A Multi-Commodity Flow Approach for Globally Aware Routing in Multi-Hop Wireless Networks*

Vinay Kolar and Nael B. Abu-Ghazaleh
CS. Department, Binghamton University
{vinkolar,nael}@cs.binghamton.edu

Abstract

Routing in multi-hop wireless networks is typically greedy, with every connection attempting to establish a path that minimizes its number of hops. However, interference plays a major role in limiting the capacity of such networks; this effect is ignored by most existing protocols. It is likely that approaches that coordinate routing to account for mutual interference will be able to achieve better performance than traditional approaches. Modeling routing with interference constraints is a complex non-linear optimization problem. We approach the problem using a Multi Commodity flow (MCF) formulation. We analyze the interaction of multiple routes and propose effective objective functions which attempt to maximize interference separation while limiting path inflation. Initial experimental results show significant improvement in performance over a traditional routing protocol. We evaluate the formulation against routes obtained using DSR under several scenarios and show that better performance is achieved in terms of throughput, goodput, and end-to-end delay.

1 Introduction

Ad hoc networks, mesh networks, and wireless sensor networks are instances of multi-hop wireless networks where nodes cooperate to forward traffic among each other. Gupta and Kumar in a seminal paper [6] derived the asymptotic capacity of such networks under the assumption of an optimal routing and packet transmission scheduling policy. The available bandwidth between a pair of communicating nodes is influenced not only by the nominal communication bandwidth, but also by ongoing communication in nearby regions of the network because of the shared nature of the medium. More specifically, other ongoing transmissions contribute interference power that can make it impossible to exchange packets between a given pair of nodes.

The majority of routing algorithms route connections greedily, taking local decisions without coordination. Typically, decisions are made for each connection considering metrics such as shortest path; such policies may lead to routing connections to mutually interfering nodes when, perhaps, other regions of the network are idle.

An ongoing aim of our research is to determine whether globally aware routing that is cognizant of the interference effect of connections on each other is capable of significantly improving the routing performance in multi-hop wireless networks. Modeling routing with a complete set of interference constraints is a complex problem. For example, it has been shown that an optimal constrained routing with just the bandwidth constraints for a multi-commodity problem is NP-hard [4]. We build on recent works that model the routing and scheduling problem in multi-hop wireless networks as a network flow problem [9, 10]. Section 2 relates our work to these previous efforts as well as others.

We model the network, including interference, as an extensible Linear Programming (LP) model and investigate objective functions that lead to routes which are *interference separated*. We identify: (1) crucial parameters that affect the overall connection health; and (2) unexpected effects from a standard formulation that arise especially in multiple connection environment. We propose alternative formulations that address these effects. The basic model is presented in Section 3, and the objective function formulation is analyzed in Section 4.

While the proposed approach is not directly usable in dynamic networks, which are better suited to distributed solutions, our study is beneficial because: (1) it provides methodology and experience with the performance penalty suffered by existing routing protocols; (2) the formulated model can serve as the starting point for developing distributed routing protocols that approximate the behavior of globally aware routing protocols. Further, design decisions in the formulation were taken with an eye for future development of distributed versions (e.g., in the selection of a node based interference model). Developing distributed globally aware protocols is a future direction for our work;

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and (3) the proposed approach may be feasible for static or slowly changing networks (for example, mesh networks).

We evaluate the formulation by simulating routes obtained from the linear programming solver against those obtained by Dynamic Source Routing. Despite differences in the assumptions made by the solver and those in the simulator, significant improvement in performance was observed for most cases. We present the experimental study in Section 5. We discuss different aspects of the model operation, as well as improvements and extensions in Section 6. Finally, Section 7 presents some concluding remarks.

2 Related work

Routing in MANETs is a well-studied topic. Most routing protocols use *hop count* as the only metric to compute the routes, thus favoring the shortest path routes. Recently, the validity of hop-count as a sole metric of path quality was brought into question because it fails to account for the link quality; link quality varies due to the quality of the wireless channel and possibly the level of interference. In follow up work, Draves et al [3] ised the expected number of retransmissions (ETX) as a measure of link quality. They proposed Link-Quality Source Routing (LQSR): a greedy routing protocol that monitors link quality continuously and changes to the path that has the lowest overall cost. While this approach incorporates a measure of coordination between interfering connections, there is no guarantee that an effective state will be found as oscillation among bad states may occur. Further, it requires continuous monitoring of connections, and is restricted to routes that are discovered via the unreliable route discovery process. Nevertheless, comparing approaches such as LQSR to globally coordinated routing is an interesting topic of future research. Since the characteristics of routes (specifically hop count) found by these routing protocols has been shown to have considerable effect the connection performance [12], it is essential to ensure good quality of the routes.

The impact of interference is studied by Kodialam et al [11, 10] and Jain et al [9]. These works are the first to model routing and scheduling in multi-hop wireless networks as a network flow problem; they form the basis of our work. The general problem of routing multiple flows in a finite capacity network with additional QoS constraints is a nontrivial integer nonlinear optimization problem. A well known network flow formulation, *Multi-commodity flow (MCF)* [1], has been used successfully in traffic-engineering of wired networks to derive the theoretic bounds, and as the basis for heuristics to develop fast-runtime approximation algorithms in these traditional networks [5].

MCF has been used in other contexts for multi-hop wireless networks. Arvind et al use an MCF formulation to derive routes that maximize the lifetime of power-constrained networks [16]. This work uses a conventional formulation that does not account for interference since it is not concerned with network performance aspects of the problem. The presence of interference and bandwidth constraints significantly complicates the problem.

Our work is most related to recent work on calculating the capacity bounds of ad-hoc networks with interference. Specifically, the wireless version of the MCF formulation must account for interference. Interference effects can be captured by extending the linear programming constraints of the MCF formulation as demonstrated by Jain et al [9]. In the same paper, the authors show that it is NP-hard to compute the path with least interference. They model interference as a conflict graph and derive the upper and lower bounds on the throughput for a static topology with a pre-specified connection set, assuming idealized scheduling. They show that theoretically optimal routes yield much better throughput than the existing routing protocols do. However, most of their analysis is carried out with a single-connection model. In contrast, our work focuses on multiple connections. Further, we use a more sophisticated model, and investigate the choice of objective functions which yield more effective routing that maximizes reuse and reduces hot-spotting.

Kodialam et al. propose a joint routing and scheduling problem in [10] which promises 67% of the optimal throughput. However, they consider a model with no interference among nodes, making the problem similar to the classical MCF formulation. In a later work, they propose a mechanism for accounting for interference in their model, but they do not study this new model [11] in detail.

The work in this paper extends these previous works that use a network flow formulation in several important ways. (1)The authors only consider aggregate throughput as a metric, focusing on feasible throughput bounds where as the proposed model focuses on deriving optimal routing configurations that are tunable to application objectives. (2) They focus mostly on the effect of interference on a single connection, with cursory treatment of the more complex case of multiple connections whereas this paper focuses on analyzing and modeling interference with the multiple connection interaction as one of the main parameters. (3) We investigate the path elongation effects and other heuristics to reduce the complexity of the objective function Gupta et al. use the basic algorithm derived by Jain et al [9] and derive a simpler means of calculating cliques and reach a distributed version of the formulation[7]. Our work is different in terms of the underlying model and solution methodology, as discussed above with regards to Jain et al's original formulation [9]. Discussion of the types of interference and its effect on scheduling has been done in past research work like [6, 18]. These types have been modeled in Kodialam et al [11]. Such a model has been used in this paper to describe various kinds of interference. Raniwala et al. introduce the definition of interference period and show that interference is one of the main limiting performance factors [15]. However, their objective is effective channel assignment in a multi-channel system.

3 Multi-Commodity Flow Formulation

Consider a static multi-hop wireless network where packets for a particular connection may flow through multiple intermediate wireless links. A node m can directly transmit to another node n if the quality of the signal received by n is above a given threshold. We denote such tuple of nodes (m, n) as an *edge*. To represent this network as a graph, let N be the set of nodes where each node constitute a *vertex* of the graph and E be the set of *directed edges*. Let $G(N, E)$ represent the graph of the network. In this section we present our formulation of the routing problem as a network flow problem and distinguish it from existing network flow formulations.

3.1 Basic Routing

The problem of routing in multi-hop wireless networks can be transmformed into a Multi-commodity flow problem [1]; we describe this basic formulation here. Let (s_n, d_n, r_n) denote source, destination and the rate of the n^{th} connection. The rate of connection, r_n, is the number of bits to be sent per unit time. Let C be the set of connections. The demand for a given node is the difference between the total outflow from the node and total amount of inflow to the node. The demand at a node for n^{th} connection is represented by b_i^n as

$$b_i^n = \begin{cases} r_n, & \textit{if } i = s_n \\ -r_n, & \textit{if } i = d_n \\ 0, & \textit{otherwise.} \end{cases} \tag{1}$$

To analyze the flow at each edge, we break the flows into a set of n disjoint flows, one for each connection. Let x_{ij}^n denote the flow at edge (i, j) for the n^{th} connection. Let the maximum capacity of an edge (i, j) be denoted by $u_{i,j}$.

3.2 Feasibility of flows

The basic feasibility test on whether all the n flows can be accomodated is the standard Multi-commodity problem. Equations 2 to 4 describe the constraints that needs to be satisfied for feasibility. Equation 2 describe the limiting bound of each flow to be the maximum rate of the connection. For a given connection, each edge can carry a maximum load corresponding to the rate of the given connection. The bundle constraint for the given graph is given by Equation 3 which limits the total flow at an edge not to exceed its capacity. The flow constraint in Equation 4 specifies the demand requirement to be met at each node as the difference between the outflow and inflow (Equation 1).

$$0 \le x_{ij}^n \le r_n \forall n \in C, \forall (i, j) \in E \tag{2}$$

$$l_{ij} \le \sum_{n \in C} x_{ij}^n \le u_{ij} \forall (i, j) \in E \tag{3}$$

$$b_i^n = \left(\sum_{(i,j) \in E} x_{ij}^n \right) - \left(\sum_{(j,i) \in E} x_{ji}^n \right) \quad \forall n \in C, \forall i \in N \tag{4}$$

The above model assumes that a flow can be split into multiple routes (multi-path routing [13]). However, a single route per connection is desirable in majority of networks to avoid some side-effects that occur due to multi-path routing. Under such conditions, the problem transforms into a integer MCF problem. Each edge can either carry the full traffic for a given connection or none of it; this constraint is represented by Equation 5. The variable y_{ij}^n is a boolean variable which is set to 1 if the edge carries the traffic for the $n\mathrm{th}$ connection and 0 otherwise.

Integer flow constraint:

$$x_{ij}^n = r_n \cdot y_{ij}^n \quad \forall n \in C, \forall (i, j) \in E \tag{5}$$

3.3 Traffic parameters and auxiliary constraints

A comprehensive model needs to determine other abstract parameters that would enable an effective traffic characterization. This section models such critical parameters and introduces supplementary constraints to account for the feasibility of the parameters. These issues and constraints have not investigated by previous studies.

3.3.1 Node Based Model of Signal and Interference

In contrast to the conflict graph model used in [9] and edge-based approach in [11], our formulation adopts a more flexible *node-based interference model* where interference at a given node is calculated. We track interference at nodes, rather than edges, since the nodes are the physical entities in the network; performance viewed by the nodes allows more effective optimization of the network as viewed by its users. An additional advantage of the node-centric formulation is simpler distributed protocols as we directly optimize performance from the communicating node's perspective.

A basic model of the flow and the interference experienced at the node is studied in this section. We first split the busy time of the node into *Signal* (flows carried by the node at incoming and outgoing edges) and *Interference* (the silent period of a node to enable the neighboring flows). Differentiating between the two would help the extensibility of the model. For example, we use the *Signal* part of the busy time to restrict the number of hops taken by the node. The amount of signal carried by a node i, denoted by S_i, is the sum of flows that enter or leave the node. This is denoted by Equation 6.

$$S_i = \sum_{n \in C} \left(\sum_{(i,j) \in E} x^n_{ij} + \sum_{(j,i) \in E} x^n_{j,i} \right) \quad \forall i \in N \qquad (6)$$

Let Γ_{ij} be a two dimensional matrix of boolean values which is set to 1 if there is an interference at node j when node i is transmitting. Γ_{ij} can be derived based on node location assuming idealized propagation, or experimentally based on observed connectivity and interference.

Receiver Conflict Avoidance(RCA) model The interference at a given node can be viewed as the amount of time the node has to be silent in deferrence to neighboring flows. If there exists a scheduling mechanism which perfectly schedules the transmissions, then the node has to be silent if none of the nodes which are currently receiving can be interfered with the node's transmission: we call this model the *Receiver Conflict Avoidance (RCA)* [11]. Under RCA, the interference at a node i ($\ddot{\text{I}}_i$) will be equal the sum of inflow to all the nodes which interfere with i, as described in Equation 7:

$$\ddot{\text{I}}_i = \sum_{n \in C, \Gamma_{iz}=1, (w,z) \in E, w \neq i, z \neq i} x^n_{wz} \quad \forall i \in N \qquad (7)$$

Transmitter-Receiver Conflict Avoidance(TRCA) model Even though RCA describes an imperative condition, it is not sufficient for protocols in which scheduling is based on contention. Generally, the two way handshake of RTS-CTS in protocols like 802.11 would extend the time for which the node i will be silent. To inform the hidden nodes around the receiver about the ongoing communication, the receiver also sends a small packet to the transmitter. This two-way communication gives rise to reception of packets at both the transmitter and receiver. To avoid interference at both the ends, the node i has to be silent if a node which is within the interference range (R_i) is either transmitting or receiving. We call such interference period as *Transmitter-Receiver Conflict Avoidance (TRCA) model of interference* [11], denoted by I_i. The value of I_i is the sum of inflows and outflows of all the nodes which interfere with node i, and is given by Equation 8. given by the equation 8.

$$I_i = \sum_{n \in C, \Gamma_{wi}=1, (w,z) \in E, y \neq i, z \neq i} x^n_{wz} + \sum_{n \in C, \Gamma_{wi}=0, \Gamma_{iz}=1, (w,z) \in E, y \neq i, z \neq i} x^n_{wz} \quad \forall i \in N \qquad (8)$$

3.3.2 Active and Passive nodes

While reducing interference at nodes is crucial, there is a need to reduce the interference at the right nodes. If a node does not carry traffic, the amount of interference it experiences is immaterial. The interference at the nodes which are a part of the some connection have to be reduced. Let us denote such nodes which have $S_i > 0$ as *Active* and other nodes as *Passive*. We introduce the concept of *Normalized interference* to differentiate between the two kinds of nodes as given in Equation 9. Let *Normalized Interference* at a node i, denoted by $\hat{\text{I}}_i$, be the interference at the node if its carrying any traffic; otherwise, it is zero. The interference in Equation 9 can be computed using either the RCA or TRCA model.

$$\hat{\text{I}}_i = \begin{cases} I_i, & \textit{if } S_i > 0, \\ 0, & \textit{otherwise} \end{cases} \qquad (9)$$

3.3.3 Commitment period of a node

In a given unit of time, the time the node spends in transmission/reception can be represented by S_i. The time that the node i has to reserve to be idle for enabling the flow of interfering traffic can be represented by $\hat{\text{I}}_i$, which we call the *Commitment Period*(A_i) of the node (Equation 10). For all the active nodes, the *Commitment Period* should be lesser than or equal to the capacity of the channel; otherwise, the node will be unable to fit all the flows as expressed by:

$$A_i = S_i + \hat{\text{I}}_i \qquad (10)$$

Interference Constraint:

$$A_i \leq U \quad \forall i \in N \qquad (11)$$

The constraints given by Equations 2,3, 4, 5 and 11 collectively state the feasibilty constraints for a single path traffic considering interference. We use *interference* to mean *TRCA interference* in the remainder of the paper.

4 Objective Function Formulation

The choice of the *optimal path set* depends on the definition of optimality as expressed by the objective function. While it is necessary for an objective function to consider the interactions between connections, the complexity of the formulation should be manageable to enable reasonable solution times. This section explores the interaction between multiple connections that need to be captured by the objective function and builds a simple, yet effective, objective function in a step-by-step manner.

4.1 Tradeoffs in Objective function selection

The combination of Normalized interference($\hat{I}_i$) and the signal(S_i) representing the time a node is communicating provide the basis to construct different objective functions that foster path separation. While the above parameters can be combined to form a basic objective function, undesirable effects can result. A simple example is an objective function that minimizes the hot-spot of interference in the network and ignores the hop count of the connections. Alternatively, an objective function may lead to an excessively difficult optimization problem. This section presents some key unintended effects that arise in the multi-connection scenario and proposes approaches to address them.

Multiple Objectives Consider the objective of trying to minimize *Commitment period* at each node as shown in Equation 12.

$$\textit{Minimize} \quad A_i \quad \forall i \in N \tag{12}$$

This equation has the drawback of *Multiple Objective Functions*, since A_i has to be minimized across all the nodes; a formulation with multiple objective functions significantly complicates the optimization task. The individual objectives, either as observed at a single node or by a single connection, should be combined into a single objective that would approximate the effect of the multiple objectives. Our goal is to find such *Pareto Optimum* by combining multiple objectives into one. Such *Multiple Objective Mathematical Programs (MOMP)* can generally be solved by either having a *Weighted Sum* or *Lexicographic* approach [17]. The *Weighted sum* approach with equal weights is well suited to our problem since the aim is to reduce the interference across all the nodes and connections without a set priority to each node or connection. We would like to investigate the effects of varying weights and *Lexicographic approaches* in the future.

We introduce two simple approaches to combining the multiple objectives: minimizing the sum of the commitment periods, and minimizing their maximum.

Commitment Period Total Minimization Equation 13 demonstrates an objective function that minimizes the sum of *Commitment Periods* of all nodes in the network.

$$\textit{Minimize} \quad \sum_{i \in N} A_i \tag{13}$$

Peak Commitment Period Minimization The commitment period is an estimate of the channel state around a node: the higher the commitment period, the greater is the bottleneck created at that node. The *Bottleneck Node* is the active node with the maximum commitment period. Under optimal scheduling, the bottleneck node is the one which dictates the end to end delay of the packet. Even in more realistic schedulers (e.g., contention based 802.11), the bottleneck node experiences maximum demand and will often be the critical link in determining properties such as the end-to-end delay and effective throughput. Accordingly, an objective function can be constructed that targets reducing the commitment period of the bottleneck node (Equation 14). Note that while max leads to a non-linear objective; however, there are well known approaches for linearlizing it.

$$\textit{Minimize} \quad max\{A_i | \forall i \in N\} \tag{14}$$

The disadvantage of the combined functions is that they collapse some aspects of the objective functions captured by the multiple objective formulation. This results in some undesirable effects. For example, in the case of the peak commitment minimization, the focus is only on the *Bottleneck node* and the other nodes are ignored. We explain such issues in the next section and motivate our final, per-connection objective function.

4.2 Problems in Combined Objective Functions

Problems may arise in the combined objective function formulation. We show examples of such problems in this section.

Conjoint node effect: Consider a topology with multiple connections. The objective functions in Eq. 12, Eq. 13 and Eq. 14 use the commitment period *Normalized Interference*. Consider Equation 13 where we minimize the sum of commitment periods. If a new node is added to carry the flow for any connection, then its commitment period would rise from zero to the sum of its signal and interference. This would increase the objective value by a significant amount. Thus, the formulation favors keeping the number of active nodes to the minimum. While this is helpful in single connection scenario to keep the number of hops to the minimum, the multiple connection scenario ends up with overloaded nodes which carry more than one connection while there exists another path with same number of hops and lesser interference. This effect is termed as *Conjoint node effect*.

Connection Coupling: Consider the Equation 14 where the *bottleneck node's* commitment periods is minimized. If there exists a node in at least one of the connection with a very high value of commitment period and which cannot be reduced, then the other connections are unoptimized. This problem is termed, *Connection coupling*.

Path Inflation: Most MANET routing protocols attempt to minimize the hop count of a connection; it is well known that the performance of an isolated multi-hop connection is directly related to the number of hops under idealized propagation assumptions [12]. Even though a longer route may be prefreable to avoid the interference hot-spots, some objective functions fail to take the shorter path when one is available at the same or lower cost. Objective functions which ignore the hop-count metric may suffer from *Path Inflation*. In many cases, this objective function fails to restrict the number of hops. Adding more nodes in the connection, adds active nodes, hop-count and the interference at the other active nodes, thus leading to a greater commitment period. Thus, a simple equation like 13 restricts the flow to the shorter number of hops. This is not the case in the objective function 14. The objective minimizes the maximum commitment period of all the nodes.

To illustrate the path inflation effect, consider a single connection between nodes *25-30* in a 6x6 grid like Figure 1(b). Once the bottleneck node of maximum commitment period is found, there is no restriction by the formulation to the number of nodes in the flow provided they have a commitment period lesser than or equal to the bottle node. Based on the approach of the solver, the routes obtained the bottleneck can be inflated; an 8 hops path in taken in the above example. Equation 14 fails to restrict the commitment periods of other nodes, which leads to path inflation.

4.3 Per-connection Objective Function

This section describes an alternative objective function that mitigates the effects observed with a single combined objective function. The *Connection Coupling* and the *Conjoint node effect* suggest the need for splitting the metrics used on a *Per-connection* basis. Let *Per-connection Signal* ($\hat{\mathrm{S}}_i^n$) be the signal carried for the n^{th} connection. Let $\hat{\mathrm{y}}_i^n$ be the boolean variable as described in Equation 15 which is set to 1 if the node i is a part of the n^{th} connection. The *Active* and *Passive* nodes can also be defined on a *per-connection* basis based on the value if $\hat{\mathrm{y}}_i^n$.

$$\hat{\mathrm{y}}_i^n = \begin{cases} 1, & \textit{if}\ \hat{\mathrm{S}}_i^n > 0 \\ 0, & \textit{otherwise.} \end{cases} \qquad (15)$$

Let *Per-connection Commitment Period* ($\hat{\mathrm{A}}_i^n$) be the commitment period of a node i for connection n which is defined as follows. Let $\hat{\mathrm{A}}_i^n$ be A_i, if it is involved in carrying the flow for the n^{th} connection; otherwise, it is zero. Once the notion of $\hat{\mathrm{A}}_i^n$ is introduced, it is easier to eliminate the effect of *Connection Coupling* since we can now minimize the per-connection based activity periods. Equation 16 shows an objective function that minimizes the peak commitment per connection.

$$\textit{Minimize} \quad max\ \{\hat{\mathrm{A}}_i^n\ |i \in N\} \quad \forall n \in C \qquad (16)$$

It can be seen that equation 16 represents a *Multiple Objective Function*. This multi-objective function can be transmformed into a single objective function as explained in Section 4.1. Let $\hat{\mathrm{A}}_{\max}^n$ be the maximum value of the Normalized Commitment Period for a given connection n. This would describe the *Bottleneck link* of the n^{th} connection. Equation 17 gives the objective function which decouples the commitment periods of connections and combines them as shown in 4.1.

$$\textit{Minimize} \quad \textstyle\sum_{\forall n \in C} \hat{\mathrm{A}}_{\max}^n \qquad (17)$$

Controlling path inflation: Even though Equation 17 avoids interference hot-spots and Connection Coupling, the Path Inflation effect may still persist. This section evaluates the balance between the shorter number of hops and the avoidance of interference and explores two schemes to overcome this problem.

For a constant number of hops h, the sum of the per connection signals at all nodes is constant and is given by:

$$\sum_{i \in N} \hat{\mathrm{S}}_i^n = 2hr_n \qquad (18)$$

The source and the destination of the connection carry signal equal to the rate of the connection r_n. The router nodes carry signal equal to $2r_n$, for receiving and forwarding the signal. The premise of both approaches is to limit the sum of $\hat{\mathrm{S}}_i^n$ across all connections and to choose the best route among the set of routes selected. The first approach tries to minimize the sum by adding it in the objective function and the latter by adding linear constraints.

1. Including the signal in objective function: To dictate the shortest number of hops in a given set of connection is relatively easier. It can be observed that the sum of normalized signals at a node for different connections is equal to the total signal carried by the node. If we assign a high weight(say, a weight of α) to this sum of signal carried, such that it is much larger than the Commitment Period experienced by the node, then, by combining this sum with equation 17 would result in a new objective function given by equation 19.

$$\textit{Minimize} \quad \alpha \textstyle\sum_{i \in N} S_i + \sum_{\forall n \in C} \hat{\mathrm{A}}_{\max}^n \qquad (19)$$

Suitable value of α would force to choose the path set which not only has the shortest hops but also minimizes the interference among the flows. For a topology where nodes are placed linearly, the minimum value of α can be shown to be bounded by the equation 21 where R_i is the Interference Range and R_r is the Reception Range of the signal.

Proof: The maximum activity period for a given node is when all the traffic flows through its interference range, R_i with maximum possible hops. Let $h_{\max}$ be the maximum number of hops in the interference region. The maximum number of hops happens in a circle of radius R_i can happen when the distance between the alternate nodes is just below R_r. Let us denote this value by R_r^-. Hence, if the nodes are placed in a straight line, $h_{\max}$ is given by the Equation 20. The node in such a region should be quiet for transmission from all the hops and for the time of reception. Thus, the lower bound for α is given by Equation 21.

$$h_{\max} = \left\lfloor \frac{2R_i}{R_r^-} \right\rfloor \quad (20)$$

$$\alpha \geq (h_{\max} + 2) \sum_{n \in C} r_n \quad (21)$$

It is to be noted that the value of α is constant for the path set consisting of shortest number of hops. Hence the equation 19 tries to find a set of shortest hops path set with minimal interference.

2. Per-connection signal constraints The other approach to avoid the *Path Inflation* effect is to add a constraint which limits the number of hops taken by each path. Although the approach is elegant, this formulation would then result into a flavor of the *Constrained Shortest Path Problem* which is proved to be NP-hard in studies like [8]. The minimum number of hops needed to reach a destination can be calculated using *Breadth First Search(BFS)* algorithm. Let $h^n_{\min}$ be the minimum number of hops in the route between the source and destination of n^{th} connection. The sum of the $\hat{y}^n_i$ across all the nodes will give the number of nodes participating in the n^{th} connection, which will be equal to the $h + 1$ (h being the number of hops) in the route. To restrict h to shortest number of hops, we have to add the constraints as given in Equation 22 where $\mathcal{P}$ is a constant termed as *Path Stretch factor*.

$$\sum_{i \in N} \hat{y}^n_i - 1 = \mathcal{P}(h^n_{\min} - 1) \quad \forall n \in C \text{ and } \mathcal{P} \geq 1 \quad (22)$$

Path Stretch Factor The amount of *stretch* in the number of hops can be restricted by appropriately setting the value of $\mathcal{P}$. *Path Stretch factor* is the ratio of the maximum allowable number of hops to the shortest hop count. If $\mathcal{P} = 1$, then it forces the route to take the shortest number of hops $h^n_{\min}$. The value of $\mathcal{P}$ is a constant in this study, however, we would like to study the effect of adaptation of $\mathcal{P}$ in the future work. This can be done by either having a per-connection path stretch factor or the value can be implied by priority of the connection.

Even though the latter approach is more simple than the former approach, it restricts the feasibility solution for traffic where the capacity can become the bottleneck. By enforcing the $\mathcal{P}$ strictly in the constraint in Equation 22, the algorithm may fail to find the path set when the required number of hops can be stretched because of the unavailibility of the capacity. However, the former formulation will overcome this disadvantage by specifying the restriction on the number of hops in the objective function which is to be minimized. The latter approach also needs to run the shortest path algorithm(BFS) before the commencement of the optimization to figure out the shortest number of hops for each connection, $h^n_{\max}$.

5 Performance Evaluation and analysis

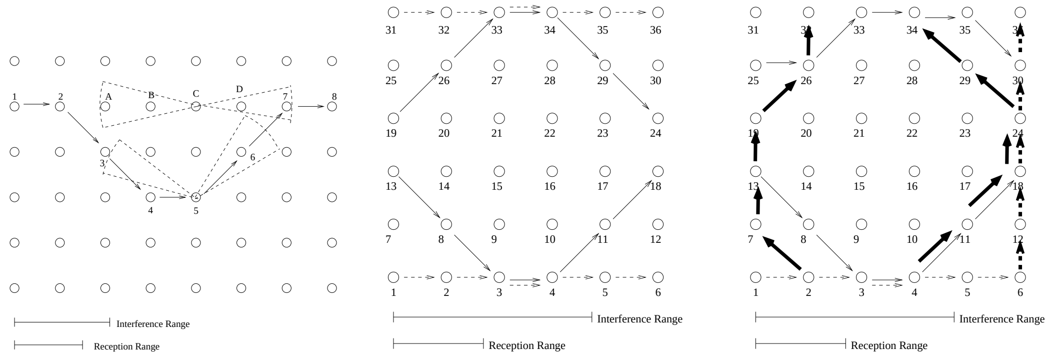
In this section, the performance of our formulation is compared with the existing routing and scheduling mechanisms. The CPLEX Linear Programming solver [2] was used to solve the LP formulation. The Qualnet simulator [14] was used to measure the performance of the proposed schemes under 802.11 protocol. We first study a grid topology of 6x6 and 8x8 nodes, and then evaluate the results of random deployment. The Qualnet simulator was modified to model the Boolean Interference Model consistent with the MCF formulation. The IEEE 802.11 MAC was used for scheduling.

To observe the behavior of the optimal routes, the solver results and the results from DSR are converted to static routes which are then used in the simulation. For each connection, the most commonly used route under DSR protocol is chosen and converted to a static route for use in the simulator. This approach is chosen to present the best possible performance obtained by DSR – always using the best path and ignoring dynamic effects and routing overhead.

5.1 Static connections in 6x6 Grid

A 6x6 grid network is studied with predefined connection patterns in order to demonstrate the characteristics of the routes given by the solver. The distance between the two adjacent nodes is set to 200m so that the a node can directly reach the immediate diagonal node. Figure 1 shows the path taken by various connections. The interference range(R_i) and the reception range(R_r) set in the solver is also shown for scaling of distances. The rate of all the connections are kept at the same value which was selected to ensure the presence of a feasible solution.

Self interference reduction A single connection from node 1 to node 8 is set up and the route taken is shown in Figure 1(a). In this scenario the reception range is set to 300m and interference range to 430m for the purpose of illustration. This is a 7 hop connection where the bottleneck



(a) 1 connection (b) 2 and 4 connections (c) 6 connections

Figure 1. Routes taken in in 6x6 Grid Topology

Metric \ Flows	2	4	6
End to End Delay	0.41	0.1	0.46
Throughput	2.44	1.17	1.24
Queue Drops	0.28	0	0.13

Table 1. Static connections in 6x6 Grid

Metric \ Flows	2	4	6
End to End Delay	0.069	0.24	0.58
Throughput	1.11	1.03	1.27
Queue Drops	0	0	0.32

Table 2. Static connections in 8x8 Grid

node would be in the middle of the connection since it experiences interference from higher number of links. Let us compare two shortest routes, *[1-2-3-4-5-6-7-8]* and *[1-2-A-B-C-D-7-8]*. The LP formulation would lead to the former route. Node 5 and node C will be the bottleneck nodes for each of these connections respectively. The arcs drawn from the bottleneck nodes denote the interference range(R_i). The commitment period of node 5 would be flows across 4 links included in the sectors where as the commitment period of node C would be 5 links, thus leading the solver to take the former route. This depicts the reduction of self-interference by the proposed model.

The remaining scenarios use the standard interference range and reception range. We first describe the shape of the routes taken and then explain the simulation results. The solid lines in Figure 1(b) shows routes taken for two connections in a 6x6 grid. The interference is reduced by the separation at the middle of the connection. In Figure 1(b), it can be seen that two connections are coupled at each edge of the grid, thus leading to interference of two connections with each other. Even if one of the connections had passed through the middle of the topology, then there would be interference between three of the connections. The maximum separation can be seen in Figure 1(c) too.

The simulation results are for the 6x6 grid shown in the Table 1. The numbers in the presented tables represent the performance obtained by the MCF obtained routes divided by that obtained by the DSR routes; thus, 1.1 performance represents a 10% increase in performance. The rates of the connections are adjusted such that when there are more connections, they each send at a higher rate. It can be seen that there is a significant improvement in throughput, end-to-end delay, and queue drops. The routes chosen by MCF formulation reduces the contention of the channel, thus leading to an increased success rate of packet transmission, helping nodes to transmit packets faster and reduce average queue size. On the other hand, if the contention success rate is lesser, then the packets accumulate in in the queue leading to packet drops. The decrease of end-to-end delay can also be attributed to the reduction in contetion. Table 2 shows the ratio of the value obtained from standard routes to that of the MCF formulation in an 8x8 grid. Significant improvement can be observed in end to end delay and queue drops. Overall, the quality of the routing is significantly better than that obtained by DSR. However, the improvement in throughput is not as high as the 6x6 case. We conjecture that this is due to the longer routes that are present in this case.

5.2 Random deployment

Table 3 shows the results when 100 nodes were randomly deployed in a 1600m x 1600m area and different number of

Metric \ Flows	4	6	8	10
End to End Delay	0.71	0.79	0.76	0.61
Throughput	1.11	1.02	1.02	1.07

Table 3. Random deployment

connections were randomly chosen. The end to end delay is considerably lower. Jitter and Queue drops also observed the same trend. However, the througput gains are not very significant.

Deeper analysis of these results, has lead us to the following observations. Low level scheduling effects play an important role in defining the effect of interference. Specifically, for some geometric configurations of interfering nodes, 802.11 was not able to successfully arbitrate the medium. In the grid scenarios, these problematic configurations did not arise due to the regular patterns. We are currently working on characterizing and incorporating the scheduling effects into our model.

5.3 Effect of the Path Stretch factor

Figure 2(a) shows a scenario where *Path Stretch* aids to reducing the contention. There are two one hop connections ($3-4$ and $9-10$) and a connection from $7-12$. The dotted semicircles shows the interference area created by the two one hop connections. The distance between the adjacent nodes of the grid is set such that the node can only reach horizontal or vertical neighbors but not the diagonal nodes. The shortest path from $7-12$ passes through the region which experiences the interference from both the one-hop connections. A larger route *[7-13-14-15-16-17-12]* would avoid the interference from the connection *3-4* but not from *9-10*. Let us denote this route by *Path-1*. Further increasing the *Path Stretch Factor* would enable the route *[7-13-19-20-21-22-23-24-18-12]* which can avoid interference by both the one-hop connection. Let this route be denoted by *Path-2*. A larger grid with a realistic interference range was constructed and the effect of the path stretch factor was observed in a similar scenario. The connection rate was adjusted such that there are no Queue drops. The end-to-end delay study in Figure 2(b) shows that when we increase the value of *Path Stretch Factor*, there is a significant decrease in the end-to-end delay. Even though the number of hops of the connection is increased, a reduced interference route would improve the end-to-end delay. Similar imrovement was also observed in the jitter too.

6 Discussion

This section presents limitations and possible extensions of the model.

Objective functions for contention based schedulers: Under a contention based scheduler, if all the active nodes of a connection have approximately the same commitment period and compete in a conflicting fashion, then the optimal route set derived from the above model may fail to deliver the expected results. Minimizing the *Average Commitment Period* of active nodes would be one of the approaches to overcome this drawback. Initial results are promising but a very high solver runtime was experienced in such cases.

Extension to complex models of interference: Interference, in reality, is not a boolean function and depends upon other factors like the cumulative power of other signals on the channel. The reception/interference power experienced by the node decreases non-linearly as the distance between them increases. Hence, a model of interference with *Distance based interference power* can be formulated without adding much complexity. BER and SINR based models can also be followed on same lines.

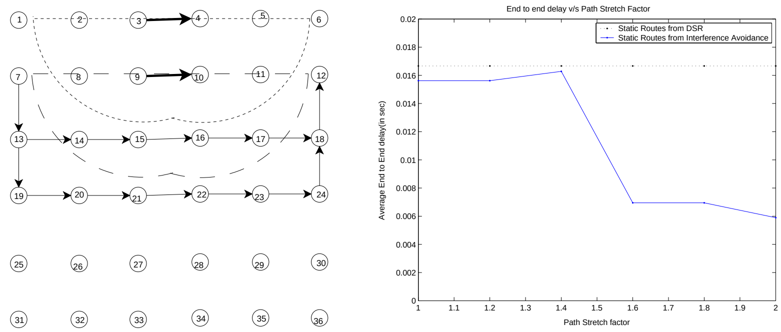
Application specific tuning and QoS Provisioning: The commitment period of the nodes(A_i) and the bottleneck node($\hat{\mathrm{A}}_{\max}$) can be used for deducing other connection parameters like inter-packet arrival time (given by $\hat{\mathrm{A}}_{\max}$), end-to-end delay(approximated by sum of A_i^n) and jitter. Extention of the model for QoS provisioning in community wireless networks like Mesh networks would benefit long-lived high-bandwidth connections that are sensitive to such parameters.

Directional Antennas and Multichannel models: A *Directional antenna* with S sectors can be modelled by applying vertex-splitting to each of node. A single vertex of $G(V,E)$ can be split into a clique of S verticies with infinite capacity edges to model intra-node flow of the packet. A similar extention to multichannel protocols can be done by edge-splitting.

7 Conclusion and Future work

In this paper we proposed a *node-based*, interference-sensitive, extensible multi-commodity flow formulation of the routing problem in multi-hop wireless networks. Multiple connection interaction were studied and *Path Separation Metrics* were abstracted. Approaches to control the *Path Stretch* and the significance of the *Commitment Period* and the *Bottle-neck node* were discussed and accounted to formulate a simple, yet effective, objective function. The extensibility of the model was shown by the ease of tuning the objective function for desired connection parameters. The results of the formulation in comparison to an existing routing protocol show promising improvement that can be achieved in connection health, despite the preliminary state of the model.

Extensions to the model by applying *Branch and price*



(a) Routes for 7-12

(b) End to End delay study

Figure 2. Path Stretch

techniques, which are well studied areas in integer multi-flow problems, would enable solving complex objectives (e.g. minimizing the average of commitment period) to be achieved in lesser run-times. In future work, we would like to evaluate such optimizations.

The effect of scheduling is not fully understood in literature. Existing approaches either assume the presence of perfect globally coordinated scheduling or ignore its effect. We have started identifying the types of scheduling interactions that occur in contention based scheduling, and will attempt to incorporate this information in the route selection process.

The ultimate goal of this work is to develop distributed protocols that achieve more effective routing than pure greedy approaches. This is also a major thrust of our future research.

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