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Measurement of Keyhole Effect in a Wireless Multiple-Input Multiple-Output (MIMO) Channel

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***Abstract*—It has been predicted theoretically that for some environments, the capacity of wireless multiple-input multiple-output systems can become very low even for uncorrelated signals; this effect has been termed "keyhole" or "pinhole." In this letter, we present the (to our knowledge) first *measurement* of this effect. The measurements are done in a controlled indoor environment, with transmitter and receiver in two adjacent rooms. One of the rooms is shielded, and propagation to the other room can occur only through a hole or a waveguide in the wall. We find that only the waveguide leads to an unambiguous keyhole, while a hole of the same size still allows multimodal propagation. Measurement of amplitude statistics also confirm theoretical predictions.**

***Index Terms*—Double complex Gaussian, eigenvalues, keyhole, measurements, multiple-input multiple-output (MIMO).**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) wireless communication systems are systems that have multi-element antenna arrays at both the transmitter and the receiver side. It has been shown that they have the potential for large information-theoretic capacities, because they provide several independent communications channels between transmitter and receiver [1]. In an ideal multipath channel, the MIMO capacity is approximately N times the capacity of a single-antenna system, where N is the smaller of the number of transmit or receive antenna elements. Correlation of the signals at the antenna elements leads to a decrease in the capacity—this effect has been investigated both theoretically [2], [3] and experimentally [4].

It has recently been predicted theoretically that for some propagation scenarios, the MIMO channel capacity can be low (i.e., comparable to the single-input single-output (SISO) capacity) even though the signals at the antenna elements are uncorrelated [5], [6]. This effect has been termed "keyhole" or "pinhole."[1] It is related to scenarios where scattering around the transmitter and receiver lead to low correlation of the signals, while other propagation effects, like diffraction or waveguiding, lead to a reduction of the rank of the transfer function matrix. The effect has been predicted and discussed theoretically but, to our knowledge, no unique measurements of a keyhole have been presented in the literature.

In this letter, we present results from a measurement campaign performed at Lund University. Several previous measurement campaigns had searched for the keyhole effect due to tunnels or corridors in real environments, but the effect has been elusive. Therefore, we have used a controlled environment with a waveguide as the only connection between rich scattering environments around the transmitter and receiver locations. In this letter, we present measured channel capacities for three different channel setups, we study the distribution of the eigenvalues and finally we investigate the received amplitude statistics to compare our measurements to theoretical studies.

II. Measurement Setup

The measurements were performed with one antenna array located in a shielded chamber, and the other array in the adjacent room. A hole in the chamber wall was the only propagation path between the rooms. We measured three different hole configurations:

1) a hole of size 47×22 mm with a 250 mm long waveguide attached (referred to as "waveguide");
2) a hole of size 47×22 mm without waveguide ("small hole");
3) a hole of size 300×300 mm without waveguide ("large hole").

The line-of-sight (LOS) between the transmitter antenna and the waveguide (or hole) and between the waveguide and the receiver antenna were obstructed by absorbing material of size 600×600 mm. Linear virtual arrays with 6 antenna positions and omnidirectional conical antennas were used both at the transmitter and the receiver; the measurements were done during night time to ensure a static environment. The measurements were performed using a vector analyzer (Rohde & Schwarz ZVC) at 3.5–4.0 GHz, where 101 complex transfer function samples spaced 5 MHz apart were recorded. The received signal was amplified by 30 dB with an external low

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[1]In this letter, we use the original definition of keyholes. Recently, some authors have called "keyhole" any scenario that shows a reduction of the rank of the transfer function matrix (compared to the independent and identically distributed (i.i.d.) complex Gaussian case). This definition would imply that any scenario with strong correlation (small angular spread) is a "keyhole."

noise power amplifier to achieve a high signal-to-noise ratio (SNR). High SNR is required to be able to measure the keyhole properties and not the properties of the noise. The measurement SNR ranged between 23 and 27 dB, where in this case noise include thermal noise, interference and channel changes during the measurements.

III. Results

We evaluate the keyhole effect by studying the channel capacity, the eigenvalues and the correlation of the measured channel matrices. Following [1], we consider the channel capacity as a random variable whose realization depends on the exact location of the transmit and receive array. Each channel realization, m, is described by a transfer function $(N_{\mathrm{Rx}} \times N_{\mathrm{Tx}})$ matrix $\mathbf{H}_m$ and its Grammian $\mathbf{H}_m\mathbf{H}_m^{\dagger}$, where N_{Rx} and N_{Tx} denotes the number of receive and transmit antennas, respectively, and $[\cdot]^{\dagger}$ denotes the matrix conjugate transpose. $\mathbf{H}_m$ is normalized as $E\left[||\mathbf{H}_m||_{\mathrm{F}}^2\right] = N_{\mathrm{Rx}}N_{\mathrm{Tx}}$ over all realizations, where $E[\cdot]$ is the expectation. The capacity for each channel realization is the well-known Shannon capacity

$$C_m = \sum_{k=1}^{K} \log_2\left(1 + \frac{P}{N_{\mathrm{Tx}}\sigma^2}\lambda_{k,m}\right) \tag{1}$$

where σ^2 is the variance of the white Gaussian noise, P is the total power and $\lambda_{k,m}$ are the eigenvalues of $\mathbf{H}_m\mathbf{H}_m^{\dagger}$; K is the number of nonzero eigenvalues. For the evaluation of the measured capacity, we insert our measured transfer functions (as described in Section II) into (1). For an "ideal multipath channel," $\mathbf{H} = \mathbf{G}$, where $\mathbf{G}$ is defined as a random matrix with independent identically distributed (i.i.d.) complex Gaussian entries. For the theoretical keyhole the channel matrix is rank one and the elements are distributed according to an i.i.d. double complex Gaussian distribution, i.e., $\mathbf{H} = \mathbf{GKG}$, where $\mathbf{K}$ f is the all ones matrix [5].

In order to cleanly separate the effects of signal correlation from the true keyhole effect, we investigate the capacity decrease in correlated Gaussian channels. We estimate the receive and transmit antenna correlation matrices as

$$\hat{\mathbf{R}}_{\mathrm{H}}^{\mathrm{Tx}} = \frac{1}{MN_{\mathrm{Rx}}} \sum_{m=1}^{M} \left[\mathbf{H}_m^{\dagger}\mathbf{H}_m\right]^T \tag{2}$$

$$\hat{\mathbf{R}}_{\mathrm{H}}^{\mathrm{Rx}} = \frac{1}{MN_{\mathrm{Tx}}} \sum_{m=1}^{M} \mathbf{H}_m\mathbf{H}_m^{\dagger} \tag{3}$$

where M is the number of transfer function samples and $[\cdot]^T$ denotes matrix transpose. In our measurements of the "waveguide" the corresponding receive and transmit antenna correlation vectors are estimated as

$$\left|\hat{r}_{\mathrm{H}}^{\mathrm{Rx}}\right| = \begin{bmatrix} 1.0 \\ 0.42 \\ 0.37 \\ 0.24 \\ 0.17 \\ 0.18 \end{bmatrix}, \quad \left|\hat{r}_{\mathrm{H}}^{\mathrm{Tx}}\right| = \begin{bmatrix} 1.0 \\ 0.41 \\ 0.087 \\ 0.22 \\ 0.46 \\ 0.59 \end{bmatrix}.$$

These values were obtained with a limited number of transfer function samples ($M = 101$) due to the long duration of the measurements. The correlation matrices allow a prediction of

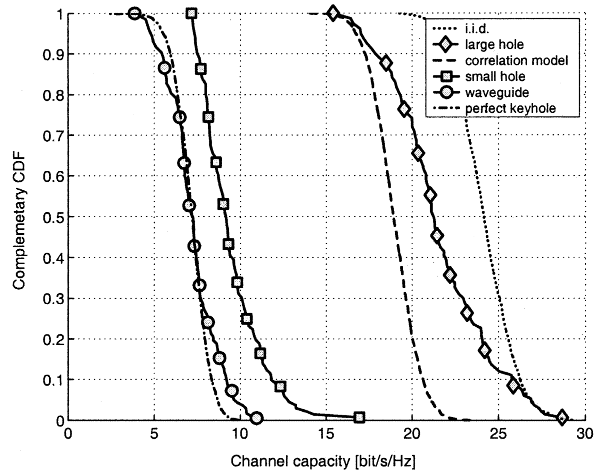

Fig. 1. Channel capacity complementary CDFs for different setups with equal number of transmitter and receiver antenna elements.

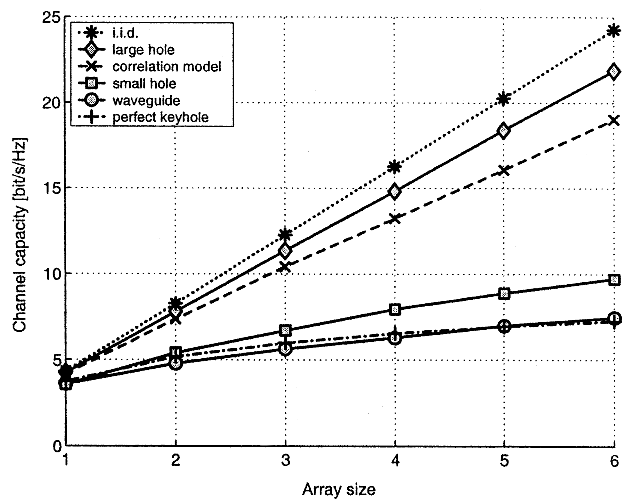

Fig. 2. Mean channel capacity versus the antenna array size for equal number of transmitter and receiver elements.

the capacity decrease due to signal correlation (i.e., an effect that is different from a keyhole effect). For the correlated capacity the channels is modeled as [2]

$$\mathbf{H} = \left(\mathbf{R}_{\mathrm{H}}^{\mathrm{Rx}}\right)^{1/2} \mathbf{G} \left[\left(\mathbf{R}_{\mathrm{H}}^{\mathrm{Tx}}\right)^{1/2}\right]^T \tag{4}$$

where the correlation matrix corresponds to measured correlations, the square root is defined as $\mathbf{R}^{1/2}\left(\mathbf{R}^{1/2}\right)^{\dagger} = \mathbf{R}$.

Fig. 1 shows the measured complementary cumulative distribution function (CCDF) of the channel capacities for the three hole configurations with an SNR of 15 dB. For comparison, the figures also presents the i.i.d. capacity, the correlated capacity and the capacity for a perfect theoretical keyhole. We see that the measured capacity for the "waveguide" setup is very close to the simulated perfect keyhole. Possible reasons for the differences include: 1) noise; 2) channel variations during the measurements; 3) residual correlations; and 4) a too small number of channel realizations. With the "large hole" the capacity is close to an i.i.d. channel and the CCDF for its measured capacity is in between the curves for the i.i.d. model and the correlated model. The difference in capacity between the measured "waveguide" and "large hole" configuration is up to 17 b/s/Hz. The CCDF

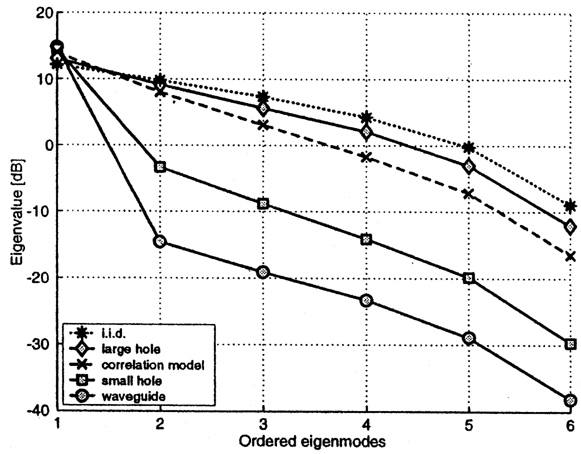

Fig. 3. Mean of the ordered eigenvalues in decibels (dB).

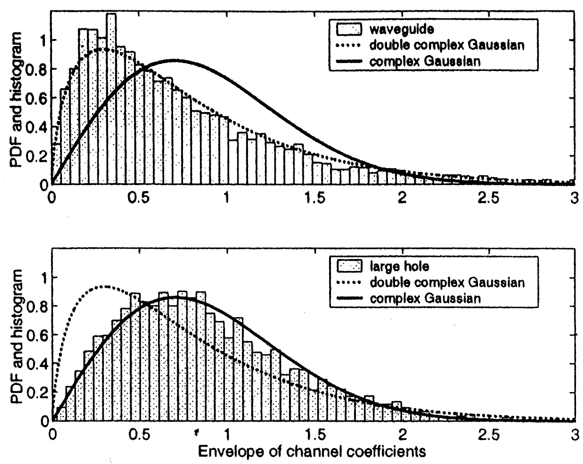

Fig. 4. Envelope distribution for the "waveguide" and for the "large hole" measurements.

for the correlation model shows the decrease in capacity related to the receive and transmit antenna correlation, and shows a capacity that is more than 11 b/s/Hz higher than the "waveguide" measurement. In Fig. 2 the mean capacities versus the number of antenna elements, $N_{\mathrm{Rx}} = N_{\mathrm{Tx}}$, are shown for an SNR of 15 dB. In this figure it can clearly be seen that the capacity for the "waveguide" setup nearly follows that of a perfect keyhole. The increase in capacity for more antenna elements is due to the combining gain of the receiver array. The capacity of the "large hole" increases almost as the capacity for the i.i.d. channel. This shows that the keyhole effect has disappeared entirely when the "large hole" of size 300×300 mm is the (only possible) path between transmitter and receiver. With the "small hole" there is some rank reduction, but the hole has a large enough cross section to allow more than one mode to propagate through it, while the length of the hole (2-mm aluminum plate) is too short to attenuate all but one waveguide mode entirely.

In Fig. 3 the mean of the ordered eigenvalues for the different channel setups is presented. It can be clearly seen that the "waveguide" channel is of low rank. The difference between the means of the largest and second largest eigenvalue is almost 30 dB for the measured keyhole. As a comparison, the difference between these eigenvalues for the ideal i.i.d. channel is around 2 dB. The difference between the largest and smallest eigenvalues (i.e., the condition number of the matrix $\mathbf{H}_m\mathbf{H}_m^\dagger$) is almost 50 dB for the "waveguide". The "small hole" setup also shows a significant difference from the "large hole" and i.i.d. setup. Except for the largest eigenvalue, the measured eigenvalues are, however, around 10 dB larger for the "small hole" compared to the "waveguide."

Finally we investigate the statistics of the received amplitudes for the "waveguide" and "large hole" setups. In Fig. 4 histograms of the received amplitudes are shown for the two cases. As a reference we have also shown the probability density functions (pdfs) for the amplitude of a complex Gaussian (i.e., Rayleigh) variable and a double complex Gaussian variable. The received amplitudes with the "waveguide" correspond well to the double Gaussian distribution, which confirms the predictions of [5]. The received amplitudes for the "large hole", however, correspond to a Rayleigh distribution since in this case the channel can be described as *one* rich scattering channel though all paths into the chamber is through the large hole.

IV. Conclusions

This letter presented the first experimental evidence of the keyhole effect in wireless MIMO systems. Using a controlled indoor environment, we found keyholes with almost ideal properties: the correlations at both the receiver and at the transmitter are low but still the capacity is very low and almost identical to a theoretical perfect keyhole. Our measurements use a waveguide, a small hole without waveguide, and a hole of size 300×300 mm as the only path between the two rich scattering environments. For the small hole there is a loss in capacity compared to the ideal case, but for the large hole the capacity is almost as large as for a theoretical Gaussian channel with independent fading between the antenna elements. We anticipate that the keyhole effect due to real-world waveguides like tunnels or corridors will usually be very weak, and thus difficult to measure, as (at typical cellular frequencies) such waveguides are heavily overmoded, and thus will not lead to rank reductions. This tallies with previous investigations that found correlation the major capacity-reducing effect.

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