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Poles, Shapes, Senses of Rotation, and Sidereal Periods of Asteroids

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A method of determination of poles, triaxial ellipsoid model shapes, senses of rotation, and sidereal periods of asteroids is presented. These parameters have been obtained simultaneously by solving a set of nonlinear equations, combining the well-known methods of photometric astrometry, amplitude–aspect, and magnitude–aspect. The results for 30 asteroids are presented. For 12 objects these parameters have been determined for the first time and for 8 others they have been improved. The remaining 10 asteroids were used mainly as test objects for the method. The rotational poles continue to show a tendency to avoid the ecliptic plane. Amplitude–solar phase angle and magnitude–phase angle relationships are obtained for most of the asteroids.

1. INTRODUCTION

A very important part of the evolution of the Solar System centers on the evolution of minor planets, most of which move around the Sun between the orbits of Mars and Jupiter. The understanding of the evolution of asteroids may be helpful in the studies of the primordial epochs of planetary history. Physical properties of asteroids such as their shapes, spin periods, and spin axes, can help in constructing our knowledge of their collisional evolution.

It has been suggested on the basis of present data that the outcome of collisions is strongly size dependent in the sense that large asteroids (more than 100 km in diameter) are most likely to be gravitationally bound "rubble piles" (Davis *et al.* 1989). One of their aims has been to seek evidence for quasi-equilibrium shapes of the asteroids due to their rapid rotations. The existence of such shapes would provide strong support for the rubble pile model and permit determination of the bulk densities of such asteroids. On the other hand, objects smaller than about 100 km in diameter may be, for the most part, multigeneration fragments, whose shapes are very irregular and controlled by solid-state forces only.

One of the rotational properties of the Solar System is the prograde rotation of most major planets. It is important to find out whether this prograde sense of rotation originates from the last stochastic collision between very large planetesimals, from collapse of circumplanetary nebulae, subsequent tidal interactions, or is a fundamental property of objects in a stage of accretional buildup. The rotation of the very large asteroids may be essentially primordial and may give us direct insight into spin properties of planetesimals. However, a higher frequency of collisions in the past may have randomized the spin periods and created an isotropic distribution of spin axes.

Synodical rotation periods and lightcurve amplitudes have been determined for several hundred asteroids (Lagerkvist *et al.* 1989), allowing statistically significant analysis of correlation between size and taxonomic type to be made. Results of this analysis were presented by Binzel *et al.* (1989). For small asteroids with diameters $D < 125$ km, we observe a decrease in their rotation rate with their increasing size. The bigger asteroids ($D > 125$ km) show an increase in the rotation rate with increasing diameter. Similar effects ensue if the above analysis is made separately for C, S, and M asteroids. Moreover, for a given diameter, C asteroids show slightly slower rotation rates than S asteroids and the M asteroids are distinctly faster rotators. Lightcurve amplitudes make a crude indicator of asteroid shapes in the sense that more spherical bodies have lower amplitudes. The observed distribution of amplitudes implies that small asteroids have higher amplitudes than bigger ones, which suggests that large asteroids are more spherical.

It should be mentioned that we know sidereal periods, senses of rotation, orientation of spin axes, and shapes (see Magnusson 1989) for only about 40 objects of the population of 5000 known asteroids. Magnusson *et al.* (1989) give a description of all techniques used to obtain poles and shapes of asteroids. One approach to learning about these properties is to study the brightness variation of asteroids as they spin about their axes. Multiple observations of lightcurves from different perspectives as an asteroid pursues its orbit provide information about spin period, axis orientation, sense of rotation, and body shape. Real shapes of asteroids are approximated by triax-

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ial ellipsoids rotating round their shortest axes. It is also assumed that the brightness of an asteroid is proportional to its cross-sectional area (geometric scattering law). This simplification is justified by numerical and laboratory simulations and is made in order to clarify the role of shape, viewing geometry, surface morphology, and composition in asteroid light variations (see Magnusson *et al.* 1989).

This paper presents the methods and results of pole and shape determinations based on amplitudes, magnitudes, and epochs of features of the observed lightcurves.

II. DESCRIPTION OF THE METHOD

The methods based on photometric lightcurves can provide the most abundant data on poles and shapes of the asteroids. Techniques based on other kinds of observations can verify the photometric results as well as give additional information (see Magnusson *et al.* 1989 for full description).

The changing relative geometry of the Sun–Earth–asteroid system causes variations in the observed synodic period of rotation. The actual size and sign of these variations depend on the orientation of the spin axis and the sense of rotation. We can deduce the coordinates of the north pole by studying changes in the synodic period. This is done by selecting a well-defined feature occurring in all lightcurves and assuming that it stays at a fixed rotational phase. There are two versions of this method, called *photometric astrometry* or *epochs* method. One of them (see, e.g., Taylor 1979, Taylor and Tedesco 1983, Michalowski 1988) requires determining the number of synodic rotations between epochs before calculating pole coordinates and sidereal period. The second one (Magnusson 1986, Drummond *et al.* 1988) uses the sidereal cycles determined during the calculations.

Let us keep the north pole of the asteroid fixed and define a nonrotating asterocentric equatorial reference frame with the longitude increasing in the direction of the asteroid's rotation. The basic formula of the *photometric astrometry* or the *epochs method* is

$$\Delta T - P_{\mathrm{sid}}(\Delta N + \Delta L/360) = 0, \qquad (1)$$

where ΔT is a time interval between two light–time-corected epochs that are an integral number of sidereal rotation cycles (ΔN) apart. P_{sid} is the sidereal period of rotation and $\Delta L/360$ is the fractional part of a cycle that a body would have to rotate in order that the same feature on the surface faced the sub-"phase angle bisector" point at both the beginning and the end of the interval. The phase angle bisector (PAB) is the bisector of the angle between the asteroid–Sun and asteroid–Earth directions. Michałowski (1988) gave an algorithm for finding the longitude L of the PAB. Because L is increasing in the direction of rotation we know the sense of the asteroid rotation by solving Eq. (1).

Most of the asteroids show two maxima and two minima per rotational cycle. Times (corrected for light travel) of both maxima are used in finding the pole and P_{sid}. All the possible pairs of epochs for which $|L^i - L^j| > 50°$ are used in the calculation. As long as the extrema are caused by shape rather than topographic features, the two maxima should be one-half period apart. Thus, the number of the sidereal cycles, counted from an arbitrary moment (e.g., from the first epoch) are

$$N_i = 0.5\ \mathrm{INT}\left[2\left(\frac{T_i - T_1}{P_{\mathrm{sid}}} - \frac{L_i - L_1}{360}\right) + 0.5\right]. \qquad (2)$$

These numbers are determined at each step in finding the pole and sidereal period.

Other sources of our knowledge of the pole positions and shapes of asteroids are amplitudes and magnitudes of their lightcurves which are used by the *amplitude* and *magnitude* methods, respectively. The standard assumption made in both methods is that the shape of an asteroid is a triaxial ellipsoid with the principal semiaxes $a > b > c$. The c-axis, which coincides with the axis of the maximum moment of inertia is assumed to be the axis of rotation.

For a given solar phase angle, the triaxial ellipsoid is the faintest and displays the largest lightcurve amplitude when viewed in its equatorial plane. When viewed from near its pole it is the brightest and has the smallest amplitude.

Let us assume that the observed brightness at zero solar phase ($\alpha = 0°$) is proportional to the area of projection of the asteroid onto the plane perpendicular to the line of sight. For a given aspect angle (angle between line of sight and c-axis), we have (cf. Magnusson 1986, Michałowski and Velichko 1990)

$$1.25 \log\left[\frac{(b/c)^2\cos^2\phi + \sin^2\phi}{(b/c)^2\cos^2\phi + (b/a)^2\sin^2\phi}\right] - A(\phi, 0) = 0, \qquad (3)$$

where $A(\phi, 0)$ is the amplitude at aspect ϕ and zero phase angle ($\alpha = 0°$). The aspect can be easily calculated from the relation

$$\cos\phi = -\sin\beta\sin\beta_p - \cos\beta\cos\beta_p\cos(\lambda - \lambda_p), \qquad (4)$$

where (λ, β) and (λ_p, β_p) are the ecliptic longitudes and latitudes of the asteroid and its north pole, respectively.

Analogously, with the same assumption, we have a relation for a maximum of the asteroid's brightness (cf. Magnusson 1986).

$$H(\phi, 0) - H(90, 0) + 1.25 \log[(b/c)^2 \cos^2\phi + \sin^2\phi] = 0, \quad (5)$$

where $H(\phi, 0)$ and $H(90, 0)$ are the reduced brightness (i.e., brightness reduced to distances 1 AU from the Sun and the Earth) for the aspects ϕ and 90°, respectively.

Because of the fact that the asteroids are not observed at zero phase angle it is necessary to reduce the observed amplitudes and magnitudes to $\alpha = 0°$. Magnusson (1986) used a linear amplitude–phase relation with the same slope for all oppositions for a given asteroid. Zappala *et al.* (1990) found a linear relationship between lightcurve amplitudes and phase angles, analyzing a set of numerical and laboratory models as well as observed lightcurves. Moreover, the slopes of these relationships appear to depend linearly on the corresponding amplitudes reduced to zero phase angle. This means that the slopes of these relations depend on the aspect angle and may be different for different oppositions. The amplitude $A(\phi, 0)$ for a given aspect and zero phase angle can be computed by means of the simple formula

$$A(\phi, 0) = A(\phi, \alpha)/(1 + m\alpha), \quad (6)$$

where $A(\phi, \alpha)$ is the amplitude measured at the aspect ϕ and phase angle α, and m is a coefficient. They also found mean values of the coefficient m for various taxonomic types of asteroids: $m(S) = 0.030$, $m(C) = 0.015$, $m(M) = 0.013$.

To reduce the brightness of the lightcurve maxima, the *HG*-magnitude system, adopted by IAU Commission 20 in 1985, is used. The maximum magnitude $H(\phi, \alpha)$ of an asteroid is calculated from the formula

$$H(\phi, \alpha) = H(\phi, 0) - 2.5 \log[(1 - G)\Phi_1(\alpha) + G\Phi_2(\alpha)], \quad (7)$$

where $H(\phi, 0)$ is the absolute magnitude (i.e., the magnitude reduced to $\alpha = 0°$ and needed in Eq. (5)). G stands for the slope parameter, indicative of the gradient of the phase curve. It has been scaled in such a way that $G \approx 0$ for steep phase curves (low-albedo bodies) and $G \approx 1$ for shallow phase curves (high-albedo bodies). Φ_1 and Φ_2 are two specified phase functions, that are normalized to unity at $\alpha = 0°$ (see Bowell *et al.* 1989 for a full description of this magnitude–phase relation).

The above-mentioned two coefficients m and G are determined from all oppositions with at least two observed lightcurves. It allows to obtain the reduced amplitudes and magnitudes before proceeding to the final determination of the pole and shape of an asteroid. This way is very useful especially for asteroids with a few observed oppositions eliminating two extra unknowns (m and G) in the pole and shape determination (see Eq. (8)).

As it was pointed out by Drummond *et al.* (1988), *photometric astrometry* takes advantage of the movement of the sub-PAB point across lines of longitude on the asteroid, while the *amplitude* and *magnitude methods* take advantage of the movement of the sub-Earth point across lines of asterocentric latitude. *Photometric astrometry* is thus orthogonal to, and independent of, the other two methods, and does not seem to depend on the shape of the body.

For a given asteroid we have three types of equations: *photometric astrometry*—Eq. (1), *amplitude method*—Eq. (3), and *magnitude method*—Eq. (5). Our problem of finding a rotation axis and a shape of an asteroid can be summarized in a form of a set of nonlinear equations,

$$f_k(P_{sid}, \lambda_p, \beta_p, a/b, b/c, H(90, 0)) = 0, \quad k = 1, \ldots, l, l+1, \ldots, l+m, l+m+1, l+m+n, \quad (8)$$

where l, m, and n denote the number of *photometric astrometry*, *amplitude method* and *magnitude method* equations, respectively.

This set of equations can be solved by minimizing the quantity

$$\chi^2 = \sum_{k=1}^{M} (w_k f_k/\sigma_k)^2, \quad (9)$$

where $M = l + m + n$, σ_k is the standard deviation of the kth equation, and w_k is weighting factor which depends on the relative number of epochs, amplitudes and magnitudes, respectively, derived from each oppositions.

Equation (9) is nonlinear function of unknown parameters (P_{sid}, λ_p, β_p, a/b, b/c, H), so the minimization of χ^2 must proceed iteratively. Using the matrix form, we can write

$$X = [P_{sid}, \lambda_p, \beta_p, a/b, b/c, H]^t$$
$$F = [w_1 f_1/\sigma_1, \ldots, w_M f_M/\sigma_M]^t, \quad (10)$$

where "t" denotes the transpose of matrix.

If X_K are approximation of parameters which should minimize the χ^2 then the iterative process of solving our problem can be written as

$$X_{K+1} = X_K - (J^t(X_K)J(X_K))^{-1}(J(X_K)F(X_K)), \quad (11)$$

where $J = [\partial F_i/\partial X_j]$ is the matrix which represents the partial derivatives of all functions in F with respect to each of the variables (see Michałowski 1988, Michałowski and Velichko, 1990).

The procedure (Eq. (11)) is repeated until χ^2 stops decreasing and $|X_{K+1} - X_K| < \varepsilon$. Using the least-square fitting, we obtain also the standard deviations of the estimated parameters X.

The *amplitude* and *magnitude methods* have natural ambiguities in the location of an asteroid's rotational pole. The low orbital inclination (which holds true for most main-belt asteroids) and Eqs. (3) and (5) give us two symmetry properties in pole coordinates. The first is connected with interchanging the north and south poles, the second with reflexion in the ecliptic plane. We thus have four possible locations of an asteroid's pole: $P_1 = (\lambda_p, \beta_p)$, $P_2 = (\lambda_p + 180, \beta_p)$, $P_3 = (\lambda_p, -\beta_p)$, and $P_4 = (\lambda_p + 180, -\beta_p)$. In the present work, the north pole is assumed as the one about which the asteroid rotates in a right-hand sense, referring to a spin vector direction south of the ecliptic as retrograde (see Drummond *et al.* 1988). This means that P_1 and P_4 refer to the same rotational axis but the sense of rotation is different, and similarly for P_2 and P_3. As is mentioned above, *photometric astrometry* identifies prograde or retrograde ends of the axes. Altogether, this means that usually there are two solutions and the sense of rotation is known (P_1 and P_2 are prograde solutions while P_3 and P_4 are retrograde solutions).

Solving the set of nonlinear equations, we initially choose as the first approximation in Equation (11) that λ_1 is equal to the longitude of the asteroid for which the observed lightcurve amplitude is minimum. A value in the range 0°–90° is chosen for the latitude of the pole that is far from the ecliptic plane for asteroids with slight variation in observed amplitudes and is close to the ecliptic plane for large amplitude variation asteroids. According to the ambiguities in the location of the asteroid's pole, there are four possibilities for the pole of a given asteroid. For the biggest observed amplitude ($\phi = 90°$), according to Eq. (3), we have $A_{max} = 2.5 \log (a/b)$ and use this as the first approximation for a/b. If the asteroid is observed at an equatorial aspect ($\phi = 90°$) then its brightness is minimum and this value is assumed for $H(90, 0)$. From Eq. (5) we easily have that $\log (b/c) = 0.4 (H_{min} - H_{max})$. In order to simplify the calculations the observed lightcurves are required to have plausible sidereal periods restricted to a given interval. This interval is chosen so that the synodic period is in the middle. From this range of periods, with a step of 0.0000001 days, only those values are extracted which give different sets of numbers of sidereal cycles in Eq. (2). Usually there are about 100–150 such periods. Equation (11) is solved for each chosen sidereal period four times, according to the number of possibilities of the pole location. Usually there are both prograde and retrograde solutions for many periods. The solutions which give the best fits (in the least squares sense) are adopted. In this way one or two solutions are obtained for most asteriods in the sample considered. However, in some cases, where there are few observations, the period ambiguities cannot be broken. These are shown in Table II, which contains all results.

In order to find solutions to Eq. (8), a minimum of three lightcurves observed during three different oppositions, well placed along an orbit of the asteroid, are needed. If these lightcurves can provide full information, i.e., it is possible to derive magnitudes, amplitudes and epochs from them, a set of nine nonlinear equations (Eq. (8)) is obtained. It is perhaps the minimum number of the lightcurves that are mathematically required, but more are necessary, especially for a realistic error analysis. This is because of noise in the lightcurves, departures from assumptions, shortcomings in the model, and many other possible factors.

Results for 35 asteroids were presented by Magnusson (1986, 1990) and Drummond *et al.* (1988, 1991). Their methods are based on the same assumptions, but different mathematical procedures. In his method, Magnusson scans the celestial sphere with a large number of trial poles, and solutions which are consistent with all available information are selected. Drummond *et al.* make independent solutions for three methods, *sidereal photometric astrometry, weighted–amplitude–aspect,* and *amplitude–magnitude–aspect methods,* and adopt the pole as a mean of the values from these methods.

These two approaches involve more time-consuming calculations than the method described in this paper.

III. RESULTS

The poles, senses of rotation, sidereal periods and shapes have been obtained for 30 asteroids. Their basic parameters are summarized in Table I. Their diameters, taken from Tedesco (1989) are in column 2, while column 3 displays their taxonomic type (Tholen 1989). Columns 4 and 5 shown the G-value of the HG-magnitude system (Bowell *et al.* 1989) taken from Tedesco (1989) and Lagerkvist and Magnusson (1990), respectively. In the cases where sufficient data are available, the magnitudes have been reduced to zero phase angle with the G values obtained in this work and shown in column 6. In some cases it is impossible to determine the G-value, so those shown in columns 4 or 5 or the average G-value for a given taxonomic type, $G(S) = 0.23$, $G(M) = 0.22$, $G(C) = 0.04$ (see Lagerkvist and Magnusson 1990), are used. The choice depends on the best fit (χ^2 is minimum—see Eq. (9)) in the final pole and shape solution. Column 7 gives the coefficient m determined during the reduction of the observed amplitudes to zero phase angle. If this is not possible, the average value for a given taxonomic type according to Zappala *et al.* (1990) is used. The final column of Table I shows two numbers. The first refers to the number of oppositions during which the asteroid has been observed, and the second to the number of lightcurves

TABLE I
Asteroid Parameters

Asteroid	Diameter (km)	Type	G-value			m	No. oppositions/ No. lightcurves
			[a]	[b]	[c]		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
10 Hygiea	429	C	−0.039	0.09			7/9
15 Eunomia	272	S	0.199	0.23	0.288	0.020	12/26
21 Lutetia	100	M	0.163			0.011	5/8
23 Thalia	111	S	0.370				7/7
28 Bellona	126	S	0.221	0.13			4/4
31 Euphrosyne	248	C	0.15			0.006	4/6
40 Harmonia	111	S	0.307				4/5
43 Ariadne	65	S	−0.048	0.16	0.197	0.025	7/17
52 Europa	312	CF	−0.004	0.25	0.215	0.0	3/6
55 Pandora	68	M	0.347	0.28	0.226	0.016	4/10
60 Echo	62	S	0.332		0.282		5/8
64 Angelina	60	E	0.369		0.459	0.020	4/10
80 Sappho	82	S	0.300				4/4
87 Sylvia	271	P	0.275	0.10	0.044	0.081	8/14
130 Ekeltra	189	G	−0.036	0.06	0.071	0.040	7/12
135 Hertha	82	M	0.194	0.18			4/4
139 Juewa	162	CP	0.15				4/4
173 Ino	159	C	0.121	0.04			3/4
196 Philomela	146	S	0.475				3/3
201 Penelope	70	M	0.139	0.28	0.211	0.035	7/16
225 Henrietta	124	F	0.15				3/3
250 Bettina	86	M	0.705		0.256	0.029	4/9
334 Chicago	170	C	−0.057		0.183		4/6
337 Devosa	63	EMP	0.25	0.21	0.267	0.029	6/10
389 Industria	82	S	−0.062				3/3
511 Davida	337	C	0.020	0.07	0.100	0.013	13/27
584 Semiramis	56	S	0.339	0.23	0.290	0.047	4/9
694 Ekard	93	CP	0.15		0.235	0.010	4/6
704 Interamnia	333	F	0.019	−0.03	−0.040	0.0	4/7
747 Winchester	178	PC	0.15		0.050	0.0	4/5

[a] Tedesco 1989.
[b] Lagerkvist and Magnusson 1990.
[c] Present work.

used in the calculations. Most of them can be found in the *Asteroid Photometric Catalogue* (hereafter APC) by Lagerkvist *et al.* (1987, 1988, 1992a). It should be mentioned that not all lightcurves provide full information needed for the computations. Many of them have relative photometry or such small amplitudes that it has not been possible to determine epochs of extrema.

The results of the pole and shape determinations are shown in Table II. This table is divided into three parts: **New results, Improved results** and **Test objects.**

The **New results** refer to the asteroids for which no poles and shapes have been published. In some cases, it has not been possible to determine unique sidereal periods, and they are not displayed in Table II. Some possible values can be found in the text describing individual objects. Additional observations are needed to improve these results.

Asteroids with improved results are shown in the second part of Table II. For most of them the senses of rotation and sidereal periods have not been given so far. All of these asteroids are described in the text.

The last part of Table II contains asteroids which have been used mainly as the test objects for the method presented in this paper.

The results are presented as follows: The number and name of the asteroid are in the header line. The sense of rotation is displayed on the same line, last column (P for prograde, and R for retrograde rotation). Column 1 gives the sidereal period expressed in days and the error (in parentheses) in the last digit. The ecliptic longitude and latitude of the north pole of the asteroid are in columns 2 and 3, respectively. The errors for the coordinates of the pole (in degrees of arc) are shown in the same columns in parentheses. The next two columns give the shape of

TABLE II
Results

Sidereal period (days) (1)	Pole λ_p (2)	Pole β_p (3)	Shape a/b (4)	Shape b/c (5)	$H(90, 0)$ (6)	Sense (7)
New results						
60 Echo						P
1.048228 (9)	95 (5)	+34 (14)	1.42 (0.07)	1.37 (0.04)	8.65 (0.04)	
1.048225 (9)	275 (5)	+42 (12)	1.57 (0.08)	1.39 (0.04)	8.68 (0.05)	
64 Angelina						P
0.3647782 (7)	119 (6)	+29 (6)	1.39 (0.02)	1.05 (0.03)	7.62 (0.06)	
0.3647785 (7)	299 (6)	+27 (6)	1.38 (0.02)	1.05 (0.03)	7.62 (0.05)	
80 Sappho						P
0.584946 (1)	46 (6)	+10 (6)	1.36 (0.20)	1.87 (0.30)	7.95 (0.14)	
0.584947 (1)	219 (6)	+7 (7)	1.36 (0.20)	1.94 (0.32)	7.97 (0.15)	
139 Juewa						P
?	117 (14)	+50 (12)	1.21 (0.20)	1.68 (0.45)	7.52 (0.47)	
173 Ino						R
?	198 (8)	−21 (8)	1.24 (0.05)	1.63 (0.08)	7.99 (0.02)	
?	356 (8)	−47 (10)	1.22 (0.06)	1.55 (0.10)	7.95 (0.04)	
196 Philomela						R
?	99 (5)	−16 (7)	1.33 (0.19)	1.17 (0.20)	6.44 (0.19)	
?	273 (6)	−22 (8)	1.33 (0.18)	1.17 (0.20)	6.44 (0.19)	
225 Henrietta						R
?	241 (11)	−56 (7)	1.27 (0.03	1.89 (0.10)	8.75 (0.04)	
334 Chicago						P
0.383246 (1)	13 (7)	+32 (6)	1.67 (0.02)	1.07 (0.02)	7.50 (0.03)	
0.383246 (1)	188 (9)	+42 (6)	1.68 (0.02)	1.06 (0.02)	7.50 (0.03)	
337 Devosa						P
0.1931106 (5)	199 (12)	+59 (11)	1.20 (0.02)	1.79 (0.17)	8.90 (0.03)	
389 Industria						R
?	98 (8)	−55 (6)	1.25 (0.02)	1.38 (0.02)	7.82 (0.02)	
?	314 (7)	−50 (7)	1.26 (0.02)	1.38 (0.02)	7.82 (0.03)	
704 Interamnia						R
?	43 (8)	−21 (9)	1.14 (0.04)	1.10 (0.05)	5.96 (0.01)	
?	224 (10)	−22 (10)	1.24 (0.08)	1.04 (0.08)	5.95 (0.02)	
747 Winchester						P
?	27 (10)	+50 (10)	1.16 (0.02)	2.60 (0.55)	7.68 (0.05)	
Improved results						
10 Hygiea						R
1.150977 (2)	117 (8)	−37 (7)	1.30 (0.06)	1.18 (0.06)	5.17 (0.10)	
1.150977 (2)	304 (9)	−35 (7)	1.30 (0.06)	1.18 (0.06)	5.17 (0.10)	
21 Lutetia						P
0.340244 (1)	33 (7)	+9 (8)	1.25 (0.01)	2.62 (0.90)	7.34 (0.02)	
0.340244 (1)	214 (6)	+15 (7)	1.25 (0.01)	2.77 (1.20)	7.34 (0.02)	
23 Thalia						P
0.5133960 (3)	198 (17)	+72 (6)	1.18 (0.01)	1.42 (0.09)	7.06 (0.02)	
0.5133959 (4)	354 (8)	+47 (8)	1.18 (0.02)	1.48 (0.13)	7.07 (0.03)	
28 Bellona						P
?	73 (7)	+17 (7)	1.24 (0.01)	1.20 (0.02)	7.07 (0.03)	
?	265 (9)	+43 (7)	1.24 (0.01)	1.20 (0.02)	7.08 (0.03)	
31 Euphrosyne						R
0.2316828 (2)	126 (4)	−31 (6)	1.14 (0.01)	1.59 (0.11)	6.73 (0.01)	
40 Harmonia						P
0.3712522 (3)	208 (13)	+21 (14)	1.24 (0.17)	2.07 (0.35)	6.96 (0.35)	
52 Europa						P
?	17 (15)	+65 (20)	1.11 (0.01)	2.79 (0.95)	6.30 (0.05)	
135 Hertha						R
0.347818 (1)	126 (6)	−28 (9)	1.35 (0.08)	1.19 (0.10)	8.08 (0.10)	
0.347818 (1)	310 (6)	−31 (9)	1.36 (0.08)	1.20 (0.10)	8.08 (0.10)	
Test objects						
15 Eunomia						R
0.25344814 (4)	102 (13)	−76 (6)	1.42 (0.01)	1.35 (0.06)	5.12 (0.01)	
0.25344814 (4)	354 (13)	−57 (5)	1.42 (0.01)	1.27 (0.08)	5.12 (0.01)	
43 Ariadne						R
0.2400824 (5)	68 (7)	−22 (12)	1.64 (0.06)	1.15 (0.06)	7.79 (0.09)	
0.2400823 (8)	253 (7)	−28 (12)	1.63 (0.06)	1.18 (0.06)	7.77 (0.09)	
55 Pandora						P
0.2001595 (9)	239 (9)	+28 (10)	1.29 (0.04)	1.32 (0.09)	7.79 (0.08)	
87 Sylvia						P
0.2159859 (6)	84 (9)	+55 (9)	1.37 (0.03)	1.29 (0.08)	6.69 (0.04)	
0.2159859 (4)	297 (5)	+50 (7)	1.37 (0.02)	1.53 (0.09)	6.72 (0.02)	
130 Ekeltra						R
0.2176942 (6)	246 (8)	−32 (10)	1.33 (0.02)	1.07 (0.09)	6.88 (0.03)	
0.2176942 (5)	344 (6)	−86 (7)	1.32 (0.02)	1.06 (0.06)	6.88 (0.03)	
201 Penelope						R
0.1561433 (4)	258 (4)	−22 (7)	1.50 (0.08)	1.22 (0.08)	8.28 (0.02)	
250 Bettina						R
0.2106223 (2)	102 (4)	−30 (6)	1.36 (0.01)	1.33 (0.02)	7.51 (0.01)	
0.2106225 (2)	272 (5)	−55 (5)	1.37 (0.01)	1.34 (0.01)	7.51 (0.01)	
511 Davida						P
0.2137234 (1)	96 (8)	+32 (8)	1.23 (0.03)	1.12 (0.03)	6.18 (0.03)	
0.2137234 (1)	303 (7)	+31 (9)	1.23 (0.03)	1.12 (0.03)	6.19 (0.03)	
584 Semiramis						R
0.2112062 (6)	112 (6)	−51 (5)	1.24 (0.01)	1.15 (0.02)	8.58 (0.01)	
694 Ekard						P
0.2467460 (5)	98 (4)	+40 (5)	1.46 (0.01)	1.73 (0.04)	9.13 (0.01)	

the triaxial ellipsoid model of the asteroid (the errors of a/b and b/c are in parentheses). The maximum brightness of the asteroid (and its error) for aspect 90° and zero solar phase angle is in column 6.

New Results

60 Echo. This asteroid has been observed during five oppositions (see APC) and eight lightcurves were used in the calculation. The solutions presented here are the first published results for Echo.

64 Angelina. Table I shows that 10 lightcurves from four apparitions (see APC) were used. No previous results have been published for Angelina.

80 Sappho. There is only one lightcurve with absolute photometry. It is probably the reason for a large uncertainty in the shape of Sappho. No previous results are available for this asteroid.

139 Juewa. No pole and shape information have been published previously. No lightcurves during four apparitions cover the whole rotational cycle. Only one epoch from each apparition can be used, so the value for the sidereal period and sense of rotation may be completely wrong. However, the value P_{sid} = 0.871343 (days) seems to be the most likely for this asteroid.

173 Ino. Ino has previously been observed during three oppositions (see APC). The results presented here are the first published parameters for this asteroid and P_{sid} = 0.256798 days is the most probable value for Ino.

196 Philomela. This asteroid has been observed during three oppositions (see APC and Erikson *et al.* 1991). Philomela seems to be an example of asteroid with three lightcurves, for which it is possible to obtain pole and shape. The retrograde sense of rotation and P_{sid} = 0.347201 days are the most likely for this asteroid.

225 Henrietta. Only three lightcurves have been used to obtain the pole and shape of this asteroid. No previous pole information has been published for Henrietta and the value P_{sid} = 0.349975 days seems to be the best.

334 Chicago. There are lightcurves from four apparitions (see APC) for this asteroid. The solutions presented are the first published results for Chicago.

337 Devosa. Table I shows that 10 lightcurves from six oppositions (see APC) have been used. No previous pole and shape information have been published for this asteroid.

389 Industria. This asteroid has been observed during three oppositions (see APC, Magnusson and Lagerkvist 1991, Lagerkvist *et al.* 1992b). A few solutions have been obtained with nearly the same pole, shape, and retrograde rotation, but for different sidereal periods. The most probable one seems to be 0.3540065 or 0.3542685 days, but other values are still possible.

704 Interamnia. This asteroid has been observed during four oppositions (see APC and Shevchenko *et al.* 1992). Unfortunately, these data have not permitted a determination of the sidereal period, but the sense of rotation seems to be retrograde. The most likely period seems to be 0.3637021 days, but other values are still possible.

747 Winchester. The results presented are the first published parameters for this asteroid. Only one lightcurve has absolute photometry, so further observations are required in order to determine b/c and the sidereal period; however, P_{sid} = 0.391737 days is the most likely.

Improved Results

10 Hygiea. Prior to the paper by Michałowski *et al.* (1991), the often-quoted period of Hygiea was about 18 hr. The new synodic period reported by them was 27.63 hr. Using the method which fitted the observed amplitudes and epochs of lightcurve extrema (Michałowski 1988, Michałowski and Velichko 1990) they found the sidereal period to be 1.152462 days; two poles—(112, −41), (299, −39); and triaxial ellipsoid shape—a/b = 1.36, b/c = 1.05. Adding the available magnitudes and including this asteroid in the present study, the poles and the shape are similar to the previous ones, but the new sidereal period is shorter than the older one by about 2 min. This is probably due to some errors in counting the synodic cycles in the old version of the *photometric astrometry method*. This shorter period fits the observed epochs better than the previous one.

21 Lutetia. Lupishko and Velichko (1987) reported that this asteroid had sidereal period 0.340277 days and prograde rotation. In another paper, Lupishko *et al.* (1987) gave the pole (42, +40) or (223, +48) and the shape (a/b = 1.25, b/c = 1.09). The results presented here confirm the sense of rotation, the pole longitude, and a/b, but to a lesser degree the period and the latitude of the pole. The b/c obtained here is completely different with a large error that is probably due to the fact that only one lightcurve of Lutetia has absolute photometry. New lightcurves are required to obtain the correct parameters of this asteroid.

23 Thalia. Lightcurves from APC, and a new one from Tancredi and Gallardo (1991), are used for the current analysis. The results presented here are the first published parameters for this asteroid. Tancredi and Gallardo (1991) give only the ranges of λ_p, β_p, and a/b while b/c varies from 1 to 3. According to their solutions the longitude of the pole is between 130° and 170°, or 325° and 345°, and the latitude between 40° and 85°, 30° and 65°, respectively. They also derived that a/b ranges from 1.12 to 1.18. The results obtained in this work are in agreement with these ranges.

28 Bellona. There are Bellona lightcurves in APC and Harris *et al.* (1992). Zappala and Knezevic (1984) used the lightcurves from three oppositions and obtained two solutions for a pole of this asteroid (93, +18), (285, +37), and a triaxial ellipsoid model with a/b = 1.31 and b/c = 1.18. Lightcurves from four apparitions confirm their pole latitude and b/c, but the new pole longitudes are about 20° less than theirs, and a/b (1.24) is also lower. Further observations are needed for an unambiguous determination of the sidereal period. Two possible values (0.653651 and 0.653619 days) have been found here.

31 Euphrosyne. Barucci *et al.* (1985) and McCheyne *et al.* (1985) obtained similar results for this asteroid, pole (317, +4) or (180, +70) and a/b = 1.12, by assuming b/c = 1.0. Only one solution, with a retrograde rotation, is obtained in the present study (126, −31). The prograde end of this axis is similar, especially in longitude (306), to their results. A different b/c (1.59) is found here. The sidereal period and sense of rotation are the first obtained for Euphrosyne. New observations are needed in order to resolve existing discrepancies in all results.

40 Harmonia. There is only one lightcurve with absolute photometry, which is probably the reason for the large error in b/c. Tancredi and Gallardo (1991) presented their results as they did for 23 Thalia. The longitude of the pole should be between 15° and 25° or 195° and 210°, while the latitude between 20° and 60° or 20° and 70°, respectively, which are in very good agreement with the solution presented here.

52 Europa. Barucci *et al.* (1986), assuming b/c = 1.0, found a/b = 1.12 and the pole (0, +37) or (203, +38). The second pole has a very large error of 50°, so only the first one seems likely. The pole and a/b presented in this work are similar to their solutions except for a different b/c with a large error. New observations are needed because Europa has been observed during only three oppositions (see APC). The sidereal period seems to be 0.234987 days, but it should be improved with new lightcurves.

135 Hertha. The lightcurves from APC and Tancredi and Gallardo (1991) are used in the present work. They also reported that the pole of Hertha should be in two ranges: longitude from 110° to 130° or from 285° to 310°, latitude from 20° to 60°, or from 10° to 60°, respectively. Retrograde solutions have been obtained in the present work, so the south pole is (306, +28) or (130, +31). These solutions are in agreement with the ranges given by them. They report that a/b should be between 1.22 and 1.9, which is consistent with an a/b = 1.35 found here.

Test Objects

15 Eunomia. The pole, shape, period, and sense of rotation have been determined by many authors (see Magnusson 1989 for a list). Magnusson (1990) reported that Eunomia has retrograde rotation with P_{sid} = 0.25344808 days and two possibilities for the pole (108, −74) or (350, −59), with a/b = 1.44, b/c = 1.0. Similar results were given by Drummond *et al.* (1991): pole (106, −73), P_{sid} = 0.25344806 days, a/b = 1.44, b/c = 1.02. The results here agree with their values but with a larger b/c of 1.34.

43 Ariadne. According to Magnusson (1990) Ariadne is retrograde rotator with P_{sid} = 0.2400828 days, pole (68, −14) or (251, −16), and shape a/b = 1.76, b/c = 1.01. Drummond *et al.* (1991) gave one solution for pole (248, −10) and a/b = 1.61, b/c = 1.24, and P_{sid} = 0.24008297 days. The b/c (1.15 or 1.18) presented here is closer to the Drummond *et al.* value than to Magnusson's. The solutions give a pole for Ariadne that is farther from the Ecliptic plane and yields a sidereal period that is a little shorter (by about 0ˢ04).

55 Pandora. Drummond *et al.* (1991) reported two solutions for the pole of this asteroid: (224, +32), and (32, +40), with a/b = 1.34, b/c = 1.48 and P_{sid} = 0.2001596 days. One of the solutions presented here is similar to one of their results. The period, the sense of rotation and the shape obtained in this work also confirm their results.

87 Sylvia. Magnusson (1990) reported the prograde solutions (66, +67) and (296, +59), with P_{sid} = 0.2159851 days and a/b = 1.44, b/c = 1.5. Drummond *et al.* (1991) obtained similar results: (89, +52) and (291, +42), P_{sid} = 0.2159853 days, a/b = 1.43, b/c = 1.17. The periods of this work are closer to the period of Drummond *et al.*, but the b/c are closer to that of Magnusson.

130 Elektra. Magnusson (1990) obtained two retrograde solutions (180, −85) and (240, −40), with P_{sid} = 0.2176942 and a/b = 1.41, b/c = 1.2. Drummond *et al.* (1991) reported only one solution with the pole near the south ecliptic pole (190, −81) and P_{sid} = 0.21769512, a/b = 1.23, b/c = 1.63. The second pole given in this work has a different longitude but is also very close to the south ecliptic pole. The period, sense of rotation, and a/b confirm previous results, but the b/c (1.06) is lower than that of Drummond *et al.* (1991).

201 Penelope. Magnusson (1990) reported two retrograde solutions (80, −35) and (260, −25), with P_{sid} = 0.1561443 days, a/b = 1.50 and b/c = 1.23. Drummond *et al.* (1991) obtained two solutions: (261, −34), P_{sid} = 0.15614397, a/b = 1.55, and b/c = 1.34; and the second (74, −2), P_{sid} = 0.15612874, a/b = 1.53, b/c = 1.24. The results presented here are in good agreement with those given by Magnusson and the first solution of Drummond *et al.* The second pole and sidereal period given by Drummond *et al.* are not confirmed by either this work or by Magnusson's results.

250 Bettina. The solutions obtained here confirm the results of Drummond *et al.* (1991). They reported P_{sid} = 0.2106225 days, a/b = 1.32, b/c = 1.38, and pole (104, −16). However, the solutions obtained here place the pole of Bettina farther from the Ecliptic plane.

511 Davida. Since several pole determinations have been published for Davida (see Magnusson 1989 for details), it has been used as the first test object of the method presented in this work. The results obtained agree well with all previous solutions. Magnusson *et al.* (1989) pointed out that "511 Davida is the object for which the various techniques have given the most consistent results and it is a good candidate for calibration of other techniques." The results presented in this paper confirm this conclusion.

584 Semiramis. Drummond *et al.* (1988) reported the poles (327, −55), (90, −49), P_{sid} = 0.2112053 days, a/b = 1.19 and b/c = 1.28. Magnusson (1990) obtained two retrograde solutions: (110, −40), (320, −30), P_{sid} = 0.211206 days, a/b 1.17, b/c = 1.1. The solution of this work confirms the previous ones except for a larger a/b = 1.24.

694 Ekard. Drummond *et al.* (1991) obtained two prograde solutions: (105, +29), P_{sid} = 0.2467461, a/b = 1.45, b/c = 1.36 and the second (267, +56), P_{sid} = 0.2467469, a/b = 1.46, b/c = 1.28. The results of this work confirm their first solution, but with a b/c = 1.73 that is larger than the one that they have determined.

IV. CONCLUSIONS

Magnusson (1986, 1990) concluded that the distribution of spin axes is quite isotropic in sin β_p, but with prograde rotating asteroids in a slim majority. He also confirmed the conclusion of Barucci *et al.* (1986) concerning the apparent lack of poles close to the ecliptic plane. He attributed this to a possible selection effect in his choice of asteroids for his study. For an asteroid with a pole at a low latitude, an asteroid may be seen nearly pole-on, producing an amplitude too small for unambiguous epoch determinations. Therefore, many more lightcurve observations are needed to derive their epochs than for asteroids with poles far from the ecliptic plane. Accordingly, the bimodality of the pole distribution is probably due to this selection effect.

Drummond *et al.* (1988, 1991) confirmed this bimodality in pole distribution and agreed, as one possible explanation, with Magnusson's selection effect. They added two other possible explanations. First, the observed distribution may simply be a statistical fluke. Second, the observed bimodality may be real, perhaps reflecting some primordial distribution.

In the current study, 16 of 30 asteroids have prograde rotations, confirming previous conclusions that "there is a slight majority of prograde rotating asteroids." It should be pointed out here that there is no selection effect on the sense of rotation, so the conclusion about departure from isotrophy in the distribution of poles may be true. The bimodality in the pole distribution is quite pronounced in the same of asteroids studied in this work, because no object has a pole with a latitude in the range from $-8°$ to $+8°$.

Drummond *et al.* (1988, 1991) and Magnusson (1990) concluded that hydrostatic equilibrium shapes for rotating bodies with no internal strength, in general, do not describe asteroids. Many asteroids therefore have internal strengths that can support the observed departures from equilibrium shapes. Drummond *et al.* (1991) suggested that asteroids 39, 45, 55, 107, 130, and 216 were the best candidates for quasi-equilibrium rubble piles. The results presented in this paper also confirm these conclusions.

A continuation of the project is planned in order to increase the number of asteroids with well-determined poles, senses of rotation, and shapes. This should help address the problem of the pole distribution and, in the future, relate the distribution of poles and shapes to various properties such as size, taxonomic type, or family membership.

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