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A logic of argumentation for specification and verification of abstract argumentation frameworks

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Abstract In this paper, we propose a logic of argumentation for the specification and verification (LA4SV) of requirements on Dung's abstract argumentation frameworks. We distinguish three kinds of decision problems for argumentation verification, called extension verification, framework verification, and specification verification respectively. For example, given a political requirement like "if the argument to increase taxes is accepted, then the argument to increase services must be accepted too," we can either verify an extension of acceptable arguments, or all extensions of an argumentation framework, or all extensions of all argumentation frameworks satisfying a framework specification. We introduce the logic of argumentation verification to specify such requirements, and we represent the three verification problems of argumentation as model checking and theorem proving properties of the logic. Moreover, we recast the logic of argumentation verification in a modal framework, in order to express multiple extensions, and properties like transitivity and reflexivity of the attack relation. Finally, we introduce a logic of

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meta-argumentation where abstract argumentation is used to reason about abstract argumentation itself. We define the logic of meta-argumentation using the fibring methodology in such a way to represent attack relations not only among arguments but also among attacks. We show how to use this logic to verify the requirements of argumentation frameworks where higher-order attacks are allowed [A preliminary version of the logic of argumentation compliance was called the logic of abstract argumentation (Boella et al. 2005)].

Keywords Abstract argumentation theory · Higher-order argumentation · Modelling

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1 Introduction

Abstract [24] and structured [39] argumentation frameworks are designed for a variety of applications in multiagent systems (MAS), and artificial intelligence (AI) [40]. This raises the question *what makes these designed argumentation frameworks correct or incorrect?* There are various ways to decide or reason about the correctness of argumentation frameworks. First, in contrast to the usual kind of models in design [34], we cannot refer to an objective reality. For example, if we think to argumentation frameworks used to model multiagent dialogues, then it may be difficult to represent the dialogue. In particular, argumentative dialogues are dynamic, and the arguments can be interactive such that the proposer and the interlocutor have a symmetrical relationship. A second possibility is to consider argumentation frameworks generated by knowledge bases, comprised of facts and rules, using an appropriate logic, and verify the correctness of these frameworks with respect to these knowledge bases. The problems which arise from adopting this approach are related to the inconsistency of the knowledge bases, and the fact that the approaches which build an argumentation framework from a knowledge base may fail to account for the rationality postulates of consistency and closure on argumentation frameworks [19]. A third possibility is to provide some alternative argumentation frameworks, and explain why the chosen argumentation framework is better than the alternatives. A fourth way to reason about the correctness of the argumentation frameworks is to specify the properties or the requirements we want to satisfy in the framework, and verify whether the argumentation framework satisfies these properties and requirements. This is what is achieved in the formal specification and verification techniques in computer science, and in particular in multiagent systems [23]. Roughly, a formal specification describes what the system should do. Given such a specification, it is possible to use formal verification techniques to demonstrate that a candidate (argumentation) system design is correct with respect to the specification. This approach has the advantage that incorrect candidate system designs can be revised before a major investment has been made in actually implementing the design. A design cannot ever be declared "correct" in isolation, but only "correct with respect to a given specification". In this paper, we choose this fourth alternative to check whether an argumentation framework is correct or not with respect to the specification. We design our argumentation framework following the requirements given as specification, and then we verify whether it satisfies these requirements. Dung et al. [25] propose three properties of the argumentation frameworks in the context of practical reasoning. Roughly, in [25], if we want the agents of a multiagent system to cooperate, we may have to check that all the partial argumentation frameworks of the agents satisfy common conditions, otherwise they cannot cooperate in a successful way. The proposed conditions are *transparency* to ensure that the arguments are simple so that people of all backgrounds can follow them easily, *relevance* to ensure that repetitions, and irrelevant details that could be a distraction and a point of attack by your opponents are avoided, and *no dismissal* to ensure that no legitimate argument is dismissed without reason. They formally define such properties in two different frameworks, assumption-based argumentation and logic-based argumentation.

In this paper we answer the following research question:

– How to define a logic of argumentation to specify and verify requirements on Dung's abstract argumentation frameworks?

Examples of requirements which can be specified and verified in such a logic of argumentation can either refer to the argumentation framework itself, for example "the argument of increasing taxes to the poor people attacks the argument of giving economic sustainment to the lower-middle class", "the arguments of increasing the taxes to poor people and of cutting the services defend the argument of acting in the interests of the higher class," or to the semantics of the argumentation framework, as for instance "the argument that in Italy there are several political parties is acceptable". Moreover, there can be more complex properties such as "the acceptance of the argument to higher taxes implies the acceptance of the argument for increasing the services" or "the argument to sustain the lower-middle class or the argument to increase the services are acceptable". We distinguish three decision problems for argumentation verification:

Argumentation extension verification Given an argumentation framework, an extension of acceptable arguments and a requirement. Do the framework and the extension satisfy the requirement?

Argumentation framework verification Given an argumentation framework and a requirement. Does the framework satisfy the requirement?

Argumentation specification verification Given a partial specification of an argumentation framework and a requirement. Do all argumentation frameworks satisfying the specification, also satisfy the requirement?

We consider two scenarios in which the logic of argumentation compliance can be used. The first scenario consists in a multiagent system where the agents may ask other agents to define an argumentation framework, and then they have to verify that the properties of the argumentation framework satisfy their requirements. An example of that kind is the construction of a political manifesto. One agent has been delegated to do the political manifesto of her own party, and the other agents have to verify if this manifesto is compliant with the properties they require for the manifesto. The second scenario consists in a single agent perspective. If an agent makes a decision then she may want to verify if the decision is compliant with the requirements of the system in order to be able to defend her position. For instance, if the agent decides not to participate to a congress of her political party, then she has to verify if the argument of being ill is compliant with the rules about congress participation governing her party. This example explains how many decisions are made without considering all the possible alternatives, and so they are often rationalized retrospectively in an attempt to justify the choice, that is why verifying the compliance is relevant also for single agents in the post-rationalization phase.

There are many ways to model argumentation specification and verification. Our idea is to formally specify, given an argumentation framework which represents our starting system, the requirements we want this system to ensure, and then verify whether the system satisfies the specified requirements. In order to specify the requirements of the system, we propose to use a logic of argumentation. This logic allows us also to verify the compliance of the system with the specified requirements. However, the definition of a logic of argumentation verification raises more issues to be addressed. For instance, there are languages in which only some properties can be expressed, and then there are more complicated languages in which more complicated properties can be checked. Alternatively, there are simple frameworks of which properties can be checked, and there are more complicated, extended, frameworks of which properties can be checked. For instance, we may want to verify a requirement like "argument a attacks argument b", or we may want to verify requirements concerning argumentation extension like "if argument a is accepted then argument b is accepted too, and argument c is rejected". In order to be able to cover these alternatives with our logic of argumentation for specification and verification, we need to define some variants of it.

The research question breaks down into the following sub-questions:

1. How to define a logic of argumentation compliance?
2. How to extend this logic with a modal operator in order to specify more complex properties?
3. How to extend this logic with a nested modal operator for specifying higher-order argumentation frameworks?

A first version of this logic of abstract argumentation has been presented in [12]. In [12], the propositional and modal variant of the logic have been introduced. In this paper, we reformulate and extend this logic to apply it for argumentation verification, and we show how to do it using a number of examples. Moreover, we introduce the logic of meta-argumentation addressed as future work in [12].

First, we propose a propositional logic for the specification and verification of the argumentation frameworks. A model of the logic of argumentation represents an argumentation framework, and such an argumentation framework satisfies formulas where arguments attack or defend each other, or satisfies the fact that specific sets of arguments are extensions of the framework. Assume a set of requirements of the kind "if $acceptable(a_1)$, ..., $\neg acceptable(a_n)$ then $acceptable(b)/\neg acceptable(b)$", and an argumentation framework AF. We want to verify whether the argumentation framework satisfies the requirements. Consider, for instance, an argumentation framework with three arguments of a political agent, a, b and c where a is the argument "I will lower taxes to rich people", b is the argument "I will help rich people to maintain their privileges" and argument c is "I want to sustain lower-middle class". Arguments a and b both attack argument c. The admissible extension of this argumentation framework expressed in our logic is $A(a \wedge b)$ and argument c

is not in the admissible extension, $\neg A(c)$. The agent has to propose these arguments to a left-center union, and she knows that a rule governing the union is "the argument of sustaining lower-middle class has to be accepted", expressed in the logic as $A(c)$. It is easy to check that the requirement is not satisfied, and the agent should find other arguments to be proposed to the union.

Second, we extend the propositional logic of argumentation compliance introducing a modal operator. We need to introduce this logic because of a number of reasons. We have that using only a propositional logic of argumentation the semantics are defined in such a way that no generalization of Dung's theory is possible. Introducing the modal variant of the logic of argumentation, we allow the expression, in the requirements, of the multiple extensions of preferred, stable and complete semantics, and of the single extension of grounded semantics. The meaning of the $\Box$ operator can be associated to the attack relation which is seen as a binary relation between worlds such as a standard accessibility relation. Given that the attack relation is interpreted as an accessibility relation between worlds, it has to be defined on sets of arguments instead of single arguments. Moreover, using the modal variant, we can express the characterizations in propositional logic provided by Besnard and Doutre [6]. Finally, using this variant of the logic, we can express properties like transitivity, reflexivity, and symmetry of the attack relations.

Third, we extend the logic with a nested modal operator which can be used to verify extended argumentation frameworks with higher-order attacks. Higher-order attacks are attacks from arguments and attack relations to arguments and attack relations thus, differently from Dung's framework [24], two binary attack relations hold and they are not only among arguments, but between arguments and attack relations. In the last years, a number of approaches to compute the extensions of such extended argumentation frameworks have been proposed. Some of them develop a technique called meta-argumentation [15, 22, 31, 37], which uses abstract argumentation to reason about abstract argumentation, while others introduce, for instance, methods to compute the extensions of these extended argumentation frameworks [2]. A logic of meta-argumentation able to capture also these higher-order attacks such as nested attacks among the arguments allows us to verify also this kind of frameworks with the specified requirements. We propose to use the fibring methodology [26], a general methodology to combine logics and use them within the same language, in order to formulate the language, and the semantics of the logic of meta-argumentation in this paper.

The first challenge is to deal with argumentation frameworks in a design perspective. We argue that the specification/verification process should be applied also to the design of argumentation frameworks. Second, the logic of argumentation may be used to solve the confusion associated to instantiated argumentation frameworks. More precisely, our logic of argumentation can solve the confusion between the logic inside the arguments and the logic of the arguments. The logic inside the arguments is the structure of the arguments, i.e., a conclusion which logically follows from the set of premises, while the logic of the arguments consists in the description of the relations among arguments.

In this paper, we do not present algorithms for compliance checking and we do not introduce particular argumentation semantics or specific application domains. We show what it is a problem in designing argumentation frameworks, such as verifying the compliance of these frameworks with the requirements of the system in which they are embedded. We use examples from the argumentation theory and multiagent systems fields in order to show the usefulness of our methodology to solve the problem. Moreover, the following observations may be addressed. The introduced logics are at the level of abstract argumentation, i.e., they do not predicate on the structure of arguments, however requirements of the kind "*a* attacks *b*" may be seen as concerned with their internal structure. A requirement of this kind can be associated to the meaning "if you use *a* and *b*, do not forget the relevant attack". However, this is not the intended meaning of our requirements because this would require to give a name to all potential arguments, a requirement that may be hard to satisfy in practice. Furthermore, we assume a fixed set of arguments A which may limit the generality of the approach. One may want to express a requirement that if argument a belongs to A then a should be accepted. This is left as future work.

The paper answers the research questions and it is organized as follows. In Section 2 we introduce the propositional logic of argumentation and we show, using a number of examples, how to verify the requirements of the argumentation frameworks. We present the logical properties of the logic of argumentation, and we introduce the modal variant of this logic. Section 3 formally defines a logic of meta-argumentation, and then it shows how this logic can be used to verify the compliance of argumentation frameworks with higher-order attacks. Related work and conclusions end the paper.

2 A logic of argumentation for specification and verification

2.1 Abstract argumentation theory

We start with Dung's theory of argumentation [24]. It is called a theory of abstract argumentation, because it ignores the internal structure of arguments. In this paper, we use the presentation of [6], who, in contrast to Dung, also define sets of arguments attacking other sets of arguments.

Definition 1 (Argumentation framework) An argumentation framework is a pair $\langle A, R\rangle$, where A is a set (of arguments), and R is a binary relation over A representing a notion of *attack* between arguments ($R \subseteq A \times A$). Given two arguments a and b, $(a, b) \in R$ means that a attacks b or that a is an attacker of b. A set of arguments S attacks an argument a if a is attacked by an argument of S. A set of arguments S attacks a set of arguments S' if there is an argument $a \in S$ attacking an argument $b \in S'$.

Example 1 Consider an agent ag_1 working for a service provider on the Web. The agent, in the post-rationalization design phase, has to specify to the guarantor of the users' privacy how the service deals with the private details of the users. We have the following arguments: a is the argument "The service does not allow the distribution of details on its input", b is the argument "The service can provide the requesters' details to a telemarketer", c is the argument "The service deals with confidential information for financial transactions" and d is the argument "The service deals with confidential information for on-line voting systems". This argumentation framework is formalized as $AF_1 = \langle A_1, R_1\rangle$ where $A_1 = \{a, b, c, d\}$ and $R_1 = \{(a, b), (b, c), (b, d)\}$. This framework is visualized in Fig. 1.

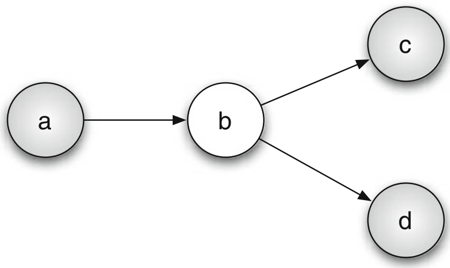


Fig. 1 The argumentation framework described in Example 1 where *white* arguments are those not acceptable and *grey* ones are those acceptable for admissible semantics

Dung [24] assumes an argumentation framework $\langle A, R \rangle$ to be given. Moreover, he gives several semantics which produce none, one or several sets of acceptable arguments called *extensions*. Most of these semantics depend on an additional notion of what is called *defence*. Instead of "S defends a", Dung says "a is acceptable with respect to S." We define also a set of arguments defending another set of arguments.

Definition 2 (Argumentation semantics) Let $\langle A, R \rangle$ be an argumentation framework.

- $S \subseteq A$ is *conflict free* if and only if there are no a and b in S such that a attacks b.
- A conflict free set $S \subseteq A$ is a *stable extension* if and only if for each argument which is not in S, there exists an argument in S that attacks it.
- An argument $a \in A$ is defended by a set $S \subseteq A$ (or S defends a) if and only if for any argument $b \in A$, if b attacks a, then S attacks b.
- A conflict free set $S \subseteq A$ is *admissible* if and only if each argument in S is defended by S.
- A *preferred extension* is an admissible subset of A, which is maximal w.r.t. set inclusion.
- An admissible $S \subseteq A$ is a *complete extension* if and only if each argument which is defended by S is in S.
- The least (with respect to set inclusion) complete extension is the *grounded extension*.

We say that $S \subseteq A$ defends $S' \subseteq A$ if and only if S defends each $a \in S'$.

Example 2 Let us analyze the argumentation framework AF_1 of Fig. 1. There are the following conflict free sets: $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, c\}$, $\{a, d\}$, $\{a, c, d\}$. The stable extension of AF_1 is $\{a, c, d\}$. Arguments c and d are defended by the set $S = \{a\}$. The conflict free sets $\{a\}$, $\{a, c\}$, $\{a, d\}$, $\{a, c, d\}$ are admissible since a defends c and d. The preferred extension of AF_1 is $\{a, c, d\}$ which is also the complete extension and the grounded extension. In this case, it is accepted that "The service does not allow the distribution of details on its input", that "The service deals with confidential information for financial transactions", and that "The service deals with confidential information for on-line voting systems".

The basic idea of the logic of argumentation for specification and verification is that there is no longer a fixed argumentation framework, in the following sense. A model of the logic represents an argumentation framework, and such a model satisfies formulas representing that the arguments attack or defend each other, or whether sets of arguments are extensions. Now, a formula is a theorem if it holds in all models, i.e., when it is true for every argumentation framework. Theorems thus quantify over argumentation frameworks. We adopt the idea of a not fixed argumentation framework, similarly to [1] who define principles of argumentation semantics. In particular, we introduce the notion of a universe of arguments U, where the set $A \subseteq U$ represents the set of arguments produced by a reasoner at a given instant of time. A is assumed to be finite independently of the fact that the underlying mechanism of argument generation admits the existence of infinite sets of arguments. It must be observed that we use a fixed set of arguments and only the attack relation changes. This is due to technical reasons: we need the names of the arguments in the definitions of the language like the names of the propositional atoms are needed in the language of propositional logic.

2.2 A simple-minded semantics

There are many ways to design a logic of argumentation verification. In this section we stay close to Dung's argumentation framework, and we generalize it in Section 2.4. We first assume a fixed signature or alphabet, which consists of the set of arguments A. L_0 is the set of conjunctions of atoms, representing sets of arguments, and L is the language that contains the notions of Dung's theory of argumentation. L_1 is the fragment of L that contains only the attack and defend connectives. Note that modalities in L cannot be nested.

Definition 3 (LA4SV language) Given a set of arguments $A = \{a_1, \ldots, a_n\}$, we define the set L_0 of argument sets, and the set L of LAA formulas as follows.

L_0: $a_i \mid (p \wedge q)$ $\quad (p, q \in L_0)$
L: $p \mid (p \rhd q) \mid (p \oslash q) \mid F(p) \mid S(p) \mid A(p) \mid P(p) \mid C(p) \mid G(p) \mid \neg\phi \mid (\phi \wedge \psi)$
$(p, q \in L_0; \phi, \psi \in L)$

We write L_1 for the fragment of L that does not contain a monadic modal operator. Moreover, disjunction $\vee$, material implication $\supset$ and equivalence $\leftrightarrow$ are defined as usual. We abbreviate formulas using the following order on logical connectives: $\neg \mid \vee, \wedge \mid \rhd, \oslash \mid \supset, \leftrightarrow$. For example, $\neg p \rhd q \wedge r$ is short for $(\neg p \rhd (q \wedge r))$.

A semantic structure consists of the binary attack relation R and the extension of acceptable arguments S.

Definition 4 (LA4SV semantics) Let A be set of arguments, let a be an element of A, let p and q be elements of L_0 and let ϕ and ψ be elements of L, let S a subset of A, and let R be a binary relation over A. We have:

$R, S \models a$ if and only $a \in S$.
$R, S \models \neg\phi$ if and only if not $R, S \models \phi$.
$R, S \models \phi \wedge \psi$ if and only if $R, S \models \phi$ and $R, S \models \psi$.
$R, S \models p \rhd q$ if and only if in argumentation framework $\langle A, R \rangle$, the set of arguments in p attack the set of arguments in q.
$R, S \models p \oslash q$ if and only if in argumentation framework $\langle A, R \rangle$, the set of arguments in p defend the set of arguments in q.

$R, S \models F(p)$ if and only if the set of arguments in p is conflict free in argumentation framework $\langle A, R\rangle$.
$R, S \models S(p)$ if and only if the set of arguments in p is a stable extension in argumentation framework $\langle A, R\rangle$.
$R, S \models A(p)$ if and only if the set of arguments in p is admissible in argumentation framework $\langle A, R\rangle$.
$R, S \models P(p)$ if and only if the set of arguments in p is a preferred extension in argumentation framework $\langle A, R\rangle$.
$R, S \models C(p)$ if and only if the set of arguments in p is a complete extension in argumentation framework $\langle A, R\rangle$.
$R, S \models G(p)$ if and only if the set of arguments in p is a grounded extension in argumentation framework $\langle A, R\rangle$.

Moreover, logical notions are defined as usual, in particular:

- $R \models \phi$ if for all S, $R, S \models \phi$
- $R \models \{\phi_1, \ldots, \phi_n\}$ if and only if $R \models \phi_i$ for $1 \le i \le n$,
- $\models \phi$ if and only if for all R, we have $R \models \phi$,
- $\Gamma \models \phi$ if and only if for all R such that $R \models \Gamma$, we have $R \models \phi$.

One natural interpretation of $R \models \phi$ if for all S, $R, S \models \phi$ is to fix a semantics, and then say "for all preferred extensions", or "for all stable extensions". Here, however, since we want to reason also about relations among semantics, we reason over all sets of arguments. With a slightly abuse of notation, we indicate in the following examples $P(a \wedge b)$ with the intended meaning that arguments a and b are in a preferred extension.

Example 3 We provide a complete description of the argumentation framework AF_1 using the $LA4SV$ language in the following way:

- $(a \rhd b) \wedge ((b \rhd c) \wedge (b \rhd d))$;

Moreover, our language allows us to provide the defence relation among arguments, as for instance we say that $(a \oslash c) \wedge (a \oslash d)$. The extensions of the framework are as follows:

- $R, S \models F(a \wedge c \wedge d)$, $R, S \models F(a \wedge c)$, $R, S \models F(a \wedge d)$;
- $R, S \models A(a \wedge c \wedge d)$, $R, S \models A(a \wedge c)$, $R, S \models A(a \wedge d)$;
- $R, S \models P(a \wedge c \wedge d)$;
- $R, S \models S(a \wedge c \wedge d)$;
- $R, S \models G(a \wedge c \wedge d)$;
- $R, S \models C(a \wedge c \wedge d)$;

Definition 5 distinguishes extension verification, framework verification, and specification verification, where the former two are defined as model checking, and the latter one as theorem proving. Compliance checking is verifying whether an argumentation framework satisfies a property expressed in LA4SV.

Definition 5 Given an abstract argumentation framework $AF = \langle A, R\rangle$, a requirement $\phi \in LA4SV$, a partial specification of argumentation frameworks $\Gamma \subseteq LA4SV$, and an extension $S \subseteq A$.

The extension S complies with requirement ϕ if and only if $R, S \models \phi$.
The argumentation framework AF complies with requirement ϕ if and only if $R \models \phi$.
The argumentation specification Γ complies with requirement ϕ if and only if $\Gamma \models \phi$.

The three kinds of argumentation compliance are illustrated in the examples below.

Example 4 Consider again argumentation framework AF_1 detailed in Example 3. We have the following requirement:

$$((a \rhd b) \wedge ((b \rhd c) \wedge (b \rhd d)) \supset ((a \oslash c) \wedge (a \oslash d))$$

The requirement means that the argument that the service does not allow the distribution of details about what is received as input (a) attacks the argument that the service provides the requesters details to a telemarketer (b), which itself attacks the arguments that the service deals with confidential information for financial transaction (c) and for on-line voting systems (d), and this implies that the argument that the service does not allow the distribution of details about what is received as input defends the arguments that the service deals with confidential information for financial transactions for on-line voting systems. The requirement is used to assess whether avoiding the distribution of input details defends the fact that the service deals with confidential information. We want to verify whether AF_1 is compliant with the requirement above. We have that it holds that $R \models ((a \rhd b) \wedge ((b \rhd c) \wedge (b \rhd d)) \supset ((a \oslash c) \wedge (a \oslash d))$, thus AF_1 is compliant with the requirement.

Example 5 Consider the following requirement $G(a \wedge b \wedge c \wedge d)$. We want to verify whether it holds for AF_1. We have that the extension does not comply with the requirement: $R, S \not\models G(a \wedge b \wedge c \wedge d)$.

Example 6 Consider now another example of use of the defence connective. Agent ag_1 prepares the political manifesto for the left-center party she is part of. She proposes the following arguments: argument a is "The health service should be denationalized to save money", argument b is "Public health is expensive for the nation" and argument c is "We want to ensure equal rights to every citizen". The requirement of the left-center party thus is $\models (a \rhd b) \wedge (c \oslash b) \supset (c \rhd a)$, where we have that a attacks b and c defends b, and this implies that c attacks a. Moreover, we have that

– $(a \rhd b), (c \oslash b) \models (c \rhd a)$

since for all $R \models (a \rhd b), (c \oslash b)$, we also have that $R \models c \rhd a$.

Given the LA4SV language and semantics and the definition of the kinds of verification, we verify whether the argumentation frameworks are compliant with the specified requirements. There are many kinds of properties which can be checked in an argumentation framework. First, there are requirements like "the argument of increasing taxes to the poor people attacks the argument of giving economic sustainment to the lower-middle class", $(a \rhd b)$, or "the arguments of increasing the taxes to poor people, and of cutting the services defend the argument of acting in the interests of the higher class", $a \wedge b \oslash c$. These requirements are more related to the structure of the argumentation framework rather than to its semantics. Second, there are requirements like "the argument claiming that in Italy there are several political parties is acceptable", $R, S \models G(\{a\})$ where we mean that argument a is in the grounded extension. Third, there are more complex requirements like "the acceptance of the argument to higher taxes implies the acceptance of the argument for increasing the services (in the preferred extension)", $R, S \models P(\{a\}) \supset P(\{b\})$, or "the argument to sustain the lower-middle class and the argument to increase the services are acceptable (in the complete extension)", $R, S \models C(\{a\}) \wedge C(\{b\})$.

Example 7 Consider the argumentation framework AF_1 introduced in Example 1. We aim to verify the compliance of AF_1 with a requirement expressed in LA4SV about extensions. The requirement is as follows: if the argument "The service deals with confidential information for financial transactions" and the argument "The service deals with confidential information for on-line voting systems" are accepted then also the argument "The service does not allow the distribution of details on its input" must be accepted too (in the stable extension). We express it in $LA4SV$ as $\models S(c \wedge d) \supset S(a \wedge c \wedge d)$. As stated above, we have that, in AF_1, $R, S \models S(a \wedge c \wedge d)$ holds.

Example 8 Some properties to be considered in the specification of an argumentation framework are the following:

- there is a stable extension;
- there is a unique stable extension: $S(p) \wedge S(q) \supset p \leftrightarrow q$;

In particular, the former is interesting because the stable semantics has a significant drawback: it is not universally defined as there are argumentation frameworks where no stable extensions exist. A simple example is provided by the argumentation framework composed by $A_2 = \{a, b, c\}$ and $R_2 = \{(a, b), (b, c), (c, a)\}$. In this three-length circle, no conflict-free set is able to attack all other arguments. The latter has to be considered because we cannot simply add it as a property: it has to hold for all p and q. Thus, it has to be added as an axiom.

2.3 Logical properties of LA4SV

The logical relations among attack formulas are characterized by the left (LD) and right distribution (RD) properties. They follow from the definition of attack among sets of arguments in terms of attacks among individual arguments. To understand this characterization, we consider two logical consequences. First, logical consequences of the distribution properties (read from right to left) are left (LS), and right strengthening (RS). Right strengthening indicates that the attack connective does not behave like a conditional connective, but it behaves in this respect like a comparative connective. Secondly, the more remarkable logical consequence of the distribution properties (read from left to right) is that if two arguments together attack another argument, then one of these arguments individually attacks the other argument (LT and RT). These splitting properties indicate room for generalizing Dung's theory.

Though for model checking these properties are not directly used, they give some insights in the concepts we use in our logic.

LD $\models (a \wedge b \rhd c) \leftrightarrow (a \rhd c) \vee (b \rhd c)$
RD $\models (a \rhd b \wedge c) \leftrightarrow (a \rhd b) \vee (a \rhd c)$
LS $\models (a \rhd c) \supset (a \wedge b \rhd c)$
RS $\models (a \rhd b) \supset (a \rhd b \wedge c)$
LT $\models (a \wedge b \rhd c) \supset (a \rhd c) \vee (b \rhd c)$
RT $\models (a \rhd b \wedge c) \supset (a \rhd b) \vee (a \rhd c)$

The logical relations among defend connectives are characterized by left additivity (LA) and right distribution (RD) properties. These properties follow from the definition of defence among sets of arguments in terms of attacks among individual arguments. The first logical consequences of these two properties (read from left to right) are left strengthening (LS) and right weakening (RW). Right weakening indicates that the defend connective behaves like a conditional connective. Secondly, we have the conjunction property RC (read from right to left).

LA $\models (a \oslash c) \vee (b \oslash c) \supset (a \wedge b \oslash c)$
RD $\models (a \oslash b \wedge c) \leftrightarrow (a \oslash b) \wedge (a \oslash c)$
LS $\models (a \oslash c) \supset (a \wedge b \oslash c)$
RW $\models (a \oslash b \wedge c) \supset (a \oslash b)$
RC $\models (a \oslash b) \wedge (a \oslash c) \supset (a \oslash b \wedge c)$

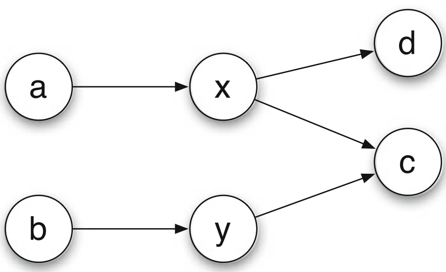
Example 9 The logical relations among defend connectives above can be applied to the argumentation framework visualized in Fig. 2: $AF_3 = \langle A_3, R_3 \rangle$ where $A_3 = \{a, b, c, d, x, y\}$, and $R_3 = \{(a, x), (x, d), (x, c), (b, y), (y, c)\}$. For instance, the RD property for the argumentation framework is as follows: $R \models (a \oslash c \wedge d) \leftrightarrow (a \oslash c) \wedge (a \oslash d)$. This is due to attacks among individual arguments such as $(a \rhd x)$ and $(x \rhd c \wedge d)$. Note that in this case a does not defend b and c.

The relation among attack and defence connectives is as follows. If a set of arguments is finite, we can simply define the defend connective in terms of attack connective.

$$(a \oslash b) \leftrightarrow \bigwedge_{c \in A} ((c \rhd b) \supset (a \rhd c))$$

An instance of this relation, which characterizes the infinite case, is the following requirement already observed in Example 6. It says that the only possible defence

Fig. 2 The argumentation framework AF_3 with attack and defence



is a direct counterattack, and thus rules out other defence tactics. This may seem counterintuitive at first sight, but it makes Dung's system effective.

$$(a \oslash b) \wedge (c \rhd b) \supset (a \rhd c)$$

Example 10 We can then express properties like "a stable extension is also a preferred extension but not viceversa", and "a preferred extension is also a complete extension but not viceversa".

- $\models S(p) \supset P(p)$
- $\models P(p) \supset C(p)$

Example 11 Consider now a political scenario where agents from different parties are arguing about the behavior of the Prime Minister. The three argumentation frameworks are composed by $A_4 = \{a, b, c, d\}$, $A_5 = \{a, b, c, d, e\}$, $A_6 = \{a, b, c, d, f\}$ and $R_4 = \{(d, a), (a, b), (c, b)\}$, $R_5 = \{(d, a), (e, c), (a, b), (c, b)\}$, $R_6 = \{(d, a), (c, b), (f, d), (a, b)\}$, visualized in Fig. 3. In a multiagent system, three agents ag_4, ag_5 and ag_6 are associated respectively to AF_4, AF_5 and AF_6. The meaning of the arguments is as follows: argument a is "A program on the public TV can be called by phone by everybody", argument b is "The Prime Minister cannot call a TV program which is criticizing it", argument c is "The Prime Minister has the right to publicly defend himself on TV", argument d is "Common citizens cannot simply call the TV program to get accepted their comments", argument e is "The Prime Minister is a certified criminal on trial" and argument f is "His opinion is more important than the one of common citizens". An external agent ag_7, who is the leader of the opposition, has to decide if ag_4, ag_5 and ag_6 can be admitted as part of his political party. In order to verify whether these agents are compliant with the specification of his party, he wants to check if the following properties hold for their argumentation frameworks:

- $\alpha_1 : a \rhd b$
- $\alpha_2 : d \oslash b$
- $\alpha_3 : P(a) \supset P(c)$
- $\alpha_4 : G(c \wedge d) \supset \neg G(b \wedge c)$
- $\alpha_5 : S(d \wedge e \wedge b) \supset \neg S(a \wedge b \wedge d)$

The compliance of the argumentation frameworks with the requirements is as follows:

- AF_4 : compliant (α_1, α_4), not compliant $(\alpha_2, \alpha_3, \alpha_5)$

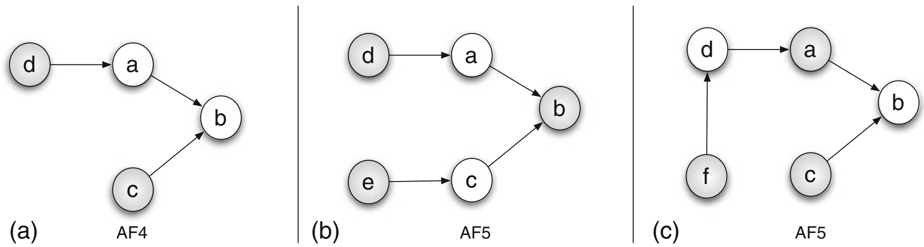


Fig. 3 The three argumentation frameworks of Example 11 to be verified

- AF_5 : compliant (α_1, α_5), not compliant $(\alpha_2, \alpha_3, \alpha_4)$
- AF_6 : compliant $(\alpha_1, \alpha_3, \alpha_4)$, not compliant (α_2, α_5)

Notice that no one of the argumentation frameworks proposed are compliant with the requirements required by the party thus these three agents will not be admitted to be part of the opposition.

Two properties to be discussed are the expressive power of the language, and the compactness of the logic.

Proposition 1 (Expressive power) *The logical language is expressive enough to distinguish two distinct argumentation systems based on the same set of arguments.*

Proof If two argumentation systems are distinct, then there are two arguments a and b such that $R_1(a, b)$ holds in one argument system $\langle A, R_1\rangle$, but $R_2(a, b)$ does not hold in the other $\langle A, R_2\rangle$—or vice versa. Then we have $R_1 \models a \rhd b$, but not $R_2 \models a \rhd b$—or vice versa. □

Proposition 2 (Compactness) *The logic is not compact, when the set of arguments A is infinite.*

Proof Follows directly from universal quantification in the definition of the semantics. For example, assume that A is infinite. We can derive that argument a defends argument b when there is an infinite set of formulas for each argument $c \in A$ that either a attacks c or c does not attack b. However, we cannot derive that a defends b from a finite set of formulas. □

A non-monotonic extension can be defined based on distinguished models and subsets minimal attack relations. Sometimes such distinguished models are called preferred models and the non-monotonic entailment is called preferential entailment.

Definition 6 A model R is a distinguished model of a set of sentences S iff

1. $R \models S$, and
2. there is no $R' \subset R$ such that $R' \models S$.

Non-monotonic entailment is defined as usual:

- $T \mid\!\sim \phi$ if and only if for all distinguished models R of T, we have $R \models \phi$.

The typical use of our logic is when an argumentation framework is specified by a set of attack statements; we call such a set an argument specification.

Proposition 3 *An argument specification is a set of attack formulas $AS = \{p_1 \rhd q_1, \ldots, p_n \rhd q_n\}$. The distinguished model of an argument specification AS is unique.*

There are some limitations to the logic proposed here. First, the semantics leave little room for generalizations of Dung's theory. Secondly, we cannot express the characterizations in propositional logic provided by [6]. Thirdly, we cannot express that stable, preferred and complete semantics admit multiple extensions whereas the grounded semantics ascribes a single extension to a given argumentation framework. In the following section, we therefore discuss an extension of LA4SV in modal logic with the aim to solve these problems.

2.4 A possible worlds semantics

To define variants and generalizations of Dung's theory, we now generalize LA4AC in a modal logic setting. We restrict ourselves to finite sets of arguments. Since sentences of the logic are finite, we cannot represent, and reason about infinite extensions. The logic therefore seems most suitable for finite argumentation frameworks.

Our generalization is based on an attack relation between sets of arguments. Such sets of arguments are called positions and represented in the semantics of the logic by worlds in a possible worlds model. The attack relation is thus a binary relation between worlds, that is, a standard accessibility relation of possible worlds semantics.

Our motivation is that Dung's assumption that the attack relation exists between individual arguments instead of sets of arguments is quite strong, and it is not warranted in cases where the cumulative weight of arguments is decisive [8, 41]. For example, in some legal cases, circumstantial evidence may be used in a cumulative way. Each piece of evidence individually would not be enough to connect a suspect to the crime scene, but many pieces of evidence taken together would be enough to conclude that the suspect was present at the crime scene. So only a set of arguments taken together would attack a position in this case:

– $(a \wedge b \rhd c) \supset (a \rhd c) \vee (b \rhd c)$

The attack relation R has to be defined on sets of arguments instead of individual arguments. The basic idea of our modal logic MLA4SV is that the attack relation R of LA4SV is now interpreted as an accessibility relation among worlds. Each world satisfies a set of arguments, such that we have that a set of arguments attacks another set of arguments.

In this section, we only consider framework and specification compliance, since these are the most relevant, and they are defined most naturally in a modal setting. For extension compliance, we need in addition a proposition indicating whether a world and thus an argument is accepted, and a set of propositions indicating the names of the world/arguments, e.g., in hybrid logic.

Formally, we define a normal bimodal semantics in which modal operator $\Box_1$ represents the attack relation, and $\Box_2$ is a universal modality used for technical reasons. Since we have right strengthening for attack connectives where normal modal operators have right weakening, we use a negation in the definition of the attack connective. Propositional formulas represent positions, i.e., sets of arguments. The logic also has negations and disjunctions in the left and right hand side of our connectives, but we do not use this in the paper. We adapt the definition of defence in terms of attack to deal with our generalized setting where s represents a set of atoms as well as a conjunction of atoms.

Definition 7 (MLA4SV language) Given a set of arguments $A = \{a_1, \ldots, a_n\}$, we define the set ML of MLA4SV formulas as follows.

ML: $a_i \mid \Box_1(\phi) \mid \Box_2(\phi) \mid F(\phi) \mid S(\phi) \mid A(\phi) \mid P(\phi) \mid C(\phi) \mid G(\phi) \mid \neg\phi \mid (\phi \wedge \psi)$ $(\phi, \psi \in ML)$.

We write ML_1 for the fragment of ML that contains only monadic modal operators $\Box_1$ and $\Box_2$. Moreover, disjunction $\vee$, material implication $\supset$, and equivalence $\leftrightarrow$ are defined as usual. We extend the modal logic with the definition of:

- $p \rhd q = \Box_2(p \supset \Box_1 \neg q)$
- $p \oslash q = \bigwedge_{s \subseteq A}(s \rhd q \supset p \rhd s)$

We abbreviate formulas using the following order on logical connectives: $\neg \mid \vee, \wedge \mid \rhd, \oslash \mid \supset, \leftrightarrow$.

For space reasons we only introduce a semantics for ML_1. The other modalities can be described by a non-normal modal semantics only, as they do not satisfy weakening nor strengthening.

Definition 8 (MLA4SV semantics) Let A be a set of arguments. A possible worlds model M is a structure $\langle W, R, V \rangle$ where W is a non-empty set (of possible worlds), R is a binary (attack) relation on W, and V is a valuation function which assigns a subset of A to each element of W. For ϕ a ML formula, we write $M, w \models \phi$ for ϕ is true or satisfied at w in M. The truth relation $\models$ is defined with induction on ϕ in the following way:

- $M, w \models a$ if and only if $a \in V(w)$ for all arguments $a \in A$,
- $M, w \models \neg\phi$ if and only if not $M, w \models \phi$,
- $M, w \models \phi \wedge \psi$ if and only if $M \models \phi$ and $M \models \psi$,
- $M, w \models \Box_1\phi$ if and only if for all w' such that $R(w, w')$ we have $M, w' \models \phi$,
- $M, w \models \Box_2\phi$ if for all $w \in W$, we have $M, w \models \phi$.

We assume that W contains exactly one world for each subset of A.

The logic MLA4SV is axiomatized as follows:

PROP	propositional tautologies
K	$\Box_1(\phi \supset \psi) \supset (\Box_1\phi \supset \Box_1\psi)$
T	$\Box_2\phi \supset \phi$
4	$\Box_2\phi \supset \Box_2\Box_2\phi$
5	$\neg\Box_2\phi \supset \Box_2\neg\Box_2\phi$
INCL	$\Box_2\phi \supset \Box_1\phi$

MLA4SV is a standard normal modal logic with universal relation [30]. The following complexity results are known for this modal logic:

- The complexity of deciding whether a formula of ML is satisfiable is EXP-complete [30].
- The complexity of checking whether a formula of ML is satisfied by a model M is P-complete [29].

Clearly, the language L is a fragment of ML. Moreover, Dung's theory can be characterized by the properties we already discussed:

- $\models (a \wedge b \rhd c) \leftrightarrow (a \rhd c) \vee (b \rhd c)$
- $\models (a \rhd b \wedge c) \leftrightarrow (a \rhd b) \vee (a \rhd c)$

We now consider some instances of Dung's theory. As far as we know, there is no a systematic study of the possible instances of Dung's theory. We consider additional axioms we can impose on the logic MLA4SV. We start with the *irreflexivity* property of R, which corresponds to the property that no argument can attack itself:

IR $\neg(a \rhd a)$

The second property we consider is *symmetry* of the attack relation, which corresponds to the property that if argument a attacks argument b, then argument b attacks argument a.

S $(a \rhd b) \leftrightarrow (b \rhd a)$

Note that symmetry is not accepted often, because a counterexample attacks a general rule, but a general rule does not necessarily attack a counterexample. For instance, if swans are white (a), but in Australia they found black swans (b) then we have $b \rhd a$ without $a \rhd b$.

If the attack relation is symmetric, then the defend relation becomes reflexive, that is, each argument defends itself:

R $a \oslash a$

Note that when we consider traditional properties of conditional logic, we do not seem to get something useful. In particular, *reflexivity* (R) does not hold for the attack relation.

R $a \rhd a$

Transitivity (T) means that if argument a attacks argument b, and argument b attacks argument c, then argument a should attack argument c. This does not hold either. Take $a = c$ for example, then we get $a \rhd a$, which conflicts with IR.

T $(a \rhd b) \wedge (b \rhd c) \supset (a \rhd c)$

By adding the accessibility relations for the monadic modal operators, we can deal with the remaining problems observed at the end of the previous section: first, the semantics leave little room for generalizations of Dung's theory; second, we cannot express the characterizations in propositional logic provided by Besnard and Doutre [6] and third, we cannot express that stable, preferred and complete semantics admit multiple extensions whereas the grounded semantics ascribes a single extension to a given argumentation framework.

We express the characterizations of conflict free sets based on the satisfiability checking condition in propositional logic provided by Besnard and Doutre [6] in the following way. First, we have that if an argument a is attacked by an argument b, then a set containing a will be conflict-free only if it does not contain b:

$$F(p) \wedge (q \rhd p) \supset \Diamond_2 F(p \wedge \neg q)$$

Second, if an argument a attacks an argument b then a set containing a is conflict-free only if it does not contain b.

$$F(p) \wedge (p \rhd q) \supset \Diamond_2 F(p \wedge \neg q)$$

Third, one can also view the concept of conflict-freedom like the fact that for any pair $(a, b) \in R$, a and b cannot both belong to a set if this set is conflict-free.

$$\Box_2(p \rhd q) \supset F(\neg p \vee \neg q)$$

A second characterization provided by Besnard and Doutre [6] aims to characterize the sets which defend all their elements. A formula has to capture the idea that if an argument a belongs to a set which defends all its elements, then for each of its attackers b, there must be in the set an element c which attacks b.

$$\Box_2(p \oslash q) \wedge (r \rhd q) \supset (p \rhd r)$$

Finally, we want be able to express that stable, preferred and complete semantics admit multiple extensions whereas the grounded semantics ascribes a single extension. These characterizations of the semantics are expressed using the MLA4SV language in the following way:

$$\models S(p) \wedge S(q) \supset \Diamond_2\neg(p \leftrightarrow q)$$

$$\models C(p) \wedge C(q) \supset \Diamond_2\neg(p \leftrightarrow q)$$

$$\models P(p) \wedge P(q) \supset \Diamond_2\neg(p \leftrightarrow q)$$

$$\models G(p) \wedge G(q) \supset \Box_2(p \leftrightarrow q)$$

As shown above, the modal logic of argumentation compliance MLA4SV is useful to express the characterizations of the notions of defence and conflict-free sets, and the characterizations of the semantics. Let us consider now how this logic can be used in the specification/verification process of the design of an argumentation framework in a multiagent system.

Example 12 Let $AF_7 = \langle A_7, R_7 \rangle$ be the argumentation framework such that $A_7 = \{a, b, c, d, e\}$ and $R_7 = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$, as introduced by Besnard and Doutre [6]. This argumentation framework is visualized in Fig. 4. We want to check whether the framework is compliant with the irreflexivity, symmetry and transitivity properties, we have discussed thus far. First, this argumentation framework is not compliant with the irreflexivity property of the attack relation: **IR** $\neg(a \rhd a)$ because it holds that that $R \models e \rhd e$. Second, the framework does not satisfy the symmetry property **S** $(a \rhd b) \leftrightarrow (b \rhd a)$. Third, as desired, transitivity does not hold for the attack relations thus it does not hold, for instance, that $M \models (c \rhd d) \wedge (d \rhd e) \supset (c \rhd e)$. In conclusion, the argumentation framework of Fig. 4 is compliant with the symmetry property but it is not compliant with the irreflexivity and transitivity properties.

We can check now the characterization of the semantics: stable, preferred and complete semantics admit multiple extensions whereas the grounded semantics

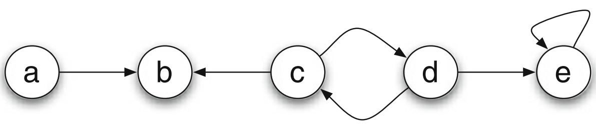


Fig. 4 The argumentation framework from [6] described in Example 12

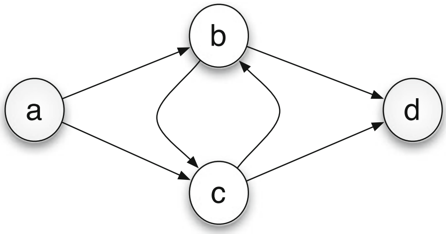


Fig. 5 The argumentation framework of agent ag_8 described in Example 13

ascribes a single extension. The extensions of the argumentation framework are as follows: $M, w \models S(a \wedge d)$, $M, w \models P(a \wedge c)$, $M, w \models P(a \wedge d)$, $M, w \models C(a \wedge c)$, $M, w \models C(a \wedge d)$, $M, w \models C(a)$, $M, w \models G(a)$. Moreover, it holds that $M \models C(a \wedge c) \wedge C(a \wedge d) \supset \Diamond_2 \neg((a \wedge c) \leftrightarrow (a \wedge d))$. In our modal characterization of the semantics, it is considered also the case in which there is only one stable extension, as in this example. The argumentation framework satisfies also the uniqueness of the grounded extension.

Example 13 Consider a multiagent system where agent ag_8 wants to check in the post-rationalization process of design whether her arguments are justified and defendable in a political debate. Let $AF_8 = \langle A_8, R_8 \rangle$ be the argumentation framework such that $A_8 = \{a, b, c, d\}$ and $R_8 = \{(a, b), (a, c), (c, b), (b, c), (b, d), (c, d)\}$, where argument a is "The leader of the party P_1 never maintains his promises", argument b is "The party P_1 will lower taxes", argument c is "The party P_1 will higher taxes" and the argument d is "The party P_1 does not do anything for the country". The argumentation framework of agent ag_8 is visualized in Fig. 5. ag_8 wants to verify the following requirement: if for every argument attacking argument d, then there exists in her framework another argument which defends d. The requirement, expressed in MLA4SV, is verified for the argumentation framework AF_8: $R \models \Box_2(a \oslash d) \wedge ((b \vee c) \rhd d) \supset (a \rhd (b \vee c))$.

Example 14 Let $\langle A_9, R_9 \rangle$ be the argumentation framework of ag_9 such that $A_9 = \{a, b, c, d, e\}$ and $R_9 = \{(a, b), (b, c), (b, d)\}$, $\langle A_{10}, R_{10} \rangle$ be the argumentation framework of ag_{10} such that $A_{10} = \{b, c, e\}$ and $R_{10} = \{(e, c), (c, b)\}$, $\langle A_{11}, R_{11} \rangle$ be the argumentation framework of ag_{11} such that $A_{11} = \{d, e\}$ and $R_{11} = \{(d, e)\}$ (Fig. 6). The arguments are as follows: where argument a is "The leader of the party P_1 never maintains his promises", argument b is "The party P_1 will lower taxes", argument c is "The party P_1 will higher taxes", argument d is "The party P_1 does not do anything good for the country", argument e is "Taxes will increase only for rich people". In the political party, in order to allow these agents to cooperate, the leader sets a number of requirements the unified argumentation framework has to satisfy.[1] Consider the following properties:

– $\alpha_1 : G(a) \supset \Box_2 G(a \wedge c)$;

[1] In this paper, we are not interested in an analysis of the techniques developed to merge the argumentation frameworks, presented for instance in [21]. Here we simply consider the merged framework as the union of the arguments and the attack relations of each partial framework.

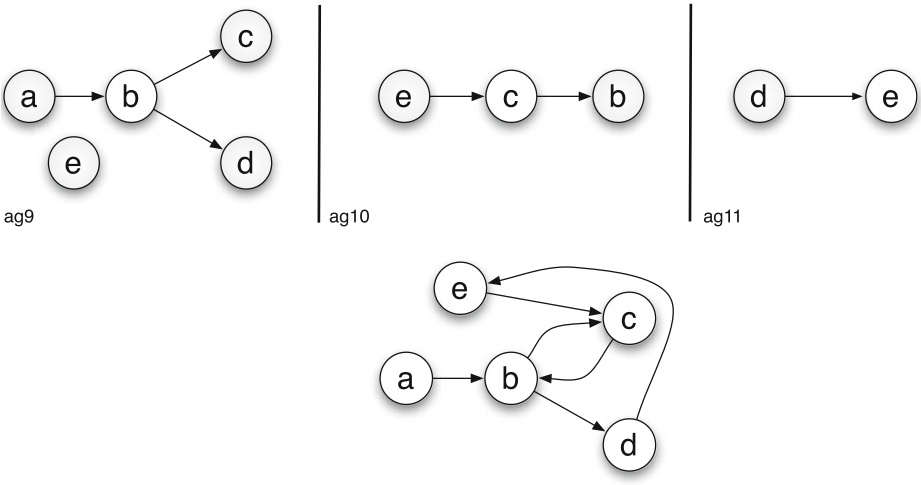


Fig. 6 The argumentation frameworks of the agents ag_9, ag_{10}, ag_{11} and the unification of their frameworks described in Example 14

- $\alpha_2 : (b \rhd (c \vee d)) \wedge (a \rhd b) \supset \Box_2(a \oslash (c \vee d))$;
- $\alpha_3 : (e \rhd c) \supset \Box_2 \neg P(e \wedge c)$;

The first requirement means that "if it is accepted that the leader of the party P_1 never maintains his promises then it is also accepted that the party P_1 will higher taxes", where again $G(a)$ means that a is part of the grounded extension. The grounded extension of the unified argumentation framework is $M, w \models G(a \wedge c \wedge d)$ thus the framework is compliant with the first property α_1. The second requirement means that if the argument b "the party P_1 will lower taxes" attacks both the argument c "The party P_1 will higher taxes", and the argument d "The party P_1 does not do anything good for the country", and argument a "The leader of the party P_1 never maintains his promises" attacks argument b then argument a defends both arguments c and d. It holds that $M \models (b \rhd (c \vee d)) \wedge (a \rhd b) \supset \Box_2(a \oslash (c \vee d))$ thus property α_2 is satisfied. The third requirement means that if argument e attacks argument c, the argument "Taxes will increase only for rich people" and the argument "The party P_1 will higher taxes" cannot be accepted together (in the preferred extension). The preferred extension is $M, w \models P(a \wedge c \wedge d)$. It holds that $M \models (e \rhd c) \supset \Box_2 \neg P(e \wedge c)$ thus the requirement is satisfied by the unified argumentation framework. The unified framework is compliant with the requirements, and the agents can cooperate profitably for improving the party.

3 A logic of meta-argumentation for higher-order attacks

In the analysis of argumentation, reasoning about arguments has been restricted to meta-rules such as, for example, the order of arguments, or the choice of words.

However, Dung [24] has shown that reasoning about arguments can also be based on concepts such as attack and defence. A typical example may be:

A: I think p and q defend r.
B: But s attacks r.
C: No problem, since p attacks s.

Note that the agents do not enumerate the complete argumentation framework, that is, they do not list the complete attack relation R of the argumentation framework $\langle A, R\rangle$. To formalize this example we therefore cannot assume a fixed argumentation framework $\langle A, R\rangle$, as Dung does. We need the logical language to quantify over argumentation frameworks.

In this example, the agents make arguments like "p and q defend r" which themselves refer to arguments p, q and r. The former may therefore be called meta-arguments. The logic that formalizes or characterizes the reasoning of agents about arguments, when they construct meta-arguments, is therefore at first sight quite different from the logic typically used in argumentation. We believe that the confinement of the logic to the internal structure of arguments is too limited; there is also a role of logic in the formalization of reasoning about the arguments.

In the last years, numerous approaches to meta-argumentation, such as using abstract argumentation theory, to talk and reason about abstract argumentation theory have been provided [15, 17, 42, 43]. In particular, some of these approaches [4, 5, 14, 28, 37, 38] deal with the problem of how to model higher order attack relations. The basic idea consists in extending Dung's abstract framework with an additional binary attack relation which is called higher order attack. An higher order attack relation is a binary relation $(A \cup R) \times (A \cup R)$ which holds not only among arguments, but among attack relations and arguments. This kind of extended argumentation framework would require new argumentation semantics in order to assess the set of acceptable arguments in presence of an higher order attack. Modgil and Bench-Capon [38] firstly apply a meta-level approach to higher order attacks. Roughly, an higher order attack of the kind $(a \rhd (b \rhd c))$ at the object level is represented in the meta-level as an attack from argument a to the meta-argument representing the attack relation $(b \rhd c)$. Other approaches to deal with meta-arguments have been developed by [2, 3, 14, 22].

The aim of this section is to present a logic of meta-argumentation in order to provide a formal way to represent also higher order attacks between arguments and attack relations. The sole approach aiming at formalizing a logic of meta-argumentation is provided by Wooldridge et al. [44] where they observe that even the most superficial study of argumentation and formal dialogue indicates that, not only there are arguments made about object-level statements, they are also made about arguments. They introduce argumentation in meta-level reasoning, such as reasoning about reasoning, observing that logical approaches to meta-level reasoning have been widely studied. They adopt a first-order meta-logic which is a first-order logic whose domain (the set of entities that may be referred to in the language) includes sentences of another language (the object language).

We agree with Wooldridge et al. [44] and Boella et al. [14] that meta-argumentation is particularly useful for agent theory. The meta-level could be used, potentially, to speed up argumentations by means of a kind of "caching" function. Just like in chess *Polish opening*, you can use patterns of arguments, give them a name, and know that such a pattern attacks or defends another pattern. If you respect your opponent, there is no need to "play out" the whole argument.

Our logic of meta-argumentation does not consider the internal structure of the arguments, as done by Wooldridge et al. [44], thus we develop a logic of meta-argumentation considering arguments as atomic elements whose internal structure is not taken into account, as in Dung [24]. Our aim is to define a logic of meta-argumentation such that we can encompass different kinds of attacks between attack relations and arguments as modeled by [2, 14, 38]. Differently from the modal logic of abstract argumentation we have introduced in Section 2.4, in the logic of meta-argumentation we introduce the attack modality by using the fibring methodology [26]. We follow the example of Boella et al. [13] where the *says* operator, used in access control logic, is introduced using the fibring methodology, and of Boella et al. [16] where a multi modal monadic second-order logic with operators based on fibring is introduced to represent and reason about coalition formation and cooperation.

In the logic of meta-argumentation, we enrich propositional logic with formulas of the kind ϕ *attacks* ψ (I) where *attacks* is a modal binary operator and ϕ and ψ are general formulas. We let ϕ and ψ belong to the same language L_0 defined in Section 2. In order to formally specify how to evaluate expressions like the one above, we formalize the *attacks* modality by using the fibring methodology [13] which provides a formal tool to combine logics in a common framework which is coherent and does not collapse. This approach offers us to iterate the *attacks* modality and to have extremely complex formulas in which free variables are shared between different levels of nesting of the *attacks* operator. In Formula (I), ϕ and ψ can share variables, i.e., arguments, and ϕ may include occurrences of the *attacks* operator. The formula ϕ is used to select the set of arguments making the assertion *attacks*.

Example 15 For selecting a single argument whose name is a, we write a *attacks* b. We can express the fact that two arguments a and b together attack c in the following way $(a \wedge b)$ *attack* c. In this view, we can express that an argument a does not attack an argument b in the following way $\neg a$ *attacks* b.

We now introduce our basic logic of meta-argumentation step by step from a semantical point of view. First, we introduce, as done by Boella et al. [13], modalities indexed by propositional atoms, then we take into account classical models for the propositional setting. The logic of meta-argumentation can be defined for any logic L as a meta-level logic based on L. In this paper, we motivate the language for the case L = classical logic. Adding the *attacks* connective to a system is like adding many modalities. In order to explain and motivate the logic of meta-argumentation technically, we analyze the options of adding the modalities to classical propositional logic. The approach we address is semantic.

Let S be a nonempty set of possible worlds. For every subset $U \subseteq S$, consider the binary relation $R_U \subseteq S \times S$. This defines a multimodal logic containing at most 2^S modalities $\Box_U$, $U \subseteq S$. The models are of the form (S, R_U, t_0, h), $U \subseteq S$. We have that if $U = \{t | t \models \phi_U\}$ for some ϕ_U we obtain a modal logic with modalities indexed by formulas of itself. We now provide some formal definitions.

Definition 9 (Language) Consider classical propositional logic with connectives $\wedge, \vee, \supset, \neg$ and a binary connective $\Box_\phi \psi$, where ϕ and ψ are formulas. The usual definition of wff is adopted.

We define Kripke models as follows:

Definition 10 (Kripke models)

– A model has the form

$$m = (S, R_U, t_0, h), U \subseteq S$$

where for each $U \subseteq S$, R_U is a binary relation on S, $t_0 \in S$ is the actual world and h is an assignment, giving for each atomic q, a subset $h(q) \subseteq S$.

– We can extend the assignment h to all formulas by structural induction:

  – $h(q)$ is already defined, for q atomic (i.e., argument)
  – $h(A \wedge B) = h(A) \cap h(B)$
  – $h(\neg A) = S - h(A)$
  – $h(A \to B) = (S - h(A)) \cup h(B)$
  – $h(A \vee B) = h(A) \cup h(B)$
  – $h(\Box_\phi \psi) = \{t | \forall s(t R_{h(\phi)} s \supset s \in h(\psi))\}$

– $m \models A$ iff $t_0 \in h(A)$.

It is our intention to read $\Box_\phi \psi$ as ϕ *attacks* ψ.

We refer to Boella et al. [13] for the completeness proof of the fibred security language. The completeness of the logic of meta-argumentation is equivalent. The further step consists in the definition of an axiomatization for this logic of meta-argumentation. A number of problematic issues arise from the definition of this generalization as, for instance, the definition of the meaning of axioms like $\Box_B X \to \Box_A X$ in argumentation theory where A and B are sets of arguments. The definition of an axiomatization for the logic of meta-argumentation is under definition, and it is left as future work.

Consider the two properties we discussed thus far for the modal variant of our logic:

– $\models (a \wedge b \rhd c) \leftrightarrow (a \rhd c) \vee (b \rhd c)$
– $\models (a \rhd b \wedge c) \leftrightarrow (a \rhd b) \vee (a \rhd c)$

These two properties do not hold for the logic of meta-argumentation as it is defined. However, it is always possible to define these properties only for a finite subset of formulae for which we are interested in having these properties satisfied. The possibility to define properties which hold only for finite subsets of formulae, i.e., arguments, gives to our meta-argumentation logic an high level of flexibility. The only property which holds for this logis is the following one:

– $\models (\phi \leftrightarrow \psi) \leftrightarrow (\Box_\phi f \leftrightarrow \Box_\psi f)$

We now want to adopt our logic of meta-argumentation in order to formally define the requirements we have to check for higher order argumentation frameworks. Consider the following example from [2], extended to provide the intuition of the use of higher order attacks for representing preference relations.

Example 16 Suppose agent Bob is deciding about his Christmas holidays together with Sue and, as a general rule of thumb, he is willing to buy cheap last minute offers. Suppose two such offers are available, one for a week in Gstaad and another for a week in Cuba. Then, using his behavioral rule, Bob can build two arguments, one, let say G, whose premise is "There is a last minute offer for Gstaad" and whose conclusion is "I should go to Gstaad", the other, let say C, whose premise is "There is a last minute offer for Cuba" and whose conclusion is "I should go to Cuba". As the two choices are incompatible, G and C attack each other, a situation giving rise to an undetermined choice. Suppose however that Bob has a preference P for skiing and knows that Gstaad is a renowned ski resort. So let us consider P as an argument whose premise is "Bob likes skiing" and whose conclusion is "If possible, Bob prefers a ski resort". P might then attack C, but this would not be sound since P is not actually in contrast with the existence of a good last minute offer for Cuba and the fact that, according to Bobs general behavioral rule, this provides him with a good reason for going to Cuba. Thus, it seems more reasonable to represent P as attacking the attack from C to G, causing G to prevail. Assume now that Sue tells Bob that there have been no snowfalls in Gstaad since one month, and from this fact he derives that it might not be possible to ski there. This argument N, whose premise is "The weather report informs that in Gstaad there were no snowfalls since one month" and whose conclusion is "It is not possible to ski in Gstaad", does not affect neither the existence of last minute offers for Gstaad nor Bob's general preference for ski, rather it affects the ability of this preference to affect the choice between Gstaad and Cuba. Thus argument N attacks the attack originated from P. Suppose that Mary tells Bob that in Gstaad it is anyway possible to ski, thanks to a good amount of artificial snow. This leads to building an argument, let say A, which attacks N. Finally suppose there is an argument M, whose premise is "It is not possible to ski in Gstaad" and whose conclusion is "It is better to go skiing in Cortina". It is not attacked directly by argument A but it is attacked by the attack relation from A to N. This example is illustrated in Fig. 7.

The argumentation framework described in the Christmas holidays example is an extended argumentation framework $\langle A, R, R^2\rangle$ where A is a set of elements called arguments, R is a binary attack relation on $A \times A$, and R^2 is a binary higher order attack relation on $(A \cup R) \times (A \cup R)$. Let $\langle A, R, R^2\rangle$ be the extended argumentation framework of the Christmas holidays example such that $A_{12} = \{g, c, p, n, a, m\}$, $R_{12} = \{(g, c), (c, g), (a, n)\}$ and $R^2_{12} = \{(p, (c, g)), (n, (p, (c, g))), ((a, n), m)\}$. We now provide a partial description of this extended argumentation framework by using the logic of meta-argumentation in which only the existing attacks are considered. Notice that for providing a complete description of the framework we should describe also which attacks does not hold. Using the *attacks* operator we can represent the argumentation framework of Fig. 7 as the formula

$$\Box_g c \wedge \Box_c g \wedge \Box_a n \wedge \Box_p(\Box_c g) \wedge \Box_n(\Box_p(\Box_c g)) \wedge \Box_{\Box_a n} m$$

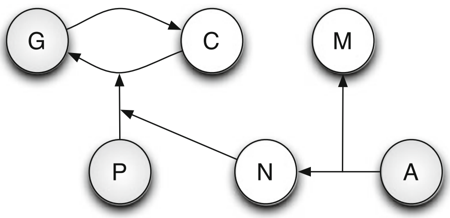


Fig. 7 The argumentation framework of the Christmas holidays example

Notice that we represent the higher order attacks from argument a and argument n to the attack from c to g, and the attack from p to the attack $c \rhd g$, respectively, as nested modalities where arguments p and n correspond to the formula ϕ, and $c \rhd g$ and $p \rhd (c \rhd g)$ correspond to formula ψ in $\Box_\phi \psi$. In particular, the attack established by the preference relation represented by argument p to the attack among c and g is an higher order attack of order 2, and the attack addressed by argument n towards a second order attack is an higher order attack with order 3. We represent, instead, the attack from an attack to an argument as a nested modality where the attack operator is the index of the modality. For example, the attack from $a \rhd n$ to argument m is represented as an higher order attack of order 2 in the following way $\Box_{\Box_a n} m$. Now we want to check the compliance of the extended argumentation framework with the requirement which specifies that "if there is an attack from argument y to argument z which attacks argument x, then argument x attacks argument y"

$$\alpha : \Box_{\Box_y z} x \supset \Box_x y$$

We match the nested attack relation $\Box_{\Box_y z} x$ with the nested attack relation $\Box_{\Box_a n} m$ of the extended argumentation framework. We verify if it holds in the framework that $\Box_m a$. This attack relation is not specified in the argumentation framework thus we return that the requirement α is not satisfied by the considered framework: $R_U \not\models \Box_{\Box_a n} m \supset \Box_m a$. Summarizing, this logic of meta-argumentation allows us to specify and verify complex requirements for extended frameworks, such as extended argumentation frameworks with higher order attacks, which we cannot express by using only the modal variant of our logic.

4 Related work

Since the logic of the attack connectives satisfies left and right strengthening, it seems that it may be related to preference logic. In particular, "argument a attacks argument b" may be interpreted as "argument a is preferred to argument b". However, this is less helpful than it may seem at first sight, because the area of preference logic is characterized by lack of consensus. Further observations concerning the relation with preference logic are provided in [12].

A logic for argumentation is presented by Bochman [11]. The objective of this study consists in a systematic development of a propositional approach to argumentation, in which arguments are represented as special kinds of propositions. By an argument theory, the author means an arbitrary set of attacks between sets of arguments. He extends the notion of attack relation to the notion of collective attack relation by considering arbitrary sets of arguments satisfying the compactness requirement. Collective argumentation can be given a four-valued semantics that stems, given $a \rightarrow b$: *If all arguments in a are accepted, then at least one of the arguments in b should be rejected.* This kind of constraints is similar to what we adopt for checking compliance but we do not introduce new semantics, we refer to standard Dung's ones. Moreover, the author extends the language of arguments with a negation connective, called a global negation, since it switches the evaluation contexts between acceptance and rejection. Bochman [11] introduces a number of stronger propositional attack relations that correspond to systems of causal inference. This probative argumentation allows reasoning by cases. The correspondence between argumentation and causal reasoning is established on the level of logical monotonic formalisms. Consider now the similarities and the differences between the logic of argumentation proposed by Bochman [11] and the one proposed in this paper. Both the approaches see the development of a logic of argumentation as a useful extension of an abstract argumentation theory that allows us to endow argumentation with logical capabilities. The perspectives are different. Bochman [11] applies the logic of argumentation to causal reasoning, investigating the relation between the logic and the four-valued semantics which is associated to the attack relation. We define a logic of argumentation where not only attack is directly represented but also defence between arguments. Moreover, we extend this logic of argumentation with the modalities, developing a modal logic of abstract argumentation. Finally, we show how to apply this logic of argumentation for checking the compliance of the argumentation frameworks.

The defend connective behaves like a standard conditional connective, with one important exception: it does not satisfy the identity rule. An argument a does not necessarily defend argument a, because when another argument b attacks argument a, there is no reason why argument a attacks argument b (unless the attack relation is symmetric, of course). Consequently, to consider the defend connective we need an identity free logic, which are rare. A possible choice is input/output logic [35, 36], which has been proposed in philosophical logic for normative or deontic reasoning, and which has been used in artificial intelligence to characterize causal reasoning [9] and logic programming [10]. To emphasize the lack of identity, Makinson and van der Torre write their conditional "if input a, then output x" as (a, x).

The defend connective behaves like so-called simple-minded output, which is defined as a closure on a set of conditionals under replacements of logical equivalents, and the following three proof rules of strengthening of the input, weakening of the output, and the conjunction rule for the output. See the above mentioned papers for semantics of this proof system.

Definition 11 Let AS be a set of defend formulas $\{p_1 \oslash q_1, \ldots, p_n \oslash q_n\}$. Simple-minded output is the closure of AS under replacement of logical equivalents, and the following three rules.

$$\frac{a \oslash x}{a \wedge b \oslash x} SI \qquad \frac{a \oslash x \wedge y}{a \oslash x} WO \qquad \frac{a \oslash x, a \oslash y}{a \oslash x \wedge y} AND$$

At this point, it is very tempting to define both attack and defend in a single conditional logic to study their interaction. This is formalized in the following definition of the input/output logic of abstract argumentation for verification.

Definition 12 Let IOLA4SV be simple minded output, together with the following two definitions for p and q conjunctions of atomic propositions.

- $p \rhd q = (p, \neg q)$
- $p \oslash q = (p, q)$

Let us now consider the relation between attack and defend in IOLAA. The characteristic axiom that a defends b implies that if c attacks b, then a also attacks

c, is given by the following unusual rule: $\frac{(a,b),(c,\neg b)}{(a,\neg c)}$. However, clearly we do not want to derive that a defends b implies that if c attacks b, then c also attacks a, that is: $\frac{(a,b),(c,\neg b)}{(c,\neg a)}$. Note that the distinction between these two inference rules is whether the formulas start with a negation symbol. Consequently we cannot accept one without the other, unless we add additional syntactic constraints. For further details about the comparison among LA4SV and I/O logic, see [12].

Note that if we give to the defend connective $\oslash$ another connotation, we can have that the two inference rules above become desirable. Let us assign the connective $\oslash$ to the notion of support. When we have that $a \oslash b$ then this means that "argument a supports argument b". The input/output logic for argumentation verification with support instead of defence is the same as defined above, in particular it holds that $p \oslash q = (p, q)$. Following existing approaches in the literature about support in abstract argumentation [17, 22], we have that when a support relation and an attack relation involve the same arguments, new attacks may arise. In particular, the following two patterns are available: if $a \oslash b$ and $c \rhd b$ then $a \rhd c$ and, if $a \oslash b$ and $c \rhd b$ then $c \rhd a$. These two patterns are not the only ones defined in this context, in particular the first one considers the support relation stronger then the attack relation. The mirror pattern where attack is stronger than support considers that if $a \oslash b$ and $c \rhd b$ then $c \rhd a$. Given the highly controversial connotation of the notion of support in argumentation theory, the development of a logic of argumentation able to characterize this notion is addressed as future research. Particularly, this logic should be able to solve the confusion between the logic inside the arguments, i.e, support intended between the premises and a conclusion, and the logic of the arguments, i.e., the support between abstract arguments.

In Krause et al. [33], the authors present the syntax and proof theory of a logic of argumentation. This logic of argumentation is the core of a proof theoretic model for reasoning under uncertainty. The starting point is that arguments have the form of logical proof, but they do not have the force of logical proof. The authors take an existing logic, (&,⊃) minimal logic, as the basis for the logic of argumentation and the arguments are seen as proofs in minimal logic. Propositions are labelled with a representation of the arguments which supports their validity. Argumentation is decomposed into two components: the construction of arguments themselves, and the reasoning about arguments at the meta-level. The aim of this proposal is not really to generate a new logic as to augment an existing logic by labeling propositions with the arguments which support those propositions.

In Gabbay [27], the connection between argumentation theory and modal logic is investigated. The logical content of an argumentation network is the sets of acceptable, not acceptable and undecided arguments. The modal logic $LN1$ is such that for a Kripke model and for any modal formula $m(P)$ for an argumentation network P such that $m(P)$ holds in the model, only three types of assignment are possible: 0, 1, ?. Any model of $m(P)$ will give truth values to the atoms in each world and the atom will acquire a type. The modal formula $m(P)$ contains the nodes of the network P as atomic propositions. The difference between the approach proposed by Gabbay [27] and our one is in the relation with argumentation theory. Gabbay proposes a correspondence between the sets of acceptable/not acceptable/undecided arguments of an argumentation network and all the models of the modal formula associated to the same argumentation network while we define a new logic with two kinds of modal operators, one representing attack and the other representing the universal modality.

Caminada and Gabbay [20], starting from the ideas of Gabbay [27], define several notions of extensions within modal logic. The authors, with a proof-theoretic point of view, characterize complete and grounded extensions in terms of modal logic entailment. The similarity with our approach consists in the denotation of propositional atoms as arguments. The difference is that we introduce the extensions as primitives of the language while Caminada and Gabbay [20] describe alternative ways to express argumentation semantics using modal logic.

Grossi [29] analyzes argumentation theory by means of logical tools developed in modal logic. The idea is that a Dung's abstract framework can be viewed as a Kripke frame where the arguments are sets of modal states and the attack relation is the accessibility relation. The author shows how a number of key notions in argumentation theory can obtain a natural formulation within appropriate modal languages. As examples of such formulation, the author studies argumentation labelings as Kripke models where a valuation is translated into a function from a vocabulary P to sets of arguments. The paper endows a calculus and dialogues games are discussed regarding the logic of argumentation. The differences with our approach are that Grossi [29] denotes propositional atoms as sets of arguments instead of arguments and the extensions are defined in the logic instead of being considered as primitives. For instance, Grossi [29] uses worlds for arguments while we use propositional atoms as arguments. However, the aims of the two logics are different: Grossi [29] tries to define some particular Dung's semantics, and we aim to define the general Dung's framework, starting to capture the notion of defence.

An approach using value-based argumentation for justifying compliance in the context of risk management has been proposed by Burgemeestre et al. [18]. In risk management, compliance is verified through an audit process to identify whether rules and procedures are present in an IT-system, are known by employees, and are actually adhered to. The burden of proof in this case lies with the company: they must decide upon and explain how they ensure compliance to the relevant regulations in their specific business. The claim of the authors is that decision support about compliance should not only help companies to make decisions but also enable external auditors to assess the quality of the decisions. They argue that argumentation theory provides a framework that can help the companies to underpin their compliance decisions and justify them to stakeholders. The structured nature of the argumentation framework with its claims and counter attacks closely resembles the audit process as it is encountered it in practice. They underline that the so called critical questions which are provided by the argumentation approach can help to make the audit process more systematic. Although Burgemeestre et al. [18] propose an approach which applies argumentation theory in order to check the compliance of a system, there are many differences with our approach. In particular, we propose a more formal approach with the definition of three logics used to specify and verify the requirements of the system, while Burgemeestre et al. [18] is more based on the dialectical view of argumentation theory with the introduction of the critical questions like in argumentation schemes [7]. However, the basic idea is shared by the two approaches and further research will be addressed in applying our logics of argumentation to the case study introduced by Burgemeestre et al. [18].

Kaci et al. [32] propose the following argumentation specification for argumentation based systems: the user specifies abstract arguments, a symmetric incompatibility relation on the arguments, and a preference relation over the arguments. The system calculates first the attack relation from the incompatibility relation and the

Table 1 Comparison between the logics of argumentation in the literature

	Bochman	Gabbay	Gabbay-Caminada	Grossi	LA4SV/MLA4SV	Meta
Propositional	Yes	No	No	No	Yes	Yes
Modal	No	Yes	Yes	Yes	Yes	Yes
Conditional	No	No	No	No	Yes	No
Kripke model	No	Yes	Yes	Yes	No	Yes
Proof-theory	No	No	Yes	No	No	No
Games	No	No	No	Yes	No	No
Defence	No	No	No	No	Yes	No
Compliance	No	No	No	No	Yes	Yes

preference relation, and thereafter the acceptable arguments using one of Dungs semantics. This proposed specification allows the authors to define the properties of the preference relation and the way to define the attack relation from the other two relations. They first introduce basic propositional argumentation, and then they extend it with a support function, and a conclusion function. Even if the basic idea of argumentation specification is close to the one developed in this paper, Kaci et al. [32] address this issue by considering an argumentation framework with an incompatibility relation together with a preference relation over arguments, and the resulting mapping is that argument A attacks argument B if and only if A and B are incompatible, and B is not preferred to A. In this paper, we do not deal with preferences in argumentation theory, and we present three kinds of logics to specify and verify the requirements of abstract frameworks.

Table 1 summarizes the comparison between the logics of argumentation presented in this section, and our one.

5 Concluding remarks and future research

In this paper, we present three variants of a logic of argumentation. We introduce this logic for specifying and verifying abstract argumentation frameworks. In a design perspective, argumentation frameworks may have to be compliant with the requirements imposed by the system designer. In this situation, we need a formal tool to specify the requirements the system imposes, and then to verify whether the specific argumentation framework is compliant or not with these requirements. In particular, we highlight three possible cases of verification: (i) verify if an argumentation framework is compliant with the requirement, (ii) verify if an extension of an argumentation framework is compliant with the requirement, and (iii) verify if all the frameworks satisfying a specification satisfy also the requirement. The three logics of argumentation allow us to specify and verify abstract frameworks. In particular, the logic of meta-argumentation allows to specify and verify also extensions of abstract argumentation frameworks dealing with higher-order attacks among the arguments.

There are several issues for further research.

First, the main extension which has to be addressed is the development of suitable algorithms for checking compliance of argumentation frameworks, providing then the complexity studies for these algorithms and applying them to case studies.

Second, we have to to consider instantiated arguments [19] instead of arguments without an explicit internal structure. In that case, is it still sensible to verify compliance at the abstract level? Is there still a role of our logic of argumentation verification? Consider the following example of knowledge base. We have the following not defeasible information $\{a, b, \neg(c \wedge d)\}$ and the following defeasible rules $a \Rightarrow c$ and $b \Rightarrow d$. We can build the following arguments where strict rules are based on classical entailment: $A : a \Rightarrow c$, $B : b \Rightarrow d$, $C : (a \Rightarrow c) \wedge \neg(c \wedge d) \rightarrow \neg d$, $D : (b \Rightarrow d) \wedge \neg(c \wedge d) \rightarrow \neg c$ and $E : \neg(c \wedge d)$. If you want to verify the compliance of this framework with the property $R, S \models F(A) \supset F(A \wedge B \wedge E)$. The framework is compliant with the property but is it sensible to consider the compliance at this level? For example, this framework yields to inconsistent conclusions. If we want to check the consistency of the framework, we should look at the defeasible and not defeasible information from which it is constructed.

The third issue for the extension of the languages used in this paper is to introduce a temporal operator, and verify the compliance of argumentation frameworks changing over time. This is a straightforward extension, but it brings the logic much closer to classical logics used in computer science for verification and specification. Moreover, with the dynamics of argumentation, more interesting properties can be specified than in the present static case

Forth, the first propositional variant of the logic introduces monadic modal operators and provides a model-based semantics where models include argumentation frameworks. Without an axiomatic system, all the proofs have still to be made at the level of argumentation frameworks. We plan to axiomatize the logic as future work.

Finally, this logic for argumentation compliance can be used to model policies in the field of policy-based management. We propose to represent the policies using argumentation theory, and then to model the conflicts between policies as attack relations between arguments. The logic can be used to express the external principles that often constrain the policies of a system. We can check in this way if a policy or a set of policies is compliant with the principles posed by the management of the policies.

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