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**Parabolic avalanche scaling in the synchronization  
of cortical cell assemblies**  
**Supplementary Information**

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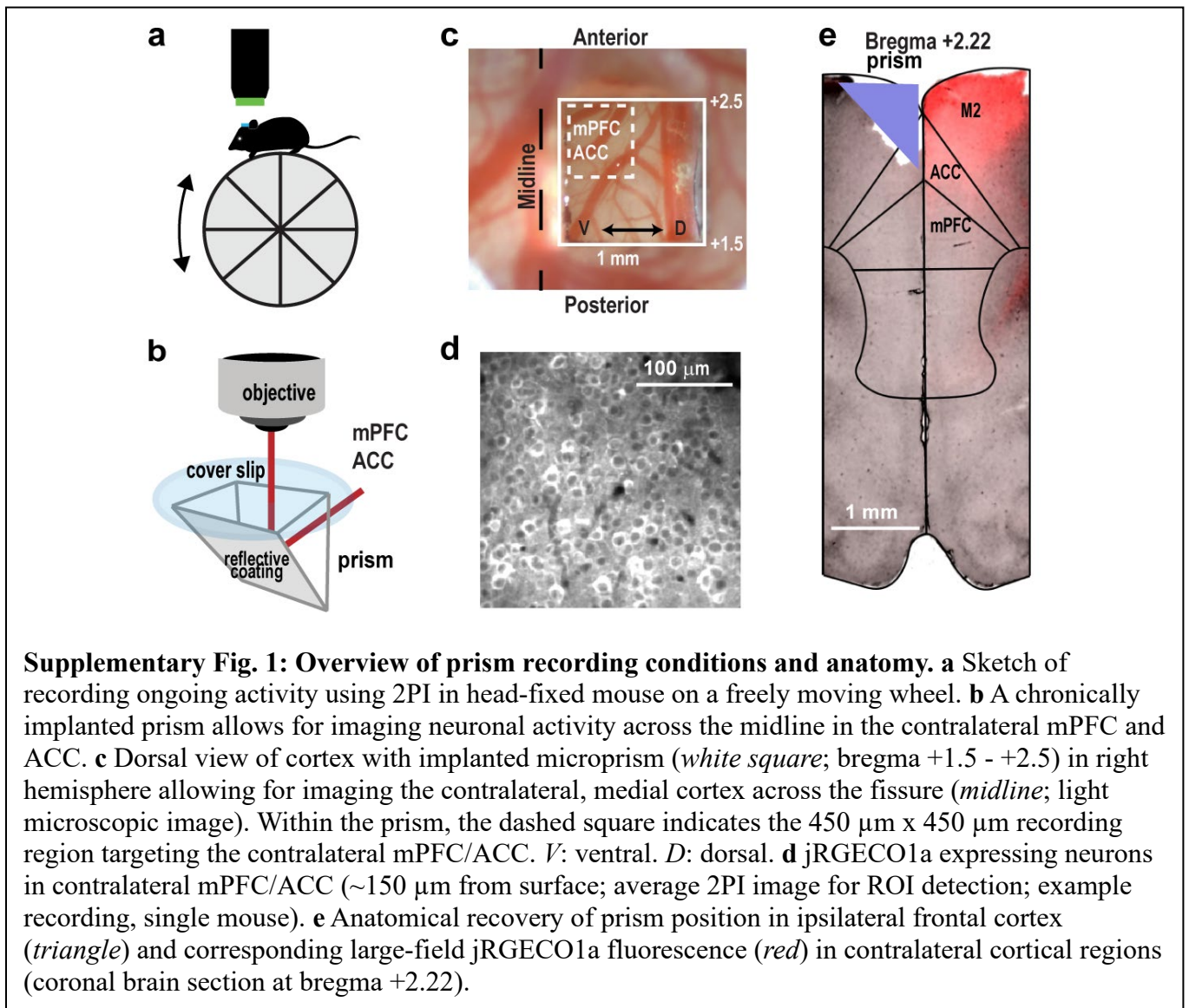
† Correspondence

Figures: 18 Supplementary Figures

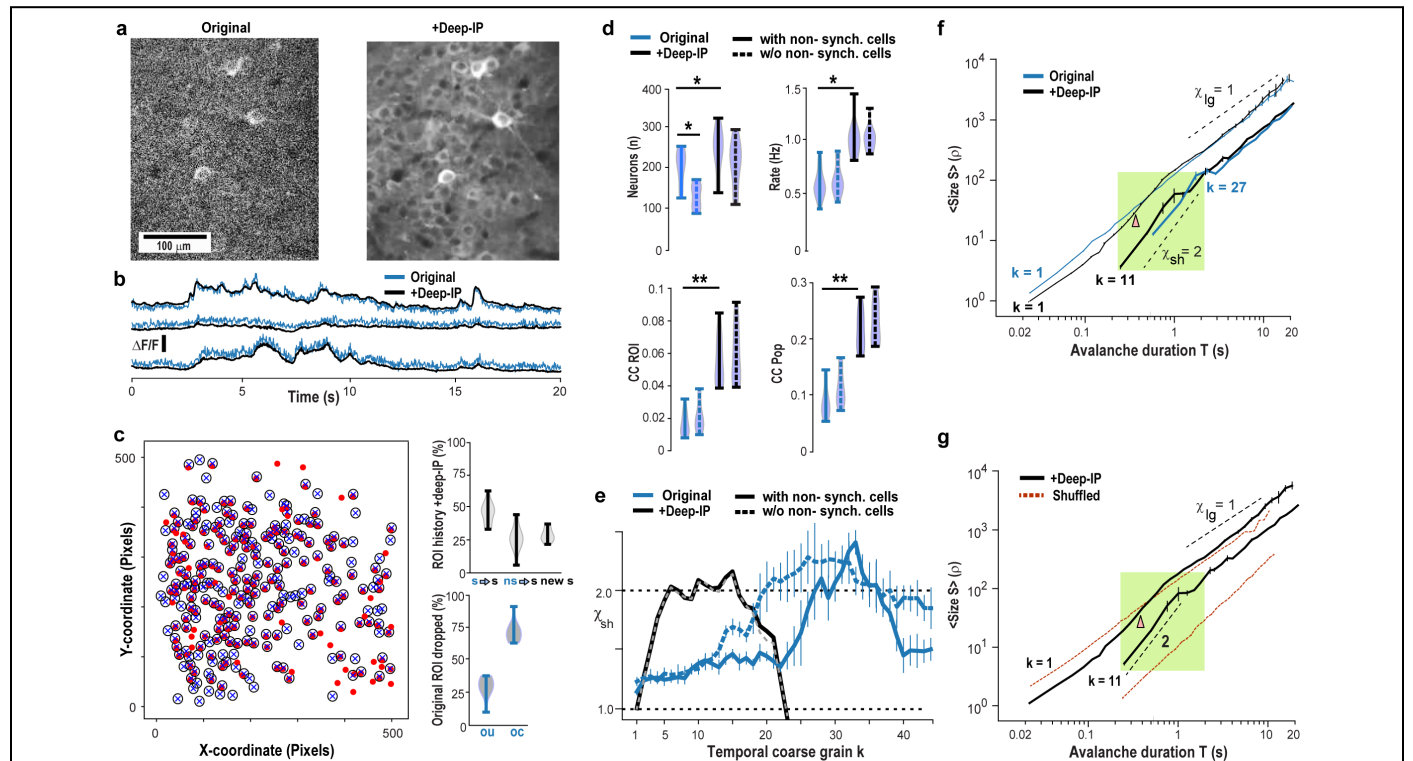
Running Title: Parabolic avalanches in cortical synchronization

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## 19 Supplementary Fig. 1

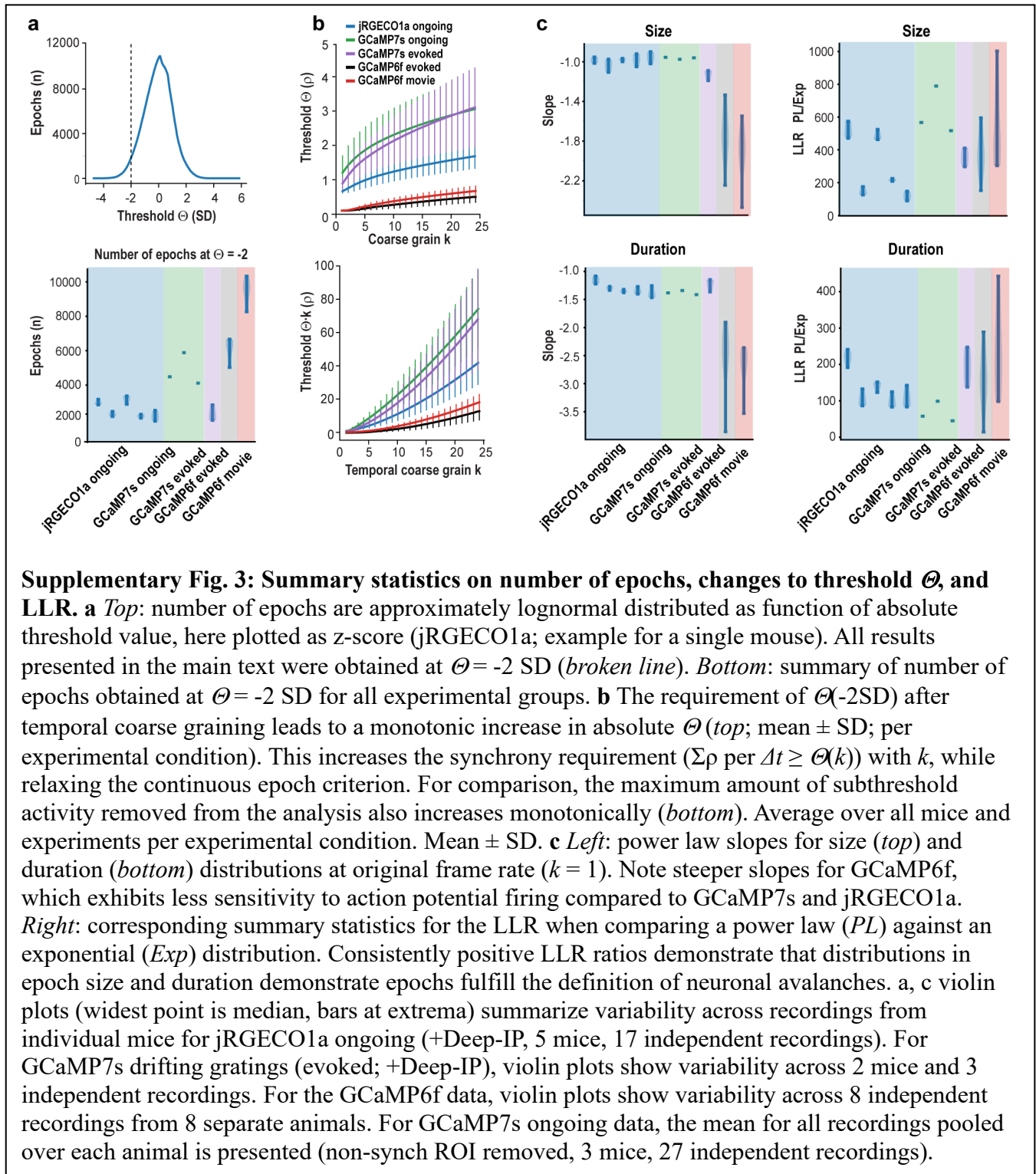


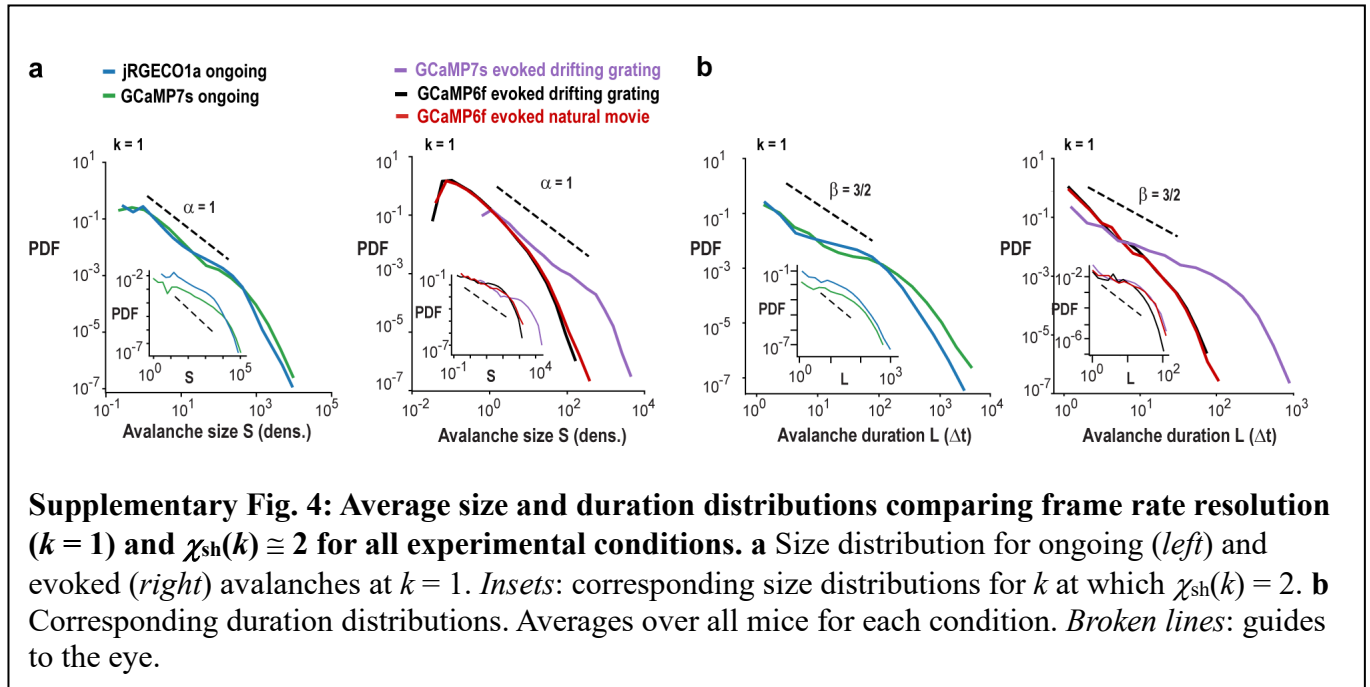
## 21 Supplementary Fig. 2

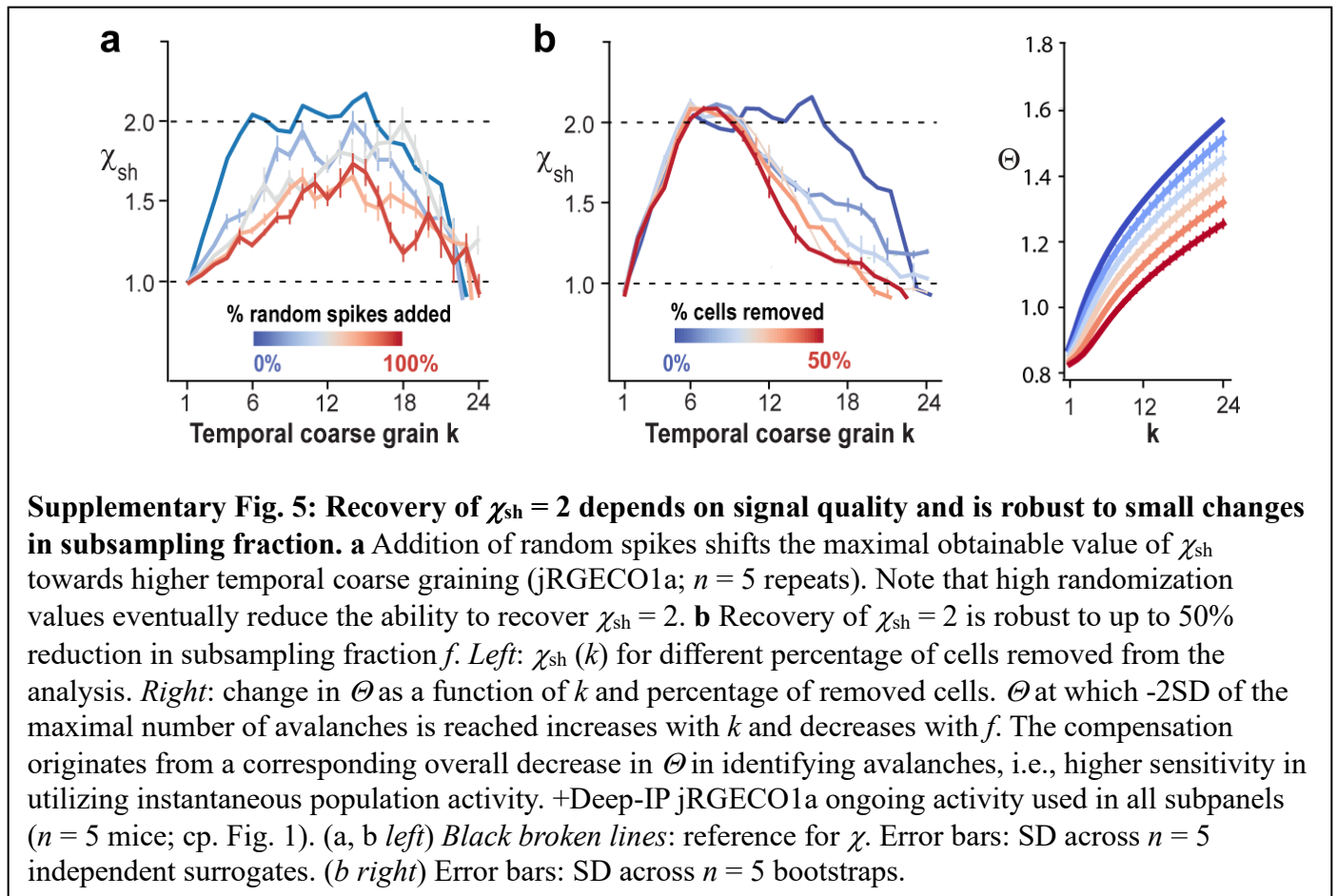


**Supplementary Fig. 2: Improvement in prism-based 2-PI of ongoing neuronal activity by adding Deep-IP and its effect on avalanche scaling.** **a** Example of a single 2PI frame (22 ms) before (*original*) and after spatiotemporal deep-interpolation (*+Deep-IP*; mPFC; prism; red-shifted jRGECO1a, single mouse). **b** Deep-IP reduces noise-fluctuations in relative fluorescent change  $\Delta F/F$  (three example ROIs, shifted for clarity). **c** Example of ROI identification in the original Field-of-View (*square*) and after applying Deep-IP. *Filled red circle*: original, population correlated ROI. *X*: +Deep-IP. *Open circle/X*: original ROI included after +Deep-IP (jRGECO1a, single mouse). *Top right*: the population of correlated neurons in +Deep-IP originate from originally synchronized ( $s \rightarrow s$ , *left*) and non-synchronized neurons ( $ns \rightarrow s$ , *middle*), as well as newly identified synchronized neurons (*new s*, *right*). *Bottom right*: the small fraction of neurons in the original analysis not recognized anymore after +Deep-IP (set to 100%) are proportionally drawn from originally uncorrelated (*OU*) and correlated (*OC*) neurons. **d** Summary statistics of the change in number of neurons (*Neurons*,  $p = 0.019$  original vs. original w/o non-synch. cells;  $p = 0.029$  original vs. +Deep-IP), mean spike rate (*Rate*;  $p = 0.045$ ), average cross correlation between neurons (*CC ROI*;  $p = 0.0032$ ), and between single neurons and the remaining population of neurons (*CC Pop*;  $p = 0.000015$ ). Data describe neuronal statistics for the original analysis (*Original*), with +Deep-IP added (*+Deep-IP*) and when non-synch cells are included (*solid lines*) or removed (*broken lines*); \* indicate  $p < 0.05$  and \*\*  $p < 0.005$ , two-sided Mann Whitney U test. Non-synchronized cells exhibited a cross-correlation with the remaining population activity significantly lower ( $p < 0.001$ ) than synchronized ones after z-scoring (see also Suppl. Fig. 8). Note that after adding Deep-IP, the prevalence and impact of non-synchronized neurons is greatly diminished, while rate and CC values are significantly increased. A minimal z-score ( $p < 0.001$ ) in the correlation with population activity was used to distinguish synchronized from non-synchronized neurons. Violin plots in c and d are widest at the median, with bars at extrema. **e** +Deep-IP greatly improves the temporal resolution (low  $k$ ) at which  $\chi_{sh}$  is obtained. Summary of  $\chi_{sh}$  as a function of  $k$  (for legend see d). **f**  $\chi_{sh} = 2$  holds for avalanche durations  $\sim 0.2 - 2$  s at  $k = 11$  (*green shaded region*) and  $k = 27$ . Note that after +DeepIP,  $\chi_{sh} \cong 2$  holds for avalanches of similar absolute duration now visible at  $k = 1$  (*arrowhead*), which was not present in the original analysis. **g** Shuffle comparison for  $k = 1$  and  $k = 11$ . As shown in f,  $\chi_{sh} = 2$  holds for avalanche durations  $T = \sim 0.2 - 1.3$  s at  $k = 11$  (*green region*), which is abolished by temporal shuffling (*red*).  $k = 1$  shown for comparison. (c – g, averages over  $n = 5$  mice; jRGECO1a). Values are mean  $\pm$  SD.

## 22 Supplementary Fig. 3



23 **Supplementary Fig. 4**

25 **Supplementary Fig. 5**

27 **Supplementary Notes & Supplementary Fig. 6**28 *Error estimates when deriving the parabolic scaling exponent using thresholding*

29 Starting with a generic inverted parabola,  $y = Ax(D - x)$ , with amplitude  $A$  and roots  $0, D$ , we next make  
 30 the requirement that the avalanche size,  $S$ , i.e., area underneath the parabola, needs to scale quadratically  
 31 with the avalanche duration,  $D$ , (Suppl. Fig. S6a)

$$32 \quad S = \int_0^D Ax(D - x) dx \propto D^2. \quad (1)$$

33 Given  $S = cD^2$ , with  $c$  being a constant, and solving the integral in (1), we obtain  $A = 6c/D$ . Therefore,  
 34  $y = 6cx(D - x)/D$  represents self-similar parabolic avalanches with duration  $D$  for any  $c$  in which size  $S$   
 35 scales quadratically with  $D$ .

36 We then proceed to apply a threshold  $\Theta$  to this original parabolic shape, which changes both our  
 37 perceived duration and size (Suppl. Fig. S6a). The thresholded duration,  $D_\Theta$ , will be given by the difference  
 38 between the roots of the thresholded parabola

$$39 \quad y_\Theta = y - \Theta = 6cx(D - x)/D - \Theta. \quad (2)$$

40 Solving for the roots, we have  $6cx(D - x)/D - \Theta = 0 \rightarrow -6cx^2/D + 6cx - \Theta = 0$ , with roots

$$41 \quad x_{0+,-} = (D/2) \left( 1 \pm \sqrt{1 - 2\Theta/3Dc} \right), \quad (3)$$

42 from which we obtain:  $D_\Theta = x_{0+} - x_{0-}$ ,

$$43 \quad D_\Theta = D \sqrt{1 - 2\Theta/3Dc}. \quad (4)$$

44 When using a *soft threshold*, the avalanche size is defined as the area of the parabola above the threshold,

$$45 \quad S_\Theta = \int_{x_{0-}}^{x_{0+}} (6cx(D - x)/D - \Theta) dx, \text{ where } x_{0+} \text{ and } x_{0-} \text{ are the roots of the thresholded parabola in (2),}$$

46 given by (3). By solving the integral, we obtain

$$47 \quad S_\Theta = -(2c/D)(x_{0+}^3 - x_{0-}^3) + 3c(x_{0+}^2 - x_{0-}^2) - t(x_{0+} - x_{0-}). \quad (5)$$

48 Using (3) and (4), we have

$$49 \quad S_\Theta = -(cD^2/4)((1 + D_\Theta/D)^3 - (1 - D_\Theta/D)^3) + (3cD^2/4)((1 + D_\Theta/D)^2 - (1 - D_\Theta/D)^2) - \Theta D_\Theta$$

$$\begin{aligned}
50 &= (cD^2/4)\{(1 + D_\Theta/D)^2[3 - (1 + D_\Theta/D)] - (1 - D_\Theta/D)^2[3 - (1 - D_\Theta/D)]\} - \Theta D_\Theta \\
51 &= (cD^2/4)\{[1 + 2D_\Theta/D + (D_\Theta/D)^2](2 - D_\Theta/D) - [1 - 2D_\Theta/D + (D_\Theta/D)^2](2 + D_\Theta/D)\} - \Theta D_\Theta \\
52 &= (cD^2/4)[2 + 4D_\Theta/D + 2(D_\Theta/D)^2 - D_\Theta/D - 2(D_\Theta/D)^2 - (D_\Theta/D)^3 - 2 + 4D_\Theta/D - 2(D_\Theta/D)^2 \\
53 &\quad - D_\Theta/D + 2(D_\Theta/D)^2 - (D_\Theta/D)^3] - \Theta D_\Theta \\
54 &= (cD^2/4)[6D_\Theta/D - 2(D_\Theta/D)^3] - \Theta D_\Theta \\
55 &= (cDD_\Theta/2)(3 - (D_\Theta/D)^2) - \Theta D_\Theta.
\end{aligned}$$

56 Again using (4), we have

$$\begin{aligned}
57 &S_\Theta = (cDD_\Theta/2)(3 - 1 + 2\Theta/3Dc) - \Theta D_\Theta \\
58 &= cD_\Theta(D + \Theta/3c) - \Theta D_\Theta. \tag{6}
\end{aligned}$$

59 From (4), isolating  $D$ , we have  $D^2 - 2\Theta D/3c - D_\Theta^2 = 0$ . Solving for  $D$ , we find

$$60 \quad D = \Theta/3c + \sqrt{D_\Theta^2 + (\Theta/3c)^2}, \tag{7}$$

61 from which we excluded the negative root given our constraints ( $D \geq 0$ ,  $D_\Theta \geq 0$ ,  $0 \leq \Theta \leq 3Dc/2$ ).

62 Applying (7) into (6), we finally arrive at

$$63 \quad S_\Theta = cD_\Theta \left( \sqrt{D_\Theta^2 + (\Theta/3c)^2} + 2\Theta/3c \right) - \Theta D_\Theta. \tag{8}$$

64 In the definition of a hard threshold,  $S_\Theta^H$  adds the  $\Theta \times D_\Theta$  rectangle below the threshold (Suppl. Fig. 6a)

65 and obtains the size scaling

$$66 \quad S_\Theta^H = cD_\Theta \left( \sqrt{D_\Theta^2 + (\Theta/3c)^2} + 2\Theta/3c \right). \tag{9}$$

67 Given that any thresholding will affect scaling estimates, we note that (8) provides a higher estimate of

68 the scaling exponent  $\chi_{sh}$ , whereas (9) provides a lower estimate. Generally, the error introduced by

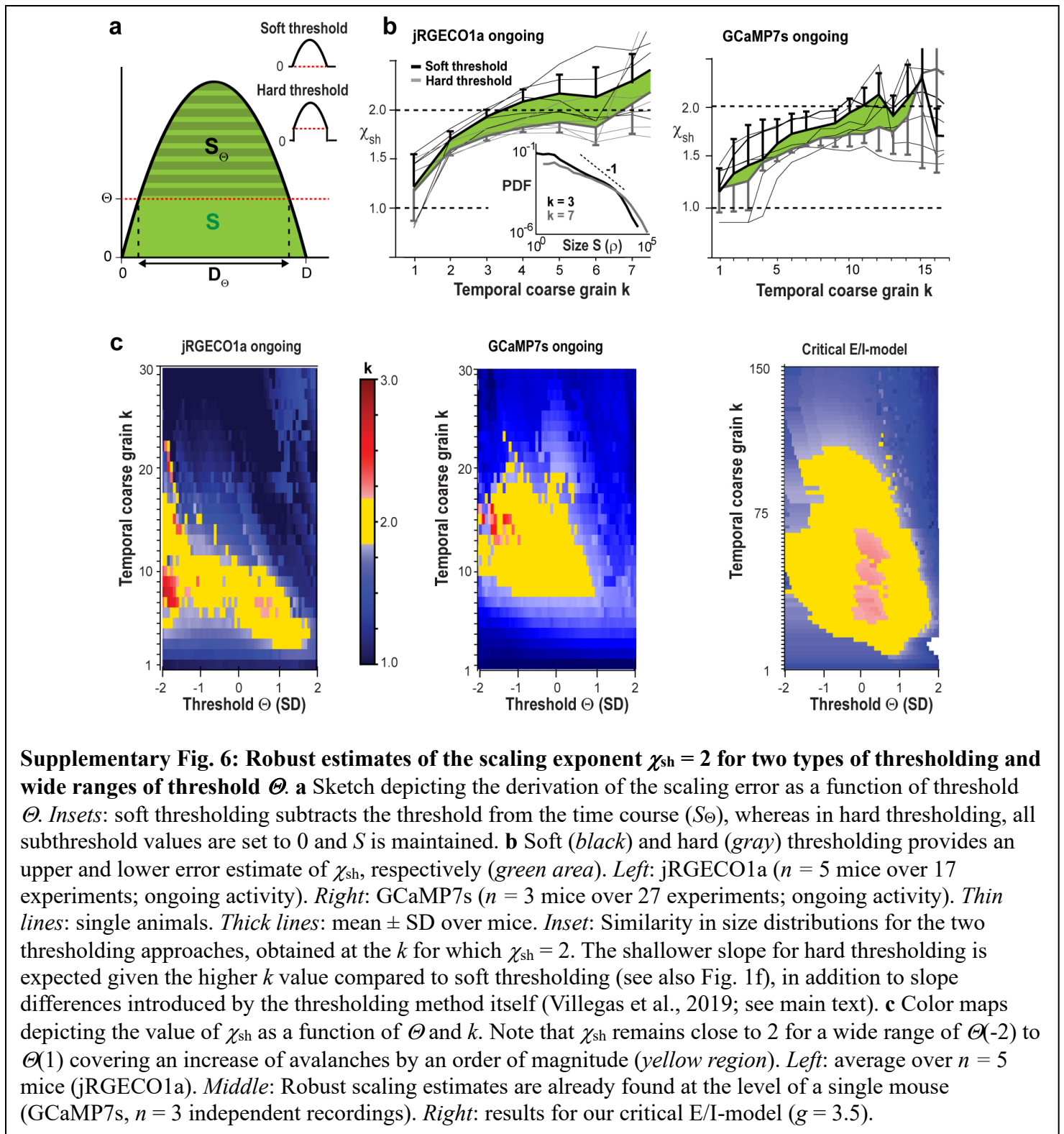
69 thresholding will be small if the threshold is small with respect to the amplitude of the parabola.

70 Accordingly, in our analysis, we set our initial threshold  $\Theta$  for  $k = 1$  small, i.e., at  $-2SD$ . We note that slope

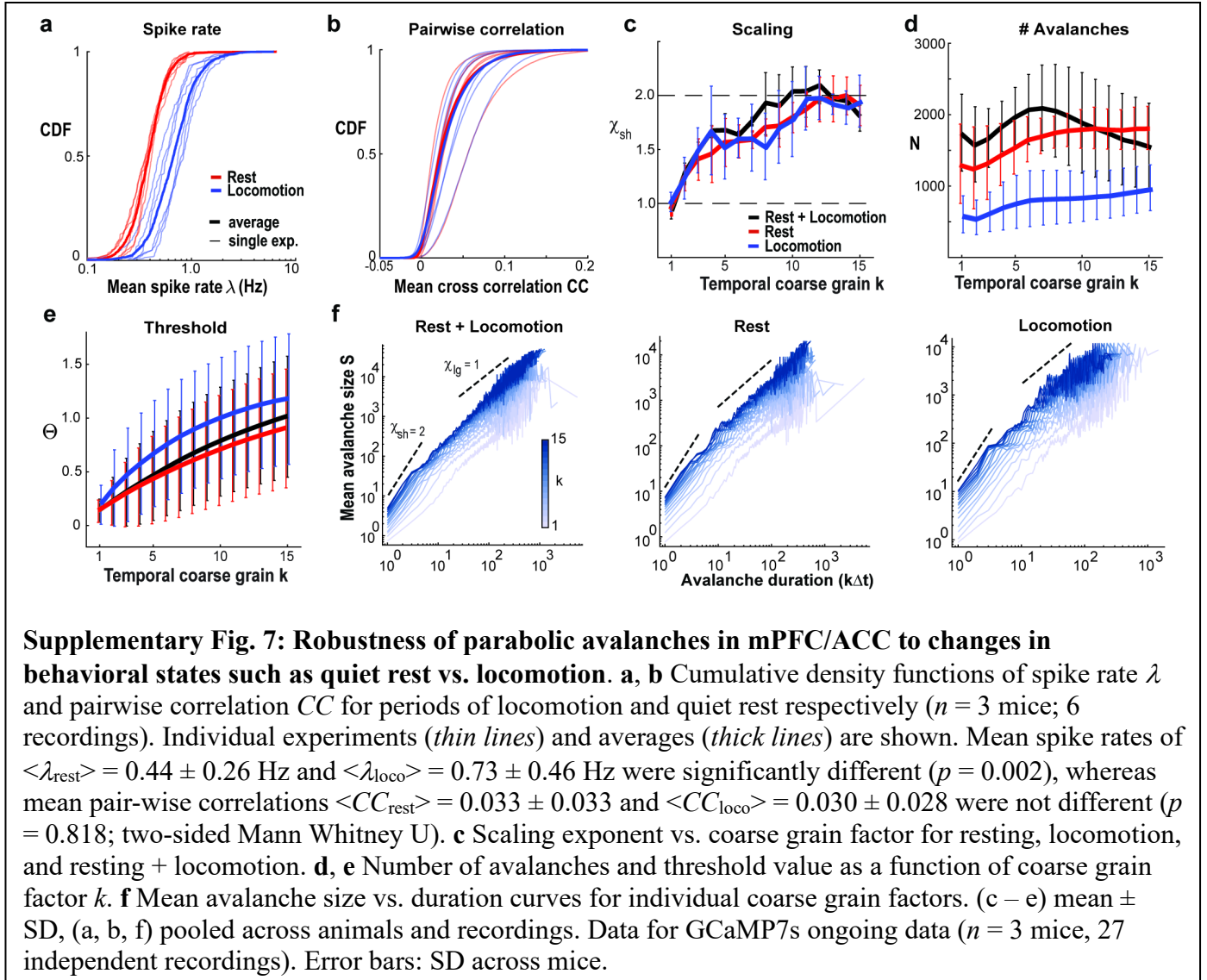
71 estimates in the corresponding size distributions can be large due to different coarse-graining values

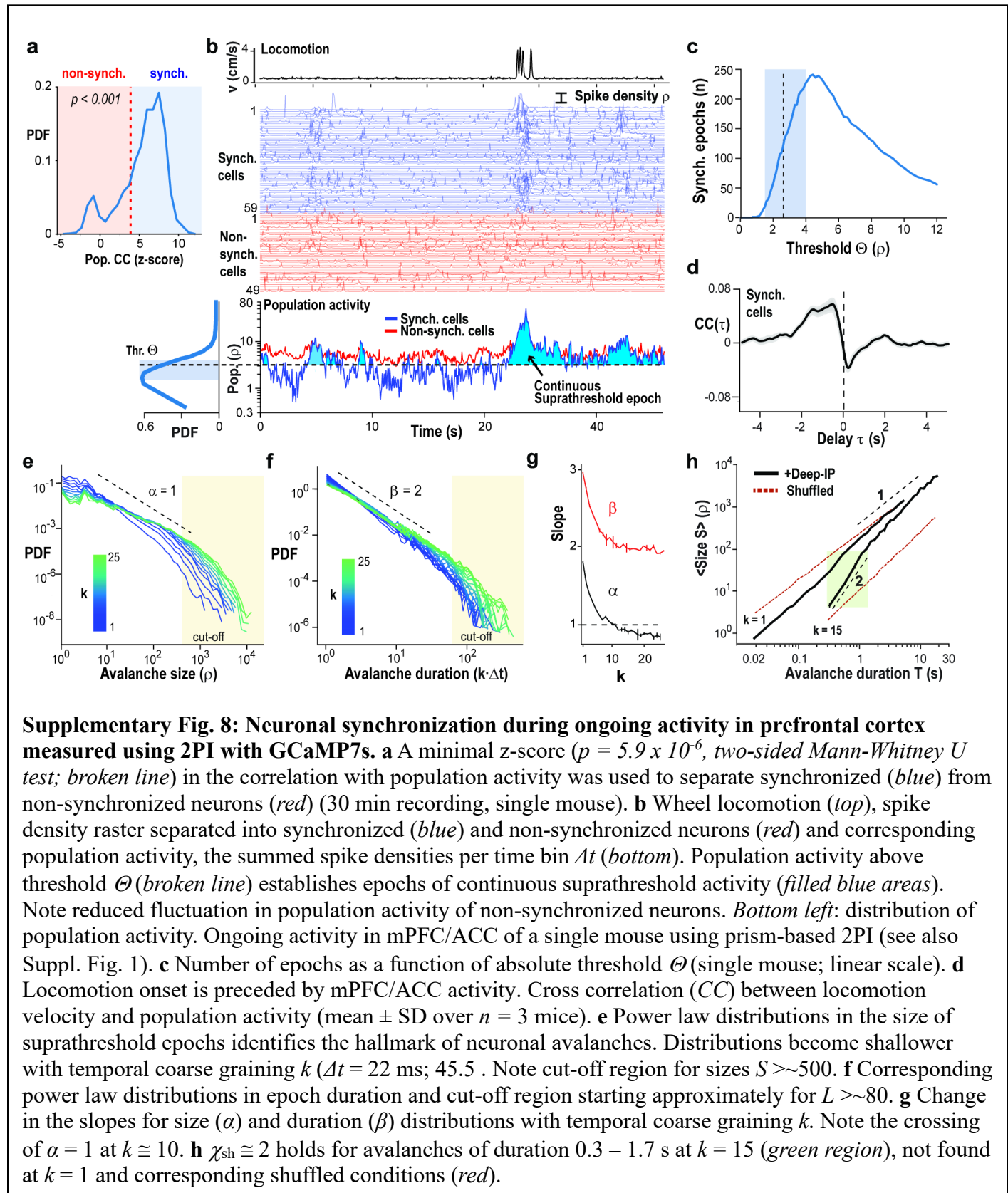


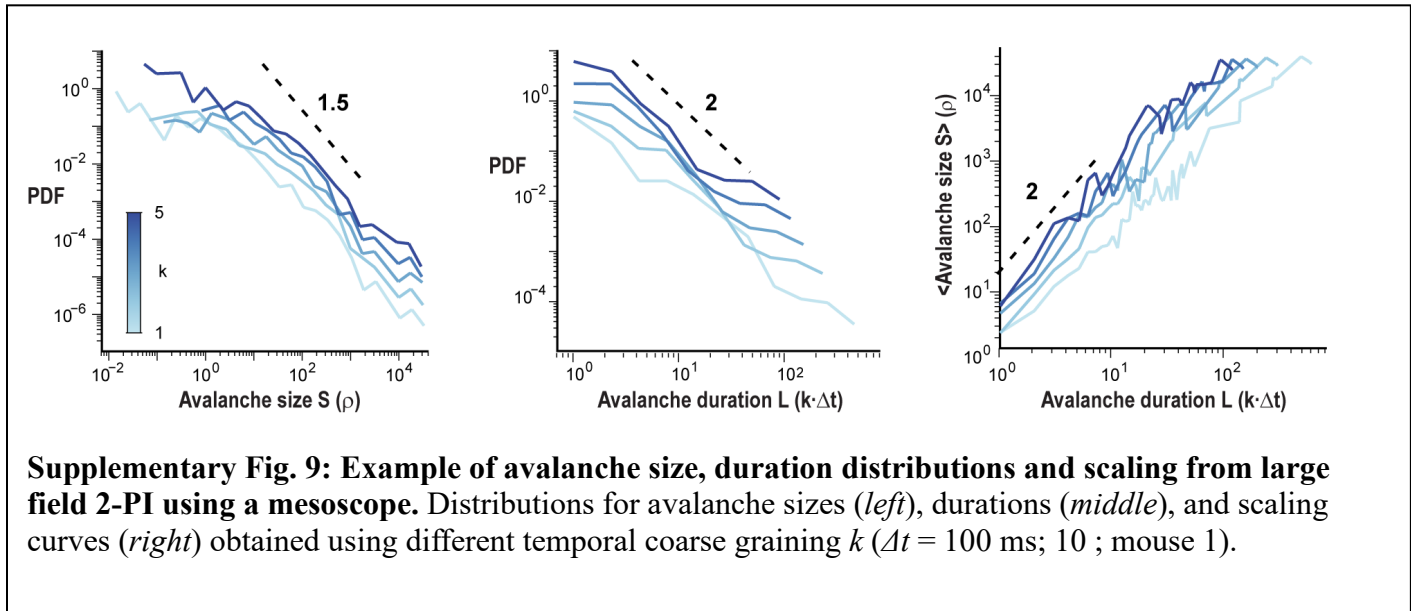
72 required to achieve  $\chi_{sh} = 2$  which leads to more shallow slopes with increasing  $k$  (Suppl. Fig. 6b, inset; see  
 73 also Figure 1).



## 74 Supplementary Fig. 7

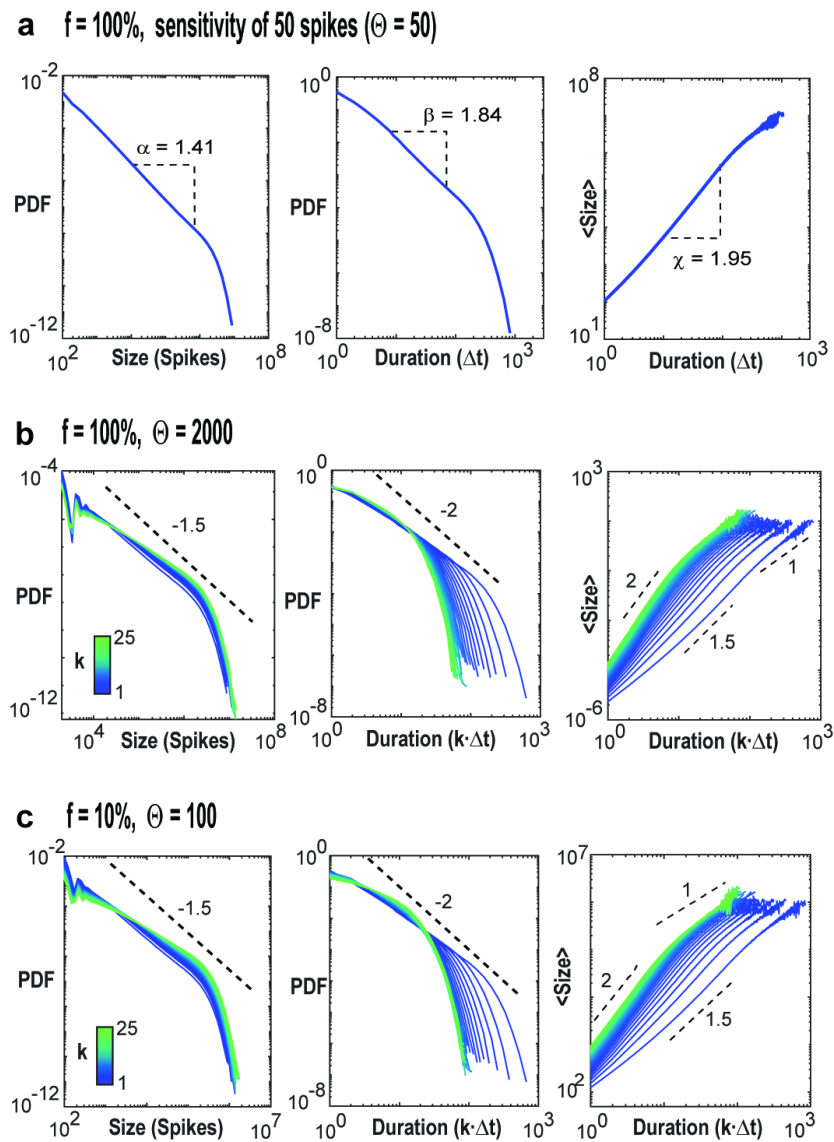


75 **Supplementary Fig. 8**

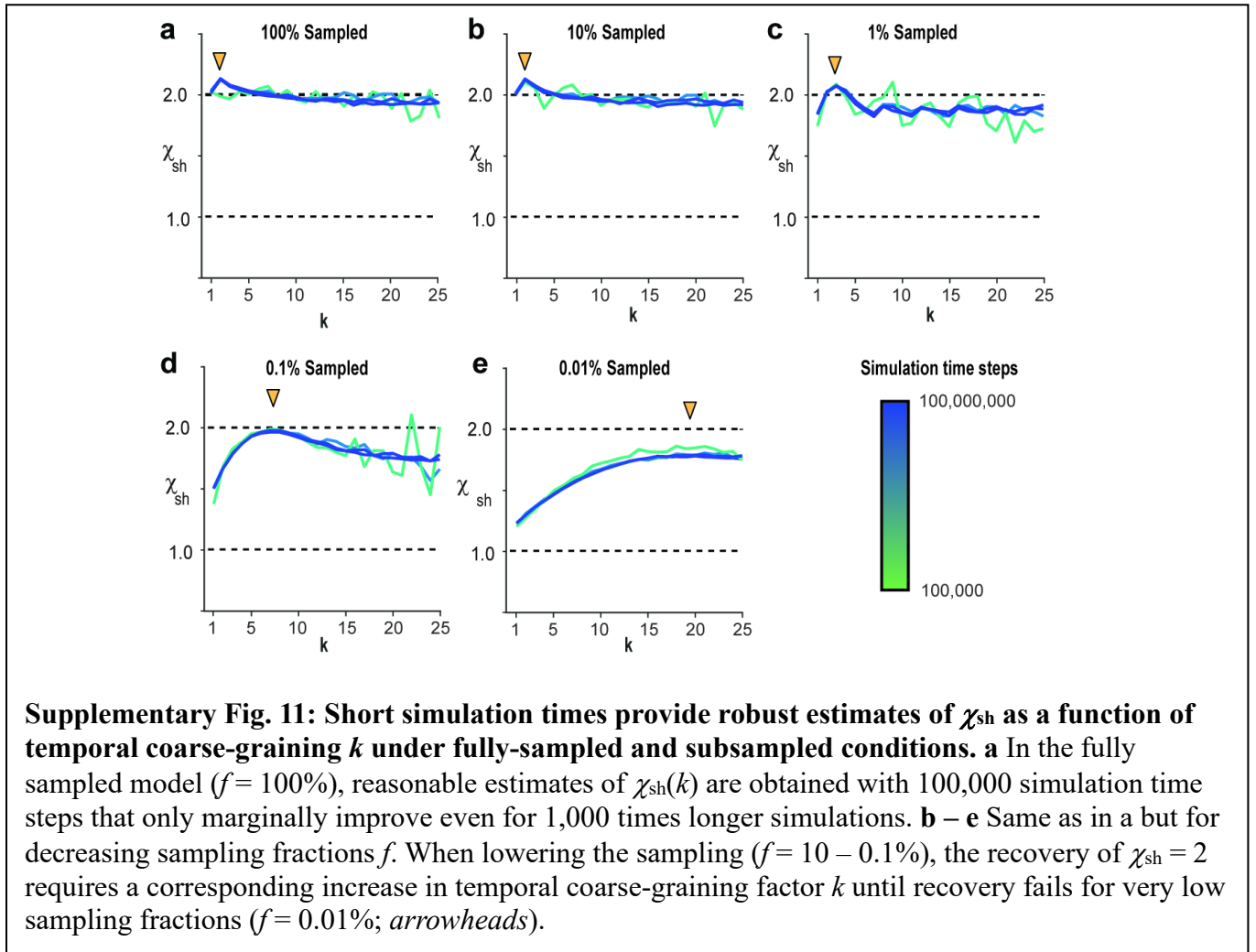
76 **Supplementary Fig. 9**

77 **Supplementary Fig. 10**

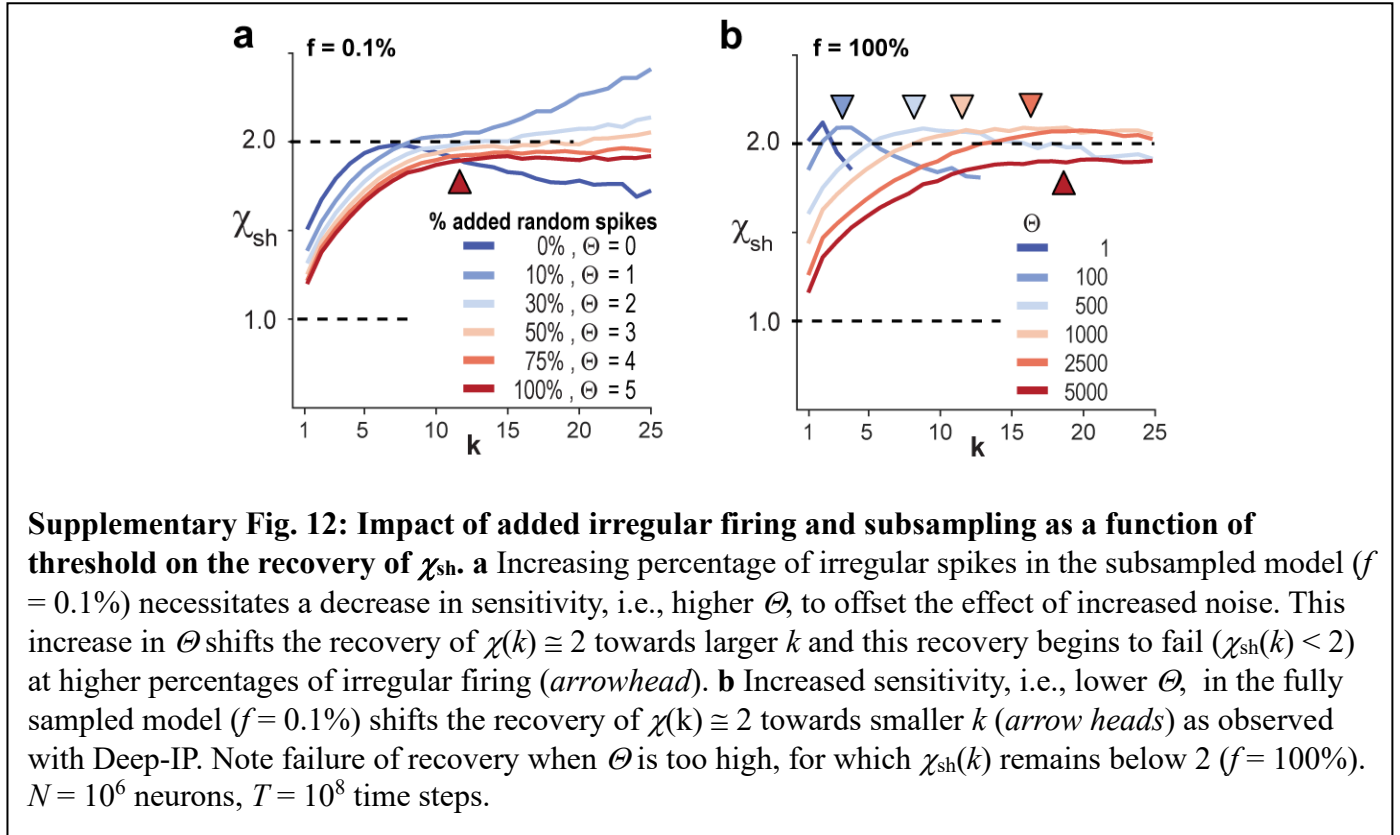
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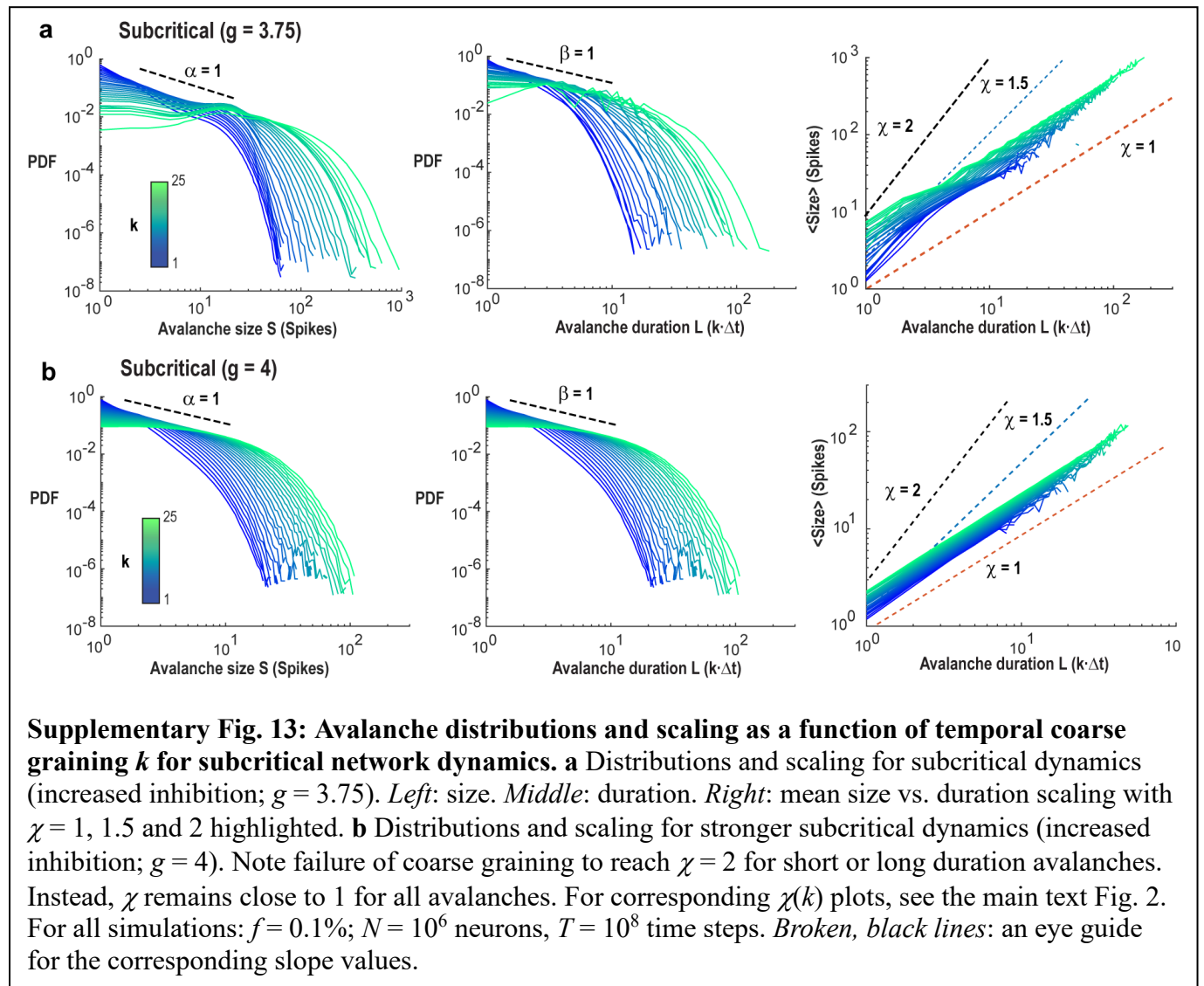


**Supplementary Fig. 10: Examples of avalanche distributions as a function of threshold  $\Theta$  and sampling fraction  $f$  for critical E/I-model.** **a** Distributions for the fully sampled model ( $f = 100\%$ ;  $N = 10^6$  neurons,  $g = 3.5$ ;  $10^8$  time steps) at population spike sensitivity of 50 spikes/time step ( $\Theta = 50$ ). *Left*: size. *Middle*: duration. *Right*: mean size vs. duration scaling with  $\chi = 1.95$ . Note that the Kinouchi model requires about a minimum of 100 spikes in  $N = 10^6$  to initiate successful propagation. The fully sampled model exhibits critical exponents close to  $\alpha = 3/2$ ,  $\beta = 2$  that fulfill the analytical prediction of  $\chi = (\beta - 1)/(\alpha - 1) = 2$ . **b** Temporal coarse graining recovers  $\chi \approx 2$  at low sensitivity, i.e., high  $\Theta$ . Example distributions and scaling for the fully sampled model ( $f = 100\%$ ) and high threshold  $\Theta = 2000$ . **c** As in **b** with 10 times reduced sampling ( $f = 10\%$ ) and 20 times reduced threshold ( $\Theta = 100$ ). For corresponding  $\chi(k)$  plots see main text Fig. 3. Note that in this model, uncorrelated propagated activity yields long-duration avalanches with  $\chi_{lg} = 1.5$ .  $T = 10^8$  time steps. *Broken, black lines*: slopes as a visual guide to the eye.

79 **Supplementary Fig. 11**

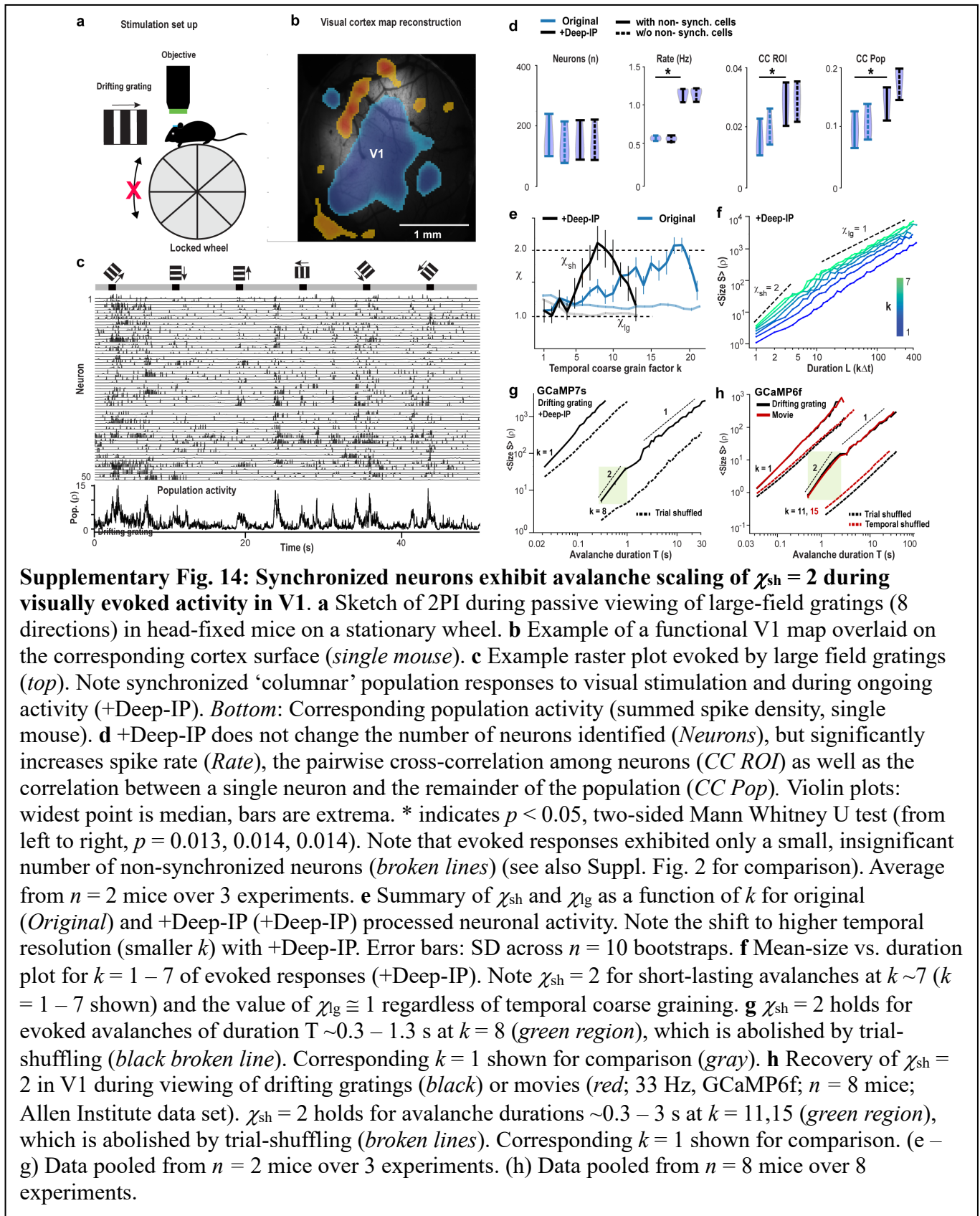
## 80 Supplementary Fig. 12

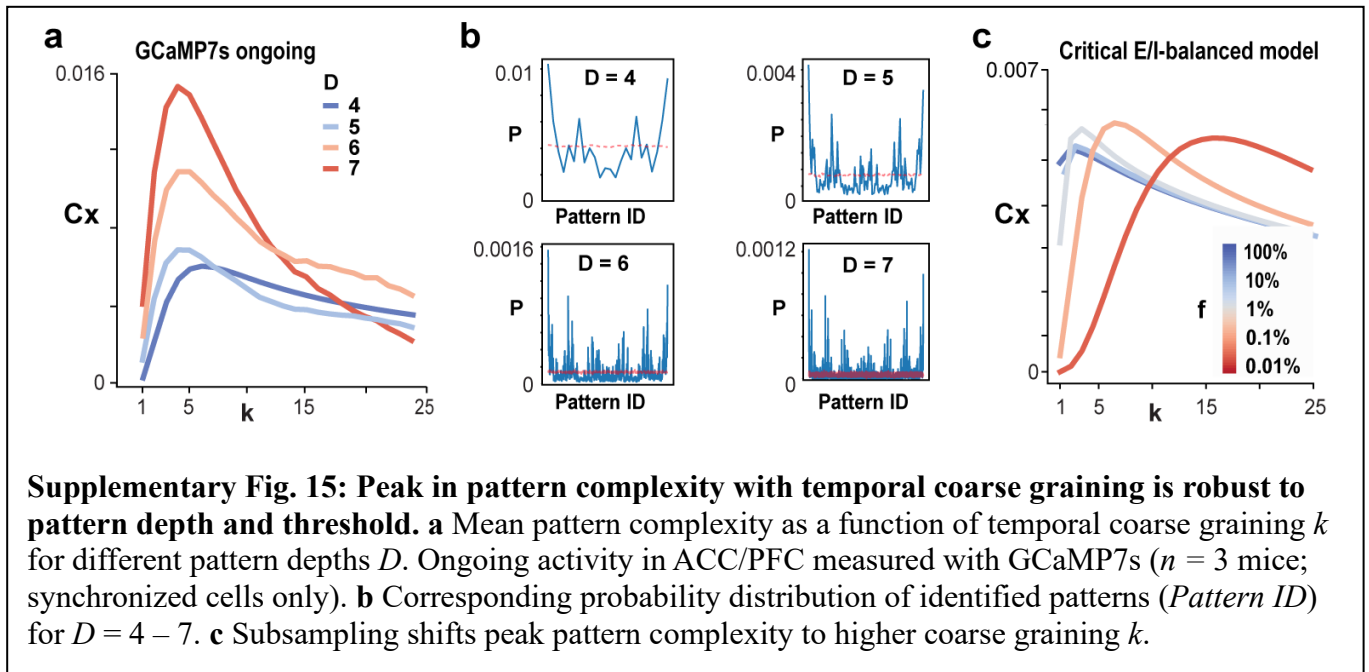


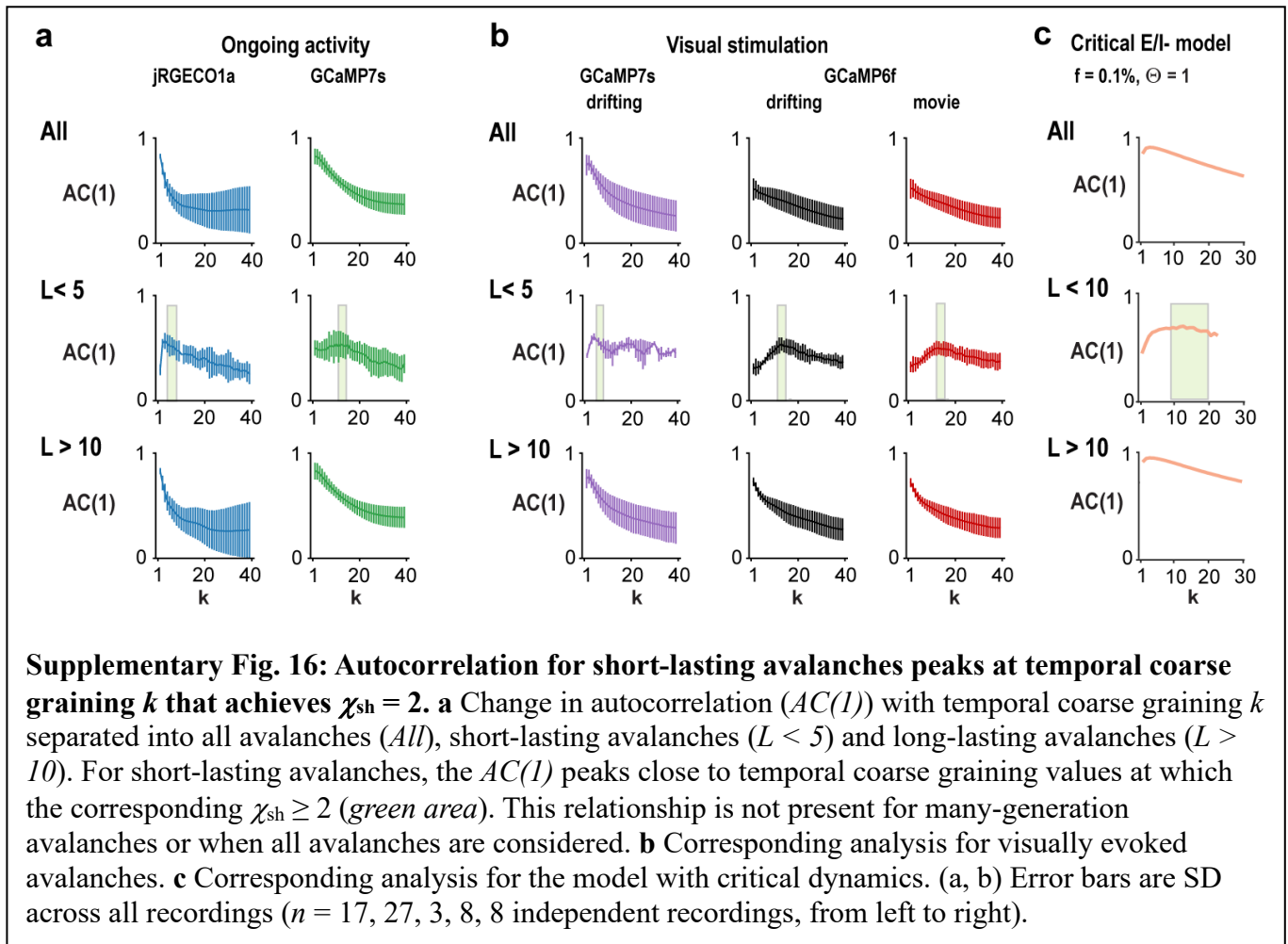
81 **Supplementary Fig. 13**

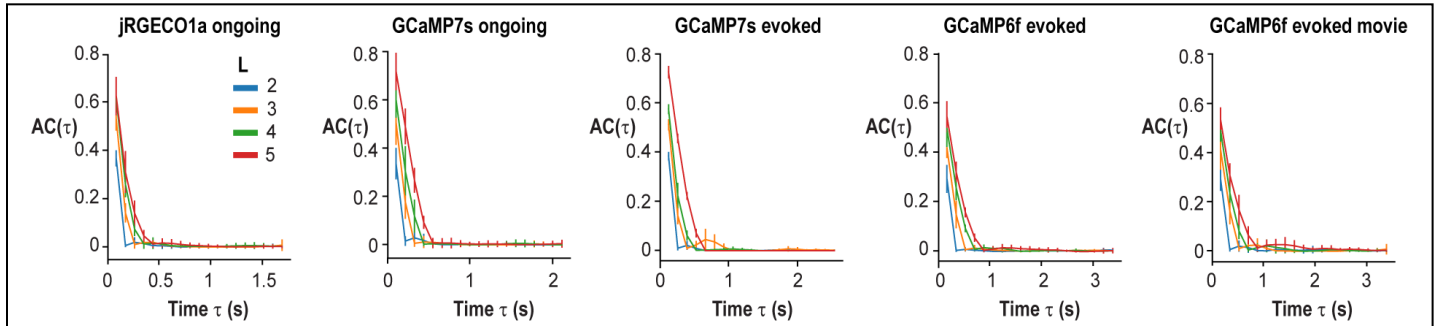


## 82 Supplementary Fig. 14



83 **Supplementary Fig. 15**

85 **Supplementary Fig. 16**

87 **Supplementary Fig. 17**

**Supplementary Fig. 17: Parabolic avalanches do not show periodic recurrence.** Autocorrelation for parabolic avalanches at temporal resolution for which  $\chi_{\text{sh}} = 2$  separated by number of generations  $L$  (mean  $\pm$  SD over all mice per experimental condition). Note the absence of recurrence supporting a non-oscillatory origin of parabolic avalanches. For jRGECO1a ongoing (+Deep-IP)  $n = 5$  mice over 17 independent recordings. For GCaMP7s ongoing data (non-synch ROI removed),  $n = 3$  mice over 27 independent recordings. For GCaMP7s drifting gratings (evoked; +Deep-IP),  $n = 2$  mice over 3 independent recordings. For the GCaMP6f data,  $n = 8$  independent recordings from 8 separate animals.

## 89 Supplementary Fig. 18

