A New Economic Generalized Particle Model for Flow Control

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Abstract

The problem of bandwidth allocation in computer networks can be likened to the supply-demand problem in economics. This paper presents the *economic generalized particle model* (EGPM) approach to intelligent allocation of network bandwidth. EGPM is a significant extension and further development of the generalized particle model (GPM) [1]. The approach comprises two major components: (1) dynamic allocation of network bandwidth based on GPM; and (2) dynamic modulation of price and demands of network bandwidth. The resulting algorithm can be easily implemented in a distributed fashion. Pricing being the network control mechanism in EGPM is carried out by a tatonnement process. We discuss the EGPM's convergence and show that the approach is efficient in achieving the global Pareto optimum. Via simulations, we test the approach, analyze its parameters and compare it with GPM and a genetic-algorithm-based solution.

Index terms: Intelligent bandwidth allocation, economic generalized particle model (EGPM), price and demands dynamic modulation, distributed and parallel algorithm, dynamical process, computer networks.

1 Introduction

1.1 Approaches to the bandwidth allocation problem

Well-known optimization approaches that have been applied to network resource allocation include: the max-min fairness (progressive filling) algorithm [2], Lagrangian multiplier approaches such as the ones by Kelly and Low et al. [3, 4, 5, 6, 7, 8, 9], the ant colony optimization approach [10, 11, 12], and genetic algorithms [13, 14, 15].

The max-min fairness algorithm has been widely used in digital networks, where it is used to allot bandwidth as equally as possible to all the users under certain transmission conditions. Although the algorithm is easy to realize, it tends to yield lower utilization of bandwidth resources than other approaches. The algorithms proposed by Kelly and Low et al. dynamically control the data

transmission rates of source nodes in the network so that the global utility of all the source nodes may be maximized. These algorithms can usually achieve a comparatively higher utilization of network resources with some degree of fairness. Using these algorithms, however, flow control is centralized, which makes the algorithms difficult to realize in real environments. Evolutionary algorithms (EA) are heuristic-based global search and optimization methods that have found their way into virtually every area of real world optimization. Ant colony optimization (ACO) and genetic algorithm (GA) are well-known examples; they belong to the class of meta-heuristics or approximate algorithms capable of obtaining fairly good solutions to hard combinatorial optimization problems in a reasonable amount of computation time. Their main limitation is that their empirical performance is unknown, and they could sometimes consume excessive amounts of computation time.

During the past several years, a number of approaches using economic models for resource allocation have been proposed, which include [16, 17, 18, 19, 20, 21]. Some of them divide the traffic into multiple priority classes, and use a fixed price for each class. To achieve greater network's efficiency, a better strategy may be to use dynamic pricing whose variation depends on network activities such as congestion. Wang and Schulzrinne [16] proposed a strategy where the price depends on the service class's average demand. The price is negotiable through a negotiation protocol. The strategy however requires resource reservation which obviously would incur some amount of overhead. In the pricing model of Baglietto et al. [17], the same amount of bandwidth is assigned to all traffic classes. Although simple, this approach may not be that reasonable as different customers may have different demands for bandwidth. Pricing in the scheme proposed in [18] is governed by a well-defined statistical model based on the source traffic. The scheme however does not take into account dynamic changes in the traffic.

Auction-based algorithms could be an effective model for solving classical assignment problems. Bitsaki et al. [19] pointed out that auctioning as an approach can outperform its main competitors by a margin for some important types of problems, and is also well suited for parallel implementation.

Pricing as an effective means to achieve economic efficiency for computer networks has attracted much attention recently. A number of pricing schemes have been proposed, such as [20, 21]. An appropriate pricing policy will provide incentives for users to behave in a way that would improve the overall utilization and performance of the network. A proper pricing policy influences the users' demands and the network revenue. There exist many papers on pricing for communication networks using the principles of microeconomics. Kelly et al., for example, use congestion price for rate control in wired networks, where elastic traffic users for throughput reasons can dynamically change their data rates [4]. Siris adopts a similar approach for CDMA networks [22]. Low presented a duality model of several TCP/AQM gate protocols [6], which treats these protocols as distributed primal-dual algorithms carried out over the Internet in real-time to maximize the aggregate utility and subject to certain capacity constraints. Kelly et al.'s and Low's methods can be regarded as directly or indirectly based on the Lagrangian multiplier method, and because of that, flow control is centralized.

1.2 The Generalized Particle Model (GPM)

Recently, Shuai and Feng proposed the *generalized particle model* (GPM) approach [1][1] which is based on hybrid energy functions. GPM can overcome some of the main deficiencies of the other approaches while retaining some of the good features of those well-known approaches just discussed. GPM was shown to be easy to realize, able to achieve comparatively higher network resource availability, and suitable for distributed implementation. GPM uses two "force-fields", the demand force-field and the resource force-field. There are numerous particles and forces in every force-field, where the particles and forces follow their own dynamic equations to represent the network entities and their behaviors and interactions respectively. The approach features high parallelism, low computational complexity, and simplicity for hardware implementation. The GPM approach, however, has the following limitations when applied to the network bandwidth allocation problem.

- The demand force-field in GPM is used to determine the pricing. Since the prices of bandwidth computed based on the demand force-field are linked to many factors such as congestion, supply and demands of bandwidth, interaction of various network entities, etc., the convergence rate to the equilibrium prices is low.

- Since many parameters in GPM need to be correctly chosen, GPM may not always produce a solution for efficient resource utilization. In order to accommodate as many connections as possible in a congested network, the resources need to be used efficiently. GPM's complex solutions (in both pricing and resource allocation) may be impractical although theoretically they can achieve better resource utilization.

- When the network traffic changes dynamically, the pricing is modulated via several or more iterative steps, and only when these steps are completely over can the new price take effect. Thus, prices modulate slowly in response to changes of the network traffic.

1.3 Economic Generalized Particle Model (EGPM)

This paper presents the EGPM approach for network bandwidth allocation. EPGM is based on GPM [1] and the excess demand function [28]. EGPM retains the advantages and overcome the disadvantages of the price-based approaches mentioned above, as follows.

- It adds congestion-dependent components to the price.

- It takes network activities and dynamic traffic changes into account.

- Its pricing depends on the excess demands of network resources.

- It does not require resource reservation.

- It uses price to reflect the dynamic situation of resource demands and the supply.

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[1] The work reported in [25] predated that of [1]; the latter incorporated several major improvements and achieved better results for the bandwidth allocation problem.

- It does not assign the same amount of bandwidth to all classes of traffic.

- Its complexity is relatively low.

Similar to the auction algorithm, EGPM rests on the belief that an appropriate pricing policy will provide incentives for the users to behave in ways that would improve the overall utilization and performance. EGPM makes use of the following ideas in the field of economics.

Yield management. By adjusting the allocation of (limited) resources, a company can optimize its total revenue or "yield." In [23], it is suggested that (1) it is expensive or impossible to store excess resources; (2) commitments need to be made when future demand is uncertain; (3) the company should differentiate among customers who may have different demand curves; (4) the same unit of capacity can be used to deliver many different products or services; and (5) producers are profit-oriented and have absolute freedom of action. Obviously, the bandwidth allocation problem has all these characteristics.

General equilibrium model. General equilibrium theory seeks to explain production, consumption and prices in an economy. Walras suggests that equilibrium can be achieved through a process of tatonnement [24].

Tatonnement process. Prices are lowered for goods with positive prices and excess supply, and should be raised for goods with excess demand [24]. EGPM applies the *tatonnement process* to GPM for the pricing of bandwidth.

Pareto efficiency. If an economic system is Pareto efficient, then no individual can be made better off without another individual being made worse off [24]. In this paper, we prove that the solution obtained by the EGPM algorithm is globally Pareto optimal.

We study particularly the relationship between price and demands in networks, and present the detailed EGPM approach which exploits the price-demands trade-off in solving the bandwidth allocation problem.

The following summarizes the differences of EGPM from GPM.

- In EGPM, the resource force-field is retained for allocating network resources; but instead of the demand force-field, the economic tatonnement process is used to determine the prices.

- The solution obtained by the EGPM algorithm is globally Pareto optimal.

- The EGPM algorithm is simpler than GPM in terms of the initial conditions and the choice of the main parameters in the algorithm, hence more practical.

- By the tatonnement process, EGPM is more effective in resource utilization than GPM. Also, EGPM's convergence rate is faster.

- In EGPM, only the value of the excess demand function is used in updating the price; hence, price modulation works faster than GPM in response to dynamic network traffic changes.

The provision of network services can be viewed as an economy where users are consumers and network resources are commodities. The price for bandwidth dynamically reflects the equilibrium between demand and supply. When the supply of bandwidth is less than the demands of bandwidth, the network becomes congested, and the price will rise; whereas when the supply of bandwidth is more than the demands of bandwidth, the price will fall. In EGPM, prices depend on the traffic demands and the available bandwidth. The bandwidth allocated to a user is equal to the demand for a price that the user can accept. Hence, the pricing scheme in EGPM is dynamic where the prices are not fixed and can change as the traffic load in the network changes.

In this paper, we consider the idealized situation where users can freely choose their shares of fixed or variable bandwidth in order to maximize their benefit. The network coordinates their choices through resource pricing. By bringing up the price at times of peak usage of bandwidth, the demands would adjust themselves, and the result is that less capacity is needed to meet the demands, and the capital utilization becomes higher. Some traditional approaches would reserve certain capacity for the users ahead of time. Reservation however neglects the efficiency gains that are possible through sharing of resources. EGPM avoids reservation so as to take advantage of statistical multiplexing.

To avoid large communication overheads in the computation, we (1) price only bandwidth (but not buffers), and (2) avoid reservation. In typical network situations, both bandwidth and buffer capacities will eventually become scarce. We believe that scarcity is best treated through pricing as price-based allocation is consistent with a competitive marketplace. We consider bandwidth to be a natural commodity to price, whereas pricing buffers is less satisfactory from a user's point of view. Furthermore, network performance is usually more sensitive to bandwidth than to buffers.

The EGPM algorithm is divided into two parts, as shown in Fig. 1: (1) dynamic allocation of network bandwidth based on GPM, and (2) dynamic modulation of prices and demands based on the tatonnement process. The tatonnement process and auctioning share some common features. Allocation based on GPM can realize the optimization of multiple objectives, including the global utility, the personal utilities, the minimal personal utility, the resource utilization, the users' satisfaction degrees, etc. Auction-based allocations cannot achieve the same.

The design of the EGPM algorithm pays special attention to the three "evaluation indices": bandwidth utilization, demand satisfaction, and fairness. These are in fact rather common measures for network performance. The former two indices are embodied in Eq. 4. Proposition 5 for the equation implies that the EGPM algorithm will be able to narrow the gap between bandwidth resources and the demand and maximize the utilization of the links' bandwidth. Max-min fairness is embodied in Eq. 3 and assured by Proposition 1.

1.4   Paper structure

Section 2 presents the model of bandwidth allocation problem. In Section 3, we give an overview of the EGPM architecture. In Sections 4 and 5, the evolution of the allocation policy and that of the pricing policy are addressed, respectively. The convergence of the EGPM algorithm is discussed and proved in Section 6. Section 7 presents the EGPM algorithm which is a synthesis of the results of the previous sections. In Section 8, we present the performance of EGPM based on simulation, and

compare it with GPM, genetic algorithm and other well-known bandwidth allocation approaches. Finally, we draw conclusions in Section 9.

2   Problem Model for Bandwidth Allocation

The following are the important issues to consider in our solution.

1. Allocation effectiveness: the customers should be satisfied, the available bandwidths should be utilized as fully as possible, and the allocation should be fair.

2. Pricing strategy: how the various bandwidths should be priced in order to maximize every link's and network's revenue.

3. Parallel allocation: the allocation method should be easy to be implemented in a distributed fashion with minimal communications among the computing entities.

The first issue above presents specific objectives which are common to most bandwidth allocation problems and solutions. In our approach, the degree of this objective being met is measured by the *demand satisfaction level* (DSL), the *bandwidth utilization level* (BUL), and a fairness term. Every link's revenue and network's revenue are two artificial objectives in our EGPM algorithm, which are means to optimizing the real objectives of the problem. EGPM purports to address all the above issues.

In the bandwidth allocation problem, a source-destination pair communicate through a channel associated with the pair, and have a certain demand for bandwidth. The channel can be realized through one or more paths into which the channel's traffic would be split. We consider the scenario with a given number of links and their capacities, a given number of channels and their demands for bandwidth, and a given number of paths per each of these channels. This is the case of a *static* environment. Our approach applies also to the *dynamic* case where these parameters may change over time. If the environment does change during the calculation of an allocation, the EGPM algorithm will allocate the bandwidths based on the current calculation and then perform a new calculation from the beginning according to the new conditions.

The network is an undirected graph of N nodes and m links, and there are J channels. The main parameters of any problem instance are as follows.

A_i: The i-th physical link $(i = \overline{1, m})$.

$T^{(j)}$: The j-th channel $(j = \overline{1, J};\ J \leq N \times (N-1)/2)$.

n_j: The number of the paths of channel $T^{(j)}$ $(n = \sum\limits_{j=1}^{J} n_j)$.

$T_k^{(j)}$: The k-th path of the j-th channel.

r_i: The maximum bandwidth of link A_i.

$d^{(j)}$: The median of the bandwidth demand of channel $T^{(j)}$.

ς_j: The mean square deviation of the bandwidth demand of channel $T^{(j)}$.

$x_{ik}^{(j)}$: $x_{ik}^{(j)} = 1$ if path $T_k^{(j)}$ passes through link A_i; otherwise 0.

$a_{ik}^{(j)}$: The bandwidth of link A_i that is allotted to path $T_k^{(j)}$.

$p_k^{(j)}$: The price per unit bandwidth that path $T_k^{(j)}$ has to pay.

$a_k^{(j)}$: The actual bandwidth obtained by path $T_k^{(j)}$—that is, $a_k^{(j)} = \min_i \{a_{ik}^{(j)} \,\big|\, \forall x_{ik}^{(j)} = 1\}$.[2]

Table 1 shows a snapshot of allocation.

Table 1: The bandwidth allocation problem

Path / Link	$T_1^{(1)}$	$\ldots$	$T_{n_1}^{(1)}$	$\ldots$	$T_1^{(j)}$	$\ldots$	$T_{n_j}^{(j)}$	$\ldots$	Link bandwidth
A_1	$x_{11}^{(1)}, a_{11}^{(1)}$	$\ldots$	$x_{1,n_1}^{(1)}, a_{1,n_1}^{(1)}$	$\ldots$	$x_{11}^{(j)}, a_{11}^{(j)}$	$\ldots$	$x_{1,n_j}^{(j)}, a_{1,n_j}^{(j)}$	$\ldots$	r_1
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$		$\vdots$
A_i	$x_{i1}^{(1)}, a_{i1}^{(1)}$	$\ldots$	$x_{i,n_1}^{(1)}, a_{i,n_1}^{(1)}$	$\ldots$	$x_{i1}^{(j)}, a_{i1}^{(j)}$	$\ldots$	$x_{i,n_j}^{(j)}, a_{i,n_j}^{(j)}$	$\ldots$	r_i
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$		$\vdots$
A_m	$x_{m1}^{(1)}, a_{m1}^{(1)}$	$\ldots$	$x_{m,n_1}^{(1)}, a_{m,n_1}^{(1)}$	$\ldots$	$x_{m1}^{(j)}, a_{m1}^{(j)}$	$\ldots$	$x_{m,n_j}^{(j)}, a_{m,n_j}^{(j)}$	$\ldots$	r_m
Allotted bandwidths (price) to paths	$p_1^{(1)}, a_1^{(1)}$	$\ldots$	$p_{n_1}^{(1)}, a_{n_1}^{(1)}$	$\ldots$	$p_1^{(j)}, a_1^{(j)}$	$\ldots$	$p_{n_j}^{(j)}, a_{n_j}^{(j)}$	$\ldots$	
Required bandwidths of channels	$d^{(1)}, \varsigma_1$			$\ldots$	$d^{(j)}, \varsigma_j$			$\ldots$	

The above parameters of the problem fall into several categories, as in Table 2.

Table 2: The notations

Category	Parameter(s)	During calculation
The network	$A_i, T^{(j)}, T_k^{(j)}, N, m, J, n_j, x_{ik}^{(j)}$	fixed
Supply & demand	$r_i, d^{(j)}(t=0), \varsigma_j$	fixed
Solution	$a_k^{(j)}$	changing
Artificial variables	$d^{(j)}(t>0), a_{ik}^{(j)}, p_k^{(j)}$	changing

The bandwidth allocation problem is defined as:

Bandwidth allocation problem:

Try to guarantee: satisfaction indicator (SI).

Maximize: demand satisfaction level (DSL), bandwidth utilization level (BUL), and fairness.

Given conditions: $N, m, J, n_j, r_i, d^{(j)}(t=0), \varsigma_j, x_{ik}^{(j)}$, and topology.

Solution: $a_k^{(j)}$.

Definition 1 *The* satisfaction indicator *is a logical variable indicating whether the demand of a channel is met or not.*

$$SI^{(j)} = \begin{cases} Yes & \sum_{k=1}^{n_j} a_k^{(j)}(t) \in [d^{(j)} - \varsigma_j, d^{(j)} + \varsigma_j] \\ No & otherwise. \end{cases}$$

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[2]If $x_{ik}^{(j)} = 0$, $a_{ik}^{(j)} = 0$.

Definition 2 *The* demand satisfaction level *of a channel (source-destination pair) shows the actual degree of satisfaction (or dissatisfaction) of the channel.*

$$DSL^{(j)} = \frac{\sum\limits_{k=1}^{n_j} a_k^{(j)}(t)}{d^{(j)}(t)} \times 100\%.$$

Definition 3 *The* bandwidth utilization level *of every link is defined as*

$$BUL_i = \frac{\sum\limits_{j=1}^{p} \sum\limits_{k=1}^{n_j} a_k^{(j)}(t) \cdot x_{ik}^{(j)}(t)}{r_i(t)} \times 100\%.$$

Definition 4 *The fairness of the allocation is measured by (according to [26])*

$$f(y_1, y_2, \cdots, y_J) = \left(\sum_{j=1}^{J} y_j \right)^2 \bigg/ J \sum_{j=1}^{J} y_j^2$$

where $y_1, y_2, \cdots, y_J$ are the bandwidths allocated to J channels respectively.

In addition, we introduce two artificial measures—the *network's revenue* and every *link's revenue*—and some artificial variables $(d^{(j)}(t > 0), a_{ik}^{(j)}, p_k^{(j)})$ into the EGPM algorithm which are useful in the derivation of the final solution.

Definition 5 *The* revenue *of link A_i is given by*

$$\sum_{j=1}^{J} \sum_{k=1}^{n_j} p_k^{(j)}(t) \cdot a_k^{(j)}(t) \cdot x_{ik}^{(j)}(t)$$

and the revenue of the network *is the sum of the revenues of all the links.*

The former two indices are embodied in Eq. 4. According to Proposition 5, the EGPM algorithm will always result in a decrease of Eq. 4—i.e., the algorithm will narrow the gap between bandwidth resources and the demand, and will increase the utilization of the links' bandwidth. Max-min fair allocation can be obtained by the EGPM algorithm, which is assured by Proposition 6 as well as Proposition 1. Max-min fairness is embodied in Eq. 3. The degree of fairness achieved is quantified by the fairness measure $f()$ defined above.

3 The EGPM Architecture

As shown in Fig. 1, our EGPM-based approach and its corresponding algorithm are divided into two parts: (1) dynamic allocation of network bandwidth based on GPM, and (2) dynamic modulation of prices and demands for network bandwidth.

GPM uses a *resource force-field* to model the interactions of the network entities [1]. A similar force-field is used here by EGPM, which consists of numerous particles and forces. A particle with its own dynamic equations represents an entity of the network being modeled, a network link in

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this case; each force with its own time-varying properties represents a certain kind social interaction among the network entities. The evolution process is iterative: at every step the allocation module would produce an allocation based on which the modulator module would generate a new set of modulated prices and demands. The iteration terminates when an equilibrium is reached. In this paper, the two modules are combined in a single EGPM algorithm.

Bandwidth allocator   **Price and demand modulator**

$$p_k^{(j)}(t+1) = p_k^{(j)}(t) + z_j(t) \cdot \theta$$

$$d^{(j)}(t+1) = p_k^{(j)}(t) \cdot d^{(j)}/p_k^{(j)}(t+1)$$

distributed parallel algorithm for bandwidth allocation

distributed asynchronous algorithm for price and demand modulation

The EGPM Algorithm

a particle and its forces

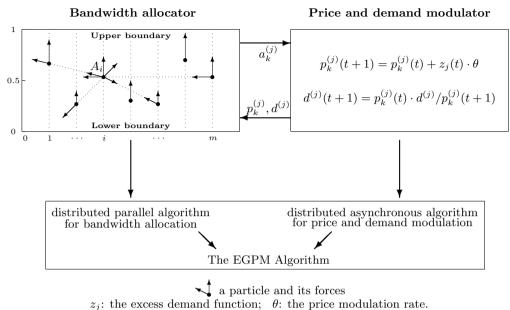z_j: the excess demand function; θ: the price modulation rate.

Figure 1: The EGPM architecture

A particle in the resource force-field would move along a certain trajectory under the influence of a composite force. The particles' behaviors are characterized by the following.

- Each particle has an autonomous self-driving force corresponding to the "personality" of the network link being modeled.

- The stimulus of a particle is a function of the particle's objective, utility, and intention, which presents a multiple objective optimization problem to be solved.

- There are a variety of interactive forces among the particles, including unilateral forces, corresponding to the various kinds of social interaction that are possible in a network.

4 Evolution of the Allocation Policy

The allocation module accepts new parameter values at every step of the iterative process to produce an allocation which is closer to equilibrium than the previous allocation. Since the evolution of the allocation policy in GPM and that in EGPM are essentially the same, we only highlight the relevant definitions and properties here. For further details and the missing proofs, we refer the reader to [1].

The mathematical model involving m links and n paths is defined as follows. We refer to a particle in the force-field that represents a link in the network a link particle.

Definition 6 *Let $u_i(t)$ be the distance of link particle A_i from the upper boundary of the force-field at time t, and let $J(t)$ be the total utility of all link particles. We define*

$$u_i(t) = \exp\left[-\sum_{j=1}^{J}\sum_{k=1}^{n_j} p_k^{(j)}(t) \cdot a_k^{(j)}(t) \cdot x_{ik}^{(j)}(t)\right]; \tag{1}$$

$$J(t) = \sum_{i=1}^{m} u_i(t) \tag{2}$$

The larger the value of the double summation in Eq. 1 (hence the smaller the value of $u_i(t)$), the more profit the ith link earns. The optimization problem here is posed as a minimization problem and so the term is negated. The exponential function makes $u_i(t)$ fall between 0 and 1. $u_i(t)$ indeed is the utility of link particle A_i. The distance between the bottom and the upper boundary is set to be 1. As for the total utility (Eq. 2), the smaller the value of $J(t)$ the better.

Definition 7 *At time t, the potential energy function $P(t)$, which is created by the upward gravitational force of the force-field, is defined by*

$$P(t) = \varepsilon^2 \ln \sum_{i=1}^{m} \exp\left[-u_i^2(t) / 2\varepsilon^2\right] \tag{3}$$

where $0 < \varepsilon < 1$ is a parameter to be tuned in the implementation, which affects directly the convergence speed.

The smaller the value of $P(t)$ the better. The decrease of this value implies the increase of the minimal utility among all the links (see Proposition 1). By this equation, we consider not only the aggregate, but also the individual personal utilities, in particular the minimum one, when trying to optimize the network bandwidth allocation. ε represents the strength of the upward gravitational force in the force-field. The larger the value of ε the faster the particles would move up; hence, ε influences the convergence speed of the allocation process. Too fast a speed however might result in some of the possible solutions being skipped over, and so ε needs to be carefully adjusted in order to maximize the users' satisfaction.

Definition 8 *At time t, the potential interaction energy function, $Q(t)$, is defined by*

$$Q(t) = \beta_1 \sum_{i=1}^{m} \left| \sum_{j=1}^{J}\sum_{k=1}^{n_j} a_k^{(j)}(t) - r_i(t) \right|^2 + \beta_2 \sum_{j=1}^{J} \left| \sum_{k=1}^{n_j} a_k^{(j)}(t) - d^{(j)}(t) \right|^2 \tag{4}$$

where $0 < \beta_1, \beta_2 < 1$.

The smaller the value of $Q(t)$ the better. β_1 and β_2 are the weights assigned to the amount of available link bandwidth and the degree of satisfaction of the path demands, respectively.

Eq. 4 quantifies the effect of interactions among the links during bandwidth allocation. The first and the second term of $Q(t)$ can be seen as penalty functions due to the constraints on the utilization of resources (i.e., link bandwidth) and the satisfaction of the users (i.e., the channels) respectively. Therefore, resource utilization and users' satisfaction can be explicitly included as part

of optimization objectives through some appropriate choices of the coefficients β_1 and β_2. There exist specific interactive forces among the particles, and these forces may cause the potential energy components represented by the first and second terms of $Q(t)$ to decrease.

Definition 9 *The dynamics of particle A_i are defined by*

$$\triangle a_{ik}^{(j)}(t) = -\lambda_1 \frac{\partial u_i(t)}{\partial a_{ik}^{(j)}(t)} - \lambda_2 \frac{\partial J(t)}{\partial a_{ik}^{(j)}(t)} - \lambda_3 \frac{\partial P(t)}{\partial a_{ik}^{(j)}(t)} - \lambda_4 \frac{\partial Q(t)}{\partial a_{ik}^{(j)}(t)} \tag{5}$$

where λ_1, λ_2, λ_3 are one-dimensional coefficient vectors, and λ_4 is a two-dimensional coefficient vector. $\lambda_1 = (\lambda_i^{(1)})_{m\times 1}$, where $\lambda_i^{(1)}$ represents the strength of A_i to pursue personal profit, $0 \le \lambda_i^{(1)} \le 1$. $\lambda_2 = (\lambda_i^{(2)})_{m\times 1}$, where $\lambda_i^{(2)}$ represents the strength of A_i to consider the aggregate profit of all the links, $0 \le \lambda_i^{(2)} \le 1$. $\lambda_3 = (\lambda_i^{(3)})_{m\times 1}$, where $\lambda_i^{(3)}$ represents the strength of A_i to increase the minimal personal profit among all the link particles, $0 \le \lambda_i^{(3)} \le 1$. $\lambda_4 = (\lambda_{iu}^{(4)})_{m\times m}$, where $\lambda_{iu}^{(4)}$ represents the strength of A_i to interact with A_u, $0 \le \lambda_{iu}^{(4)} \le 1$.

A_i updates $a_{ik}^{(j)}$ according to Eq. 5—that is, $a_{ik}^{(j)}(t+1) = a_{ik}^{(j)}(t) + \Delta a_{ik}^{(j)}(t)$. We can therefore obtain the radial velocity of link particle A_i along the vertical direction towards the upper boundary of gravitational field by the equation

$$v_i = du_i/dt = \sum_{j=1}^{J} \sum_{k=1}^{n_j} \frac{\partial u_i}{\partial a_{ik}^{(j)}} \frac{da_{ik}^{(j)}}{dt}. \tag{6}$$

Proposition 1 *The decrease of the potential energy $P(t)$ of the gravitational field will result in the increase of the profit of the link whose profit is the minimum among all the links.*

Proof. Let $U(t) = \max_i u_i^2(t)$. Because $U(t) = \max_i u_i^2(t) \le \sum_{i=1}^{m} u_i^2(t) \le m \cdot \max_i u_i^2(t) = m \cdot U(t)$, we have $\left[e^{\frac{U(t)}{2\varepsilon^2}} \right]^{2\varepsilon^2} \le \left[\sum_{i=1}^{m} e^{\frac{u_i^2(t)}{2\varepsilon^2}} \right]^{2\varepsilon^2} \le \left[m e^{\frac{U(t)}{2\varepsilon^2}} \right]^{2\varepsilon^2}$.

Simultaneously taking the logarithm of each side of the above equation leads to

$$U(t) \le 2\varepsilon^2 \ln \sum_{i=1}^{m} e^{\frac{u_i^2(t)}{2\varepsilon^2}} \le U(t) + 2\varepsilon^2 \ln m,$$

$$2\varepsilon^2 \ln \sum_{i=1}^{m} e^{\frac{u_i^2(t)}{2\varepsilon^2}} \ge U(t) \ge 2\varepsilon^2 \ln \sum_{i=1}^{m} e^{\frac{u_i^2(t)}{2\varepsilon^2}} - 2\varepsilon^2 \ln m,$$

$$2P(t) \ge \max_i u_i(t) \ge 2P(t) - 2\varepsilon^2 \ln m.$$

Since m is the number of links, $2\varepsilon^2 \ln m$ is a constant. $P(t)$ therefore represents the maximum among all the $u_i(t)$'s associated with A_i, namely, the minimum of the personal profits obtained by a link at time t. Hence, decreasing $P(t)$ implies the increase of the profit of the link whose profit is the minimum among all the links. $\square$

Definition 10 *(Max-min Fairness) [27] A feasible allocation of bandwidth a is max-min fair if and only if an increase of any bandwidth within the domain of feasible allocations will be at the cost of a decrease of some already smaller bandwidth allocation. Formally, for any other feasible distribution Y, if $y_{ik}^{(j)} > a_{ik}^{(j)}$ then there must exist some (i', k', j') such that $a_{i'k'}^{(j')} \le a_{ik}^{(j)}$ and $y_{i'k'}^{(j')} < a_{i'k'}^{(j')}$.*

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Proposition 2 *The behavior of A_i, by the third term of $\triangle a_{ik}^{(j)}$ (Eq. 5), will always bring about an increase of the minimum profit obtained by a link, and this increase is directly proportional to the coefficient vector λ_3.*

Proposition 3 *The behavior of A_i caused by the first and third terms of $\triangle a_{ik}^{(j)}$ (Eq. 5) will always result in an increase of the personal profit of A_i, and this increase is proportional to the coefficient vectors λ_1 and λ_3.*

Proposition 4 *The behavior of A_i caused by the second term of $\triangle a_{ik}^{(j)}$ (Eq. 5) will increase the global utility of the network, which is directly proportional to the coefficient vector λ_2.*

Proposition 5 *The behavior of A_i caused by the fourth term of $\triangle a_{ik}^{(j)}$ (Eq. 5) will narrow the gap between bandwidth resources and the demand, and the strength is proportional to the coefficient vector λ_4.*

Proposition 6 *(Max-min fair allocation) Max-min fair allocation can be obtained by updating the allotted a by Eq. 5.*

5 Evolution of the Pricing Policy

In this section, we present the evolutionary model that describes mathematically the dynamic modulation of prices and demands of network bandwidth. We introduce an important function, the *excess demand function*, which is a key element of the price modulation process.

5.1 Excess demand function

Definition 11 *The price vector $p^* = p_1^{*(1)}, \cdots p_k^{*(j)}, \cdots p_{n_J}^{*(J)}$, a solution of the bandwidth allocation problem $a = a_1^{(1)}, \cdots a_k^{(j)}, \cdots a_{n_J}^{(J)}$ is in equilibrium if and only if a is a feasible solution, and*

$$z_j(p^*) = \sum_{k=1}^{n_j} a_k^{(j)} - d^{(j)} = 0. \tag{7}$$

The mapping z is called the excess demand function and it has the following properties:

P1: z is single valued and continuous for all $p > 0$.

P2: z satisfies Walras' law: $p \cdot z(p) = 0$ for all $p > 0$.

P3: z is homogeneous of degree zero: $z(\alpha p) = z(p)$ for all $\alpha > 0$.

P4: There is a scalar $\nu < 0$ such that $z_j(p) > \nu$ for all j and $p > 0$.

P5: $\lim_{p(t) \to p} \left[\max_{j,k \in JK_p} z_j(p(t)) \right] = \infty$, when $p(t) > 0$, $p \neq 0$ and $JK_p = \{j, k : p_k^{(j)} = 0\} \neq \emptyset$.

Homogeneity is an elementary property of the excess demand function because the channels' price vector stays the same when the budget constraints are multiplied by a positive constant. Walras' law and the continuity property result from the channels' minimization problems when the preferences are strictly concave and locally non-satiated. P4 means that all the component functions of z are bounded from below on $\mathbb{R}^n_+$; this is because the net supply of bandwidth to the channels cannot exceed the channels' total demand. By P5, all the bandwidths of the paths are desirable in the sense that when some of these bandwidths become free of charge, there will exist at least some channels whose excess demand functions become infinitely large. When z has the properties P1–P5, the economy has at least a ray of equilibrium prices.[3]

The absolute value of the excess demand function is inversely proportional to the demand satisfaction level (Def. 2). EGPM's pricing policy aims at maximizing the demand satisfaction levels of the channels, and therefore the iterative price modulation processes in EGPM have to satisfy the conditions that (1) the absolute value of the excess demand function will be minimized, and (2) the modulation processes will converge.

5.2   Iterative price modulation processes

The dynamic modulation of price in the EGPM algorithm is based on the following. The price will rise when the supply of bandwidth is less than the demands of bandwidth ($z_j < 0$), whereas the price will fall when the supply of bandwidth is more than the demands of bandwidth ($z_j > 0$). The tendency to increase or decrease in price is directly proportional to $|z_j|$. In economics, such a dynamic modulation of price is known as the *tatonnement process*. The continuous time tatonnement process, introduced by Samuelson [29], is described by the differential equation

$$\dot{p}(t) = z(p(t)) \tag{8}$$

where $\dot{p}(t)$ is the time derivative of $p(t)$. This process is usually interpreted as an "auction" run by a fictitious agent, which sets the prices until an equilibrium is reached. It can be shown that under the following stability condition C1, the process of Eq. 8 is globally stable, i.e., it converges to an equilibrium for any positive initial prices.

C1: There exists $p^* > 0$ that solves Eq. 7 and satisfies $p^* \cdot z(p) > 0$ for all $p > 0$ for which $z(p) \neq 0$.

The convergence condition C1 can be interpreted as the weak axiom of revealed preferences between the equilibrium p^* and any disequilibrium price vector. The simplest discrete time alternative for the process of Eq. 8 is the fixed-point iteration

$$p(t+1) = p(t) + z(p(t)) \tag{9}$$

where t is the iteration index corresponding to the time instants at which the prices are modulated. Analyzing Eq. 9 instead of Eq. 8 suits the bandwidth allocation problem better, for which the price modulation process proceeds over discrete time instants. Unfortunately, Eq. 9 is not suitable for

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[3]See for example Chapter 17 of [28] for more about the properties of excess demand functions.

solving Eq. 7, because some of the prices may become negative during the iteration, for which z is usually not defined. One way to obtain non-negative prices is to update $p_k^{(j)}(t)$ as follows.

$$p_k^{(j)}(t+1) = max\{0, p_k^{(j)}(t) + \theta \cdot z_j(p_k^{(j)}(t))\} \qquad (10)$$

where θ is called the price modulation rate which is a positive constant. The convergence of this process restricts the choice of θ and $p(t=0)$. The drawback of the process of Eq. 10 is that due to P5 the excess demand function is not finite if some prices become zero. It can be easily seen that the discrete time process of Eq. 10 does not converge under the same assumptions as the continuous time process of Eq. 8. For example, the convergence of process of Eq. 10 depends on the choice of the parameter θ. Moreover, normalized discrete time processes tend to exhibit chaotic behavior. We give an alternative discrete time process which converges under assumptions that are very close to those required for the process of Eq. 8 to converge. This process can be seen as a variation of the fixed-point iteration of Eq. 9 and it is defined by the following formula.

$$p(t+1) = p(t) + \mu_t \cdot z(p(t)) \qquad (11)$$

where the parameter μ_t is updated as follows:

Step 1: A scalar $\gamma_t > 0$ is chosen such that $p(t) + \gamma_t \cdot z(p(t)) > 0$, and $\gamma_t = \gamma_{t-1}$ for $t \geq 1$ if $p(t) + \gamma_{t-1} \cdot z(p(t)) > 0$.

Step 2: $\mu_t = \min\{\gamma_t, M/\|z(p(t))\|\}$, where $M > 0$.

The first step above is to guarantee that the new prices are positive. When $p(t) > 0$, there is a positive number γ_t such that $p(t) + \gamma_t \cdot z(p(t)) > 0$. It follows that when the initial prices are positive, i.e., $p(0) > 0$, all the prices obtained during the process are positive as well. The second step guarantees that $\mu_t \cdot z(p(t))$ is bounded in the Euclidean norm $\|\cdot\|$. As a result, the magnitude of the change of the price vector is bounded, namely $\|p(t+1) - p(t)\| = \|\mu_t \cdot z(p(t))\| \leq M$, where M is an arbitrarily chosen positive number. This property is needed in the convergence analysis of the process. Note that by these two steps, μ_t is updated only when it is necessary, to obtain positive prices or to bound the changes by M.

The process of Eq. 11 satisfies the law of demand in the sense that the price of bandwidth would rise with excess demands (of the channels), and the price would fall with excess supply. Moreover, the prices are adjusted in proportion to the excess demands in a fashion similar to that in the process of Eq. 8.

6 Convergence Analysis of EGPM

6.1 Convergence analysis

We prove in this section that the process of Eq. 11 converges when z has the properties P1–P5 and satisfies C1 as well as C2 which is stated below. In the condition C2, vector p^* is the same equilibrium vector for which C1 holds, for $E_\varsigma = \{p \in \mathbb{R}_+^n : \|z(p)\| < \varsigma\}$.

C2: There are positive scalars ς and σ such that $p^* \cdot z(p) \geq \sigma \|z(p)\|^2$ for all $p \in E_\varsigma$.

Let us examine the geometrical interpretation of the conditions C1 and C2. The condition C1 means that the hyperplane $\{a \in \mathbb{R}^n : p^* \cdot a = 0\}$ supports the set $\{a \in \mathbb{R}^n : a = z(p), p > 0\}$ (see Fig. 2). The condition C2 means that this set is at least located locally around the origin, inside a ball which has its center at the ray of solutions $\{p : p = \lambda p^*, \lambda > 0\}$. Hence we can write $p^* \cdot z \geq \sigma \|z\|^2$ equivalently as $\|p^*/(2\sigma) - z\| \leq \|p^*/(2\sigma)\|$.

The way in which the parameter μ_t is updated guarantees that the norm of the scaled excess demand, $\mu_t z$, is bounded by the constant M. As a result, the scaled excess demand is for all $p > 0$ inside a ball centered at the ray of solutions, as illustrated in Fig. 2, where $\sigma = 1/2$ and $\lambda = 1$.
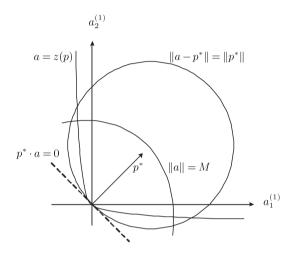

Figure 2: Illustration of the convergence conditions

Theorem 1 *Let z have the properties P1–P5 and satisfy the conditions C1–C2. Then the process of Eq. 11 converges to an equilibrium for any $p(0) > 0$. If there is a unique ray of equilibria, then there exists an $N \geq 0$ such that the convergence is monotonical when $t \geq N$.*

The monotonical convergence of the sequence $\{p(t)\}_t$ to $\widetilde{p}$ means that $\|p(t) - \widetilde{p}\| \to 0$, when $t \to \infty$, and if $p(t) \neq \widetilde{p}$, then $\|p(t+1) - \widetilde{p}\| < \|p(t) - \widetilde{p}\|$. The following lemmas are used in the proof of Theorem 1 which can be found in the appendix. Here we let $B(p^*, \varsigma)$ denote the closed ball with radius $\varsigma > 0$ and centered at p^*, i.e., $B(p^*, \varsigma) = \{a \in \mathbb{R}^n : \|a - p^*\| \leq \varsigma\}$. The proofs of Lemmas 1–3 are in the appendix.

Lemma 1 *Let the continuous mapping $z : B(p^*, r) \to \mathbb{R}^n$ satisfy Walras' law for all $p \in B(p^*, r)$, and let the inequality $p^* \cdot z(p) \geq \|z(p)\|^2$ hold for all $p \in B(p^*, r)$. If $p(0) \in B(p^*, r)$ and $\mu_t \leq 1$ for all t, then the iterative process $p(t+1) = p(t) + \mu_t \cdot z(p(t))$ converges. When $0 < \mu_L \leq \mu_t$, the process converges to the solution of $z(p) = 0$.*

Note that Lemma 1 does not assume that p^* solves $z(p) = 0$. It can be shown, however, that under the conditions of Lemma 1 vector p^* is a solution. The following lemma shows that convergence is monotonical when there is a unique ray of equilibria.

Lemma 2 *Let z satisfy the same conditions as in Lemma 1 and let the iteration $p(t+1) = p(t) + \mu z(p(t))$, $\mu > 0$, converge to a solution $\widetilde{p}$ for which there exists $\alpha > 0$ such that*

$$\|z(p)\|^2 \leq 2\alpha z(p) \cdot \widetilde{p} \tag{12}$$

for all $p \in B(p^*, r)$. Then the convergence is monotonical.

Lemma 3 *If z has the properties P1, P3–P5, and satisfies C1–C2, then there exists $\sigma > 0$ such that $p^* \cdot \hat{z}(p) \geq \sigma \|\hat{z}(p)\|^2$ for all $p > 0$, where*

$$\hat{z}(p) = \begin{cases} Mz(p)/\|z(p)\| & if \ \|z(p)\| \geq M, \\ z(p) & otherwise. \end{cases}$$

Theorem 2 *When there is a unique equilibrium for the model of bandwidth allocation, the equilibrium prices solve the bandwidth optimization problem, i.e., the solution is globally Pareto optimal.*

Proof. Assume that each path uses the process of Eq. 11 to update its price. The best price for a path is calculated from $\sum_{k=1}^{n_j} a_k^{(j)} = d^{(j)}$. At equilibrium, this is the condition that is satisfied. Furthermore, the links use Eq. 5 to calculate $a_{ik}^{(j)}$, which guarantees local optimality of the solution and the feasibility of the condition $\sum_{j=1}^{J} \sum_{k=1}^{n_j} a_{ik}^{(j)}(t) \leq r_i(t)$, $i = \overline{1, m}$. Uniqueness of the equilibrium guarantees that the locally feasible optimum solution is also the global optimum. Under these conditions, the outcome of the proposed approach is a solution to the bandwidth optimization problem. $\qquad\square$

Pareto optimality is a relevant criterion in a multiobjective problem setting. At Pareto optimum, we can find no other feasible solution that can improve some objectives without depriving other objectives. Theorem 2 states that, if the equilibrium is unique, then the outcome of the proposed approach is the optimal solution to the problem of maximizing individual revenues. If there are multiple equilibria, however, the resulting bandwidth allocation a is only locally optimal.

From Theorem 2, if a solution a is in equilibrium when the price vector is p, a is an optimal solution, which implies that every link can obtain as large a profit as possible and so can the network for the total profit. As a result, individual optimization (which implies fairness) is consistent with global optimization and contributes to the efficiency of the latter.

Generally speaking, the utility function should satisfy the two conditions that (1) u_i is a concave function, and (2) u_i is monotonic.

In this paper, the utility function is defined by

$$u_i(t) = \exp\left[-\sum_{j=1}^{J}\sum_{k=1}^{n_j} p_k^{(j)}(t) \cdot a_k^{(j)}(t) \cdot x_{ik}^{(j)}(t)\right];$$

$$u_i'(t) = -p_k^{(j)}(t) \cdot \exp\left[-\sum_{j=1}^{J}\sum_{k=1}^{n_j} p_k^{(j)}(t) \cdot a_k^{(j)}(t) \cdot x_{ik}^{(j)}(t)\right] \leq 0;$$

$$u_i''(t) = \left[p_k^{(j)}(t)\right]^2 \cdot \exp\left[-\sum_{j=1}^{J}\sum_{k=1}^{n_j} p_k^{(j)}(t) \cdot a_k^{(j)}(t) \cdot x_{ik}^{(j)}(t)\right] \geq 0.$$

When $p_k^{(j)}(t) \neq 0$, $u_i'(t) < 0$ and $u_i''(t) > 0$. When $p_k^{(j)}(t) \neq 0$, u_i is monotonic and concave.

The properties mentioned above have a deeper meaning. u_i being a concave function implies that the improvement of the satisfaction level for bandwidth between two end nodes should be realized by means of balancing the bandwidth among several links. u_i being monotonic implies that the more links there are and the more bandwidth each link has, the better.

6.2   Convergence rate analysis

Theorem 3 *The iteration process of Eq. 11 converges to a solution p^* in 2-rank.*

Proof. Let x_t denote $p(t)$; then x_{t+1} denotes $p(t+1)$, and x^* denotes p^*. Supposing that $x_{t+1} = f(x_t)$, the iteration process $p(t+1) = p(t) + \mu_t \cdot z(p(t))$ becomes

$$f(x) = x + \mu_t \cdot z(x) \tag{13}$$

And we also have $f(x^*) = x^* + \mu_t \cdot z(x^*)$. Because $z(x^*) = 0$, we have $f(x^*) = x^*$.

Since $z_j(p(t)) = \sum_{k=1}^{n_j} a_k^{(j)}(t) - d^{(j)}(t)$, we have

$$\mathbf{a}_{ik}^{(j)}(t+1) = \mathbf{a}_{ik}^{(j)}(t) + \triangle\mathbf{a}_{ik}^{(j)}(t);$$

$$\triangle\mathbf{a}_{ik}^{(j)}(t) = u_i(\lambda_1\mathbf{p}_k^{(j)} + \lambda_3 w_i) - \lambda_2 - 2\lambda_4\beta_1|\sum_{j=1}^{p}\sum_{k=1}^{n_j}\mathbf{a}_k^{(j)} - \mathbf{r}_i| - 2\lambda_4\beta_2|\sum_{k=1}^{n_j}\min_i\mathbf{a}_{ik}^{(j)} - \mathbf{d}^{(j)}|.$$

We can obtain $z'(x^*) = A \neq 1$ and $A \neq 0$. $z(x)$ is continuously differentiable for two times. From the Taylor's equation, when x sufficiently approaches x^*, we get

$$z(x) = z(x^*) + A(x - x^*) + 0.5z''(\xi)(x - x^*)^2.$$

Let $x^* = 0$, we obtain

$$z(x) = Ax + O(x^2).$$

From Eq. 13 and $A \neq 1$, we get

$$f(x) = x + \mu_t \cdot [Ax + O(x^2)] = x + A\mu_t \cdot x + O(x^2).$$

Therefore

$$\lim_{t\to\infty}\frac{x_{t+1} - x^*}{(x_t - x^*)^n} = c \Rightarrow \lim_{t\to\infty}\frac{x + A\mu_t \cdot x + O(x^2) - x^*}{(x - x^*)^n} = c \Rightarrow n = 2.$$

So the iteration process of Eq. 11 converges to a solution p^* in 2-rank.

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7   The EGPM Algorithm

Here we present the EGPM algorithm which consists of a component for distributed parallel bandwidth allocation and a component for distributed asynchronous price-demands modulation.

0.   **Input:**

$r_i, \varsigma_j, d^{(j)}, x_{ik}^{(j)}$

1.   **Initialization:**

$t \leftarrow 0$

$\triangle t, \theta, \varepsilon, \beta_1, \beta_2, a_{ik}^{(j)}(t), p_k^{(j)}(t)$

2. $a_k^{(j)}(t) \leftarrow \min_i a_{ik}^{(j)}(t)$

$z_j \leftarrow \sum_{k=1}^{n_j} a_k^{(j)}(t) - d^{(j)}(t)$

Compute $v_i(t)$ according to Eq. 6

3.   **while** $(\mid z_j \mid > \varsigma_j$ and $v_i(t) \neq 0)$ **do**

3.1) $p_k^{(j)}(t + \triangle t) \leftarrow p_k^{(j)}(t) + z_j(t) \cdot \theta$

   if $(p_k^{(j)}(t + \triangle t) \leq 0)$

   $\theta \leftarrow \theta/2$

   $p_k^{(j)}(t + \triangle t) \leftarrow p_k^{(j)}(t) + z_j(t) \cdot \theta$

$d^{(j)}(t + \triangle t) \leftarrow p_k^{(j)}(t) \cdot d^{(j)}(t)/p_k^{(j)}(t + \triangle t)$

$t \leftarrow t + \triangle t$

3.2)   Compute $u_i(t)$ according to Eq. 1

Compute $v_i(t)$ according to Eq. 6

Compute $da_{ik}^{(j)}(t)/dt$ according to Eq. 5

$a_{ik}^{(j)}(t) \leftarrow a_{ik}^{(j)}(t - \triangle t) + da_{ik}^{(j)}(t)/dt$

if $(\sum_{j=1}^{l} \sum_{k=1}^{h} a_{ik}^{(j)}(t) \leq r_i$ and $\sum_{j=1}^{l} \sum_{k=1}^{h+1} a_{ik}^{(j)}(t) > r_i)$

   $a_{ik}^{(j)}(t) = 0$

$a_k^{(j)}(t) \leftarrow \min_i a_{ik}^{(j)}(t)$

$z_j \leftarrow \sum_{k=1}^{n_j} a_k^{(j)}(t) - d^{(j)}(t)$

3.3)   **if** $(\mid z_j(t) \mid \geq \mid z_j(t - \triangle t) \mid)$

   $\theta \leftarrow \theta/2$

   $p_k^{(j)}(t) \leftarrow p_k^{(j)}(t - \triangle t)$

   $d^{(j)}(t) \leftarrow d^{(j)}(t - \triangle t)$

ε is a parameter to be tuned in the implementation, which affects directly the rate of convergence. θ is the price modulation rate. ς_j is the desired threshold of z_j, which should be input as σ^2 (variance) of $d^{(j)}$, and $d^{(j)}$ should be input as μ (mean) of $d^{(j)}$. Substeps 3.1 and 3.3 together implement the price and demands modulation, and Substep 3.2 implements the bandwidth allocation policy.

The process of network bandwidth allocation comprises real-time variation of the supply and demands of bandwidth. In the network, links are appended to or deleted from, the bandwidth

demands between two end nodes arise and end, and the bandwidth supply or demands increase or decrease. The time quantum $\triangle t$ used by the iterative process is adjusted and modulated based on the speed of change of the supply and demands of bandwidth. $\triangle t$ should be made smaller when the supply and demands vary rapidly, and larger when the supply and demands vary tardily.

The $(a)_{m \times n}$ and $(p)_{1 \times n}$ can be initialized as: (1) some average values between 0 and 1, or (2) random numbers between 0 and 1. In fact, based on the experiments we have done, we found that the final results are not affected by how these variables are initialized. β_1 and β_2, the weights used in Eq. 4, can be initialized as any constants between 0 and 1.

In Substep 3.1, when $p_k^{(j)}(t + \triangle t) \leq 0$ and $p_k^{(j)}(t) > 0$, we perform "$\theta \leftarrow \theta/2$" one or more times to make $p_k^{(j)}(t + \triangle t) > 0$. In Substep 3.3, if $\mid z_j(t) \mid \geq \mid z_j(t - \triangle t) \mid$, which means the result of the t-th iteration is not better than the last iteration, we perform "$\theta \leftarrow \theta/2$". It is possible to leap over better convergence points because θ is too big and convergence speed is too fast. By making θ smaller (a half), we can avoid this. In Substep 3.2, if $(\sum\limits_{j=1}^{l} \sum\limits_{k=1}^{h} a_{ik}^{(j)}(t) \leq r_i$ and $\sum\limits_{j=1}^{l} \sum\limits_{k=1}^{h+1} a_{ik}^{(j)}(t) > r_i)$, we let all $a_{ik}^{(j)}(t) = 0$ in order to make all $a_{ik}^{(j)}$ of the i-th row of the allocation matrix to be not larger than r_i.

Since the "actual bandwidth" obtained by a path is the smallest $a_{ik}^{(j)}$ over all the links that make up the path, we let $a_k^{(j)}(t) \leftarrow \min\limits_{i} a_{ik}^{(j)}(t)$ in Substep 3.2. The individual $a_{ik}^{(j)}$'s are often larger than $a_k^{(j)}$. The wastage due to over-allocation however will not be present in the final solution as the $a_{ik}^{(j)}$'s exist only in the computing process.

The EGPM algorithm can be executed to allocate network bandwidth in parallel, by dynamically modulating price and demands of network bandwidth in an asynchronous fashion. Although the solution at equilibrium as worked out by this asynchronous modulation of price and demands is a local equilibrium solution within a small time frame, it will approach the globally optimal solution within a large time frame. What is more, the asynchronous nature of the algorithm makes the algorithm adaptable to complex and fast changing network environments.

8 Simulations and Comparisons

8.1 Bandwidth allocation by EGPM

8.1.1 The parameters

The main parameters of the EGPM algorithm are: ε, β_1, β_2, θ, m, n, J, $\lambda_1 = (\lambda_i^{(1)})_{m \times 1}$, $\lambda_2 = (\lambda_i^{(2)})_{m \times 1}$, $\lambda_3 = (\lambda_i^{(3)})_{m \times 1}$, $\lambda_4 = (\lambda_i^{(4)})_{m \times m}$. ε and θ influence the convergence speeds of bandwidth allocation and price modulation, respectively. β_1, β_2 are the weights assigned to the available bandwidth and the satisfaction degree of the channels, respectively. m, n, J are fixed given values reflecting the dimension of the problem.

Fig. 3 shows the evolutionary trajectories of 4 of 38 link particles in the force-field during problem solving. The simulation uses the following parameter values: $\varepsilon = 0.8$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $\theta = 0.5$, $m = 38$, $n = 32$, $J = 10$, $\lambda_1 = (\lambda_i^{(1)})_{m \times 1}$, $\lambda_2 = (\lambda_i^{(2)})_{m \times 1}$, $\lambda_3 = (\lambda_i^{(3)})_{m \times 1}$, $\lambda_4 = (\lambda_i^{(4)})_{m \times m}$, and $\lambda_i^{(1)}$, $\lambda_i^{(2)}$, $\lambda_i^{(3)}$, $\lambda_{iu}^{(4)}$ are chosen at random from [0,1]. The relatively fast speed of convergence can be

observed in Fig. 3. The y-axis is the dynamic state, u_i, of a particle, as defined by Eq. 1.
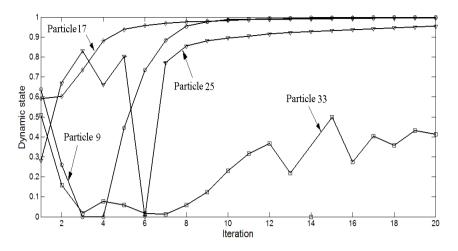

Figure 3: The kinematics and dynamic trajectories of four selected particles in the force-field during bandwidth allocation

In Fig. 3, there is a huge dive in the trajectory of Particle 25. Modulated prices and demands may make bandwidth allocation of several links huge change at next iteration. When allocated bandwidth $a_k^{(j)}$ and prices $p_k^{(j)}$ are updated, dynamic state u_i of link particles will change according to Eq. 1.

In addition, in Fig. 3, there is a decrease trend in the trajectory of Particle 33. That is, the dynamic state (representing link's revenue) of most link particles increases while that of a few of link particles decreases. Every link's revenue and network's revenue are two artificial objectives in our EGPM algorithm, which are means to optimizing the real objectives (SI, DSL, BUL, f) of the problem. Every link's revenue and network's revenue are all given attention to by EGPM. The dynamic state u_i represents every link's revenue. When a link's revenue conflicts with network's revenue, the link will give in and the link's revenue will decrease.

λ_1, λ_2, λ_3, and λ_4 can be adjusted to suit the problem's objectives. λ_1 and λ_3 control the links' autonomy and self-interest; λ_2 reflects the links' interest to improve the global state. λ_4 reflects the strength of the interaction among the links. For instance, when $\lambda_1 = 0$, $\lambda_3 = 0$ and $\lambda_2 = 1$, the links care for global profits only; when $\lambda_1 = 1$, $\lambda_3 = 1$ and $\lambda_2 = 0$, they care only for their own profits; and when $0 \leq \lambda_1, \lambda_3, \lambda_2 \leq 1$, they opt for a balance.

Table 3: Allocation performance from different degrees of link selfishness

The links' disposition	Supply > Demand	Supply ≈ Demand	Supply < Demand
Initial allocation	0.2	0.5	0.4
Global profits only	0.3	0.5	0.6
Own profits only	0.67	0.66	0.62
Global profits as well as own profits	0.76	0.72	0.8

Table 3 shows how different choices of λ values influence the performance of EGPM. The numbers in the table are the average of two indices—bandwidth utilization (BUL) and the channels' satisfaction level (DSL). The simulation comprises more than 100 problems. We can see that when

the link particles care for both global profits and their own profits, the performance of EGPM is the best.

8.1.2 Effectiveness

To see how EGPM reacts to the varying level of available bandwidth, we tested it using the bandwidth allocation problem instance in [14]. The network has 7 nodes, 11 links, 42 source-destination pairs (channels) and 84 virtual paths (two paths per channel), whose topology is shown in Fig. 4. In this problem, we set $n_j = 2$ (for $j = \overline{1, 42}$), the same as in [14]. Table 4 shows the channels and their bandwidth demands.
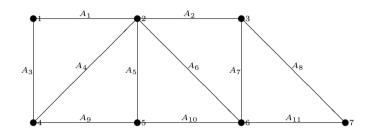

Figure 4: A 7-node network

Table 4: The 42 channels and their bandwidth demands

Channel $T^{(j)}$	1:①-②	2:①-③	3:①-④	4:①-⑤	5:①-⑥	6:①-⑦	7:②-①	8:②-③	9:②-④	10:②-⑤	11:②-⑥	12:②-⑦	13:③-①	14:③-②
$d^{(j)}$: μ	110	80	120	70	95	115	120	80	100	130	100	85	100	50
$d^{(j)}$: σ^2	50	25	35	30	40	20	40	45	30	50	65	16	70	55

Channel	15:③-④	16:③-⑤	17:③-⑥	18:③-⑦	19:④-①	20:④-②	21:④-③	22:④-⑤	23:④-⑥	24:④-⑦	25:⑤-①	26:⑤-②	27:⑤-③	28:⑤-④
$d^{(j)}$: μ	110	80	55	145	60	90	100	100	130	115	60	70	50	120
$d^{(j)}$: σ^2	35	60	40	20	40	45	30	50	65	10	55	40	50	10

Channel	29:⑤-⑥	30:⑤-⑦	31:⑥-①	32:⑥-②	33:⑥-③	34:⑥-④	35:⑥-⑤	36:⑥-⑦	37:⑦-①	38:⑦-②	39:⑦-③	40:⑦-④	41:⑦-⑤	42:⑦-⑥
$d^{(j)}$: μ	90	100	80	130	50	70	130	115	50	30	150	90	110	105
$d^{(j)}$: σ^2	20	60	30	65	60	30	45	60	10	25	80	30	60	40

For the links' capacity (r_i), two types of cases are considered: limited capacity ($r_i = 240$ Mbits/sec and $r_i = 480$ Mbits/sec), and abundant capacity ($r_i = 720$ Mbits/sec and $r_i = 960$ Mbits/sec). We also include a "heterogeneous" case where link capacity differs from link to link.

Table 5: Allocation results for different link capacities

	(SI=Yes)/J	Average BUL	Fairness f	Average DSL
$r_i = 240 Mbits/s$ (extremely scarce)	32/42	0.9134	0.4304	0.3619
$r_i = 480 Mbits/s$ (scarce)	42/42	0.9646	0.7385	0.7449
$r_i = 720 Mbits/s$ (plentiful)	42/42	0.9084	0.8601	1.0000
$r_i = 960 Mbits/s$ (extremely plentiful)	42/42	0.7340	0.8972	1.0000
r_i=[560,480,320,480,560,480,400,480,560,480,480](Mbits/s) (heterogeneous)	42/42	0.8036	0.7555	0.5233

The results are summarized in Table 5. From the table, we observe the following.

- The demands of all source-destination pairs (channels) are satisfied (according to SI, the satisfaction indicator) in all cases except the case of extreme scarce capacity (240 Mbits/sec) of which 75% of the demands are satisfied.

- The average BUL, average DSL, and fairness are reasonably high in all five cases, especially in the cases of scarce capacity.

- When the link capacity is plentiful, the satisfaction level (DSL) of all source-destination pairs (channels) can reach 100%.

- When network capacity decreases rapidly, such as from 960 to 480 Mbits/sec, the performance indices do not seem to follow suit.

- Whether the network is homogeneous or heterogeneous in terms of link capacities is not a key factor for SI, BUL, DSL and fairness.

8.1.3 Efficiency and parallelism

In this section, we show the actual times used to compute the solutions, on a cluster. The machines of the cluster each has a Pentium 4 2.0 GHz CPU with 512 Kbytes of L2 cache and 512 Mbytes of DDR SDRAM, and they are interconnected via Fast Ethernet.

The network of the problem is such that if there are N nodes in the network, the number of source-destination pairs (channels) is $N \times (N-1)$. We use the number of nodes and channels to represent the "scale" of the problem. Convergence times and iterations for different scales using EGPM are shown in Table 6. The data come from using 1, 4, and 8 computing nodes of the cluster. "Increase" represents the percent of increase of the current scale from the previous scale.

Table 6: Convergence times and iterations of EGPM

Scale		sequential				4 parallel nodes				8 parallel nodes			
nodes	channels	iterations	increase	time (s)	increase	iterations	increase	time (s)	increase	iterations	increase	time	increase
4	12	103		0.008		134		0.175		112		0.301	
5	20	112	8.7%	0.018	125.0%	150	11.9%	0.2	14.3%	135	20.5%	0.429	42.5%
6	30	130	16.1%	0.029	61.1%	171	14.0%	0.245	22.5%	164	21.5%	0.5	16.6%
7	42	172	32.3%	0.076	162.1%	221	29.2%	0.363	48.2%	205	25.0%	0.69	38.0%
8	56	230	33.7%	0.141	85.5%	297	34.4%	0.386	6.3%	304	48.3%	0.87	26.1%
9	72	363	57.8%	0.317	124.8%	446	50.2%	0.551	42.7%	397	30.6%	1.043	19.9%
10	90	544	49.9%	0.69	117.7%	563	26.2%	0.843	53.0%	622	56.7%	1.564	50.0%
11	110	734	34.9%	1.235	79.0%	797	41.6%	1.324	57.1%	796	28.0%	2.374	51.8%
12	132	979	33.4%	2.101	70.1%	968	21.5%	1.611	21.7%	1029	29.3%	2.571	8.3%
13	156	1358	38.7%	3.782	80.0%	1360	40.5%	2.51	55.8%	1370	33.1%	3.847	49.6%
14	182	1618	19.1%	5.616	48.5%	1934	42.2%	4.21	67.7%	1857	35.5%	5.327	38.5%
15	210	2260	39.7%	9.821	74.9%	2213	14.4%	5.11	21.4%	2724	46.7%	7.708	44.7%
16	240	2865	26.8%	15.12	54.0%	3221	45.5%	8.58	67.9%	2806	3.0%	6.567	-14.8%
17	272	3627	26.6%	23.578	55.9%	4159	29.1%	13.85	61.4%	3928	40.0%	12.801	94.9%
18	306	4941	36.2%	37.534	59.2%	4885	17.5%	17.37	25.4%	4298	9.4%	13.898	8.6%
19	342	5442	10.1%	50.188	33.7%	5342	9.4%	21.17	21.9%	5328	24.0%	18.618	34.0%
20	380	7528	38.3%	78.089	55.6%	7547	41.3%	31.913	50.7%	6897	29.4%	25.627	37.6%

As shown in Fig. 5, the convergence time of the sequential version increases exponentially with the scale, which is similar to all other exact methods. When parallelized, the convergence time drops significantly across the larger scale problems. The times for the smaller scale problems are dominated by the message exchange times, and the sequential version appears to be more efficient in that range.

The number of iterations for all the cases, however, is more or less the same for either the sequential or the parallel versions.
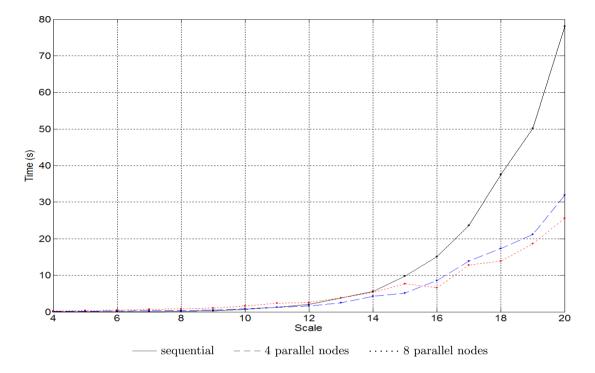

Figure 5: Convergence times of EGPM using one or more processors

The EGPM algorithm has the ability to carry out the allocation in a distributed manner with little communication overheads. The bandwidth allocation variables, $a_{ik}^{(j)}$ (see Table 1), can be computed and updated in parallel without any information exchange, which is the foundation of EGPM's parallelism. There is some information exchange between entries in a column (in Table 1) to compute and update the actual bandwidth allocation to the paths $(a_k^{(j)})$, the price $(p_k^{(j)})$ and the demand variable $(d^{(j)}(t > 0))$; as well as some information exchange between entries in a row in order to satisfy the links' capacity restrictions. These exchanges of information in a row or column are responsible for the less than ideal speedup that can be achieved using parallel implementation.

8.2   Comparison with GPM

Fig. 6 shows an example highlighting the difference between demand and allocation in a *dynamic* environment where the bandwidth demands may change with time. In Fig. 6(a), the curved line corresponds to the demand and the rectangles the supply (allocated bandwidth) over a period of time. T in the figure denotes the length of a run of the GPM algorithm to calculate the equilibrium allocation. Because the allocation will not change until the the equilibrium is reached in a run of the GPM algorithm, the allocation stays unchanged between any two consecutive runs in the figure.

The EGPM algorithm allows links to trade resources to settle excess demand at every iteration. In Fig. 6(b), the two curved lines represent the change of the demand and the allocated bandwidth, respectively. For both GPM and EGPM, the shaded regions represent the "excess demand function"
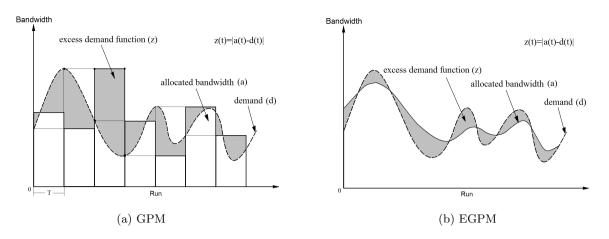
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(a) GPM

(b) EGPM

Figure 6: Excess demands in a dynamic environment

(i.e., over- or under-allocation). The shaded regions of both GPM and EGPM shrink in size when the interval of environmental changes becomes longer. When that interval is small (i.e., the changes are frequent) the excess demand function of EGPM is smaller and EGPM is preferable. However, because EGPM needs to renew the allocation at every iteration step in a run before reaching equilibrium, it takes a longer time than GPM. Hence, when the interval of change is excessively long, GPM might have an advantage.

Next, we compare the satisfaction level. There is a direct relationship between the satisfaction level and the excess demand function: when the level is low, the excess demand function is large and the allocation is not preferable, and if the level is high, the excess demand function is small and the allocation is well done. The level reaches 1 if and when the allocation matches exactly the demand.
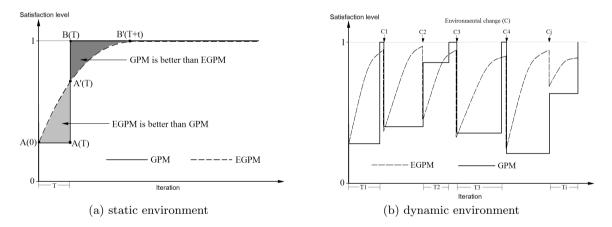

(a) static environment

(b) dynamic environment

Figure 7: Satisfaction level of demand in static environment and in dynamic environment

In a static environment where the demands of the paths do not change, the satisfaction level changes in a fashion which is as shown in Fig. 7(a). The figure shows that the GPM algorithm takes shorter time (T) to complete its run than the iterative EGPM algorithm. It can be easily seen that, for a static environment, if T is small, GPM is preferable, and if T is large, EGPM is preferable.

Fig. 7(b) shows an example of a changing satisfaction level in a dynamic environment. As can

be seen in the figure, because the environment changes soon after or even before GPM reaches equilibrium, the allocation by GPM is effective only for a short pinch of time. On the other hand, the allocation by EGPM adapts to the changing environment more responsively.

8.3   Comparison with genetic algorithm

Here we compare the experimental results of EGPM with that of a genetic algorithm for the same problem of bandwidth allocation which can be found in [14]. Table 7 shows the results for the 7-node network with links running at 480 Mbits/sec. The experimental results of GA come from [14] and those of EGPM come from our own experiments.

Table 7: Traffic demands and bandwidth allocations for the 7-node network using EGPM vs. GA for the case of capacity = 480 Mbits/s

Channel $T^{(j)}$	1:①-②	2:①-③	3:①-④	4:①-⑤	5:①-⑥	6:①-⑦	7:②-①	8:②-③	9:②-④	10:②-⑤	11:②-⑥	12:②-⑦	13:③-①	14:③-②
$d^{(j)}$: μ	110	80	120	70	95	115	120	80	100	130	100	85	100	50
$d^{(j)}$: σ^2	50	25	35	30	40	20	40	45	30	50	65	16	70	55
EGPM BA	110	55	120	70	55	95	120	36	70	107	65	69	70	24
GA BA	143.47	42.02	185.94	57.55	35.28	43.29	148	59.62	118.45	164.31	129.81	61.42	19.14	47.59
EGPM SI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
GA SI	Yes	No	Yes	Yes	No	No	Yes	Yes	Yes	Yes	Yes	No	No	Yes
EGPM DSL	1	0.6875	1	1	0.5789	0.8261	1	0.45	0.7	0.8231	0.65	0.8118	0.7	0.48
GA DSL	1	0.5252	1	0.8221	0.3714	0.3764	1	0.7452	1	1	1	0.7226	0.1914	0.9518

Channel $T^{(j)}$	15:③-④	16:③-⑤	17:③-⑥	18:③-⑦	19:④-①	20:④-②	21:④-③	22:④-⑤	23:④-⑥	24:④-⑦	25:⑤-①	26:⑤-②	27:⑤-③	28:⑤-④
$d^{(j)}$: μ	110	80	55	145	60	90	100	100	130	115	60	70	50	120
$d^{(j)}$: σ^2	35	60	40	20	40	45	30	50	65	10	55	40	50	10
EGPM BA	75	80	55	125	60	90	70	50	65	105	17	70	34	110
GA BA	41.91	40.84	86.40	156.09	132.43	149.83	53.84	130.66	76.39	8.20	49.91	99.68	45.88	133.09
EGPM SI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
GA SI	No	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	Yes	Yes	Yes	Yes
EGPM DSL	0.6818	1	1	0.8621	1	1	0.7	0.5	0.5	0.9130	0.2833	1	0.68	0.9167
GA DSL	0.381	0.5105	1	1	1	1	0.5384	1	0.5876	0.0713	0.8318	1	0.9176	1

Channel $T^{(j)}$	29:⑤-⑥	30:⑤-⑦	31:⑥-①	32:⑥-②	33:⑥-③	34:⑥-④	35:⑥-⑤	36:⑥-⑦	37:⑦-①	38:⑦-②	39:⑦-③	40:⑦-④	41:⑦-⑤	42:⑦-⑥
$d^{(j)}$: μ	90	100	80	130	50	70	130	115	50	30	150	90	110	105
$d^{(j)}$: σ^2	20	60	30	65	60	30	45	60	10	25	80	30	60	40
EGPM BA	70	56	50	65	50	40	85	72	40	25	69	61	50	105
GA BA	93.96	57.83	83.48	153.34	97.72	50.79	131.47	132.25	18.82	27.7	165.03	30.07	72.18	139.24
EGPM SI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
GA SI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	Yes	Yes
EGPM DSL	0.7778	0.56	0.625	0.5	1	0.5714	0.6538	0.6261	0.8	0.8333	0.46	0.6778	0.4545	1
GA DSL	1	0.5783	1	1	1	0.7256	1	1	0.3764	0.9233	1	0.3341	0.6562	1

BA: Bandwidth allocation;   SI: satisfaction indicator;   DSL: Demand satisfaction level.

As shown in Table 7, 10 (out of 42) channels failed to be satisfied using GA, whereas the EGPM algorithm satisfies all the channel demands. The average demand satisfaction levels of EGPM and GA are 0.7449 and 0.7890, respectively; EGPM is slightly below because it gives preference to the SI index. In fact, EGPM tries to maximize all three objectives: DSL, BUL and fairness.

As shown in Table 8, EGPM is several orders of magnitude faster than GA in computing the solutions. And the convergence time of EGPM increases more steadily with the problem scale than GA.

Other than genetic algorithm, the comparison Lagrangrian multiplier approach, the max-min

Table 8: Convergence times and iterations of EGPM vs. GA

Scale		EGPM: 4 parallel nodes				EGPM: sequential				GA			
nodes	channels	iterations	increase	time	increase	iterations	increase	time	increase	iterations	increase	time	increase
4	12	134		0.175		103		0.008		2000		1353	
5	20	150	11.9%	0.2	14.3%	112	8.7%	0.018	125.0%	2000	0.0%	3024	123.5%
6	30	171	14.0%	0.245	22.5%	130	16.1%	0.029	61.1%	2000	0.0%	4761	57.4%
7	42	221	29.2%	0.363	48.2%	172	32.3%	0.076	162.1%	10000	400.00%	36971	676.5%

algorithm, and the ant colony algorithm are also popular choices for solving the bandwidth allocation problem. We give a succinct comparison of these different approaches with EGPM/GPM in Table 9.

Table 9: EGPM vs. GPM and other well-known algorithms

	EGPM	GPM	Lagrangrian multiplier	Ant colony	Max-min	GA
Flow control	Decentralized /adaptive	Decentralized /adaptive	Centralized /static	Decentralized /adaptive	Centralized /static	Decentralized /adaptive
Adaptive to topology changes	Fast	Middle	Slow	Middle	Fast	Middle
Routing overhead	Low	Low	High	Low	Low	Low
Routing preference	By Excess demand function	By Hybrid energy functions	By Utility function	By Pheromone laid	By Transmission time/delay	By Chromosomes
Information exchange	Can by piggybacked in data packets	Can by piggybacked in data packets	Separate routing entries transmission	Can by piggybacked in data packets	Separate routing entries transmission	Can by piggybacked in data packets

9 Conclusion

In this paper, an approach based on the economic generalized particle model (EGPM) for intelligent allocation of network bandwidth is proposed. The approach transforms the complicated network bandwidth allocation problem into efficient, parallel allocation of network bandwidth. The mechanism is based on asynchronous modulation of prices of network bandwidth. The special features of the proposed model and its algorithm include: (1) high parallelism and real-time computational performance; (2) a market-oriented mechanism between the demands and service; (3) the microscopic characterization of an individual link and the macroscopic properties of the whole network being combined to achieve both fairness and efficiency; and (4) better adaptation to the real-time dynamic network environment.

The EGPM method can be applied to other optimization problems, such as resource allocation in grid, cache optimization in CDN, etc. When solving grid computing problems by the EGPM algorithm, because the problem model of grid will be simpler than that of bandwidth allocation, only two of the ordinal numbers i, j, k in the of bandwidth allocation model will be needed for the model of grid; moreover, the logic variable $x_{ik}^{(j)}$ will always be equal to 1, and can be left out.

In the future, we plan to improve EGPM so that it will become a generic algorithm for NP-hard combinatorial optimization problems, such as the traveling salesman problem (TSP), the quadratic assignment problem (QAP), etc., as well as for multi-objective optimization problems. We hope to develop the EGPM algorithm by a new approach, giving it the ability to deal with difficult problems that feature non-homogeneous, noisy, incomplete or obscured information, constrained resources, and massive processing of large amounts of data.

Here are some specific suggestions on future work.

1. The choices of coefficients in the EGPM algorithm and their impact should be further studied.

2. Communication times among computing entities should be reduced to improve the parallel execution of EGPM.

3. The present approach treats all demands to be equally important and tries to maximize the overall satisfaction. To support true QoS, priority classes with preferential treatment by the allocation mechanism need to be introduced. How to prove that the guarantee of a certain QoS can be met if certain conditions are satisfied "in theory" will be a key issue.

4. Our work in this paper is predominantly theoretical and based on a number of simplifying assumptions. Between this and a practical solution for real-life deployment, much work needs to be done.

5. Capacity planning including priority classes and reservation is not addressed in the current design of EGPM, which however is very important in practical bandwidth allocation.

Appendix: Convergence Proofs

Proof of Lemma 1. Observe that when $\mu_t \leq 1$,

$$\mu_t \cdot p^* \cdot z(p) \geq \|\mu_t \cdot z(p)\|^2. \tag{14}$$

This can be seen by multiplying both sides of $p^* \cdot z(p) \geq \|z(p)\|^2$ with μ_t^2 and noticing that $\mu_t^2 \cdot p^* \cdot z(p) \leq \mu_t \cdot p^* \cdot z(p)$ because $\mu_t \leq 1$. From Eq. 14 and Walras' law we have

$$\|p(t+1) - p^*\|^2 = \|p(t) + \mu_t z(p(t)) - p^*\|^2 = \|\mu_t z(p(t))\|^2 - 2\mu_t z(p(t)) \cdot p^* + \|p(t) - p^*\|^2$$
$$\leq \|p(t) - p^*\|^2.$$

Note that $p(t)$ belongs to $B(p^*, r)$ for all $t = 0, 1, \ldots$, when $p(0) \in B(p^*, r)$. Therefore, the sequence $\{\|p(t) - p^*\|\}_t$ converges, and as a result, the sequence $\{\|p(t)\|\}_t$ is bounded. From Walras' law it follows that

$$\|p(t)\|^2 = \|p(0)\|^2 + \sum_{i=0}^{t-1} \mu_i^2 \|z(p(i))\|^2,$$

so that $\{\|p(t)\|\}_t$ is a growing and bounded sequence and hence convergent. The iteration formula yields

$$p(t) = p(0) + \sum_{i=0}^{t-1} \mu_i z(p(i)).$$

Hence, $\|p(0) + \sum_{i=0}^{t-1} \mu_i z(p(i))\|$ converges, too. From the triangular inequality we get

$$\|p(0) + \sum_{i=0}^{t+l} \mu_i z(p(i))\| \geq \left| \|p(0) + \sum_{i=0}^{t} \mu_i z(p(i))\| - \|\sum_{i=t+1}^{t+l} \mu_i z(p(i))\| \right|$$

and

$$\|p(t + l) - p(t)\| = \|\sum_{i=t+1}^{t+l} \mu_i z(p(i))\| \to 0, \tag{15}$$

when $t \to \infty$ and $l \geq 1$. Thus, $\{p(t)\}_t$ is a Cauchy sequence and hence convergent. Let $\widetilde{p}$ denote the limit point of this Cauchy sequence.

We now show that when $0 < \mu_L \leq \mu_t$ the sequence $\{p(t)\}_t$ converges to the solution of $z(p) = 0$. By setting $l = 1$ it follows from Eq. 15 that $\mu_t \|z(p(t))\| \to 0$. Because $\mu_L \|z(p(t))\| \leq \mu_t \|z(p(t))\|$ holds and z is continuous, $\widetilde{p}$ is a solution of $z(p) = 0$. □

Proof of Lemma 2. If Eq. 12 holds for $\alpha > 0$ then it holds for any $\widetilde{\alpha} > \alpha$. Specifically, we can choose $\widetilde{\alpha} > 0$ such that Eq. 12 holds for $p^* = \widetilde{\alpha}\widetilde{p} - 2\widetilde{p}$ instead of $\alpha\widetilde{p}$. Moreover we can take α such that $\|z(p)\|^2 < 2\alpha z(p)\widetilde{p}$ if p is not a solution. We can deduce that $\|p(t+1) - p^*\|^2 < \|p(t) - p^*\|^2$, and $\|p(t+1) - \alpha\widetilde{p}\|^2 < \|p(t) - \alpha\widetilde{p}\|^2$ when $p(t)$ is not a solution. By the parallelogram law we get

$$\|p(t) - \alpha\widetilde{p}\|^2 + \|p(t) - p^*\|^2 = 2\|p(t) - \widetilde{p}\|^2 + 2(\alpha - 1)\|\widetilde{p}\|^2.$$

By rearranging the terms we have

$$\begin{aligned}
2\|p(t) - \widetilde{p}\|^2 &= 2(\alpha - 1)\|\widetilde{p}\|^2 - \|p(t) - \alpha\widetilde{p}\|^2 - \|p(t) - p^*\|^2 \\
&> 2(\alpha - 1)\|\widetilde{p}\|^2 - \|p(t+1) - \alpha\widetilde{p}\|^2 - \|p(t+1) - p^*\|^2 \\
&= 2\|p(t+1) - p^*\|
\end{aligned}$$

and hence $\{p(t)\}_t$ converges monotonically to $\widetilde{p}$. □

Proof of Lemma 3. Let z satisfy C2 on $E_{\bar{\varsigma}} = \{p \in \mathbb{R}_+^n : \|z(p)\| < \bar{\varsigma}\}$ with constant $\bar{\sigma}$. By the homogeneity of excess demand we know that $\hat{z}$ obtains all its values on the unit simplex $\triangle = \{p \in \mathbb{R}_+^n : \sum_{j=1}^{J} \sum_{k=1}^{n_j} P_k^{(j)} = 1\}$. Because of P4 and P5 it can be seen that $p^* \cdot z(p(t)) \to \infty$ when $p(t) \to p$ and $JK_p \neq \emptyset$. As a result, we have

$$\lim_{p(t) \to p} p^* \cdot \hat{z}(p) > 0$$

when $JK_p \neq \emptyset$. From this property, the continuity property, and C1, it follows that there exists $\delta > 0$ such that $p^* \cdot \hat{z}(p) \geq \delta$ for all $p \in \triangle \backslash S$, where $S = \{p \in \triangle : p_k^{(j)} > \varsigma', \forall j = \overline{1, J}, k = \overline{1, n_p}, \varsigma' > 0$ is chosen such that $E_{\bar{\varsigma}} \cap \triangle \subset S$.

Clearly, the infimum of $p^* \cdot \hat{z}(p)$ over $S \backslash E_{\bar{\varsigma}}$ is positive, since otherwise $\hat{z}$ would violate C1. Let $\alpha > 0$ denote this infimum. We have $p^* \hat{z}(p) \geq \min\{\delta, \alpha\}$ for all $p \in \triangle \backslash E_{\bar{\varsigma}}$. Because $\|\hat{z}(p)\| \leq M$ we get $p^* \cdot \hat{z}(p) \geq \hat{\sigma}\|\hat{z}(p)\|^2$ for all $p \in \triangle \backslash E_{\bar{\varsigma}}$ by choosing $\hat{\sigma} < \min\{\delta, \alpha\} \backslash M^2$. The result follows by setting $\sigma = \min\{\bar{\sigma}, \hat{\sigma}\}$. □

Proof of Theorem 1. The process of Eq. 11 can be expressed as

$$p(t + 1) = p(t) + \lambda_t \hat{z}(p(t))$$

where $\lambda_t = \min\{\gamma_t, 1\}$, and $\hat{z}$ is as defined in Lemma 3. When z has the properties P1–P4, so does $\hat{z}$, and P5 implies that $\hat{z}$ has the following property.

P5': $\lim_{p(t) \to p} \left[\max_{j,k \in JK_p} \hat{z}_j(p(t)) \right] > 0$, when $p \neq 0$, and $JK_p = \{j, k : p_k^{(j)} = 0\} \neq \emptyset$.

Moreover, we know from Lemma 3 that $p^* \cdot \hat{z}(p) \geq \sigma\|\hat{z}(p)\|^2$ holds for all $p > 0$ when z satisfies C1–C2. Due to homogeneity p^* can be replaced by p^*/σ in C1 and C2; hence, without loss of

generality we may suppose that $\sigma = 1$. It follows then from Lemma 1 that the iteration converges.

Let us show that due to P5' the parameter λ_t is updated only finitely many times, from which it follows that λ_t has a positive lower bound that is required by Lemma 1 to obtain convergence to a solution of Eq. 7. On the contrary, suppose that $\lambda_t \to 0$. It then follows that $p_t \to p$, where some components of p are zero, i.e., $JK_p \neq \emptyset$. Namely, if the sequence $\{p(t)\}_t$, $p(t) > 0$ for all t, converges to some point in $\mathbb{R}^n_+$, then the parameters γ_t and λ_t would be updated only finitely many times and λ_t could not converge to zero. Hence, we have $p_k^{(j)}(t) \to 0$ for all $j, k \in JK_p$, i.e., for all $\varsigma > 0$ we find $N_\varsigma \geq 0$ such that $p_k^{(j)}(t) < \varsigma$, when $t \geq N_\varsigma$ and $j, k \in JK_p$. Thus, by the continuity property and P5' there are $l, h \in JK_p$ and $N \geq 0$ such that $p_h^{(l)} \to 0$, and $z_l(p(t)) > 0$ for all $t \geq N$. Now we get from the iteration formula that $p_h^{(l)}(t+1) > p_h^{(l)}(t)$ for all $t \geq N$, which contradicts $p_h^{(l)} \to 0$. Hence, λ_t is updated finitely many times, i.e., there exists $\bar{N}$ such that $\lambda_t = \lambda_{\bar{N}}$ for all $t \geq \bar{N}$. Convergence to a solution of Eq. 7 follows from Lemma 1.

Let us assume that there is a unique ray of solutions for Eq. 7. Then the process of Eq. 11 converges to a point $\tilde{p} = \beta p^*$, where $\beta > 0$. From Lemma 3 we see that there exists $\alpha > 0$ such that for $\alpha\tilde{p}$ we have $2\alpha\tilde{p} \cdot \hat{z}(p) \geq \|\hat{z}(p)\|^2$ for all $p > 0$. Lemma 2 then implies monotonical convergence.$\square$

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