric methods for comparing the efficiency of FM-based controllers that do not have a normal distribution that can be addressed in future research.

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