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Gluon condensates in a cold quark–gluon plasma

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ABSTRACT

The quark–gluon plasma which has been observed at RHIC is a strongly interacting system and has been called sQGP. This is a system at high temperatures and almost zero baryon chemical potential. A similar system with high chemical potential and almost zero temperature may exist in the core of compact stars. Most likely it is also a strongly interacting system. The strong interactions may be partly due to non-perturbative effects, which survive after the deconfinement transition and which can be related with the non-vanishing gluon condensates in the sQGP. In this work, starting from the QCD Lagrangian we perform a gluon field decomposition in low ("soft") and high ("hard") momentum components, we make a mean field approximation for the hard gluons and take the matrix elements of the soft gluon fields in the plasma. The latter are related to the condensates of dimension two and four. With these approximations we derive an analytical expression for the equation of state, which is compared to the MIT bag model one. The effect of the condensates is to soften the equation of state whereas the hard gluons significantly increase the energy density and the pressure.

1. Introduction

One of the most interesting results of the RHIC program is the discovery of an extremely hot and dense state of matter made of quarks and gluons in a deconfined phase and which behaves like an ideal fluid [1]. While the production of such a plasma of quarks and gluons had been predicted, it was a surprise to find that this system is strongly interacting and very different from the originally expected gas of almost non-interacting quarks and gluons, described by perturbative QCD. This state has been called strongly interacting quark–gluon plasma (sQGP) and there are many approaches to study its properties. The most fundamental approach is provided by lattice QCD [2]. Since lattice QCD has still some limitations, such as the difficulty in dealing with systems with large baryon chemical potential, there are several models (see, for example, [3–5]) which incorporate the essential features of the full theory and which can be employed to study the sQGP. In some of them [3,4] the sQGP is treated as a gas of quasi-particles, in which the quarks and gluons have an effective mass. In some works, such as in [3,6], the sQGP was treated with semi-classical methods. In [3] the color charges were assumed to be large and classical obeying Wong equations of motion. In this approach the quantum effects in the QGP are basically reduced to generate thermal-like masses and cause the effective coupling to run to larger values at smaller values of the temperature.

The medium created in heavy ion collisions has high temperature and zero baryon chemical potential. On the other corner of the phase space, we find the QGP at zero temperature and high baryon number. Presumably, this kind of system exists in the core of dense stars. This cold QGP has a richer phase structure and at high enough chemical potential we may have a color superconducting phase. Because of the limitations of lattice calculations in this domain and also because of the lack of experimental information, the cold QGP is less known than the hot QGP. Nevertheless it is quite possible that it shares some features with the hot plasma, being also a strongly interacting and semi-classical system.

In this work we shall study the non-perturbative effects in the cold QGP generated by the residual dimension-two and dimension-four condensates, using a mean field approximation.

In the vacuum, non-perturbative effects have been successfully understood in terms of the QCD condensates, i.e., vacuum expectation values of quark and gluon "soft" (low momentum) fields. The best known are the dimension-three quark condensate and the dimension-four gluon condensate [7]. These condensates can, in principle, be computed in lattice QCD or with the help of models. In practice, since they are vacuum properties and therefore universal, they can be extracted from phenomenological analyses of hadron masses, as it is customary done in QCD sum rules [8]. The condensates are expected to vanish in the limit of very high temperature or chemical potential. However, it has been suggested that they may survive after the deconfinement transition both in the high temperature [9,10] and in the high chemical potential cases [11]. For our purposes the relevant gluon condensates are

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those of dimension four [7], $\langle 0|\frac{\alpha_s}{\pi}F^2|0\rangle$ $(=\langle F^2\rangle)$, and of dimension two [12–15], $\langle 0|g^2A^2|0\rangle$ $(=\langle g^2A^2\rangle)$.

We shall derive an equation of state (EOS) for the cold QGP, which may be useful for calculations of stellar structure. Our EOS can be considered an improved version of the EOS of the MIT bag model, which contains both the non-perturbative effects coming from the residual gluon condensates and the perturbative effects coming from the hard gluons, which are enhanced by the high quark density. As it will be seen, the effect of the condensates is to soften the EOS whereas the hard gluons significantly increase the energy density and the pressure.

2. The equation of state

In this section we develop a mean field approximation for QCD, extending previous works along the same line [16–20]. The Lagrangian density of QCD is given by:

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{q=1}^{N_f}\bar{\psi}^q_i\left[i\gamma^\mu\left(\delta_{ij}\partial_\mu - igT^a_{ij}G^a_\mu\right) - \delta_{ij}m_q\right]\psi^q_j \tag{1}$$

where

$$F^{a\mu\nu} = \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + gf^{abc}G^{b\mu}G^{c\nu} \tag{2}$$

The summation on q runs over all quark flavors, m_q is the mass of the quark of flavor q, i and j are the color indices of the quarks, T^a are the SU(3) generators and f^{abc} are the SU(3) antisymmetric structure constants. For simplicity we will consider only light quarks with the same mass m. Moreover, we will drop the summation and consider only one flavor. At the end of our calculation the number of flavors will be recovered. Following [16,17], we shall start writing the gluon field as:

$$G^{a\mu} = A^{a\mu} + \alpha^{a\mu} \tag{3}$$

where $A^{a\mu}$ and $\alpha^{a\mu}$ are the low ("soft") and high ("hard") momentum components of the gluon field respectively. The former will be responsible for the long range and low momentum transfer, non-perturbative processes whereas the latter will be relevant in the short distance perturbative processes. The field decomposition made above requires the choice of an energy scale defining the frontier between soft and hard. This energy scale, E, lies in the range $\Lambda_{QCD} < E < 1$ GeV. In principle, the dependence of the results on this choice can be studied with the renormalization group techniques. Accurate results would also require the knowledge of the scale dependence of the in-medium gluon condensates, which in our case is poor. Therefore, in order to keep the simplicity of our approach, we will not specify the separation scale and will assume that $A^{a\mu}$ represents the soft modes which populate the vacuum and $\alpha^{a\mu}$ represents the modes for which the running coupling constant is small.

Inserting (3) into (2) we obtain:

$$F^{a\mu\nu} = \left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + gf^{abc}A^{b\mu}A^{c\nu}\right) + \left(\partial^\mu\alpha^{a\nu} - \partial^\nu\alpha^{a\mu} + gf^{abc}\alpha^{b\mu}\alpha^{c\nu}\right) + gf^{abc}A^{b\mu}\alpha^{c\nu} + gf^{abc}\alpha^{b\mu}A^{c\nu} \tag{4}$$

2.1. The mean field approximation

In a cold quark–gluon plasma the density is much larger than the ordinary nuclear matter density. These high densities imply a very large number of sources of the gluon field. With intense sources the bosonic fields tend to have large occupation numbers at all energy levels, and therefore they can be treated as classical fields. This is the famous approximation for bosonic fields used in relativistic mean field models of nuclear matter [21]. It has been applied to QCD in the past [18] and amounts to assume that the "hard" gluon field, α^a_μ, is simply a function of the coordinates [21]:

$$\alpha^a_\mu = \alpha^a_0\delta_{\mu 0} \tag{5}$$

In fact, for cold nuclear matter, it is further assumed that α^a_μ is constant in space and time [21]:

$$\partial_\nu\alpha^a_\mu = 0 \tag{6}$$

As a consequence of this approximation, the term $gf^{abc}\alpha^b_0\alpha^c_0$ will vanish because of the color symmetry. We also assume that the soft gluon field $A^{a\mu}$ is independent of position and time and thus:

$$\partial^\nu A^{a\mu} = 0 \tag{7}$$

Substituting (5), (6) and (7) into (4) we have $F^{a\mu\nu} = f^{abc}(gA^{b\mu}A^{c\nu} + gA^{b\mu}\alpha^c_0\delta^{\nu 0} + g\alpha^b_0\delta^{\mu 0}A^{c\nu})$. Inserting this into (1), the QCD Lagrangian simplifies to:

$$\begin{aligned}\mathcal{L}_{QCD} = -\frac{f^{abc}f^{ade}}{4}\big[&g_s^2A^b_\mu A^c_\nu A^{d\mu}A^{e\nu} \\ &+ g^2A^b_\mu A^c_\nu A^{d\mu}\alpha^e_0\delta^{0\nu} + g^2A^b_\mu A^c_\nu\alpha^d_0\delta^{0\mu}A^{e\nu} \\ &+ g^2A^b_\mu\alpha^c_0\delta_{0\nu}A^{d\mu}A^{e\nu} + g^2\alpha^b_0\delta_{0\mu}A^c_\nu A^{d\mu}A^{e\nu} \\ &+ g^2A^b_\mu\alpha^c_0\delta_{0\nu}\alpha^d_0\delta^{0\mu}A^{e\nu} + g^2A^b_\mu\alpha^c_0\delta_{0\nu}A^{d\mu}\alpha^e_0\delta^{0\nu} \\ &+ g^2\alpha^b_0\delta_{0\mu}A^c_\nu A^{d\mu}\alpha^e_0\delta^{0\nu} + g^2\alpha^b_0\delta_{0\mu}A^c_\nu\alpha^d_0\delta^{0\mu}A^{e\nu}\big] \\ &+ \bar{\psi}^q_i\left\{i\gamma^\mu\left[\delta_{ij}\partial_\mu - iT^a_{ij}\left(g_sA^a_\mu + g_h\alpha^a_0\delta_{0\mu}\right)\right] - \delta_{ij}m\right\}\psi^q_j\end{aligned} \tag{8}$$

In the above expression the coupling is running. In the first line g^2 connects four soft fields and is therefore large. Accordingly we call it $g = g_s$. In the fourth line, the interaction between the field A^a_μ and the quarks is dominated by low momenta, the coupling is large and hence $g = g_s$. In the same line the interaction between α^a_0 and the quarks is dominated by high momenta, the hard coupling is small and we call it $g = g_h$. At this point, in the second and third lines the couplings could be soft or hard.

We shall now replace the soft gluon field A^b_μ and its powers by the corresponding expectation values in the cold QGP. The product of four fields in the first line of the above equation can be related to the gluon condensate through the relations similar from [16,17]:

$$\left\langle g_s^2A^a_\mu A^b_\nu A^{c\rho}A^{d\eta}\right\rangle = \frac{{\phi_0}^4}{(32)(34)}\left[g_{\mu\nu}g^{\rho\eta}\delta^{ab}\delta^{cd} + g_\mu{}^\rho g_\nu{}^\eta\delta^{ac}\delta^{bd} + g_\mu{}^\eta g_\nu{}^\rho\delta^{ad}\delta^{bc}\right] \tag{9}$$

and

$$-\frac{1}{4}\left\langle F^{a\mu\nu}F_{a\mu\nu}\right\rangle = -\frac{\pi^2}{g_s^2}\left\langle\frac{\alpha_s}{\pi}F^{a\mu\nu}F^a_{\mu\nu}\right\rangle = -b\phi_0^4 \tag{10}$$

where the constant b is given by:

$$b \equiv \frac{9}{4(34)} \tag{11}$$

In the second and fourth lines of (8) we have odd powers of $A^{a\mu}$ which have vanishing expectation values:

$$\langle A^{a\mu} A^{b\nu} A^{c\rho} \rangle = 0 \tag{12}$$

$$\langle A^{a\mu} \rangle = 0 \tag{13}$$

In the third line of (8) we approximate the products of the type $g^2 A^{a\mu} A^{b\nu}$ by the expectation value of two soft fields given by [16,17]:

$$\langle g_s^2 A^{a\mu} A^{b\nu} \rangle = -\frac{\delta^{ab} g^{\mu\nu}}{32} \mu_0^2 \tag{14}$$

Since the above matrix element involves only soft fields we have $g = g_s$. The $\langle g_s^2 A^2 \rangle$ condensate is associated with a dynamical gluon mass [16,17] which is defined as:

$$m_G^2 \equiv \frac{9}{32} \mu_0^2 \tag{15}$$

In spite of the recent progress in the field, still very little is known about $\langle A^2 \rangle$ at finite (and high) density. In our approach, as in [22], we have $\langle g_s^2 A^2 \rangle < 0$ in (14) so $m_G{}^2$ is positive.

Using expressions (9), (12), (13), (14) and (15) in (8) we arrive at the following effective Lagrangian:

$$\mathcal{L} = -b\phi_0^4 + \frac{m_G^2}{2} \alpha_0^a \alpha_0^a + \bar{\psi}_i^q \left(i\delta_{ij} \gamma^\mu \partial_\mu + g_h \gamma^0 T_{ij}^a \alpha_0^a - \delta_{ij} m \right) \psi_j^q \tag{16}$$

This Lagrangian describes a system with quarks, soft gluons and hard gluons. The quarks couple only to the hard gluons. The hard gluons couple to the quarks and to the soft gluons. The field α propagates (on a background with the field A) with large momentum but does not exchange hard momenta with the background. The only effect of this interaction is to give an effective mass to the hard gluon.

2.2. Pressure and energy density

From the Lagrangian (16) we can derive the equations of motion:

$$m_G^2 \alpha_0^a = -g_h \rho^a \tag{17}$$

$$\left(i\delta_{ij} \gamma^\mu \partial_\mu + g_h \gamma^0 T_{ij}^a \alpha_0^a - m \right) \psi_j = 0 \tag{18}$$

where ρ^a is the temporal component of the color vector current given by:

$$j^{a\nu} = \bar{\psi}_i \gamma^\nu T_{ij}^a \psi_j \tag{19}$$

From the Lagrangian we can obtain the energy–momentum tensor and the energy density of the system through:

$$\varepsilon = \langle T_{00} \rangle \tag{20}$$

In the present case the energy–momentum tensor is given simply by:

$$T^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi_i)} (\partial_\nu \psi_i) - g^\mu{}_\nu \mathcal{L} \tag{21}$$

and consequently:

$$\varepsilon = \frac{\partial \mathcal{L}}{\partial(\partial_0 \psi_i)} (\partial_0 \psi_i) - g_{00} \mathcal{L} \tag{22}$$

which, with the use of (16) gives:

$$\varepsilon = i\bar{\psi}_i \gamma^0 (\partial_0 \psi_i) - g_{00} \left\{ -b\phi_0^4 + \frac{m_G^2}{2} \alpha_0^a \alpha_0^a + \bar{\psi}_i^q \left(i\delta_{ij} \gamma^\mu \partial_\mu + g_h \gamma^0 T_{ij}^a \alpha_0^a - \delta_{ij} m \right) \psi_j^q \right\} \tag{23}$$

Using (18) in the expression above we find

$$\varepsilon = b\phi_0^4 - \frac{m_G^2}{2} \alpha_0^a \alpha_0^a + i\bar{\psi}_i \gamma^0 (\partial_0 \psi_i) \tag{24}$$

Multiplying (18) by $\bar{\psi}_i$ from the left we find:

$$i\psi_i^\dagger (\partial_0 \psi_i) = \psi_i^\dagger \left(-i\vec{\alpha} \cdot \vec{\nabla} + \gamma^0 m \right) \psi_i - g_h \rho^a \alpha_0^a \tag{25}$$

From the usual Dirac theory applied to the study of nuclear matter we have [21]:

$$\psi_i^\dagger \left(-i\vec{\alpha} \cdot \vec{\nabla} + \gamma^0 m \right) \psi_i = 3 \frac{\gamma_Q}{2\pi^2} \int_0^{k_F} dk\, k^2 \sqrt{\vec{k}^2 + m^2} \tag{26}$$

In the last two expressions we have:

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \qquad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where $\vec{\sigma}$ are the standard 2×2 Pauli matrices, the unit entries in γ^0 are 2×2 unit matrices and γ_Q is the quark degeneracy factor $\gamma_Q = 2(\text{spin}) \times 3(\text{flavor})$. The sum over all the color states was already performed and resulted in the pre-factor 3 in the expression above. k_F is the Fermi momentum defined by the quark number density ρ:

$$\rho = \langle N | \psi_i^\dagger \psi_i | N \rangle = \frac{3}{V} \sum_{\vec{k},\lambda} \langle N | N \rangle = \frac{3}{V} \sum_{\vec{k},\lambda}$$

$$= 3 \frac{\gamma_Q}{(2\pi)^3} \int d^3k = 3 \frac{\gamma_Q}{2\pi^2} \int_0^{k_F} dk\, k^2$$

which gives:

$$\rho = \frac{\gamma_Q}{2\pi^2} k_F{}^3 \tag{27}$$

In the above expression $|N\rangle$ denotes a state with N quarks. Inserting (26) into (25) and then (25) into (24) we find:

$$\varepsilon = b\phi_0^4 - \frac{m_G^2}{2} \alpha_0^a \alpha_0^a - g_h \rho^a \alpha_0^a + 3 \frac{\gamma_Q}{2\pi^2} \int_0^{k_F} dk\, k^2 \sqrt{\vec{k}^2 + m^2} \tag{28}$$

Using (17) we can eliminate the field α_0^a in the above expression:

$$\varepsilon = b\phi_0^4 + \left(\frac{g_h^2}{2m_G{}^2} \right) \rho^a \rho^a + 3 \frac{\gamma_Q}{2\pi^2} \int_0^{k_F} dk\, k^2 \sqrt{\vec{k}^2 + m^2} \tag{29}$$

We can relate the color charge density ρ^a and the quark number density ρ. To do this we shall use the notation of [23] and write the quark spinor as $\psi_i = \psi c_i$, where c_i is a color vector. We have:

$$\begin{aligned} \rho^a \rho^a &= \left(\bar{\psi}_i \gamma^0 T_{ij}^a \psi_j \right) \left(\bar{\psi}_k \gamma^0 T_{kl}^a \psi_l \right) \\ &= \left(\psi_i^\dagger T_{ij}^a \psi_j \right) \left(\psi_k^\dagger T_{kl}^a \psi_l \right) \\ &= \left(c_i^\dagger T_{ij}^a c_j \right) \psi^\dagger \psi \left(c_k^\dagger T_{kl}^a c_l \right) \psi^\dagger \psi = 3\rho^2 \end{aligned} \tag{30}$$

where we used the relations $\psi^\dagger \psi = \rho$ and $(c_i^\dagger T_{ij}^a c_j)(c_k^\dagger T_{kl}^a c_l) = 3$. Performing the momentum integral we arrive at the final expression for the energy density:

$$\varepsilon = \left(\frac{3g_h^2}{2m_G^2}\right)\rho^2 + b\phi_0^4 + 3\frac{\gamma_Q}{2\pi^2}\left\{\frac{k_F^3\sqrt{k_F^2+m^2}}{4}\right\} + \frac{3m^2}{8}\frac{\gamma_Q}{2\pi^2}\left[k_F\sqrt{k_F^2+m^2} - m^2\ln\left(\frac{k_F+\sqrt{k_F^2+m^2}}{m}\right)\right] \tag{31}$$

The pressure is given by

$$p = \frac{1}{3}\langle T_{ii}\rangle \tag{32}$$

Repeating the same steps mentioned before and using:

$$\psi_i^\dagger(-i\vec{\alpha}\cdot\vec{\nabla})\psi_i = 3\frac{\gamma_Q}{(2\pi)^3}\int d^3k\left\{\frac{\vec{k}^2}{\sqrt{\vec{k}^2+m^2}}\right\} = 3\frac{\gamma_Q}{2\pi^2}\int_0^{k_F} dk\,k^2\left\{\frac{\vec{k}^2}{\sqrt{\vec{k}^2+m^2}}\right\} \tag{33}$$

we arrive at:

$$p = \frac{m_G^2}{2}\alpha_0^a\alpha_0^a - b\phi_0^4 + \frac{\gamma_Q}{2\pi^2}\int_0^{k_F} dk\,k^2\left\{\frac{\vec{k}^2}{\sqrt{\vec{k}^2+m^2}}\right\} \tag{34}$$

Performing the momentum integral, using (17) and the relation for ρ^a and the quark number density ρ in (34) we obtain the final expression for the pressure:

$$p = \left(\frac{3g_h^2}{2m_G^2}\right)\rho^2 - b\phi_0^4 + \frac{\gamma_Q}{2\pi^2}\left\{\frac{k_F^3\sqrt{k_F^2+m^2}}{4}\right\} - \frac{3m^2}{8}\frac{\gamma_Q}{2\pi^2}\left[k_F\sqrt{k_F^2+m^2} - m^2\ln\left(\frac{k_F+\sqrt{k_F^2+m^2}}{m}\right)\right] \tag{35}$$

The speed of sound c_s is given by:

$${c_s}^2 = \frac{\partial p}{\partial \varepsilon} \tag{36}$$

In the expressions above, g_h is small, since it comes always from the coupling between the hard gluons and the quarks.

Both (31) and (35) have three terms. The first term, proportional to ρ^2, comes from the purely hard gluonic term appearing in the Lagrangian and from the hard gluon term appearing in the quark equation of motion. The second term, proportional to ϕ_0^4, comes exclusively from the soft gluon terms and it has opposite signs in the energy and in the pressure. This is precisely the behavior of the bag constant term in the MIT bag model which has the same origin. The third term comes from the quarks. In short, we can say the both the energy density and the pressure are the sum of three contributions: the hard gluons, the soft gluons and the quarks.

3. Numerical results and discussion

We now compare our results (31), (35) and (36), with the corresponding results obtained with the MIT bag model for a gas of quarks at zero temperature [21,24]:

$$\varepsilon_0 = \left(\frac{9}{4}\right)\pi^{2/3}\rho_B^{4/3} + \mathcal{B} \tag{37}$$

and

$$p_0 = \frac{1}{3}\left(\frac{9}{4}\right)\pi^{2/3}\rho_B^{4/3} - \mathcal{B} \tag{38}$$

and

$${c_0}^2 = \frac{\partial p}{\partial \varepsilon} = \frac{1}{3} \tag{39}$$

We choose $\mathcal{B} = 110\ \text{MeV}\,\text{fm}^{-3}$, which lies in the range $(50 < B < 200\ \text{MeV}\,\text{fm}^{-3})$ used in calculations of stellar structure [25–27]. For the comparison we must rewrite (27), (31) and (35) as functions of the baryon density, which is $\rho_B = \frac{1}{3}\rho$.

If we neglect the gluonic terms and choose the quark mass m to be zero in (31) and (35) we can show that they coincide with (37) and (38) with $\mathcal{B} = 0$. In this limit, our model reduces to the MIT bag model.

We next consider the MIT bag model with finite $\mathcal{B}$ and our model with massless quarks and soft gluons but no hard gluons. This comparison is meaningful because with these ingredients both models describe the dynamics of free quarks under the influence of a soft gluon background. In this case we can identify our gluonic term with the gluonic component of the MIT bag model, represented by the bag constant. We then obtain an expression for the bag constant in terms of the gluon condensate:

$$\mathcal{B}_{QCD} = b\phi_0^4 = \left\langle \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu}\right\rangle \tag{40}$$

where in the last equality we have used (10) and (11). The above relation has been found in previous works, such as, for example, [9]. Fixing $\mathcal{B}$ and choosing a reasonable value of the coupling of the soft gluons, g_s, appearing in (10) we can infer the value of the dimension-four condensate, $\langle F^2\rangle$, in the deconfined phase. For $\mathcal{B}_{QCD} = \mathcal{B} = 110\ \text{MeV}\,\text{fm}^{-3}$ and $g_s = 2.7$ (which would correspond to $\alpha_{s\,(soft)} = g_s^2/4\pi = 0.6$) we find:

$$\langle F^2\rangle = \left\langle \frac{\alpha_s}{\pi}F^{a\mu\nu}F^a_{\mu\nu}\right\rangle = \frac{g_s^2}{\pi^2}\mathcal{B}_{QCD} = 0.0006\ \text{GeV}^4 \tag{41}$$

In the lack of knowledge of the in-medium dimension-two condensate, we use the factorization hypothesis, which, in the notation of Refs. [16] and [17], implies the choice $\mu_0^2 = g_s\phi_0^2$. As a consequence, (9), (10), (14) and (15) are related and we obtain:

$$\langle g_s^2A^2\rangle = -\sqrt{\frac{(4)(34)\pi^2}{9}\langle F^2\rangle} = -0.3\ \text{GeV}^2 \tag{42}$$

which corresponds to a dynamical mass of $m_G = 290$ MeV. This number is consistent with the values quoted in recent works [28–30], which lie in the range $200 < m_G < 600$ MeV. Finally, the numerical evaluation of (31), (35) and (36) requires the choice of g_h, the coupling of the hard gluons, and of the quark mass, m. We choose them to be $g_h = 0.35$ (corresponding to $\alpha_{s\,(hard)} = g_h^2/4\pi = 0.01$) and $m = 0.02$ GeV.

For Figs. 1–3 we use the values from (41) and (42). In Fig. 1 we show the energy density, pressure and speed of sound obtained with (31), (35) and (36) divided by the corresponding MIT values: ε_0, p_0 and c_0. We observe that, for this set of parameters, our EOS is harder than the MIT one. This can also be seen in the plot of the pressure as a function of the energy density, shown in Fig. 2. In the same range of baryon densities, we have more energy, much more pressure and consequently a larger speed of sound. This behavior can be attributed to the first term of Eqs. (31) and (35), which comes from the hard gluons. This term is exactly the same both in (31) and (35) and in the limit of high densities becomes dominant yielding $p \simeq \varepsilon$ and hence $c_s \to 1$. Physically, this term represents the perturbative corrections to the MIT approach. Since the quark

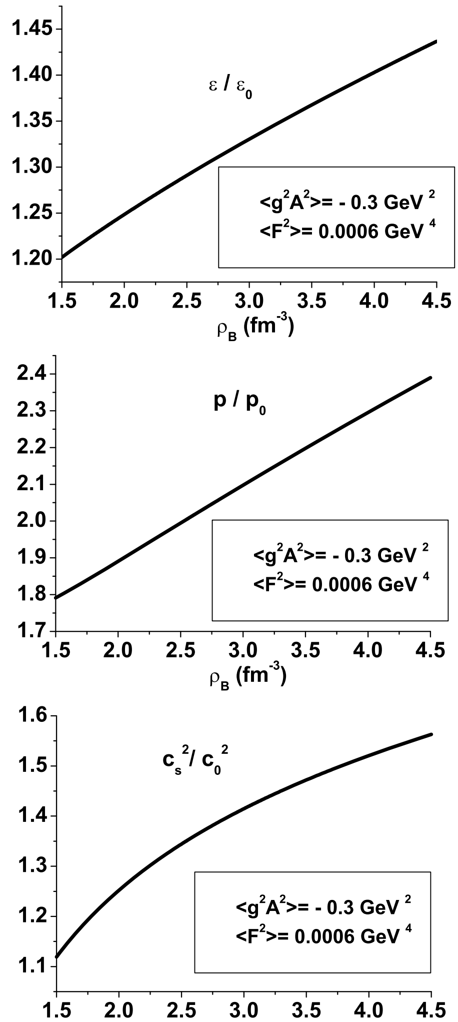

Fig. 1. Energy density, pressure and speed of sound, as functions of the baryon density, divided by the corresponding MIT values: ε_0, p_0 and c_0. The values for the condensates came from (41) and (42).

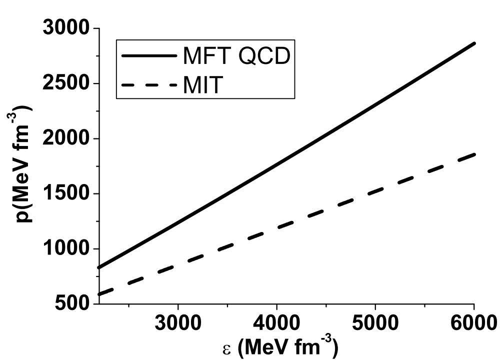

Fig. 2. Pressure as a function of the energy density.

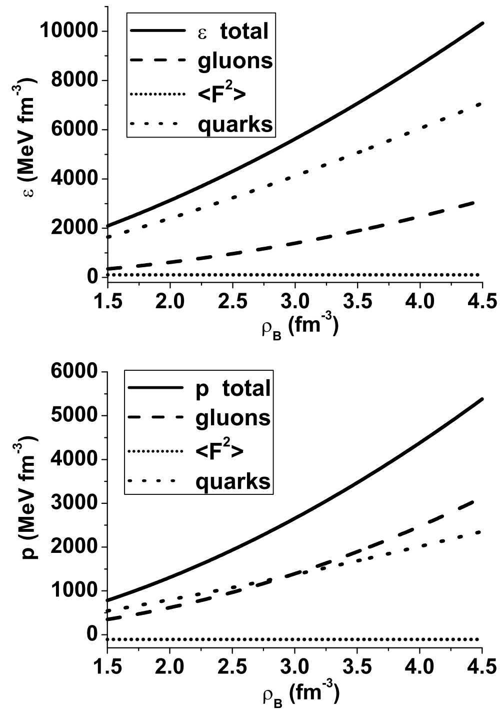

Fig. 3. Individual contributions to the energy density and to the pressure: hard gluons, quarks, soft gluons and the sum of the three components.

density is extremely large, even in the weak coupling regime (typical of the hard gluons) the field α_0^a is intense. A similar situation occurs in the color glass condensate (CGC). In that context, a proton (or nucleus) boosted to very high energies becomes the source of intense gluon fields generated in the weak coupling regime. Also in that case semi-classical methods were applied to study this gluonic system.

In Fig. 3 we plot the energy density (31) (upper panel) and the pressure (35) (lower panel) as a function of the baryon density ρ_B. We take ρ_B always starting at 1.5 fm^{-3}. We can observe that the quarks and hard gluons give the dominant contributions both to the energy and to the pressure. Looking at the pressure we see that the hard gluons give a repulsive contribution whereas the soft gluon contribution is attractive. It is interesting to see that our curves follow closely those of Refs. [26] and [27], computed with slightly different versions of the MIT bag model.

In Fig. 4 we show the EOS for different choices of the condensates, which are now treated as independent from each other. In the upper panel we fix $\langle F^2\rangle$ and vary $\langle g_s^2 A^2\rangle$, starting from the central value -0.3 GeV^2 (corresponds to a $m_G = 290$ MeV) and increasing its magnitude up to the lattice result -2.56 GeV^2 [31] (or $\langle g_s^2 A^2\rangle = -(1.6\ \text{GeV})^2$ which corresponds to $m_G = 848.5$ MeV). In the lower panel we perform the complementary study keeping $\langle g_s^2 A^2\rangle$ and increasing the magnitude of $\langle F^2\rangle$. As it can be seen, increasing the condensates reduces the pressure and, in the case of $\langle g_s^2 A^2\rangle$, softens the equation of state. This behavior could be anticipated from Eqs. (31), (35) and from equation of motion (17). Indeed, keeping fixed the coupling and the quark density, when we increase the gluon mass, the field becomes weaker. In a more accurate treatment, with the inclusion of spatial inhomogeneities, the equation of motion (17) would contain a Laplacian term and its solution would show a Yukawa behavior, with the mass m_G controlling the screening of the field α_0^a.

In Fig. 5 we show the energy per particle as a function of the baryon density for different values of the gluon condensates. As in the previous figure, in the upper panel we fix $\langle F^2\rangle$ and vary $\langle g_s^2 A^2\rangle$. Increasing $\langle g_s^2 A^2\rangle$ the energy per particle grows slower with baryon density. The system becomes more compressible. In the lower panel we keep $\langle g_s^2 A^2\rangle$ fixed and increase the magnitude of $\langle F^2\rangle$. Increasing $\langle F^2\rangle$ leads, as before, to a more compressible system but the total energy is now larger. For the central values of $\langle F^2\rangle$ and $\langle g_s^2 A^2\rangle$ we obtain values of ε/ρ_B which are compatible with those found in Ref. [32] for equivalent baryon densities. As it can be seen in all curves, the energy per particle is always much

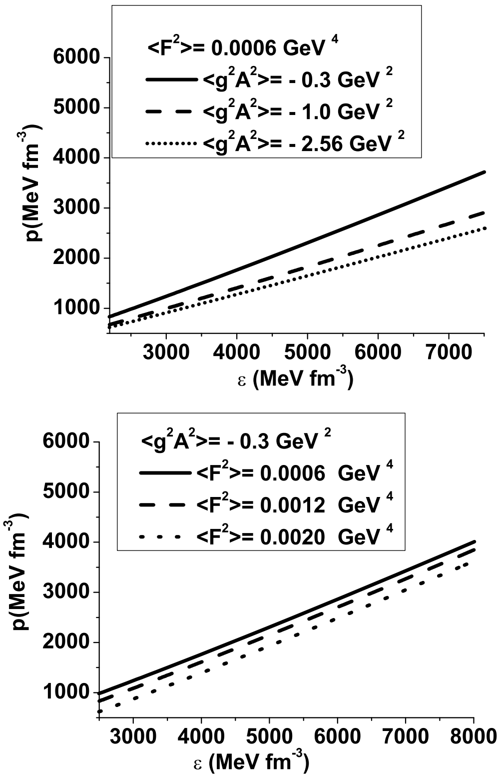

Fig. 4. EOS for different values of dimension-two and dimension-four gluon condensates.

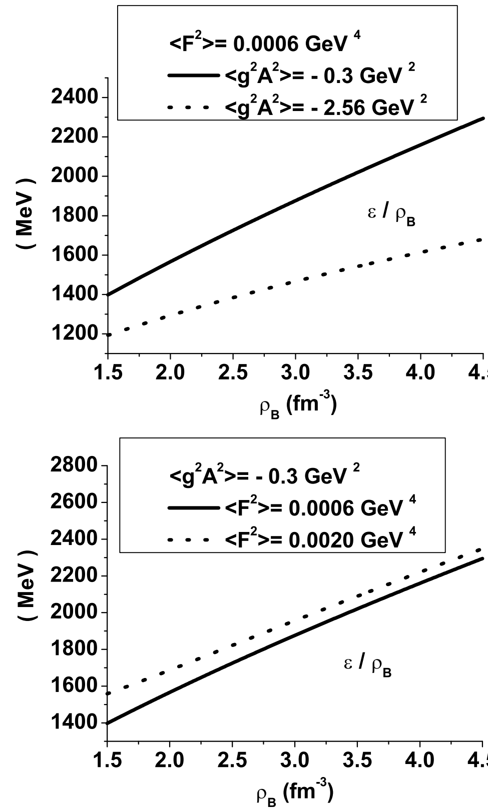

Fig. 5. Energy per particle as a function of the baryon density for different values of the gluon condensates.

larger than the nucleon mass (939 MeV) and hence the system under consideration can decay into nuclear matter.

Before concluding an important remark is in order. We started the derivation of the QGP equation of state from the QCD Lagrangian (1), which is gauge invariant. We ended up obtaining the energy density (31) and the pressure (35), which are physical observables and therefore should be also gauge invariant. However, along the way we made some approximations and gauge invariance is no longer manifest. It might even have been broken. We have neglected the derivatives of the gluon fields in (6) and (7). While from a physical point of view, this seems quite reasonable, since we are addressing an infinite and uniform system, from a more formal point of view, these approximations imply the loss of gauge invariance. We are currently investigating the quantitative effects of including the derivatives and the results will be reported elsewhere. The dimension-two condensate $\langle g^2 A^2 \rangle$ is gauge dependent, being most often defined in the Landau gauge. However this gauge dependence does not seem to be so severe. In fact, it has been conjectured [14] that this condensate is dominated by a gauge-invariant dimension-two contribution which is local only in the Landau gauge, being otherwise non-local.

To summarize, we have derived an equation of state for the cold QGP, which may be useful for calculations of stellar structure. The derivation is simple and based on three assumptions: (i) decomposition of the gluon field into soft and hard components; (ii) replacement of the soft gluon fields by their expectation values ("in-medium condensates") and (iii) replacement of the hard gluon fields by their mean-field (classical) values. Our EOS can be considered an improved version of the EOS of the MIT bag model, which contains both the non-perturbative effects coming from the residual gluon condensates and the perturbative effects coming from the hard gluons, which are enhanced by the high quark density. It is reassuring to observe that our EOS has the correct limits, where we recover the MIT bag model results. The parameters are the usual ones in QCD calculations: couplings, masses and condensates. The effect of the condensates is to soften the EOS whereas the hard gluons significantly increase the energy density and the pressure.

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