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Journal
IEEE COMMUNICATIONS LETTERS, 18(9)

ISSN
1089-7798

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Publication Date
2014

DOI
10.1109/LCOMM.2014.2340872

Resource Allocation in OFDMA Networks With Imperfect Channel State Information

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***Abstract*—This letter investigates the effect of imperfect channel state information (CSI) on the performance of radio resource management for a downlink multi-user OFDMA system. It is shown that the imperfect CSI introduces a signal-and-power dependent noise term, which makes the symbol error rate (SER) constellation-point dependent, and yields an error floor on the SER. In addition, a two-step multi-user channel, power and constellation allocation algorithm is proposed to maximize the total network throughput.**

***Index Terms*—Imperfect CSI, channel assignment, power control, OFDMA, throughput.**

I. Introduction

In recent years, high data rate becomes one of most urgent demands for the development of radio communication systems. Restricted by the spectral resource limitation, how to increase the spectral efficiency becomes an important research topic. The OFDMA system has attracted significant interest due to its ability to achieve high spectral efficiency. However, one challenge in the design of OFDMA systems is the efficient radio resource management among multiple users.

Multi-user resource management in terms of assigning powers, subchannels, and rates in OFDMA systems has been extensively studied over the past several years [1]–[5]. Various objectives such as throughput/rate maximization and power minimization under certain constraints are considered. However, a common assumption in these works is that the perfect channel state information (CSI) is available. In practical systems, the CSI is always imperfect.

The effect of the imperfect CSI on the utility maximization problem in multiuser wireless communication systems has been studied in [6]–[10]. The traditional approach to deal with the imperfect CSI is to first obtain the distribution of the SNR, and then obtain the utility function. However, there is an assumption that the noise is not affected by the imperfect CSI and the variance of the noise keeps constant.

This letter addresses the effect of the imperfect CSI on the system performance of the multi-user resource management for a downlink OFDMA wireless network. A signal-and-power dependent noise term caused by the imperfect CSI is shown, which causes the symbol error rate (SER) to be constellation-point dependent and also leads to an error floor on the SER. Then, a two-step channel, power and constellation allocation algorithm is proposed to solve the optimization problem of adaptive resource allocation with imperfect CSI. Numerical simulations illustrate that, by taking into account the channel estimation error in the resource allocation algorithm, it can reduce the performance degradation caused by the imperfect CSI, especially when the channel estimation error is large.

Manuscript received January 19, 2014; accepted July 2, 2014. Date of publication July 18, 2014; date of current version September 8, 2014. The associate editor coordinating the review of this paper and approving it for publication was N.-D. Dao.

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Digital Object Identifier 10.1109/LCOMM.2014.2340872

II. System Description and Problem Formulation

We consider a downlink multi-user OFDMA wireless network in which one base station (BS) communicates with K users. The available frequency band is equally divided into N orthogonal subchannels, where the fading gain of each subchannel is assumed to be constant during a frame period. Assuming user k receives its data from the l-th subchannel, then the received signal by user k is given by [7]

$$y_{k,l} = \sqrt{P_{k,l}} h_{k,l} x_{k,l} + n_{k,l}, \tag{1}$$

where $x_{k,l}$ is the transmitted signal with unit variance, $P_{k,l}$ is the transmit power, $h_{k,l}$ is the channel gain, and $n_{k,l}$ is the Guassian noise with zero mean and variance $\sigma^2_{n,k,l}$.

If the channel $h_{k,l}$ is known, that the following matched filtering operation is performed by user k:

$$z_{k,l} = y_{k,l} \frac{h^*_{k,l}}{|h_{k,l}|^2} = \sqrt{P_{k,l}} x_{k,l} + \tilde{n}_{k,l}, \tag{2}$$

which yields the SNR $\gamma_{k,l} = P_{k,l}|h_{k,l}|^2/\sigma^2_{n,k,l}$, and $\tilde{n}_{k,l} = n_{k,l}/h_{k,l}$.

In this letter, we focus on the square M-QAM modulation. Then, the transmitted signal can be expressed as

$$x_{k,l} = \frac{1}{2}(2\alpha - \sqrt{M-1})d_{\min} + j\frac{1}{2}\left(2\beta - \sqrt{M-1}\right)d_{\min}, \tag{3}$$

where $\alpha, \beta \in \{1, 2, \ldots, \sqrt{M}\}$, and $d_{\min} = \sqrt{3/(M-1)}$ is the minimum distance among constellation points.

Then, the throughput of a data frame, defined as the number of bits successfully transmitted, is given by [5], [10]

$$T^M_{k,l}(P_{k,l}) = L \log_2 M \times \left[\left[1 - \frac{2\left(\sqrt{M}-1\right)}{\sqrt{M}} Q\left(\sqrt{\frac{3P_{k,l}|h_{k,l}|^2}{(M-1)\sigma^2_{n,k,l}}}\right)\right]^{2L} - \frac{1}{M^L}\right], \tag{4}$$

where L is the number of symbols contained in a frame, $T^M_{k,l}(P_{k,l})$ is the throughput of user k on l-th subchannel with a constellation size of M, $1/M^L$ is used to guarantee the throughput to be zero when no transmit power is assigned [11], and $1/M^L \approx 0$ when L is large.

The objective of the multi-user resource management is to maximize the total throughput by assigning each user's subchannel and transmit power with the power constraint, i.e.,

$$\begin{aligned} \max_{\rho_{k,l}, P_{k,l}} \quad & \sum_{k=1}^{K}\sum_{l=1}^{N} \rho_{k,l} T_{k,l}(P_{k,l}) \\ s.t. \quad & \sum_{k=1}^{K}\sum_{l=1}^{N} \rho_{k,l} P_{k,l} \le P_{tot}, \\ & \sum_{k=1}^{K} \rho_{k,l} = 1, \qquad \forall\, l, \end{aligned} \tag{5}$$

where $\rho_{k,l} \in \{0, 1\}$ indicates whether or not the l-th subchannel is assigned to user k, P_{tot} is the total power constraint, and $T_{k,l}(P_{k,l})$ is the throughput of the optimal constellation size, i.e., $T_{k,l}(P_{k,l}) = \max_{\{M\}}\{T_{k,l}^{M}(P_{k,l})\}$, where $\{M\}$ denotes the set of available constellation sizes. Note that $T_{k,l}(P_{k,l})$ is a nonconvex and nondifferentiable function of $P_{k,l}$ [5], [10], [11].

III. Imperfect CSI Analysis

In general, the imperfect CSI is modeled as

$$h_{k,l} = \hat{h}_{k,l} + e_{k,l}, \tag{6}$$

where $\hat{h}_{k,l}$ is the estimated channel gain, and $e_{k,l}$ is the Gaussian distributed estimation error with zero mean and variance $\sigma_{e,k,l}^2$.

Using $\hat{h}_{k,l}$, (2) can be rewritten as

$$\hat{z}_{k,l} = y_{k,l}\frac{\hat{h}_{k,l}^{*}}{|\hat{h}_{k,l}|^2} = \sqrt{P_{k,l}}x_{k,l} + \frac{\sqrt{P_{k,l}}x_{k,l}e_{k,l}}{\hat{h}_{k,l}} + \frac{n_{k,l}}{\hat{h}_{k,l}}. \tag{7}$$

Compared to (2), we can observe that the imperfect CSI introduces a power-and-signal dependent noise term $\sqrt{P_{k,l}}e_{k,l}x_{k,l}/\hat{h}_{k,l}$, which is not considered in previous works. Below, we show that the power-and-signal dependent noise term results the SER constellation-point dependent, and yield an error floor on the SER.

Define $\Pr(x_{k,l}|\hat{h}_{k,l}, \alpha, \beta)$ as the conditional probability that $x_{k,l}$ is received correctly, which can be derived for different values of α and β as follows.

1) When both $\alpha, \beta \in \{2, 3, \ldots, \sqrt{M}-1\}$, $x_{k,l}$ is an internal constellation point. Based on (3) and (7), we can write

$$\begin{aligned} &\Pr(x_{k,l}|\hat{h}_{k,l}, \alpha, \beta) \\ &= \Pr\left(\left|\mathrm{Re}\left\{\hat{z}_{k,l} - \sqrt{P_{k,l}}x_{k,l}\right\}\right| < \sqrt{P_{k,l}}\frac{d_{\min}}{2}\right) \\ &\quad \times \Pr\left(\left|\mathrm{Im}\left\{\hat{z}_{k,l} - \sqrt{P_{k,l}}x_{k,l}\right\}\right| < \sqrt{P_{k,l}}\frac{d_{\min}}{2}\right) \\ &= \left[1 - 2Q\left(\frac{d_{\min}|\hat{h}_{k,l}|}{\sqrt{2}\eta_{k,l}}\right)\right]^2, \end{aligned} \tag{8}$$

where $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real and imaginary parts, respectively, $Q(x) = 1/\sqrt{2\pi}\int_x^{\infty}\exp(-u^2/2)\,du$, and

$$\eta_{k,l} = \sqrt{\frac{\sigma_{n,k,l}^2}{P_{k,l}} + \frac{\sigma_{e,k,l}^2 d_{\min}^2 \lambda}{4}}, \tag{9}$$

where $\lambda = (2\alpha - \sqrt{M} - 1)^2 + (2\beta - \sqrt{M} - 1)^2$.

2) When both $\alpha, \beta \in \{1, \sqrt{M}\}$, i.e., $x_{k,l}$ is a corner point,

$$\Pr(x_{k,l}|\hat{h}_{k,l}, \alpha, \beta) = \left[1 - Q\left(\frac{d_{\min}|\hat{h}_{k,l}|}{\sqrt{2}\eta_{k,l}}\right)\right]^2. \tag{10}$$

3) When $\alpha \in \{1, \sqrt{M}\}$, $\beta \in \{2, 3, \ldots, \sqrt{M}-1\}$, or vice versa, i.e., $x_{k,l}$ is a point on the constellation border, then

$$\begin{aligned} \Pr(x_{k,l}|\hat{h}_{k,l}, \alpha, \beta) = &\left[1 - 2Q\left(\frac{d_{\min}\hat{h}_{k,l}|}{\sqrt{2}\eta_{k,l}}\right)\right] \\ &\times \left[1 - Q\left(\frac{d_{\min}|\hat{h}_{k,l}|}{\sqrt{2}\eta_{k,l}}\right)\right]. \end{aligned} \tag{11}$$

Then, the SER is given by

$$SER(x_{k,l}|\hat{h}_{k,l}) = 1 - \frac{1}{M}\sum_{\alpha=1}^{\sqrt{M}}\sum_{\beta=1}^{\sqrt{M}} \Pr(x_{k,l}|\hat{h}_{k,l}, \alpha, \beta). \tag{12}$$

Note that, under the perfect CSI, the error rate of each constellation point is determined by the parameters $d_{\min}$, channel gain, transmit power, and the noise variance. However, from (8), (10), and (11), it is seen that the power-and-signal dependent noise makes the error rate of each constellation point not only dependent on these parameters, but also dependent on α and β related to the constellation point itself, which causes the SER constellation-point dependent. In addition, when the transmit power $P_{k,l} \to +\infty$, $d_{\min}|\hat{h}_{k,l}|/\sqrt{2}\eta_{k,l} \to |\hat{h}_{k,l}|/\sigma_{e,k,l}\sqrt{\lambda/2}$, which indicates that the power-and-signal dependent noise introduces an error floor to the SER.

For a given MQAM modulation, the throughput under the imperfect CSI is derived as

$$\begin{aligned} \hat{T}_{k,l,M}(P_{k,l}) &= L\log_2 M\left(1 - SER(x_{k,l}|\hat{h}_{k,l})\right)^L \\ &\approx L\log_2 M\left[\left(\frac{1}{M}\sum_{\alpha=1}^{\sqrt{M}}\sum_{\beta=1}^{\sqrt{M}}\Pr(x_{k,l}|\hat{h}_{k,l}, \alpha, \beta)\right)^L - \frac{1}{M^L}\right], \end{aligned} \tag{13}$$

where $1/M^L$ is used to allow $\hat{T}_{k,l,M}(P_{k,l}) = 0$ when no transmit power is assigned as in (4).

Then, with the imperfect CSI, the total power constraint is same as that in (5), and the objective should be modified to

$$\max_{\rho_{k,l}, P_{k,l}} \sum_{k=1}^{K}\sum_{l=1}^{N} \rho_{k,l}\hat{T}_{k,l}(P_{k,l}), \tag{14}$$

where $\hat{T}_{k,l}(P_{k,l}) = \max_{\{M\}}\{\hat{T}_{k,l,M}(P_{k,l})\}$.

IV. Adaptive Resource Management

In this section, we propose a two-step algorithm to solve the optimization problem in (14). For simplicity, the equal noise variance and equal error estimation variance are assumed here as in [8].

From (12) and (13), we can see that, for given transmit power and modulation, $\hat{T}_{k,l,M}(P_{k,l})$ is an increasing function of $\hat{h}_{k,l}$. Hence, the throughput under imperfect CSI $\hat{T}_{k,l}(P_{k,l})$ increases with $\hat{h}_{k,l}$. In addition, in the multi-user OFDMA system, each subchannel can only be assigned to one user. Thus, if each subchannel is assigned to the user with the largest $\hat{h}_{k,l}$, then the total throughput in (14) is maximized. Hence we have the following channel assignment rule

$$\rho_{k,l}|_{k\neq k^*(l)} = 0, \qquad \forall\, l, \tag{15}$$

where $k^*(l) = \arg\max_{1\leq k\leq K}\{\hat{h}_{k,l}\}$.

Then, the optimization problem in (14) can be simplified as

$$\begin{aligned} \max_{P_{k^*(l),l}} \quad & \sum_{l=1}^{N} \hat{T}_{k^*(l),l}\left(P_{k^*(l),l}\right) \\ s.t. \quad & \sum_{l=1}^{N} P_{k^*(l),l} \leq P_{tot}, \\ & P_{k^*(l),l} \geq 0, \qquad \forall\, l, \end{aligned} \tag{16}$$

Since $\hat{T}_{k^*(l),l}(P_{k^*(l),l})$, same as $T_{k,l}(P_{k,l})$, is a nonconvex and nondifferentiable in the transmit power [5], [10], [11], to solve the optimization problem in (16), the dual problem is considered [12]:

$$\begin{aligned} g^* = & \max_{\mu\geq 0} \hat{g}(\mu) \\ s.t. & \sum_{l=1}^{N} \hat{P}_{k^*(l),l}(\mu) \leq P_{tot}, \end{aligned} \tag{17}$$

where

$$\hat{g}(\mu) = -\mu P_{tot} + \sum_{l=1}^{N} \min_{P_{k^*(l),l}\geq 0} \left(-\hat{T}_{k^*(l),l}\left(P_{k^*(l),l}\right) + \mu P_{k^*(l),l}\right), \tag{18}$$

$$\hat{P}_{k^*(l),l}(\mu) = \arg \min_{P_{k^*(l),l}\geq 0} \left(-\hat{T}_{k^*(l),l}\left(P_{k^*(l),l}\right) + \mu P_{k^*(l),l}\right). \tag{19}$$

Similar to *Proposition* 1 and *Proposition* 2 in [10], it can be shown that $\hat{g}(\mu)$ and $\hat{P}_{k^*(l),l}(\mu)$ are both non-increasing functions of the variable μ. Based on the properties of $\hat{g}(\mu)$ and $\hat{P}_{k^*(l),l}(\mu)$, the optimal μ^* can be obtained when $\sum_{l=1}^{N} \hat{P}_{k^*(l),l}(\mu) = P_{tot}$, and then we obtain the optimal power control $P^*_{k^*(l),l} = \hat{P}_{k^*(l),l}(\mu^*)$. Thus, the iterative bisection algorithm proposed in [10] can be used to solve (17), which is summarized in Fig. 1.

In the iterative bisection algorithm, we initialize a small threshold ε, set $\mu_{\min} = 0$, and find a $\mu_{\max}$ satisfying $\sum_{l=1}^{N} \hat{P}_{k^*(l),l}(\mu_{\max}) \leq P_{tot}$. The iteration will stop when the optimal solutions μ^* and $P^*_{k^*(l),l} = \hat{P}_{k^*(l),l}(\mu^*)$ are found. Based on $P^*_{k^*(l),l}$, the best constellation size is determined by $M^* = \arg\max_{\{M\}}\{\hat{T}_{k^*(l),l,M}(P^*_{k^*(l),l})\}$.

Finally, the multi-user resource management algorithm to solve the optimization problem in (14) can be summarized as follows:

Step 1. Perform subchannel assignment according to the estimated channel fading $\hat{h}_{k,l}$ based on (15).

Step 2. Allocate the transmit power to users using the iterative bisection algorithm as shown in Fig. 1. Then, the optimal constellation size is determined by the transmit power.

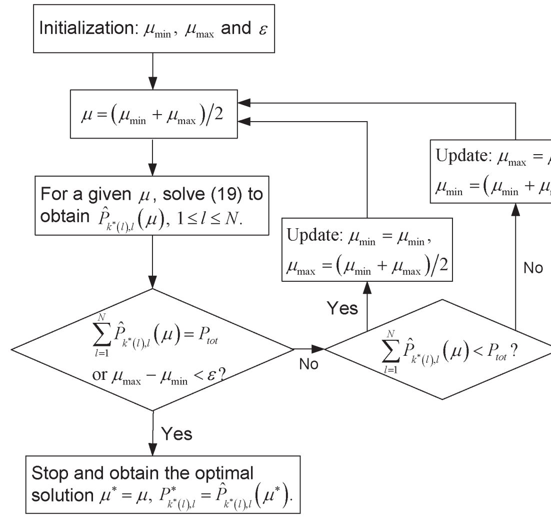

Fig. 1. Iterative bisection algorithm.

Note that, since $T_{k,l}(P_{k,l})$ has the same properties as $\hat{T}_{k^*(l),l}(P_{k^*(l),l})$, the above algorithm also works for the optimization problem with perfect CSI in (5).

The computational complexity of the step 1 is $O(KN)$. In the second step, the major complexity is introduced by the iterative bisection algorithm, which mainly contains two parts: the computation of $\hat{P}_{k^*(l),l}(\mu)$ from (19) for N separate optimization problems in the third box of Fig. 1 and the number of the iterations. Thus, the complexity order of the second step can be expressed as $O(NVW)$, where V is the average number of function evaluations required for computing $\hat{P}_{k^*(l),l}(\mu)$, and W is the average number of iterations. Since V and W are just finite constants independent of N, the complexity of second step is $O(N)$. Based on the complexity analysis above, we can see that the proposed two-step algorithm has linear complexity.

In addition, since the dual problem is used in (17), there is a duality gap that causes the proposed two-step algorithm be suboptimal. Fortunately, the duality gap is relatively small and can often be shown to diminish to zero as the KN increases [12, pp. 494], and many articles have proved its effectiveness [7]–[10]. In this letter, we find that the duality gap using the proposed algorithm is practically zero (smaller than one percent of the real optimal solution) through the numerical simulations. However, to avoid the crowd in figures, it is not shown in simulation results.

V. Simulation Results

In this section, numerical simulations are presented to illustrate the effect of the imperfect CSI on the system performance of multi-user resource management in the downlink OFDMA system. The system parameters are set as: $\{M\} = \{4, 16, 64\}$, $L = 100$, $K = 5$, $N = 5$, and $\varepsilon = 0.001$.

To show the effect of the imperfect CSI, Figs. 2 and 3 show the performance comparison for three cases: the proposed algorithm for the optimization problem in (14) considering the imperfect CSI, the proposed algorithm and the classical water-filling algorithm (WFA) [2] in which both simply treat the imperfect CSI as perfect. In Fig. 2, the total power constraint $P_{tot} = 40$ and the channel error ratio (CER), the ratio between the variance of channel estimation error and noise variance, is

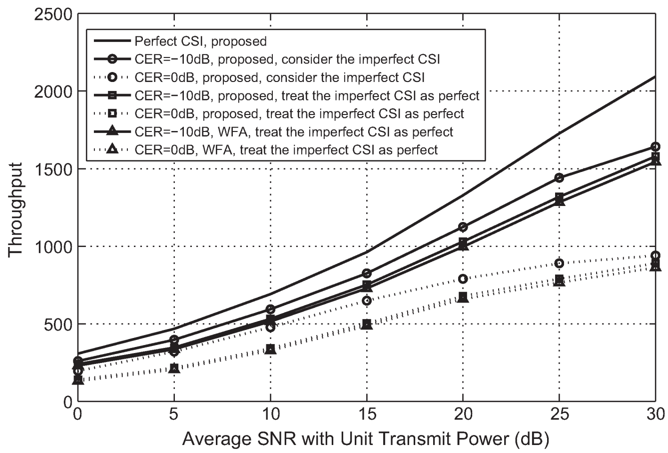

Fig. 2. Performance comparison with channel error ratios 0 dB and −10 dB.

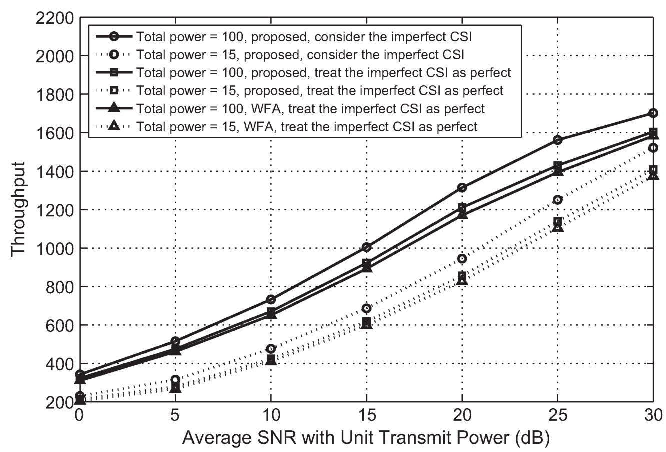

Fig. 3. Performance comparison with total transmit powers 15 and 100.

set to be 0 dB and −10 dB. In Fig. 3, the CER is −10 dB and the values of total power are 15 and 100.

From Figs. 2 and 3, we observe that the system performance with imperfect CSI is worse than that under perfect CSI, especially when the CER is large. This is reasonable because, from (7), CER is able to enhance the effect of the power-and-signal dependent noise. It also shows that the system performance with imperfect CSI saturates when both the CER and total transmit power are large, which shows the effect of the error floor of SER introduced by the imperfect CSI. In addition, the system performance will be improved significantly if we take the imperfect CSI into account in the optimization problem, especially when the CER is large, which reveals that the effect of the power-and-signal dependent noise introduced by the imperfect CSI should be considered in the radio resource management in the downlink OFDMA system. Note that, treating the imperfect CSI as perfect, it is seen that the performance of the proposed algorithm is slightly better than that of the classical WFA. This is because the practical throughput (nonconvex and nondifferentiable function) is considered in the proposed algorithm, while the approximated one (convex and differentiable function) is used in the classical WFA.

The average number of iterations required for the convergence of the iterative bisection algorithm is investigated in Fig. 4, where the values of total power are set to be 40 and 100. Fig. 4 shows that, for various average SNRs from 0 dB to 30 dB, the average number of iterations is smaller than 14.

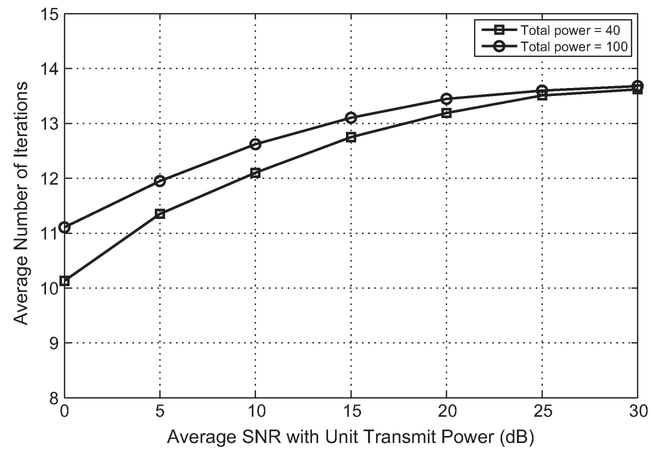

Fig. 4. The average number of iterations.

In the simulation, we average the number of iterations over 200 realizations for each average SNR.

VI. Conclusion

This letter investigates the effect of the imperfect CSI on the multi-user resource management in a downlink OFDMA system. It shows that the imperfect CSI introduces an extra signal-and-power dependent noise term, which leads to an error floor on the SER and makes the SER constellation-point dependent. By taking into account the channel estimation error in the resource allocation algorithm, the system performance degradation caused by the imperfect CSI can be reduced.

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