Performance of Quasi-Constant Envelope Phase Modulation through Nonlinear Radio Channels

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***Abstract*— This paper studies the performance of phase modulation through a fully saturated power amplifier and AWGN channel for broadband wireless systems. A nonlinear model for the fully saturated power amplifier is derived. The equivalent impulse response method is provided to analyze the SNR degradation. Both convolutional code and turbo code are used. The Viterbi decoding and the iterative decoding are employed, respectively. Near optimal BER performance is achieved employing either convolutional code or turbo code. The simulation results match the derivation. Therefore, it is practical to employ phase modulation and fully saturated power amplifiers in broadband wireless/satellite networks and sensor networks for ideal battery life, low cost and high terminal reliability.**

I. INTRODUCTION

Recent practice in wireless and satellite communications has brought new challenges to modulation and coding for the nonlinear radio channel [1]. In a broadband wireless/satellite communications network, a very large number of user terminals communicate in a very large bandwidth centered at a very high carrier frequency [2]. The power amplifiers in broadband systems consume more power than that in narrow band systems. The battery life is determined by the DC-to-AC power conversion efficiency η of the power amplifier. This is also true in all wireless communications systems employing portable terminals operated on battery, including cellular phone systems and wireless sensor networks [3]. Linear power amplifiers which are used in narrow band systems are not power efficient. For example, the linear power amplifiers for portable devices employing the class A output stage can hardly reach $\eta = 30\%$. Linear power amplifiers are too hard to be designed and manufactured for the broad bandwidth and the high radio frequency.

A common practice to combat the radio distortion is to employ predistortion [4]. In [4], it is shown that at the BER$= 10^{-5}$ the SNR degradation is 1 dB after employing the predistortion. In other words, the predistortion can not completely compensate for the radio distortion. One open topic is to achieve optimal or near optimal BER in nonlinear radio channels.

The class D and the class F power amplifiers have the ideal DC-to-AC power conversion efficiency, i.e., $\eta_D = 100\%, \eta_F = 91\%$ [5]. The circuit for these two types of power amplifiers are much simpler than that for the class A [5]. This means a reduced cost and increased reliability. These advantages motivate us to study modulation and coding for broadband wireless/satellie networks and sensor networks employing practical power amplifiers of high DC-to-AC power conversion efficiency.

In this paper we study the performance of communications systems employing $\pi/4$ QPSK or OQPSK amplified by fully saturated power amplifiers and transmitted through the AWGN channel.

II. SYSTEM MODEL

The block diagrams of the communication systems studied in this paper are shown in Fig. 1.

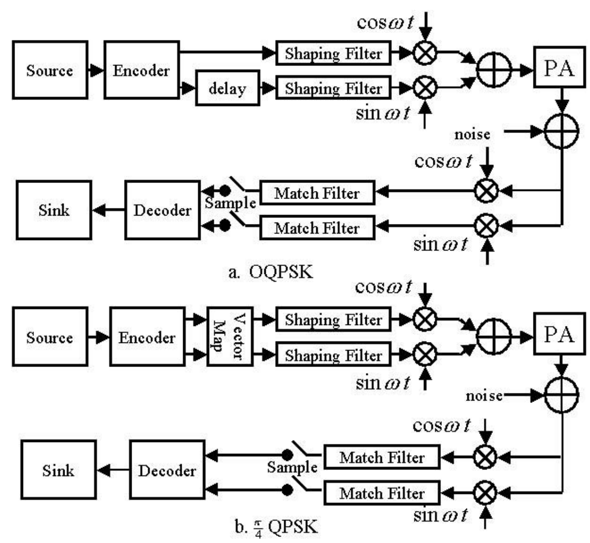

Fig. 1. Block diagram for the communication system employing OQPSK or $\pi/4$ QPSK and fully saturated power amplifiers.

Fig. 1(a) shows the block diagram for a communication system employing OQPSK. The data sequence is feeded into the encoder. The encoder can be either a convolutional encoder or a turbo encoder. The quadrature output of the encoder is delayed for one bit time before entering the shaping filter, and the in-phase output is fed into the shaping filter directly. The shaping filters output signals are multiplied with carriers respectively and combined together. After amplified by a fully saturated power amplifier, the modulated signal is transmitted through the AWGN channel. In the receiver, the down-converted signal plus noise is filtered by a matched filter and sampled. The sampled sequence is fed into the decoder.

Fig. 1(b) is the block diagram for communication systems employing $\pi/4$ QPSK. The system uses a vector mapping coder to avoid $\pm\pi$ phase shift in modulated signal. The maximal phase shift of $\pi/4$ QPSK is $\pm\frac{3\pi}{4}$, and the minimal phase shift is $\pm\frac{\pi}{4}$.

Both the convolutional code and the turbo code are employed. The convolutional code is of the rate $R = 1/2$, constraint length $K = 7$ and polynomials 133 and 171. For turbo code, we used the same encoding scheme as in CDMA2000 [6]. The polynomials are 13 and 15. The constraint length is $K = 4$. The coding rate is $R = 1/2$. The encoder employs two systematic, recursive, convolutional encoders connected in parallel, with an interleaver which is in front of the the second recursive encoder. The encoder output is punctured to achieve the designated data rate. During encoding, a tail sequence is added to make sure that at the end of each frame these two convolutional encoders go back to all-zero state. If the total number of information bits is N, the output include $2N$ encoded data symbols followed by 12 tail output symbols. For the rate $R = 1/2$, the puncture patterns for the coded information bits and tail bits are shown in Table I.

TABLE I

THE PUNCTURE PATTERN FOR THE TURBO CODE WITH $R = 1/2$ IN CDMA2000.

Information Sequence		Tail Sequence	
Output	Pattern	Output	Pattern
X	11	X	111 000
Y_0	10	Y_0	111 000
X'	00	X'	000 111
Y'_0	01	Y'_0	000 111

In Table I, a '0' means that the symbol will be deleted and a '1' means that a symbol will be passed.

The interleaver in the turbo encoder performs block interleaving of the data. It plays an important role in the encoder. The interleaving algorithm is described below:

- Step 1 Determine the turbo interleaver parameter n where n is the smallest integer such that $N_t \leq 2^{n+5}$;
- Step 2 Initialize an (n+5)-bit counter to 0;
- Step 3 Extract the n most significant bits (MSBs) from the counter and add one to form a new value. Then discard all except the m least significant bits (LSBs) of this value;
- Step 4 Obtain the n-bit output of a lookup table;
- Step 5 Multiply the value obtained in Step 3 and Step 4 and discard all except the n LSBs;
- Step 6 Bit-reverse the five LSBs of the counter;
- Step 7 Form a tentative output address that has its MSBs equal to the value obtained in Step 6 and its LSBs equal to the value obtained in Step 5;
- Step 8 Accept the tentative output address as an output address if it is less than N_t; Otherwise discard it;
- Step 9 Increase the counter and repeat Step 3 through Step 8 until all N_t interleaver output addresses are obtained.

The modified BCJR algorithm is implemented in the decoder [7], [8]. The performance is simulated using three iterations.

For the shaping filter and the match filter, we use the root raise cosine FIR filter. The sampling rate is 16 and the duration is 8 symbols. Filters with different roll-off factor β have different effects on the performance [1].

III. NONLINEAR MODEL FOR FULLY SATURATED POWER AMPLIFIERS

We derive the nonlinear model for fully saturated power amplifiers. When the input signal is $s_i(t) = a\sin(\omega_c t + \phi)$, the output can be written as

$$s_o(t) = \sum_{i=1}^{\infty} C_i \sin(i\omega_c t + i\phi)$$

where $\{C_i\}$ are the sine transformation coefficients of the output signal. Because the power amplifier is working in the fully (or almost fully) saturated state, $\{C_i\}$ can be treated as constants and do not depend on the amplitude of the input signal. The responses of class D and class F power amplifiers with the sinusoid input are shown in Fig. 2 and Fig. 3 [5].

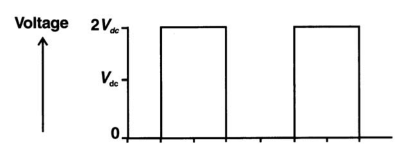

Fig. 2. The output waveform of the class D power amplifier for the input signal of the sine wave.

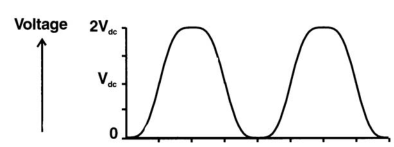

Fig. 3. The output waveform of the class F power amplifier for the input signal of the sine wave.

In phase modulation systems, the input signal to the power amplifier $s_i(t)$ can be written as $s_i(t) = a(t)\sin(\omega_c t + \phi(t))$, where $a(t) = \sqrt{(s_I^2(t) + s_Q^2(t)}$, $\phi(t) = \arctan\frac{s_Q(t)}{s_I(t)}$, $s_I(t)$ is the in-phase baseband signal, $s_Q(t)$ is the quadrature baseband signal and ω_c is the carrier frequency. The output signal can be written as $s_o(t) = C_1\sin(\omega_c t+\phi(t))+\sum_{i=2}^{\infty} C_i \sin(i\omega_c t+ i\phi(t))$.

Let

$$\tilde{s_o}(t) = \sum_{i=2}^{\infty} C_i \sin(i\omega_c t + i\phi(t)) \tag{1}$$

$$
\begin{aligned}
s_o(t) &= C_1 \sin(\omega_c t + \phi(t)) + \widetilde{s_o}(t) \\
&= \frac{C_1 s_I(t)}{\sqrt{s_I^2(t) + s_Q^2(t)}} \cos \omega_c t \\
&\quad + \frac{C_1 s_Q(t)}{\sqrt{s_I^2(t) + s_Q^2(t)}} \sin \omega_c t \\
&\quad + \widetilde{s_o}(t).
\end{aligned}
$$

In practice there is a low pass network at the output end of the power amplifier to eliminate harmonics [5]. The last term can be ignored in the transmitted signal. The transmitted signal can be written as

$$s_o(t) = \frac{C_1 s_I(t) \cos \omega_c t}{\sqrt{s_I^2(t) + s_Q^2(t)}} + \frac{C_1 s_Q(t) \sin \omega_c t}{\sqrt{s_I^2(t) + s_Q^2(t)}}. \tag{2}$$

For BPSK, the output of the power amplifier can be written as

$$s_o(t) = \begin{cases} Kcos(\omega_c t) & \text{if } s_i(t) > 0 \\ 0 & \text{if } s_i(t) = 0 \\ -Kcos(\omega_c t) & \text{if } s_i(t) < 0 \end{cases}$$

For OQPSK and $\pi/4$ QPSK, we can write the transmitted signal as

$$s_o(t) = s_{o_I}(t) + s_{o_Q}(t) \tag{3}$$

where

$$s_{o_I}(t) = \frac{K \cdot s_I(t)}{\sqrt{s_I^2(t) + s_Q^2(t)}} \cos \omega_c t \tag{4}$$

$$s_{o_Q}(t) = \frac{K \cdot s_Q(t)}{\sqrt{s_I^2(t) + s_Q^2(t)}} \sin \omega_c t \tag{5}$$

and K is the gain of the fully saturated power amplifier.

The fully saturated power amplifier does not change the phase of the input signal. It completely removes the amplitude information.

IV. DEMODULATION

The nonlinearity makes it very difficult to analyze the performance of a communication system employing a fully saturated power amplifier. Convolution can not be applied in a nonlinear system. With the nonlinear model , we can ignore the up-converter, treat the fully saturated power amplifier as a baseband power amplifier, and analyze the system in baseband. Let the baseband input signal to the power amplifier be

$$s_i(t) = s_I(t) + j \cdot s_Q(t) \tag{6}$$

where $s_I(t)$ and $s_Q(t)$ are the in-phase and the quadrature baseband signals respectively and $j^2 = -1$. The output of the power amplifier can be written as

$$s_o(t) = s_{o_I}(t) + j \cdot s_{o_Q}(t) \tag{7}$$

where

$$s_{o_I}(t) = \frac{K s_I(t)}{\sqrt{s_I^2(t) + s_Q^2(t)}} \tag{8}$$

$$s_{o_Q}(t) = \frac{K s_Q(t)}{\sqrt{s_I^2(t) + s_Q^2(t)}}. \tag{9}$$

Define $d(t) = s_o(t) - K s_i(t)$ as the distortion. The system is of the following properties: (1) The nonlinearity of the system is time invariant. Both the input signal and the output signal are stationary random processes; (2) The input signal and distortion are dependent. The distortion is the complement of the linear part $K s_i(t)$ to make the envelope constant; (3) The shaping filter plays a key role in the relation among $s_i(t)$, $s_o(t)$ and $d(t)$.

For BPSK, Eq. (7) can be simplified as $s_o(t) = sign(s_i(t))$. For M-ary PSK with $M \geq 4$, the in-phase signal and the quadrature signal cannot be treated independently. Without the knowledge of the shaping filter, it leads to unpredictable envelope variation of the power amplifier output. The in-phase signal and the quadrature signal interfere with each other more for those not symbol-aligned modulations such as OQPSK.

We introduce a model to analyze the nonlinear system. The equivalent linear model is derived from the statistic characteristics of the nonlinear power amplifier output. In the receiver, one cares only about the SNR at the sampling point. Since $s_o(t)$ is stationary and correlated with $s_i(t)$, we can take it as an equivalent output of a linear system. The equality is valid at the sampling points. The equivalent linear model of a communication system employing a fully saturated power amplifier depends on the shaping filter and the modulation scheme.

For OQPSK, we assume the source generates a sequence of symmetric binary impulses

$$x(t) = \sum_{i=-\infty}^{\infty} A_i \delta(t - iT_b) \tag{10}$$

where $A_i = \pm 1$ and T_b is the bit time. The autocorrelation $R_x(t) = \delta(t)$ and the power spectral density $P_x(f) = 1$. The shaping filter is a finite impulse response low-pass filter with the frequency response $H_s(f)$ and the time response $h(t)$. Suppose the duration of $h(t)$ is $2n$ symbols, the baseband signal can be written as

$$s_I(t) = \sum_{j=-\infty}^{\infty} \sum_{i=-n}^{n} A_{I_{i+j}} h(t - iT_s - jT_s) \tag{11}$$

$$s_Q(t) = \sum_{j=-\infty}^{\infty} \sum_{i=-n}^{n} A_{Q_{i+j}} h(t - iT_s - jT_s - T_s/2) \tag{12}$$

where $\{A_{I_i}\}$ and $\{A_{Q_i}\}$ are sequences of elements in $\{+1, -1\}$ and T_s is the symbol time. By (7), (8) and (9) the output signal $y(t)$ can be written as

$$y(t) = \frac{K s_I(t)}{\sqrt{s_I^2(t) + s_Q^2(t)}} + j \frac{K s_Q(t)}{\sqrt{s_I^2(t) + s_Q^2(t)}}. \tag{13}$$

By the statistic characteristics of the binary impulse sequence, we can calculate the autocorrelation $R_y(t)$ of the output signal. The power spectral density

$$P_y(f) = \int_{-\infty}^{\infty} R_y(t)e^{-j2\pi ft}dt. \tag{14}$$

Since $R_x(t) = \delta(t)$ and $P_x(f) = 1$, we have the equivalent impulse response $H'_s(f)$

$$H'_s(f) = \sqrt{P_y(f)/P_x(f)} = \sqrt{P_y(f)}. \tag{15}$$

The frequency response of the matched filter is $H_m(f)$. Therefore, the output of the matched filter is

$$y'(t) = \int_{-\infty}^{\infty} H'_s(f) \cdot H_m(f) \exp(2\pi jtf) df \tag{16}$$

the sample value can be written as

$$\begin{aligned} y'(0) &= \int_{-\infty}^{\infty} H'_s(f) \cdot H_m(f) \exp(2\pi jft) df|_{t=0} \\ &= \int_{-\infty}^{\infty} H'_s(f) \cdot H_m(f) df. \end{aligned}$$

Let the output of a communication system employing the linear power amplifier be

$$So(t) = K_0 \int_{-\infty}^{\infty} H_s(f) \exp(2\pi jft) df \tag{17}$$

where K_0 is the gain for the linear system, which keeps the transmitted power being the same as that in the compatible nonlinear system. The output of the matched filter in the receiver can be written as

$$y(t) = K_0 \int_{-\infty}^{\infty} H_s(f) \cdot H_m(f) \exp(2\pi.jft) df. \tag{18}$$

Then,

$$y(t = 0) = K_0 \int_{-\infty}^{\infty} H_s(f) \cdot H_m(f) df.$$

Define the degradation caused by the nonlinearity as

$$D = -20 \lg \frac{y'(0)}{y(0)} = -20 \lg \frac{\int_{-\infty}^{\infty} H'_s(f) \cdot H_m(f) df}{K_0 \int_{-\infty}^{\infty} H_s(f) \cdot H_m(f) df}. \tag{19}$$

Table II shows the degradation caused by the nonlinear power amplifier to the modulated waveforms using $\pi/4$ QPSK or OQPSK. The shaping pulse is the square root raised cosine function with the roll off factor β.

TABLE II

THE SNR DEGRADATION FOR $\pi/4$ QPSK AND OQPSK.

β	1.0	0.9	0.8	0.7	0.6
$\pi/4$ QPSK	0.296	0.285	0.292	0.308	0.339
OQPSK	0.061	0.081	0.108	0.145	0.204
β	0.5	0.4	0.3	0.2	0.1
$\pi/4$ QPSK	0.400	0.467	0.578	0.710	0.825
OQPSK	0.277	0.382	0.504	0.641	0.763

V. NUMERICAL RESULTS

We study the effect of the nonlinear radio distortion to the performance of convolutional code and turbo code.

Fig. 4 is the simulated BER for the coded $\pi/4$ QPSK. When the rate 1/2 convolutional code with $K = 7$ and the Viterbi decoding are employed, the SNR degradation is about 0.4 dB at BER=10^{-6}. The system employing $\pi/4$ QPSK and the rate $1/2$ turbo code with $K = 4$ is also simulated. The turbo decoding algorithm is the modified BCJR algorithm [7], [8]. When the 3-iteration decoder is implemented, the performance is much better than that of the convolutional code.

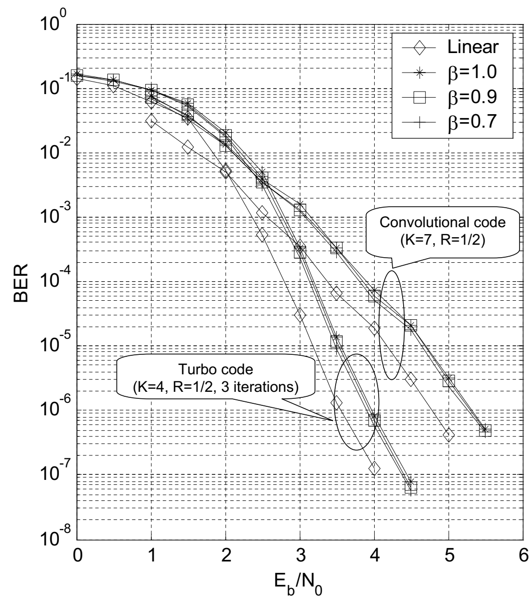

Fig. 4. The bit error rate of the communication system using $\frac{\pi}{4}$ QPSK. The power amplifier is fully saturated.

Fig. 5 shows the simulation results for commuincation systems employing OQPSK. There is almost no difference between the BER when a linear amplifier is employed and the BER when a fully saturated power amplifier is employed.

The simulated SNR degradation matches the theoretical results in the previous section.

For $\pi/4$ QPSK, the input of the shaping filters can be written as

$$\begin{aligned} x_I(t) &= \sum_{i=-\infty}^{\infty} A_i \delta(t - iT_s) \\ x_Q(t) &= \sum_{i=-\infty}^{\infty} B_i \delta(t - iT_s) \end{aligned}$$

where $A_i, B_i \in \{0, \pm 1, \pm\sqrt{2}\}$, $A_i^2 + B_i^2 = 2$ and T_s is the symbol time. During one symbol time, if the in-phase input

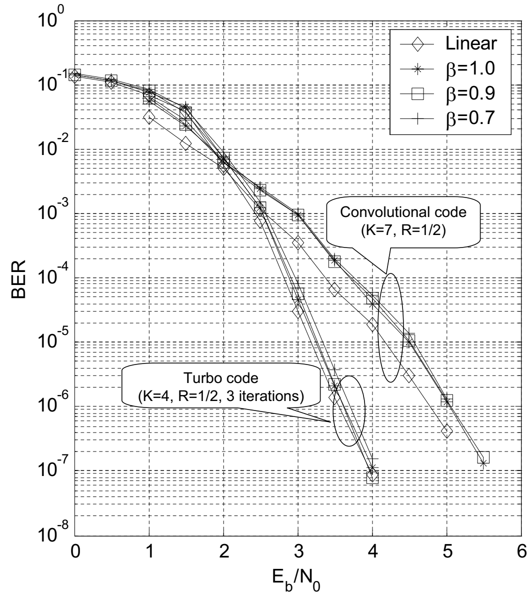

Fig. 5. The bit error rate of the communication system using OQPSK. The power amplifier is fully saturated.

is 0 and the quadrature input is $\pm\sqrt{2}$, the in-phase output of the fully saturated power amplifier is almost 0 and the quadrature output is almost fully clipped. If the in-phase and quadrature inputs are ± 1, the two outputs are all clipped. So the distortion disturbs the power distribution of the filtered $\pi/4$ QPSK signal. This causes severe degradation on the BER performance. The in-phase and quadrature baseband signals of OQPSK have $T_s/2$ phase shift. The main lobes of the filtered in-phase signal are in the middle of the main lobes of the filtered quadrature signal. The input of the fully saturated power amplifier has an envelope closer to constant than that of $\pi/4$ QPSK. Therefore, the nonlinearity of the power amplifier has less effect on filtered OQPSK signal. But when β is decreased, the side lobes of $h(t)$ become higher. It cause unpredicted change on the envelope of the filter signal, which will definitely impair the BER performance of both $\pi/4$ QPSK and OQPSK.

VI. Conclusions

The baseband equivalent model is derived for fully saturated power amplifiers. The performance is studied for $\pi/4$ QPSK and OQPSK transmitted by a fully saturated power amplifier. The SNR degradation is derived. The bit error rate is simulated for communication systems employing $\pi/4$ QPSK or OQPSK with the rate 1/2 convolutional code of the constraint length K=7 and the rate 1/2 turbo code. For $\pi/4$ QPSK, the degradation caused by the fully saturated power amplifier is 0.3 dB at BER=10^{-6}. For OQPSK, the SNR degradation is less than 0.1 dB at BER=10^{-6}. The simulation results match the derived results. When a fully saturated power amplifier is employed, both the convolutional code and the turbo code can help to achieve near optimal bit error performance compared with the compatible system employing the ideal linear power amplifier. Therefore, the quasi-constant envelope phase modulation proposed in [1] can be employed to achieve the ideal battery life, low cost and high reliability in wireless/satellite networks and wireless sensor networks.

TABLE III
The degradation of SNR at BER=10^{-6}.

	Convolutional Code (R=1/2, K=7)	Turbo Code (R=1/2, K=4, 3 iterations)
$\pi/4$ QPSK	0.5 dB	0.3 dB
OQPSK	0.2 dB	< 0.1 dB

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References

[1] Q. Liu and J. Li, "Quasi-constant envelope OQPSK through nonlinear radio and AWGN channel," *Proc. 2002 IEEE Military Commun. Conf.*, Vol. 1, pp 715 -720, Oct. 2002, Anaheim, CA.

[2] F. Garione, E. Iida, F. Valdoni, and F. Vatalaro, "Services, technologies, and systems at Ka band and beyond - A survey," IEEE J. Select. Areas Commun., vol. 17, pp. 133-144, Feb. 1999.

[3] G. J. Pottie and W. J. Kaiser, "Wireless integrated network sensors," *Communications of the ACM*, 43(5): 51-58, May 2002.

[4] S. Vaughn and R. Sorace, "Demonstration of the TDRS Ka-band transponder," *Proc. 2000 IEEE Military Commun. Conf.*, vol. 2, pp 1055-1065, Oct., 2000, Los Angeles, CA.

[5] S. R. Cripps, *RF Power Amplifier for Wireless Communications.* Boston, MA: Artech House, 1999.

[6] *Standards for CDMA2000 Spread Spectrum Systems*, EIA/TIA IS-2000.1-6, 2000.

[7] C. Berrou, A. Glavieux and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: turbo-codes," *Proc. of ICC 1993*, pp. 1064-1070, May 1993, Geneva, Switzerland.

[8] L. R. Bahl, J. Cocke, F. Jelinek and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, pp. 284-287, Mar. 1974.