Nonlinear Estimator-Based Funnel Tracking Control for A Class of Perturbed Euler Lagrange Systems

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Nonlinear Estimator-Based Funnel Tracking Control for A Class of Perturbed Euler Lagrange Systems

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Abstract

In this paper, a nonlinear estimator-based perturbation rejection funnel control method is investigated for a class of Euler-Lagrange (EL) systems to deal with the trajectory tracking problem against perturbations. To reinforce the perturbation rejection ability, perturbation estimators with nonlinear dynamics are established by employing a filtering operation, which results in asymptotic error convergence. Besides, by devising funnel variables with an exponential decaying function, a funnel control strategy is constructed to ensure tracking errors converging into a prescribed region. The tracking errors of Euler-Lagrange systems are concluded to be ultimately uniformly bounded via Lyapunov stability theory, as well as the controlling deviations are ensured to be restricted into the funnel boundary. Finally, simulations validate the effectiveness of the developed control technology.

Keywords: Nonlinear estimators, funnel control, perturbation rejection, Euler-Lagrange systems.

1 Introduction

As we know, the systems that are established by Euler-Lagrange equations of motion can be collectively referred to as Euler Lagrange (EL) systems. Therefore, many dynamic systems including wheeled mobile

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robots, pendulum systems, quadrotor aircraft, robot manipulators are reckoned as EL systems, whose have advantages of good mobility, low risk, high adaptability, high reliability and so on [1]-[4]. Among a large number of existing studies in EL systems, the trajectory tracking control of systems is one of the most popular studies to drive the system with satisfactory performance. Note that tracking dynamics of the EL system is characterized by multiple-input and multiple-output and nonlinearity, so that the operation of the EL system is sensitive to perturbations, such as parametric uncertainties and external disturbances. Therefore, for the sake of realizing high-quality working of EL systems, it is pivotal and rewarding to ensure the high-precision trajectory tracking under perturbations.

In order to deal with unknown perturbations and enhance the robustness of systems, a number of advanced control technologies have been exploited for systems by withstanding the negative impacts of perturbations. Generally, two kinds of advanced control methods for addressing perturbations have been carried out in the existing literatures, i.e., the direct compensation control methods and the indirect estimation methods. The first method directly designs control strategies owning some self-regulation and intelligent characteristic to compensate for the effects of perturbations in a feedback manner, such as adaptive compensation control methods [5, 6, 7], sliding mode control methods [8, 9], neural network-based control methods [10, 11, 12], etc. On the other hand, the second method based on indirect estimation technique in a feedforward manner have also been widely investigated by designing various disturbance observers in recent years, including sliding mode observers [13, 14, 15], extended state observers [16, 17], function approximators [18, 19], and unknown system dynamics estimators [20, 21]. To reinforce the tracking performance for quadrotors, a fixed-time sliding mode observer-based control policy was proposed in [13]. In [16], with the aid of the estimation results by using extended state observers, a super-twisting-based adaptive control method was developed to govern the trajectory tracking behaviors of quadrotors. In [18], by designing a neural network-based disturbance approximator, a robust backstepping control was exploited to recover the unknown perturbations. Based on the studies of the above observers, it seems excellent disturbance identification shall be well achieved and the robustness of the system can be effectively improved. It should be noticed that certain limitations including tedious iteration calculations in [18, 19] and unexpected chattering phenomenon in [13, 14, 15] inevitably increase computational burden and deteriorate control effects. To avoid the limitations, unknown system dynamics estimators were proposed in [20, 21] by using linear filtering calculations for available states, which a more concise design frame and a lower calculation burden can be guaranteed. However, another limitation such as assuming the differential of perturbations restricted in a know boundary should be available. Moreover, during the above control developments, the key metrics, that is, convergence time, overshoot and steady-state errors, have not been actively considered in the above studies.

It should be mentioned that many asymptotic and ultimately uniformly bounded tracking results have been achieved by using a variety of control methods (see e.g., [22]-[23]). However, the tracking results are not taken the predetermined performance into consideration. For the sake of regulating steady-state and transient profiles of systems to obey predetermined performance boundaries, funnel control methods in [24] are always exploited by defining tracking errors of systems as funnel variables so that the funnel boundaries can be devised. In some related studies, a funnel tracking control method with a low complexity was proposed in [25] for a nonlinear system by regulating tracking errors within the predefined funnel boundary. A funnel control-based feedback control strategy was developed in [26] for the hydraulic systems subject to model uncertainties, which results in a remarkable tracking performance. In [27], by introducing a novel funnel variable, a differentiator-based adaptive control scheme was implemented to achieve asymptotic convergence results of servosystems. Obviously, the tracking profiles can be governed to the predefined funnel region with a satisfied performance of overshoot, convergence time and precision by utilizing funnel control technique. Hence, it is imperative and urgent to develop a funnel control-based trajectory tracking control strategy to ensure tracking errors reducing into predetermined performance boundaries.

Inspired by the above statements, a nonlinear dynamics estimator-based perturbation rejection attitude funnel control strategy is developed to deal with the trajectory tracking of EL systems with external disturbances and parametric uncertainties. A novel nonlinear estimator is firstly investigated to estimate the unknown perturbations accurately. Then, by using the estimations, the funnel control strategy is developed to ensure the asymptotic tracking of the EL system via Lyapunov stability theory, and the tracking errors are reduced into the predetermined performance boundaries. Finally, simulations of attitude trajectory tracking of a quadrotor are given to validate the efficiency of the developed nonlinear estimator-based perturbation rejection attitude funnel control method. The highlights of our paper are generalized as follows: 1) Contrasting to the linear estimators such as unknown system dynamics estimators proposed in [20, 21], a novel nonlinear dynamics estimator is investigated to estimate the perturbations with higher estimation accuracy. 2) Comparing with the conventional controllers in [28, 29], in which ultimately uniformly bounded tracking results are delivered, a funnel control is constructed to govern the transient and steady-state profiles as a prior, such that the overshoot, convergence time and steady-state errors can be regulated within the appropriate range by predefining the funnel bound-based variables.

The arrangement of this paper is as follows. The dynamic model of the EL system is given in Section 2. The nonlinear dynamics estimator and perturbation rejection funnel control strategy of the EL system are constructed in Section 3. The simulations of attitude trajectory tracking control of a quadrotor aircraft system are conducted in Section 4 to illustrate the superior of the proposed nonlinear estimator-based perturbation rejection attitude funnel control method. Section 5 gives the conclusions of this paper.

2 Problem statement and preliminaries

Consider the dynamics of the EL system as

$$D(\eta)\ddot{\eta} + V(\eta, \dot{\eta})\dot{\eta} + G(\eta) + d_\eta = \tau_\eta, \tag{1}$$

where $\eta(t) \in R^n$ stands for the position state of the EL system, $\{D(\eta), V(\eta, \dot{\eta})\} \in R^{n\times n}$ are the inertial and Centrifugal/Coriolis force matrices of the EL system, respectively; $G(\eta) \in R^n$ is the gravity vector of the EL system; τ_η is the EL system's control input; d_η represents the disturbance.

From (1), the second-order differential equations of the EL system can be expressed by

$$\begin{cases} \dot{\eta} = \omega_\eta \\ \dot{\omega}_\eta = u + \Delta, \end{cases} \tag{2}$$

where ω_η is the velocity signal of the EL system, $u = D(\eta)^{-1}\tau_\eta$, $\Delta = -D(\eta)^{-1}(V(\eta, \dot{\eta})\omega_\eta + G(\eta) + d_\eta)$.

Assumption 1: The acceleration signal $\dot{\omega}_{\eta,i}$, $i = 1, 2, \dots, n$ of the EL system may not be measured accurately, but it is measurable with acceptable errors.

Assumption 2: The perturbation is differentiable and and it is bounded by $|\dot{\Delta}_i| \le \delta_{i1}|\dot{\omega}^*_{\eta,i}| + \delta_{i2}|\omega^*_{\eta,i}| + \delta_{i3}$, where δ_{i1}, δ_{i2}, δ_{i3} are positive constants, $\dot{\omega}^*_{\eta,i}$ and $\omega^*_{\eta,i}$ is the measured acceleration and velocity of the EL system.

Remark 1: Note that in practice, the acceleration signal $\dot{\omega}_{\eta,i}$ of the EL system can be measured by some sensors but the measured signal is indeed inaccurate due to the limitation of sensors. The acceleration can be detected suitably by $\dot{\omega}^*_{\eta,i}$ with some measurement errors comparing with $\dot{\omega}_{\eta,i}$. Thus, it seems that Assumption 2 is suitable to describe the differential of perturbation $\dot{\Delta}_i$.

This work aims to realize a nonlinear estimator-based perturbation rejection funnel control for EL systems, which is capable of precisely driving the state η to follow the pregiven reference in face of external disturbances and parametric uncertainties.

3 Main results

3.1 Nonlinear dynamic estimator design

The following filtering manipulations for the available states ω_η and u are introduced:

$$\begin{cases} k\dot{\omega}^f_\eta + \omega^f_\eta = \omega_\eta, \quad \omega^f_\eta(0) = [0, 0, 0]^T \\ k\dot{u}^f + u^f = u + \operatorname{sgn}(\zeta)\bar{k}_u, \quad u^f(0) = [0, 0, 0]^T \end{cases} \tag{3}$$

where ω_η^f and u^f denote the auxiliary filtered variables, k represents the filtering argument to be adjusted, $\bar{k}_u := [\bar{k}_{u,1}, \bar{k}_{u,2}, \ldots, \bar{k}_{u,n}]$, $\bar{k}_{u,i} = \bar{\delta}_{i1}|\dot{\omega}^*_{\eta,i}| + \bar{\delta}_{i2}|\omega_{\eta,i}| + \bar{\delta}_{i3}$, where $i = \{1, 2, \ldots, n\}$, $\bar{\delta}_{i1} \geq \delta_{i1}$, $\bar{\delta}_{i2} \geq \delta_{i2}$, $\bar{\delta}_{i3} \geq \delta_{i3}$ are positive constants, ζ is an auxiliary variable defined by

$$\zeta = \frac{1}{k}(\omega_\eta - \omega_\eta^f) - (u^f + \Delta). \tag{4}$$

Note that it is reasonable to assume that the signum function of ζ is known based on Assumption 1.

Lemma 1. Consider the auxiliary variable ζ in (4), the variable ζ is bounded and satisfies the following condition, given that the filtering constant holds $k \in (0, +\infty)$

$$\lim_{t\to\infty}[\frac{1}{k}(\omega_\eta - \omega_\eta^f) - (u^f + \Delta)] = 0. \tag{5}$$

Proof. The deviation of ζ can be expressed by

$$\begin{aligned}\dot{\zeta} =& \frac{1}{k}(\dot{\omega}_\eta - \dot{\omega}_\eta^f) - (\dot{u}^f + \dot{\Delta})\\ =& \frac{1}{k}(u + \Delta - \frac{1}{k}(\omega_\eta - \omega_\eta^f) - (\frac{u - u^f + \mathrm{sgn}(\zeta)\bar{k}_u}{k} + \dot{\Delta})\\ =& \frac{1}{k}(u^f + \Delta - \frac{1}{k}(\omega_\eta - \omega_\eta^f) - \frac{1}{k}\mathrm{sgn}(\zeta)\bar{k}_u - \dot{\Delta}\\ =& -\frac{1}{k}\zeta - \frac{1}{k}\mathrm{sgn}(\zeta)\bar{k}_u - \dot{\Delta}.\end{aligned} \tag{6}$$

Select a Lyapunov function as $V_\zeta = \frac{1}{2}\zeta^T\zeta$ such that

$$\begin{aligned}\dot{V}_\zeta =& -\frac{1}{k}\|\zeta\|^2 - \frac{1}{k}\zeta^T\mathrm{sgn}(\zeta)\bar{k}_u - \zeta^T\dot{\Delta}\\ \leq& -\frac{1}{k}\|\zeta\|^2 - \frac{1}{k}\sum_{i=1}^{n}\zeta_i\mathrm{sgn}(\zeta_i)\bar{k}_{iu} + \sum_{i=1}^{n}|\zeta_i||\dot{\Delta}_i|\\ =& -\frac{1}{k}\|\zeta\|^2 - \frac{1}{k}\sum_{i=1}^{n}|\zeta_i|\bar{k}_{iu} + \sum_{i=1}^{n}|\zeta_i||\dot{\Delta}_i|.\end{aligned} \tag{7}$$

In terms of Assumption 2 and $\bar{k}_{u,i}$ selected below (3), (7) can be further expressed by

$$\begin{aligned}\dot{V}_\zeta \leq& -\frac{1}{k}\|\zeta\|^2 - \frac{1}{k}\sum_{i=1}^{n}|\zeta_i|(\bar{\delta}_{i1}|\dot{\omega}_i^*| + \bar{\delta}_{i2}|\omega_i| + \bar{\delta}_{i3}) + \sum_{i=1}^{n}|\zeta_i|(\delta_{i1}|\dot{\omega}_i^*| + \delta_{i2}|\omega_i| + \delta_{i3})\\ \leq& -\frac{1}{k}\|\zeta\|^2.\end{aligned} \tag{8}$$

It can be easily seen from (8) that $\dot{V}_\zeta(t) < 0$ for any $\zeta \neq 0$. Moreover, (8) also implies that

$$\int_0^t \|\zeta(\tau)\|^2 d\tau \leq k(V_\zeta(0) - V_\zeta(t)). \tag{9}$$

Since $\zeta(t)$ is uniformly continuous and the right hand side of (9) is bounded, it can be deduced that $\lim_{t\to\infty} \|\zeta(t)\| = 0$ by employing Barbalat lemma. Thus, the asymptotic convergence is achieved and the error state ζ can be ensured to be reduced to zero. $\square$

It is worth emphasizing that the auxiliary variable ζ reveals a mapping between the perturbation Δ and filtered variables ω_η^f and u^f. Thus, an estimator can be constructed by

$$\hat{\Delta} = k^{-1}(\omega_\eta - \omega_\eta^f) - u^f, \tag{10}$$

where $\hat{\Delta}$ defines the estimation of the perturbation.

Define the perturbation error as

$$\tilde{\Delta} = \Delta - \hat{\Delta}. \tag{11}$$

Then we have the following conclusion of the estimation error signal.

Theorem 1: The perturbation estimation error $\tilde{\Delta}$ can be governed to converge to the following small residual regions:

$$\|\tilde{\Delta}\| \leq \sqrt{\tilde{\Delta}_i(0)e^{-t/k}}. \tag{12}$$

Proof. Considering the dynamic model (2) and filtering operation (3), we have

$$\dot{\omega}_\eta^f = k^{-1}(\omega_\eta - \omega_\eta^f) = u^f + \hat{\Delta}. \tag{13}$$

It follows from (10) that the derivation of $\hat{\Delta}$ with respect to time is derived as

$$\begin{aligned}\dot{\hat{\Delta}} =& \frac{1}{k}(u + \Delta - \frac{1}{k}(\omega_\eta - \omega_\eta^f)) - \frac{u - u^f + \mathrm{sgn}(\zeta)\bar{k}_u}{k} \\ =& \frac{1}{k}(\Delta - \frac{1}{k}(\omega_\eta - \omega_\eta^f) + u^f - \mathrm{sgn}(\zeta)\bar{k}_u) \\ =& \frac{1}{k}\tilde{\Delta} - \frac{1}{k}\mathrm{sgn}(\zeta)\bar{k}_u,\end{aligned} \tag{14}$$

which leads to

$$\dot{\tilde{\Delta}} = -\frac{1}{k}\tilde{\Delta} + \dot{\Delta} + \frac{1}{k}\mathrm{sgn}(\zeta)\bar{k}_u, \tag{15}$$

based on (11).

Considering the estimation errors $\tilde{\Delta}$, a Lyapunov function is designed as

$$V_1 = \frac{1}{2}\tilde{\Delta}^T\tilde{\Delta}. \tag{16}$$

Calculating the derivative of (16), it yields

$$\begin{aligned}\dot{V}_1 =& -\frac{1}{k}\tilde{\Delta}^T\tilde{\Delta} + \tilde{\Delta}^T(\dot{\Delta} + \frac{1}{k}\text{sgn}(\zeta)\bar{k}_u) \\ =& -\frac{1}{k}\tilde{\Delta}^T\tilde{\Delta} + \tilde{\Delta}^T\dot{\Delta} + \frac{1}{k}\tilde{\Delta}^T\text{sgn}(\zeta)\bar{k}_u.\end{aligned} \tag{17}$$

Based on the formulations in (4), (10) and (11), we know that $\text{sgn}(\zeta) = -\text{sgn}(\tilde{\Delta})$. Thus, we have

$$\begin{aligned}\dot{V}_1 =& -\frac{1}{k}\tilde{\Delta}^T\tilde{\Delta} + \tilde{\Delta}^T\dot{\Delta} - \frac{1}{k}\tilde{\Delta}^T\text{sgn}(\tilde{\Delta})\bar{k}_u \\ \leq& -\frac{1}{k}\tilde{\Delta}^T\tilde{\Delta} + \sum_{i=1}^{n}|\tilde{\Delta}_i||\dot{\Delta}_i| - \frac{1}{k}\sum_{i=1}^{n}\tilde{\Delta}_i\text{sgn}(\tilde{\Delta}_i)\bar{k}_{u,i} \\ =& -\frac{1}{k}\tilde{\Delta}^T\tilde{\Delta} + \sum_{i=1}^{n}|\tilde{\Delta}_i||\dot{\Delta}_i| - \frac{1}{k}\sum_{i=1}^{n}|\tilde{\Delta}_i|\bar{k}_{u,i}.\end{aligned} \tag{18}$$

In terms of Assumption 2 and $\bar{k}_{u,i}$ selected below (3), (18) can be further expressed by

$$\begin{aligned}\dot{V}_1 \leq& -\frac{1}{k}\tilde{\Delta}^T\tilde{\Delta} + \sum_{i=1}^{n}|\tilde{\Delta}_i|(\delta_{i1}|\dot{\omega}_i^*| + \delta_{i2}|\omega_i| + \delta_{i3}) - \frac{1}{k}\sum_{i=1}^{n}|\tilde{\Delta}_i|(\bar{\delta}_{i1}|\dot{\omega}_i^*| + \bar{\delta}_{i2}|\omega_i| + \bar{\delta}_{i3}) \\ \leq& -\frac{2}{k}V_1.\end{aligned} \tag{19}$$

One has

$$V_1(t) \leq V_1(0)e^{\frac{2t}{k}}. \tag{20}$$

In conclusion, it can be derived as

$$\|\tilde{\Delta}\| = \sqrt{2V_1(t)} \leq \sqrt{\tilde{\Delta}_i(0)e^{-2t/k}}. \tag{21}$$

Obviously, when time approximates to be infinite, the error dynamics $\tilde{\Delta}$ can be ensured to converge to the origin in an exponential sense. □

Remark 2. It should be mentioned that the existing unknown system dynamics estimators in [20, 21] and extended state observers in [30, 31, 32] can also ensure the error dynamics $\tilde{\Delta}$ converging to zero. However, a strong condition that $\|\dot{\Delta}\| \leq \delta$ should be satisfied. The proposed method can reduce the condition to Assumptions 1 and 2 of this paper. Moreover, the existing unknown system dynamics estimators also need the filtering constant k approaching to zero for the asymptotic estimation, which is also unrealistic for estimator design. Distinguishing from [20, 21], the limitations can be removed by the nonlinear estimator designs in this paper.

Remark 3. Although the rough values of the perturbations are assumed to be measured by sensors based on Assumption 1, the proposed nonlinear estimator design method can ensure the accurate estimation of perturbation, which deals with the inaccurate measurement issue from the sensor.

3.2 Controller Design

In this section, giving the definition of angle tracking error as

$$e = \eta - \eta_d, \tag{22}$$

where $e = [e_1, e_2, \dots, e_n]^T$. $\eta_d = [\eta_{d,1}, \eta_{d,2}, \dots, \eta_{d,n}]^T$ denotes the reference signal. Then, similar to [33], to regulate the error e within the predetermined funnel envelop $-\rho_i(t) \le e_i(t) \le \rho_i(t)$, a funnel function is selected as

$$\rho_i(t) = \alpha e^{-lt} + \beta, i = 1, 2, \dots, n \tag{23}$$

where α, β, l are the design parameters and satisfy $\alpha \ge \beta > 0$, $|e_i(0)| < \rho_i(0) = \alpha + \beta$.

Therefore, the funnel variable $\lambda_i(t)$ is selected as

$$\lambda_i(t) = \frac{e_i(t)}{\rho_i(t) - |e_i(t)|}. \tag{24}$$

The differential of (24) can be derived as

$$\dot{\lambda}_i(t) = \frac{1}{\rho_i(t) - |e_i(t)|}(\dot{e}_i(t) - \frac{(|e_i(t)|\dot{\rho}_i(t) - e_i(t))e_i(t)}{|e_i(t)|(\rho_i(t) - |e_i(t)|)}) = \chi_i(\dot{e}_i + \kappa_i), \tag{25}$$

where $\chi_i = \frac{1}{\rho_i(t) - |e_i(t)|}$, $\kappa_i = -\frac{(|e_i(t)|\dot{\rho}_i(t) - e_i(t))e_i(t)}{|e_i(t)|(\rho_i(t) - |e_i(t)|)}$.

Here, the velocity errors are denoted by

$$z = \omega_\eta - \omega_{\eta d}, \tag{26}$$

where $z = [z_1, z_2, \dots, z_n]^T$, $\omega_{\eta d} = [\omega_{\eta d,1}, \omega_{\eta d,2}, \dots, \omega_{\eta d,n}]^T$ represents the reference velocity signal to be designed to ensure position tracking error e convergence.

Note that the funnel variable $\lambda_i(t)$ is definitely stable if the signal satisfies the dynamics $\dot{\lambda}_i(t) = -k_{\eta,i}\lambda_i$ where $k_{\eta,i} > 0$. On the other hand, from (22), (25) can be furtherly rewritten as

$$\dot{\lambda}_i(t) = \chi_i(\omega_{\eta,i} - \dot{\eta}_{d,i} + \kappa_i). \tag{27}$$

Based on the above analysis, to stabilize the funnel variable $\lambda_i(t)$, the reference rate signal $\omega_{\eta d,i}$ can be constructed by

$$\omega_{\eta d,i} = -k_{\eta,i}\chi_i^{-1}\lambda_i + \dot{\eta}_{d,i} - \kappa_i, \tag{28}$$

where $k_{\eta,i}$ denotes the subsystem gain that needs to be regulated.

According to the attitude model (2), the differential of (26) with respect to time can be computed as

$$\dot{z} = u + \Delta - \dot{\omega}_{\eta d}. \tag{29}$$

Substituting the disturbance estimation (10) into (29), the control input results in

$$u_i = -k_{\omega,i} z_i + \dot{\omega}_{\eta d,i} - \hat{\Delta}_i, \tag{30}$$

where $k_{\omega,i}$ represents the controller gain to be governed.

3.3 Stability Analysis

Based on (28), the differential of funnel variables (24) can be derived as

$$\dot{\lambda}_i(t) = -k_{\eta,i}\lambda_i + \chi_i z_i. \tag{31}$$

Similarly, the derivation of angular rate errors z can be derived by invoking (30):

$$\dot{z}_i(t) = -k_{\omega,i} z_i + \tilde{\Delta}_i. \tag{32}$$

Theorem 2. For the attitude dynamics (2), nonlinear dynamics estimators (10), and control laws (30), all the signals in the EL system can guarantee the ultimately uniformly bounded results, especially, and the angle errors $e_i(t)$ can be regulated to converge to predefined funnel boundary when the initial angle errors satisfy $-\rho_i(t) < e_i(0) < \rho_i(t)$, on condition that the following restrictions on controller gains hold:

$$\begin{aligned} &\lambda_{\min}(k_{\eta,i}) - \frac{1}{2}\lambda_{\max}(\chi_i) > 0, \\ &\lambda_{\min}(k_{\omega,i}) - \frac{1}{2}\lambda_{\max}(\chi_i) - \frac{1}{2} > 0. \end{aligned} \tag{33}$$

Proof. Considering the entire attitude kinetics, a Lyapunov function is selected as

$$V_2 = \frac{1}{2}\sum_i (\lambda_i^2(t) + z_i^2(t)). \tag{34}$$

The derivation of (34) can be derived by combining with (31) and (32):

$$\begin{aligned} \dot{V}_2 &= \sum_i (\lambda_i \dot{\lambda}_i + z_i \dot{z}_i) = \sum_i (\lambda_i(-k_{\eta,i}\lambda_i + \chi_i z_i) + z_i(-k_{\omega,i} z_i + \tilde{\Delta}_i)) \\ &= \sum_i (\lambda_i(-k_{\eta,i}\lambda_i^2 + \lambda_i \chi_i z_i - k_{\omega,i} z_i^2 + z_i \tilde{\Delta}_i)). \end{aligned} \tag{35}$$

Utilizing Young's inequality, it is derived as

$$\sum_i |\lambda_i \chi_i z_i| \leq \frac{1}{2}\lambda_{\max}(\chi)(\|\lambda\|^2 + \|z\|^2),$$
$$\sum_i |z_i \tilde{\Delta}_i| \leq \frac{1}{2}\|z\|^2 + \frac{1}{2}\|\tilde{\Delta}_i\|^2, \tag{36}$$

where $\chi = diag\{\chi_\psi, \chi_\theta, \chi_\phi\}$, $\lambda_{\max}$ denotes the maximum value of the matrix.

Thus, (35) can be computed as

$$\begin{aligned}\dot{V}_2 \leq & -\lambda_{\min}(k_\eta)\|\lambda\|^2 + \frac{1}{2}\lambda_{\min}(\chi)\|\lambda\|^2 + \frac{1}{2}\lambda_{\min}(\chi)\|z\|^2 - \lambda_{\min}(k_\omega)\|z\|^2 + \frac{1}{2}\|z\|^2 + \frac{1}{2}\|\tilde{\Delta}_i\|^2 \\ = & -(\lambda_{\min}(k_\eta) - \frac{1}{2}\lambda_{\min}(\chi))\|\lambda\|^2 - (\lambda_{\min}(k_\omega) - \frac{1}{2}\lambda_{\max}(\chi) - \frac{1}{2})\|z\|^2 + \frac{1}{2}\|\tilde{\Delta}_i\|^2,\end{aligned} \tag{37}$$

where $\lambda_{\min}$ represents the minimum value of a matrix.

According to Theorem 1, we have

$$\lim_{t\to\infty} \|\tilde{\Delta}_i\| = 0. \tag{38}$$

It can be derived by

$$\dot{V}_2 \leq -(\lambda_{\min}(k_\eta) - \frac{1}{2}\lambda_{\min}(\chi))\|\lambda\|^2 - (\lambda_{\min}(k_\omega) - \frac{1}{2}\lambda_{\max}(\chi) - \frac{1}{2})\|z\|^2. \tag{39}$$

Thus, it can be integrated as

$$\dot{V}_2 \leq -\mu V_2, \tag{40}$$

where $\mu = \min\{2\mu_1, 2\mu_2\} > 0$.

Solving the inequality (40), one has

$$0 \leq V_2 \leq V_2(0)e^{-\mu t}. \tag{41}$$

From (41), note that the error dynamics λ, z converge to zero when time goes to infinity. Furthermore, by incorporating the funnel variable $\lambda_i(t)$ in (24), it can be derived as

$$\frac{e_i^2}{(\rho_i(t) + |e_i(t)|)(\rho_i(t) - |e_i(t)|)} \leq \frac{e_i^2}{(\rho_i(t) - |e_i(t)|)^2}. \tag{42}$$

Then, it can be furtherly calculated as

$$e_i^2 \leq \rho_i^2. \tag{43}$$

Therefore, we have

$$-\rho_i \leq e_i \leq \rho_i, \tag{44}$$

where completes the proof. □

Remark 4. Note that the asymptotic tracking results can be obtained by using the proposed method from Theorem 2. Furthermore, the error profiles can be always limited in a prescribed region by using the funnel control strategy. Moreover, it should be pointed out that the conventional controllers [28, 29] can also achieve the ultimately uniformly bounded tracking results for the quadrotor. However, different from the conventional control methods, the funnel control can take the transient and steady-state performance (i.e., the overshoot, convergence time and steady-state errors) into consideration by predefining the funnel bound-based funnel variables.

4 Simulation example

In this section, as a typical EL system, the attitude tracking control of quadrotor aircraft systems is given as in [3] to show the effectiveness of our proposed method. The coordinates of the quadrotor aircraft could be expressed by:

$$\eta := (\phi, \theta, \psi)^T$$

where (ϕ, θ, ψ) denotes the roll angle, the pitch angle, and the yaw angle of the quadrotor along with the X-, Y-, Z-axes, respectively, which is utilized to depict the quadrotor attitude.

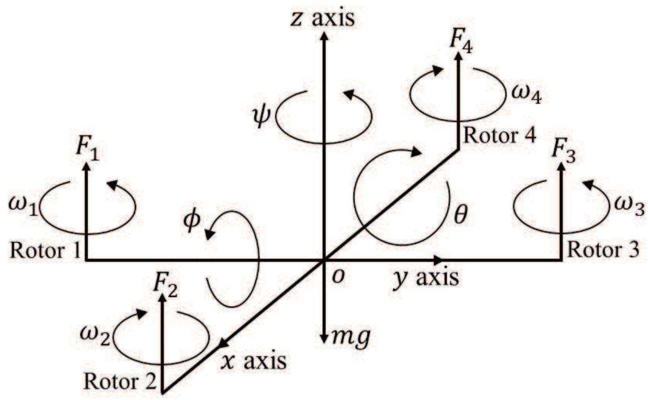

Figure 1: Quadrotor structure

The structure of the quadrotor is illustrated in Figure 1. As seen in Figure 1, an inertial-fixed coordinate is firstly established to promote the description of quadrotor attitude dynamics, where F_i represent the rotor thrusts produced by the rotor actuations. By resorting to [34, 35], the mathematical attitude model with external disturbances is typically expressed as

$$\begin{cases} \dot{\eta} = W(\eta)\omega \\ J\dot{\omega} = -\omega^{\times} J\omega + \tau + d, \end{cases} \tag{45}$$

where $\eta := (\phi, \theta, \psi)^T$, and ϕ, θ, ψ being the roll, pitch and yaw angles. $\omega := (\omega_\phi, \omega_\theta, \omega_\psi)^T = (\dot{\phi}, \dot{\theta}, \dot{\psi})^T$, $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$ represent the angular rates. The definite positive diagonal matrix $J = \mathrm{diag}[J_\phi, J_\theta, J_\psi]$, thereinto J_ϕ, J_θ, J_ψ stand for moments of inertia; τ is the control inputs denoting the torque of the aircraft; d describes the external environmental disturbances in attitude kinetics. Besides, the matrix relative to the available states in attitude loop is defined by

$$W(\eta) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}, \quad \omega^\times = \begin{bmatrix} 0 & -\omega_\psi & \omega_\theta \\ \omega_\psi & 0 & -\omega_\phi \\ -\omega_\theta & \omega_\phi & 0 \end{bmatrix}.$$

In line with (45), it can be furtherly derived as

$$\ddot{\eta} = \dot{W}(\eta)\omega + W(\eta)J_\eta^{-1}(-\omega^\times J\omega + d) + J_\eta^{-1}W(\eta)\tau, \tag{46}$$

where $J_\eta = \mathrm{diag}[J_\phi, J_\theta, J_\psi]$.

Giving the definition of $\Delta = \dot{W}(\eta)\omega + W(\eta)J_\eta^{-1}(-\omega^\times J\omega + d)$ and $u = J_\eta^{-1}W(\eta)\tau$, it can be simplified as follows:

$$\begin{cases} \dot{\eta} = \omega_\eta \\ \dot{\omega}_\eta = u + \Delta, \end{cases} \tag{47}$$

where $\omega_\eta = W(\eta)\omega$.

In the simulation, the desired trajectory of the aircraft is given as

$$[\phi_d(t), \theta_d(t), \psi_d(t)]^T = [0.5 \times \sin(0.25t), 0.5 \times \cos(0.25t), 1]^T. \tag{48}$$

The initial conditions for the quadrotor aircraft are:

$$[\phi(t_0), \theta(t_0), \psi(t_0)]^T = [2, 0.1, 0.4]^T. \tag{49}$$

The disturbances for the quadrotor aircraft are:

$$[d_\phi(t), d_\theta(t), d_\psi(t)]^T = [0.5, 0.5 \times \sin(0.2 \times t), -0.2 \times \sin(2 \times t)]^T. \tag{50}$$

Simulations are given with the following parameters and initial conditions:

$$\begin{aligned} &k = 0.01,\ \bar{\delta}_{i1} = 1,\ \bar{\delta}_{i2} = 1,\ \bar{\delta}_{i3} = 1,\ \alpha = 6,\ \beta = 0.3,\ l = 0.4, \\ &k_{\eta,i} = 1.2,\ k_{\omega,i} = 5, i = \{\phi, \theta, \psi\}. \end{aligned}$$

To embody the effectiveness and practicability for the proposed nonlinear estimator-based funnel control schemes, simulations are offered in Figures 2-5. Figure 2 plots the angle tracking curves under

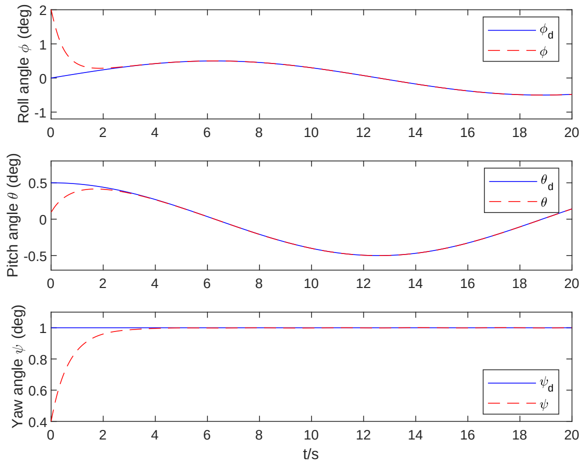

Figure 2: Response curves of the angles $\{\psi, \theta, \phi\}$ and the desired angles $\{\psi_d, \theta_d, \phi_d\}$ of the quadrotor.

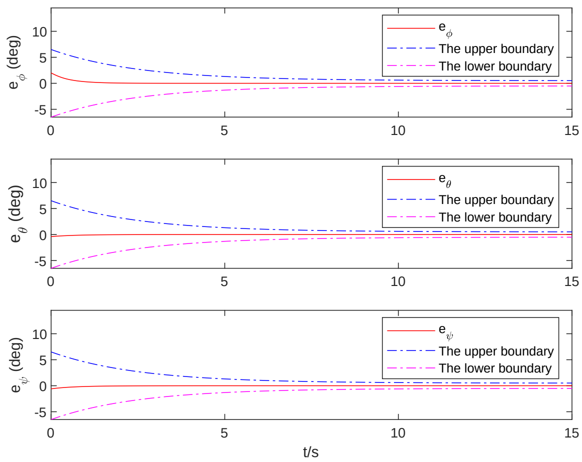

Figure 3: Response curves of the attitude tracking errors $\{e_\psi, e_\theta, e_\phi\}$ with funnel control.

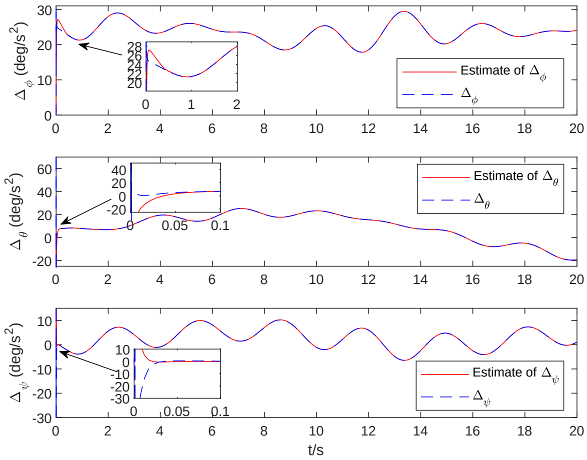

Figure 4: Response curves of estimations of perturbations by using the nonlinear estimators.

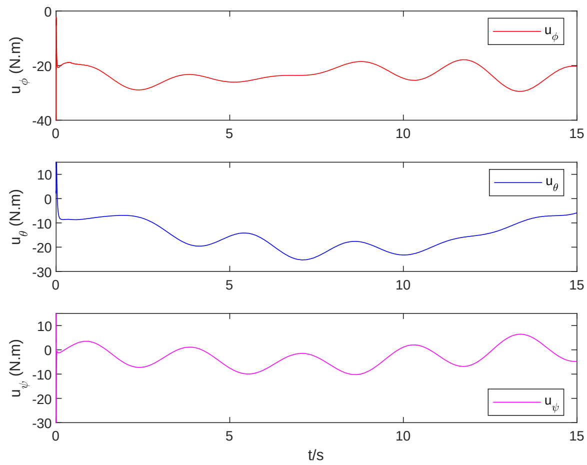

Figure 5: Response curves of the control inputs.

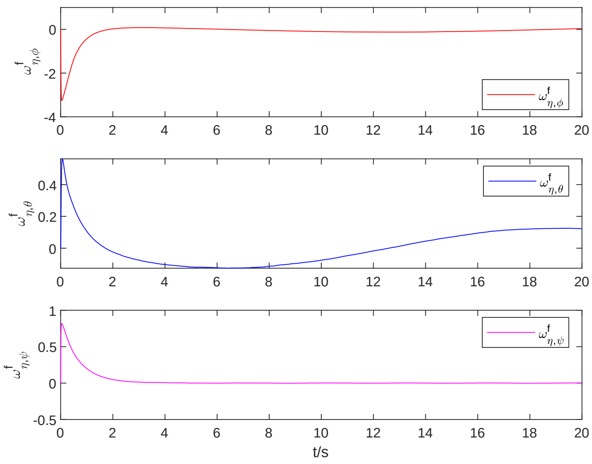

Figure 6: Response curves of estimator states $\{\omega^f_{\eta,\psi}, \omega^f_{\eta,\theta}, \omega^f_{\eta,\phi}\}$.

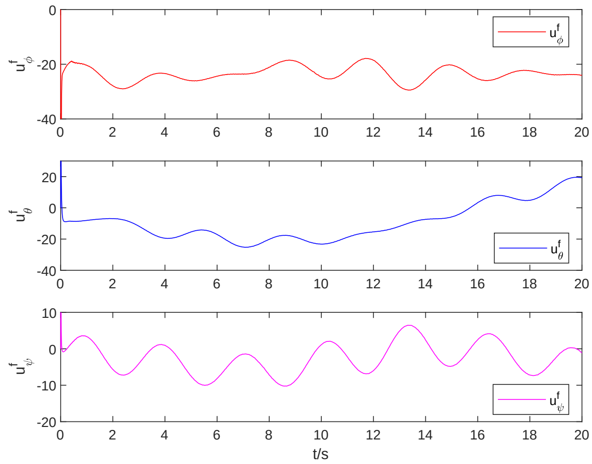

Figure 7: Response curves of estimator states $\{u^f_{\psi}, u^f_{\theta}, u^f_{\phi}\}$.

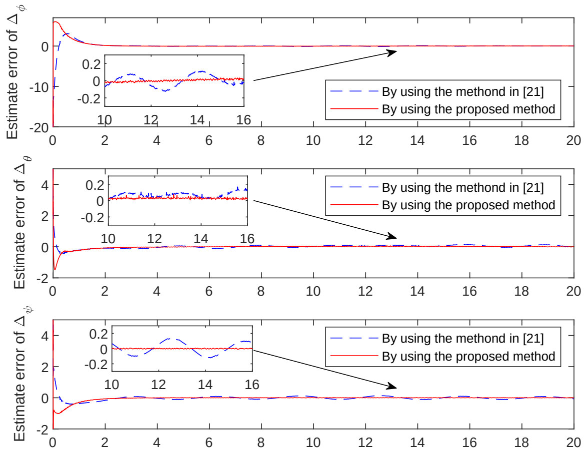

Figure 8: Comparisons of the estimation errors $\{\tilde{\Delta}_{\psi}, \tilde{\Delta}_{\theta}, \tilde{\Delta}_{\phi}\}$ by using the proposed method and the method in [21].

the developed control scheme, and it is intuitively indicated that the angle states can track the desired angels asymptotically. Figure 3 illustrates the curves of tracking errors of the quadrotor, and it is clearly shown that the errors converge into the predefined region within the funnel control boundary. In Figure 4, it shows that the proposed nonlinear estimator can promptly observe and capture the unmeasurable perturbations with an improved accuracy. Figure 5 exhibits the control actions that steer quadrotor attitude dynamics. The estimator states ω_{η}^{f} and u^{f} are demonstrated in Figures 6 and 7, respectively, which shows the stability of the estimator signals.

In order to verify efficiency of the proposed nonlinear estimator design method, comparisons results with the existing linear estimators, i.e., unknown system dynamics estimator in [21] are illustrated in Figure 8. It is indicated by Figure 8 that the perturbation estimation performances of the quadrotor by using the proposed methods are better than that of by using the method proposed in [21]. The comparative results substantiate the superiority of the proposed estimation method.

5 Conclusions

In this paper, a nonlinear estimator-based perturbation rejection funnel control method has been proposed to deal with the trajectory tracking control problem for a class of EL system with external disturbances and parametric uncertainties. The nonlinear estimator has been constructed to online identify the unknown perturbations by establishing the corresponding relationship between filtered dynamics and total perturbations. In addition, by designing a funnel function and funnel variable, a funnel control strategy has been exploited to govern the angle angle errors within a predefined region, where the overshoot, convergence time and steady-state accuracy are all restricted to an appropriate range. Finally, simulations validate the practicability and effectiveness of the developed nonlinear estimator-based funnel control method.

Conflict of interest

The authors declare that they have no conflict of interest.

Data availability statement

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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