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Frequency-division multiplexing in the terahertz range using a leaky-wave antenna

Nicholas J. Karl,[1] Robert W. McKinney,[1] Yasuaki Monnai,[2]
Rajind Mendis,[1] and Daniel M. Mittleman[1]

[1]*Department of Electrical and Computer Engineering, Rice University, MS-378, 6100 Main St., Houston TX 77005, USA*

[2]*Department of Complexity Science and Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*

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Here, we describe an implementation of the waveguide as a multiplexer, in which we treat a specific example to demonstrate the design flexibility afforded by the arbitrary variation of the plate separation $b(z)$ along the axis of the leaky waveguide's slot.

First, we note that, for any plate separation function $b(z)$, the fractional bandwidth of a channel is given by the ratio of equation (3) to equation (2): $\Delta\nu/\nu = \Delta\phi/\tan\phi$, independent of z_0 for a given input angle. Thus, by choosing $b(z)$ appropriately, one can engineer the range of frequencies that couple into or out of the waveguide, while maintaining a fixed fractional bandwidth. However, in realistic systems, it is possible that one would require a constant bandwidth for all of the multiplexed channels, rather than a fixed fractional bandwidth (which would imply a different bandwidth for each channel). This too can be achieved, by varying not only $b(z)$ but also the input angle ϕ for each channel. The following example illustrates this principle.

For this example, we suppose that we wish to multiplex a series of channels, equally spaced between ν_L = 150 GHz and ν_H = 300 GHz, with a channel spacing of 30 GHz (thus, in this example there are six channels). We require that the multiplexer have a fixed acceptance bandwidth of $\Delta\nu$ = 20 GHz for all six channels. To compute the required plate separation $b(z)$ and input angles for the six channels, we assume that the illumination location (the center of the illuminated spot) is shifted along the z axis linearly with frequency (here, z = 0 corresponds to one end of the waveguide). Assuming a waveguide length L = 50 mm, we have

$$\nu(z) = \frac{\nu_H - \nu_L}{L} z + \nu_L \tag{S1}$$

such that the channels impinge upon the waveguide at equally spaced intervals, 10 mm apart. We note that this spacing is larger than the free-space wavelength for all relevant frequencies, so there would be no problem with channel cross-talk due to spatial mode overlap. With this relation between ν and z, plus equations (2) and (3) and an assumed constant acceptance angle of $\Delta\phi = 0.25$ (determined by the input coupling optics), we have two equations in the two unknowns b and ϕ. The solutions take the form:

$$b(z) = \frac{c_0}{2\nu(z)^2 \Delta\phi}\sqrt{\Delta\nu^2 + \left(\nu(z)\Delta\phi\right)^2} \tag{S2}$$

$$\tan\left(\phi(z)\right) = \frac{\nu(z)\Delta\phi}{\Delta\nu} \tag{S3}$$

These two equations are plotted in Fig. S1, for the specified values of L, ν_H, ν_L, $\Delta\phi$, and $\Delta\nu$. Figure S2 illustrates the linearly varying frequency and fixed spectral bandwidth as a function of position along the waveguide axis. The leftwards-pointing arrows indicate the frequencies of the six channels (150, 180, 210, 240, 270, and 300 GHz), while the legend indicates the angle of

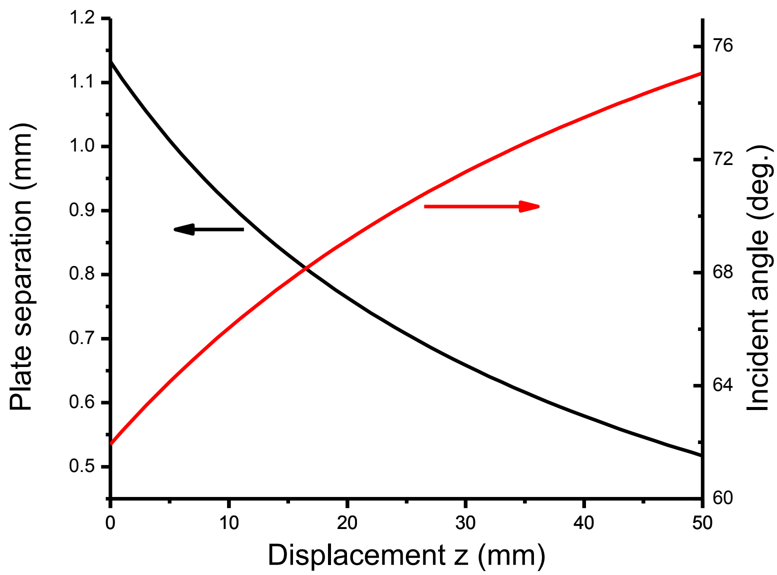

Figure S1: Analytical curves for the plate separation and the incident angle. Variation of the plate separation b (black curve) and the angle of incidence ϕ (red curve) as a function of position z along the waveguide [equations (S2) and (S3)], for the particular desired configuration described in the text.

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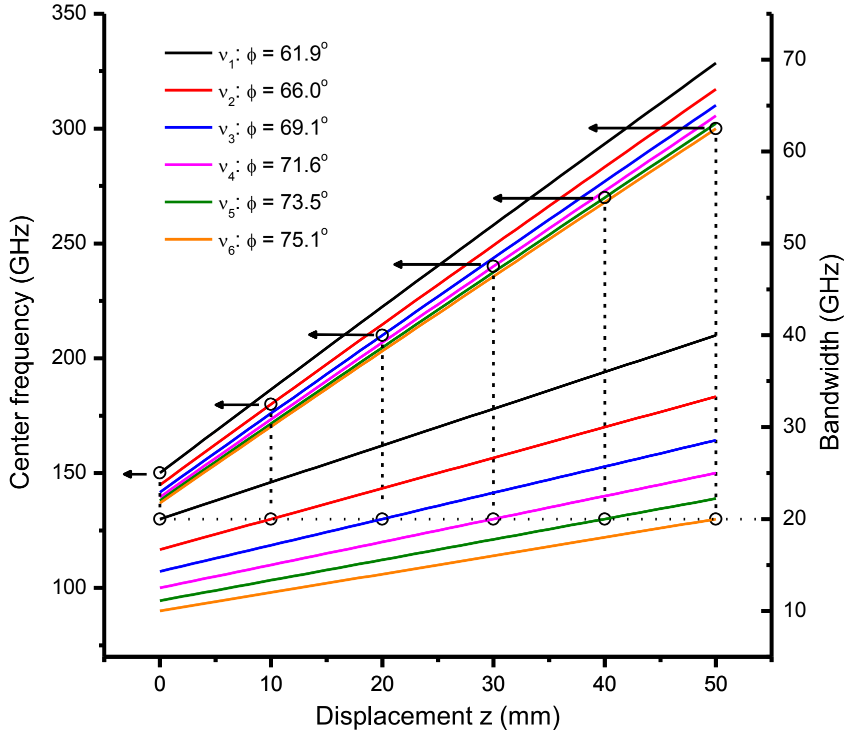

Figure S2: Analytical curves for the center-frequency and bandwidth. Variation in the incoupled frequency (upper group of lines) and bandwidth (lower group of lines), for a waveguide with the plate separation $b(z)$ shown in Fig. S1, plotted for six specific input angles as shown in the legend. For these angles, an equally spaced set of input beams will give rise to six equally spaced frequency channels from 150 to 300 GHz (six vertical dashed lines), all with a 20 GHz bandwidth (horizontal dotted line).

incidence for each channel (numbered in order from lowest frequency to highest). These six specified channels all exhibit the same 20 GHz bandwidth, as indicated by the vertical dashed lines.

Various issues could potentially limit the operation of this type of multiplexer. One possible point of concern is group velocity dispersion, which could cause time-multiplexed pulses within a given spectral channel to temporally overlap. However, for realistic channel bandwidths, this effect is likely to be negligible. Even if the waveguide is fairly dispersive, the change in the group delay across the channel bandwidth is still quite small for a waveguide of reasonable length. For example, consider a waveguide with a (constant) plate separation of 0.85 mm (this is about the average value of b for the example configuration discussed here). For this value of b, assuming a channel centered at 300 GHz with a 20 GHz bandwidth, and assuming a waveguide length of 5 cm, the group delay only varies by about 7 picoseconds across the bandwidth of the channel. Given that the maximum data rate is limited by the bandwidth to one bit per 50 picoseconds, this value is certainly tolerable. Obviously, this is only an estimate, as there is no closed-form expression for the dispersion of a waveguide in the case where the plate separation varies along the propagation direction. For much broader channel bandwidths or for operation very close to the waveguide cutoff frequency, a more detailed and elaborate calculation would be required.

Another potentially problematic issue may arise from ringing – multiple reflections inside the structure which give rise to interference effects at the measurement location. Fortunately, these effects are also likely to be small. Such effects could arise from reflections of the wave inside the waveguide due to impedance mismatch between the guided mode and free space, for example at the lateral (open) edges. As in the case of group-velocity dispersion, the relatively large (wavelength-scale) plate separation limits the impedance mismatch to a fairly low value. As a result, we have been unable to observe any manifestation of these multiple reflections in our experiments so far. Of course, a different multiplexer configuration, for example using a smaller plate spacing, could give rise to a large impedance mismatch and therefore a more significant ringing effect. In this case, it would be possible to flare the lateral edges of the waveguide, such that the multiplexed modes along the central axis are unaffected, but the impedance mismatch to empty space at the lateral edges is completely removed.