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A weathervane ship under wave and current action: an experimental verification of the wave drift damping formula

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Abstract

In this paper, the stability of the equilibrium of a tanker, free to rotate around a vertical axis while being displaced with constant velocity in following sea, is experimentally analyzed. The configuration emulates a turret system and the only bifurcation parameter is the articulation (turret) position. The post-critical behavior depends strongly on the wave-current interaction and the experiments were used not only to display this fact but also to provide a direct verification of the wave drift damping formula derived in [J Fluid Mech 275 (1994) 147; J Fluid Mech 313 (1996) 39]. Although restricted to following sea, the experimental setup was able to evaluate the adequacy of the formula in predicting the coupled action of the lateral drift force and steady yaw moment, the two force components that apparently have not been yet subjected to any experimental investigation. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Wave-current interaction is an important topic in oceanography and has received considerable attention in the literature. Among several relevant works one could select, by the boldness and elegance of the approach, the works by Bretherton and Garret [1], who extended to this field the classical idea of the 'adiabatic invariant' proving the conservation of the 'wave action', by Whitham [2], who used the variational principles of the Mechanics and geometric optics to obtain the pertinent equations in a Hamiltonian form.

In the specific field of the dynamics of floating bodies the importance of the wave-current interaction has been discovered experimentally by Wichers [3], who observed in a free oscillation of a ship that the damping increases in the presence of waves. For this reason, the influence of the current (or ship) velocity U on the second order steady wave forces has been called 'wave drift damping' in the specialized literature. The basic set of equations can be obtained from the classical ship motion theory by disregarding terms of order U^2, see Ref. [4], and the numerical integration of the resulting equations is a relatively difficult task, see Ref. [5]. Using some plausible assumptions and conservation of the wave action, Aranha [6,7] derived a simple formula expressing the steady wave forces influenced by the velocity U directly in terms of the standard drift forces coefficients. Furthermore, it has been shown, from the basic set of equations, that the proposed formula is exact within the context of the pertinent theory, although this conclusion has been questioned based on observed discrepancies with some numerical results.

Trassoudaine and Naciri [8], however, have recently analyzed Wichers original experiment, obtaining a very close agreement between the experimental results and the theoretical ones derived from the wave drift damping formula. The concordance between both results was observed in the frequency range $0 < KL < 15$, covering not only the resonant regime but also holding in a much wider frequency range than the one observed in a direct numerical computation, where the agreement with the formula is apparently restricted to the range $0 < KL < 5$ for a ship like structure, see Refs. [9,10]. More important than to display the expected numerical ill behavior as the frequency increases, the agreement with the wave drift damping formula is a strong indication of the correctness of Wichers explanation for the observed increase of the damping in the presence of waves. In high frequency ($KL > 15$) the difference between the formula and the

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experimental results is relatively large, indicating either a possible experimental inaccuracy or else that a different physical mechanism could be playing a role in this frequency range. The mismatch should not be due to the formula, which can be confirmed by an analytical result in the limit $\omega \rightarrow \infty$ as commented next. In fact, consider a half submerged sphere and a circular cylinder, both with radius a and being exposed to a wave with amplitude A. If $\tau = U\omega/g$, the drift force in high frequency is given, for both geometry, by the standard geometric optics result $D_x = 2/3\rho g A^2 a$ when $\tau = 0$. In the same frequency range, the wave drift damping formula predicts that the derivative $\mathrm{d}D_x/\mathrm{d}\tau$ at $\tau = 0$ is the same for both geometry and is given by $8/3\rho g A^2 a$. In an interesting (although rather critical) paper, Sierevogel and Hermans [11] determined numerically, from the geometric optics equations, the function $D_x(\tau)$ for a sphere and circular cylinder, obtaining two curves that not only coalesce in the limit when $\tau \rightarrow 0$ but also have the tangent $8/3\rho g A^2 a$. As a matter of fact, the geometric optics limit predicted by the wave drift damping formula has been apparently suggested by Bessho a long time ago, as quoted in Ref. [12].

This paper presents yet another experimental verification of the wave drift damping formula. The basic motivation was an actual problem in ocean engineering and the obtained results not only reaffirms the adequacy of the formula to predict the desired answer but also it indicates, strongly, the importance that the wave-current interaction may have in the design of a floating production system. This more practical problem is described below and it will be further elaborated in the main body of this paper.

A weathervane ship (turret) free to rotate around a vertical axis is being used by the oil industry as a floating production unit. The basic idea is to provide the ship with a mechanism that allows it to be aligned with the resultant of the environmental action, minimizing the loads on the mooring lines. Supposing initially that only the ocean current is present, the trivial equilibrium position, characterized by the yaw angle $\psi = 0$, is stable when the articulation (turret) is far ahead the midship; as the articulation approaches the midship section the equilibrium configuration bifurcates and the stable equilibrium position is then characterized by a non-zero value of the yaw angle ($\psi \neq 0°$).

The second order steady wave forces can stabilize or not the trivial equilibrium position, depending on the wave direction: head waves have a stabilizing effect while in following sea the destabilizing moment in the turret increases and, with it, the value of ψ at the equilibrium. A naive approach, where the wave-current interaction is ignored, would show, in some circumstances, a dramatic increase in the angle ψ. For example: if L is the ship length and the turret is placed $0.3L$ ahead the midship then $\psi \cong 5°$ when only a current with intensity $U \cong 1.9$ m/s is acting; on the other hand, if superposed to this current there is a following sea with amplitude $A \cong 2.7$ m and frequency KL $\cong 16$, the value predicted by the naive approach would be $\psi \cong 80°$. If, however, the standard drift forces are corrected by the effect of the wave-current interaction, in the way predicted by the formula proposed in Refs. [6,7], the actual value of the yaw angle would be much smaller, of the order $\psi \cong 20°$. These numbers give an idea about the practical importance of the wave-current interaction in analyzing the equilibrium of a turret system and, at the same time, it indicates a clear experimental setup to test the adequacy of the proposed formula to predict the wave-current interaction. Besides, this is apparently the first experimental work to check indirectly how the wave-current interaction affects the lateral drift force and the steady yaw moment, since Wichers decay test in surge is an indirect check only for the longitudinal drift force.

In Section 2 of this work the theoretical background is described and some numerical results, concerning the ship that was tested, are presented; in Section 3 the experimental setup is defined and the obtained results are presented and discussed. In Section 4 a discussion concerning the interaction between ocean current and irregular waves is briefly addressed.

2. Theoretical background

Suppose a ship with length L, beam B, draft T and block coefficient C_B with an articulation (turret) distant aL from the midship being towed with uniform velocity U along a wave tank; if ψ is the angle between the longitudinal ship axis and the towing direction, see Fig. 1, the moment in the turret due to the current can be expressed in the form

$$\hat{N}_C(\psi) = \frac{1}{2}\rho U^2 T L^2 [C_{6C}(\psi) - aC_{2C}(\psi)], \tag{1a}$$

where $C_{6C}(\psi)$ is the coefficient of the yaw moment and $C_{2C}(\psi)$ is the coefficient of the lateral force. Heuristic models, based on the low aspect wing theory, can be used

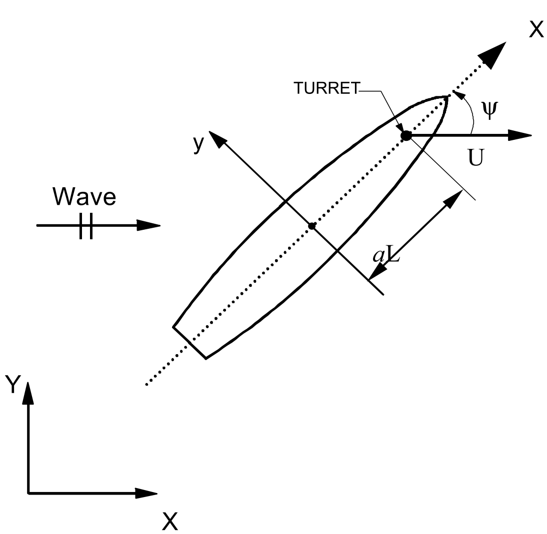

Fig. 1. Sketch of the model in the bifurcation experiment.

to express these coefficients in terms of the main ship dimensions, the adherence with experimental results being quite reasonable in general, see Ref. [13]; if C_Y is the lateral force coefficient for $\psi = 90°$ and $-l/L(C_Y - \pi T/2L)$ is the related yaw moment coefficient, $\{C_{2C}(\psi);\ C_{6C}(\psi)\}$ can be expressed as

$$C_{2C}(\psi) \cong \left(C_Y - \frac{\pi T}{2L}\right)\sin\psi|\sin\psi| + \frac{\pi T}{2L}\sin^3\psi + \frac{\pi T}{L}\left(1 + 0.4\frac{C_B B}{T}\right)\sin\psi|\cos\psi|, \tag{1b}$$

$$C_{6C}(\psi) \cong -\frac{l}{L}\left(C_Y - \frac{\pi T}{2L}\right)\sin\psi|\sin\psi| + \frac{\pi T}{L}\sin\psi\cos\psi - \left(\frac{1+|\cos\psi|}{2}\right)^2\frac{\pi T}{L}\left(\frac{1}{2} - 2.4\frac{T}{L}\right)\sin\psi|\cos\psi|.$$

As it will be discussed in the following section, Eqs. (1a) and (1b) provide a reasonably accurate prediction for the yaw angle ψ at equilibrium when only the current is present, although a small adjustment was necessary in one of the cases analyzed. Also, the hydrodynamic coefficients $\{C_Y;\ l/L\}$ can be approximated from some known sectional results, as discussed in Ref. [13].

Suppose now that a wave with frequency unit amplitude, frequency ω and celerity $c = g/\omega$ is incident in a direction that makes an angle β with the towing direction. As it is known, the existence of the current introduces a Doppler shift in the frequency, the 'frequency of encounter' ω_e being

$$\omega_e = \left(1 - \frac{U}{c}\cos\beta\right)\omega. \tag{2a}$$

The incident wave is also refracted by the current (aberration effect), the new incidence direction being given by β_1 with

$$\beta_1 = \beta + 2\frac{U}{c}\sin\beta. \tag{2b}$$

To make shorter the notation, the two components of the steady wave forces in the horizontal plane and the steady yaw moment may be grouped into a single generalized steady wave force vector $\mathbf{D}(\omega,\beta)$. If now $\mathbf{D}_0(\omega,\beta)$ is the generalized steady wave force vector in the standard problem, where the current velocity is zero ($U = 0$), and $\mathbf{D}_U(\omega,\beta)$ is the related force vector for $U \neq 0$, where the effect of the wave-current interaction is incorporated, the following formula relates these two vectors (see Ref. [7]):

$$\mathbf{D}_U(\omega,\beta) = \left(1 - 4\frac{U}{c}\cos\beta\right)\cdot\mathbf{D}_0(\omega_e,\beta_1). \tag{2c}$$

In following sea $\beta = 0°$ and the incidence angle with respect to the ship axis is ψ. In the naive approach, where the wave-current interaction is ignored, the second order wave moment in the turret would be given by (A: wave amplitude)

$$\hat{N}_W^{(WO)}(\omega,\psi) = \frac{1}{2}\rho g A^2 L^2\left[N_Z(\omega,\psi) - aD_y(\omega,\psi)\right], \tag{3a}$$

where $N_Z(\omega,\psi)$ is the standard steady yaw moment normalized by $1/2\rho g A^2 L^2$ and $D_y(\omega,\psi)$ is the lateral drift force normalized by $1/2\rho g A^2 L$. The correct value of this moment, incorporating the wave-current interaction, is given by (see Eqs. (2a)–(2c) with $\beta = 0°$)

$$\hat{N}_W(\omega,\psi) = \frac{1}{2}\rho g A^2 L^2\left(1 - 4\frac{U}{c}\right)\left[N_Z(\omega_e,\psi) - aD_y(\omega_e,\psi)\right], \tag{3b}$$

$$\omega_e = \left(1 - \frac{U}{c}\right)\omega; \qquad c = g/\omega.$$

The total moment in the turret is the sum of the moment due to the current, given in Eqs. (1a) and (1b), with the moment due to the wave, given in Eqs. (3a) and (3b); normalizing this value by $1/2\rho U^2 T L^2$ one obtains

$$N^{(WO)}(\omega,\psi) = \left[C_{6C}(\psi) - aC_{2C}(\psi)\right] + \frac{gT}{U^2}\left(\frac{A}{T}\right)^2\left[N_Z(\omega,\psi) - aD_y(\omega,\psi)\right], \tag{4a}$$

If the wave-current interaction is ignored; for a given ship the wave effect is invariant if both the frequency and A/U are kept constant. On the other hand, if the wave-current interaction is accounted for by means of the formula (2a)–(2c), the following expression can be derived:

$$N(\omega,\psi) = \left[C_{6C}(\psi) - aC_{2C}(\psi)\right] + \frac{gT}{U^2}\left(\frac{A}{T}\right)^2\left(1 - 4\frac{U}{c}\right) \times\left[N_Z(\omega_e,\psi) - aD_y(\omega_e,\psi)\right]. \tag{4b}$$

Notice that, besides gT/U^2 and $(A/T)^2$, the moment depends also on U/c, the wave-current interaction parameter. Using Eq. (1b) for $\{C_{2C}(\psi);\ C_{6C}(\psi)\}$ and computing the steady force coefficients $\{N_Z(\omega,\psi);\ D_y(\omega,\psi)\}$ by means of a standard linear frequency domain program one can determine, for a wave with amplitude A and frequency ω, the function $N(\omega,\psi)$; the stable equilibrium position ψ_E is defined by the following conditions:

$$N(\omega,\psi_E) = 0, \tag{4c}$$

$$\left(\frac{\partial N}{\partial\psi}\right)_{\psi=\psi_E} < 0.$$

The experimental determination of the equilibrium angle ψ_E was conducted at the IPT wave tank using a model of a tanker with main dimensions defined in Table 1. In the last two columns of this table the hydrodynamic coefficients $\{C_Y;\ l/L\}$, introduced in Eq. (1b) and related to a current incident in the direction $\psi = 90°$, are also given for this ship.

Table 1
Dimensions of the VLCC in meters and hydrodynamic coefficients for lateral current

Ship	Lab.	C_B	L	B	T	C_Y	l/L
VLCC	IPT	0.832	320.0	54.5	21.6	0.84	5.9%

These coefficients were determined experimentally in a former work, see Ref. [13].

A linear frequency domain code (WAMIT) was employed to determine numerically, from the known far field expressions, the lateral drift force and the steady yaw moment in the tanker. The ship was discretised into 1832 panels, the convergence of the results being tested from three different discretisations, and the forces were determined for waves with frequencies $\omega_k = k(0.05)$ rad/s $k = 1,\ldots,16$ and incidence angles (degrees) $\beta_k = k(10°)$ $k = 0,1,\ldots,18$. For different values of the frequency and incidence angles, within the range $0 \le \omega \le 0.80$ rad/s ($0 \le \mathrm{KT} \le 1.4$) and $0° \le \beta \le 180°$, the steady forces were determined by linear interpolation.

In Fig. 2 the normalized lateral drift coefficient and yaw moment for this ship are presented as functions of the wave frequency and for three different values of the incidence angle ψ. With the numerical results and Eq. (1b) the stable equilibrium angle ψ_E was determined from Eq. (4a), without the wave-current interaction, and from Eq. (4b), when this effect is taken into account by means of the wave drift damping formula.

3. Experimental results

The wave tank at IPT is 280 m long, 6.60 m wide and 4 m deep, with a wavemaker in one end and a beach in the other one. The VLCCs model has a geometric scale in the proportion 1:90 to the full scale and it was towed with controlled velocity by an instrumented car. An articulation was placed at the turret position, leaving the model free to rotate in yaw while preserving the freedom of motion in the vertical plane (heave, roll and pitch). The displacements in the horizontal plane were restricted by three coils, with stiffness 5.82 N/m each (model scale), anchored in the car and in a spatially fixed vertical cylinder ending in the articulation; one of the coils was in the longitudinal tank direction and the two others were disposed symmetrically in relation to the tank axis making an angle of 120° with it. The maximum stiffness R of the anchoring system was in the longitudinal tank direction, with $R = 71$ kN/m in full scale, leading to a maximum natural frequency of the order $K_N T \cong (RT/\rho g\Delta) = 0.0005$, T being

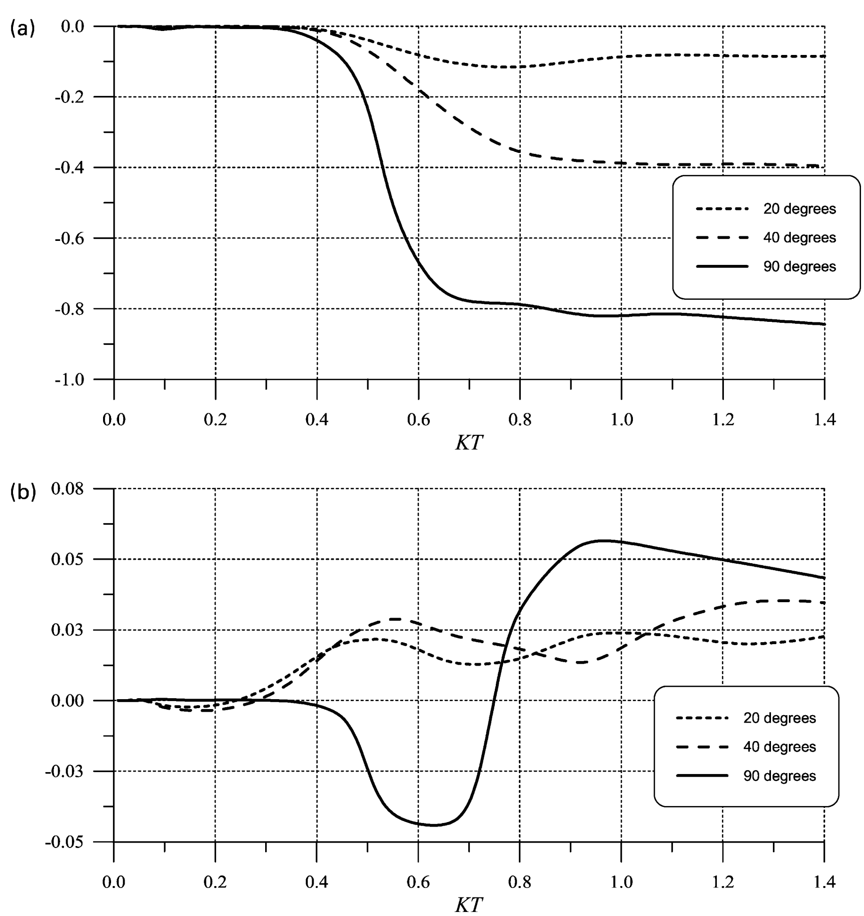

Fig. 2. (a) Lateral drift force of the VLCC normalized by $1/2\rho gA^2L$ (WAMIT). (b) Steady yaw moment of the VLCC normalized by $1/2\rho gA^2L^2$ (WAMIT).

Table 2
Correction from the actual to the nominal values of A and U. $\psi_{(4b)}$: computed value using the actual values of A and U; Ψ_{EXP}: measured value; $\Delta\psi = \psi_{EXP} - \psi_{(4b)}$; $\psi_{(4b),N}$: computed value using the nominal values $A = 2.7$ m, $U = 1.90$m/s; $\psi_{EXP,N} = \psi_{(4b),N} + \Delta\psi$. Turret position: 0.3$L$

KT	A (m)	U (m/s)	$\psi_{(4b)}$ (degrees)	ψ_{EXP} (degrees)	$\Delta\psi$ (degrees)	$\psi_{(4b),N}$ (degrees)	$\psi_{EXP,N}$ (degrees)
0.2	2.50	1.91	4.5	5.7	1.2	4.4	5.6
0.4	2.85	1.91	11.2	7.2	−4.0	10.6	6.6
0.6	2.56	1.85	15.0	14.9	−0.1	15.2	15.1
0.8	2.81	1.95	17.3	17.5	0.2	18.4	18.6
1.0	2.63	1.85	19.2	20.5	1.3	18.4	19.7

the ship draft; in the range KT ≥ 0.2, where the tests were made, the anchoring system can be ignored for the linear wave motion in the horizontal plane, as it is usual in seakeeping problems.

The wave period was gauged by a wave probe, the instrumented car velocity (towing velocity) by an odometer and the yaw angle by a potentiometer. Roller bearings were placed between the model's structure and the articulation to minimize the influence of the rotational friction but several other factors, including imperfections of the model, can interfere with the experimental error. By comparing the equilibrium angle in repeated experiments and also the value of the angle when the model bifurcates to one side or to the other, the data obtained in Ref. [13] allows one to estimate the global error as being of the order ±3°.

Two turret positions were analyzed, one distant 0.3L ($a = 0.3$) from the midship and the other 0.2L ($a = 0.3$) and five different wave frequencies were tested, trying to cover a relatively wide spectrum from KT = 0.2 (KL ≅ 3.2, $2\pi/\omega \cong 20$ s) to KT = 1 (KL ≅ 16, $2\pi/\omega \cong 8.9$ s) In all experiments, the intention was to keep the same wave amplitude $A = 2.70$ m ($A = 3.00$ cm in the model scale) and the same towing velocity $U = 1.90$ m/s ($U = 0.20$ m/s in the model scale). Waves much higher than this one could not be maintained truly harmonic in the high frequency range (KT ≈ 1) and towing velocities smaller than 0.2 m/s in the model scale could not be controlled with confidence; on the other hand, the wave effect, proportional to $(A/U)^2$, would be small if either A is relatively smaller than 3.0 cm or U is greater than 0.2 m/s in the model scale. Although the basic motivation was to select values of A and U that would enhance the wave effect in the bifurcation phenomenon, it is worth to mention that the final result of this study was very much consistent with the results obtained for the same tanker exposed to the centenary wave/decennary current condition in Campos Basin, as discussed in Section 4.

In each run both the wave amplitude and the towing velocity were strictly constant but it was difficult to set them exactly equal to the nominal values $A = 2.70$ m and $U = 1.90$ m/s. However, in order to display the equilibrium angle ψ_E as a function of KT, the parameter A/U should be kept constant and the following procedure was used with this purpose: at the actual values of A and U the theoretical value of ψ_E, obtained from Eq. (4b), was compared with the actual measured value and the difference $\Delta\psi$ was determined; the theoretical value $\psi_{E,N}$ was then computed from Eq. (4b) with the nominal values $A = 2.70$ m and $U = 1.90$ m/s and the nominal value of the experimental result was then defined by the equality $\psi_{EXP,N} = \psi_{E,N} + \Delta\psi$. Tables 2 and 3 synthesizes this computation for the results related to the turret placed at 0.3L and 0.2L.

Besides the five frequencies analyzed, the bifurcation experiment was also run only in the presence of the current, corresponding to the point KT = 0 (no waves). As observed in Ref. [13], the heuristic model predicts the critical turret position (namely, the value of a below which the trivial equilibrium position becomes unstable) with an error of order 10%; specifically, $(a_{CR})_{EXP} \cong 0.39$ while $(a_{CR})_{TH} = 0.36$. For the case where the turret is at 0.3L from the midship this imprecision is not relevant since then the turret is near the critical position and the moment due to the current is small; as a consequence, there is a very close adherence between Eq. (4b) and the experiments when KT = 0 and $a = 0.3$. In the case where the turret is at 0.2L from the midship the imprecision in the heuristic model is more relevant: for $a = 0.2$ the theoretical value

Table 3
Correction from the actual to the nominal values of A and U. $\psi_{(4b)}$: computed value using the actual values of A and U; Ψ_{EXP}: measured value; $\Delta\psi = \psi_{EXP} - \psi_{(4b)}$; $\psi_{(4b),N}$: computed value using the nominal values $A = 2.7$m, $U = 1.90$m/s; $\psi_{EXP,N} = \psi_{(4b),N} + \Delta\psi$. Turret position: 0.2$L$

KT	A (m)	U (m/s)	$\psi_{(4b)}$ (degrees)	ψ_{EXP} (degrees)	$\Delta\psi$ (degrees)	$\psi_{(4b),N}$ (degrees)	$\psi_{EXP,N}$ (degrees)
0.196	2.50	1.96	20.1	23.4	3.3	20.0	23.3
0.397	2.84	1.94	27.2	26.3	−0.9	26.6	25.7
0.605	2.65	1.99	32.2	30.8	−1.4	32.2	30.8
0.799	2.75	1.94	36.2	31.6	−4.6	36.7	32.1
1.000	2.64	1.87	39.3	38.0	−1.3	38.7	37.4

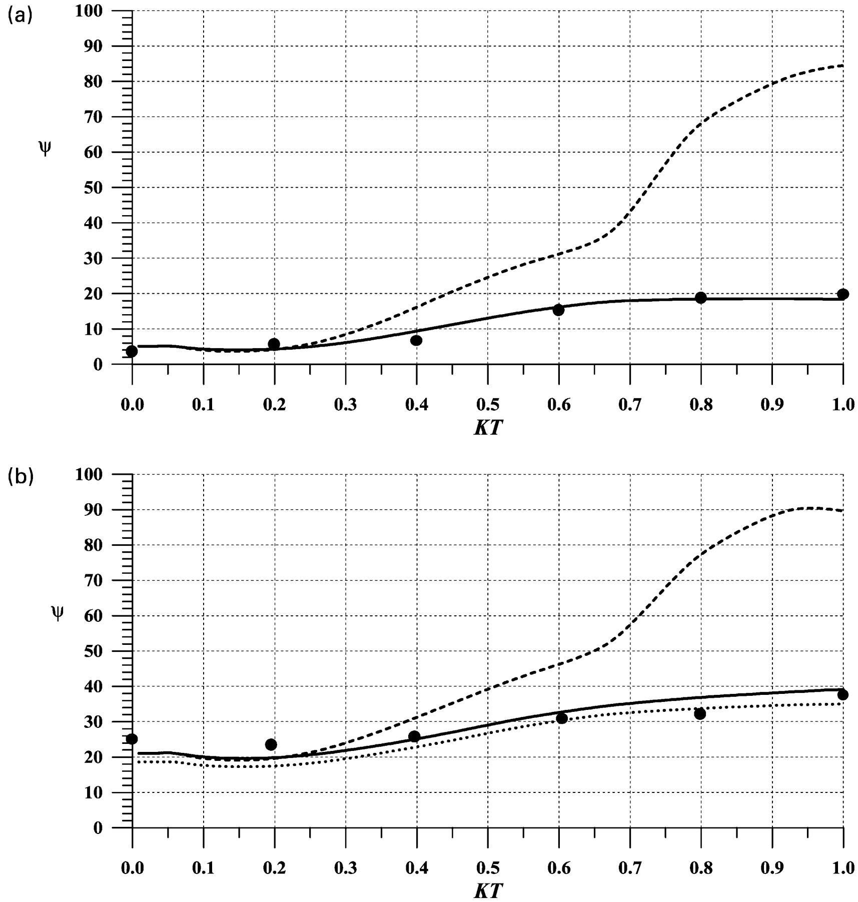

Fig. 3. (a) Yaw angle at the stable equilibrium as function of the wave frequency KT. Turret at $0.3L$, $A = 2.70$ m, $U = 1.90$ m/s. Experiments: (●); theory Eq. (4b): (—); theory w/o wave-current interaction, see Eq. (4a): (- - -). (b) Yaw angle at the stable equilibrium as function of the wave frequency KT. Turret at $0.2L$, $A = 2.70$m, $U = 1.90$ m/s. Experiments: (●); theory Eq. (4b): (—); theory w/o wave-current interaction, see Eq. (4a): (- - -); theory Eq. (4b) w/o ad hoc correction in Eq. (1a) (⋯).

at $\mathrm{KT} = 0$ is now $\psi_{(4b)} = 18°$ while the experimental value is $\psi_{\mathrm{EXP}} = 25°$ (this experimental point, however, was obtained in a single trial and it has a confidence a bit smaller than the others, obtained in repeated trials).

If the value $(a_{\mathrm{CR}})_{\mathrm{EXP}} \cong 0.39$ is imposed in the heuristic model in the way indicated in Ref. [13], the theoretical value of ψ for $a = 0.2$ would be $\psi = 21°$, closer to the experimental value than the original one. For this reason, an ad hoc correction in the heuristic model was introduced in this case: instead to use $a = 0.2$ in Eq. (1a) the value $a_1 = 0.187$ was employed in this expression (and only there) to enforce the result $\psi = 21°$ when $\mathrm{KT} = 0$. Furthermore, a similar correction as the one shown in Table 2 was also introduced in this case, see Table 3.

Figs. 3a,b display the final result for both positions of the turret. In the case where the turret is at $0.3L$ (Fig. 3a) the agreement is very good, as already seen in Table 2; also, by comparing the results obtained from Eq. (4a), where the wave-current interaction is ignored, with the ones obtained from Eq. (4b), where the wave drift damping formula is used, one can directly assess the practical relevance of the wave-current interaction. In the case where the turret is at $0.2L$ (Fig. 3b) the agreement is again very good although less impressive than in the former case. For the sake of completeness, the original curve Eq. (4b), without the above-mentioned ad hoc correction in the heuristic model, is also plotted, showing that the prediction based on the original heuristic model Eqs. (1a) and (1b) is reliable from a more practical point of view even when $a = 0.2$. In both cases ($a = 0.3$; 0.2) the difference $\Delta\psi$ is in general below the estimated experimental error, of order $\pm 3°$.

Two known effects were not considered in the theoretical analysis since they were supposedly small within the scope of this study: first, the forces caused by the Kelvin's waves generated by the body in uniform translation; second, a Munk's like moment related with the interaction between the second order steady wave flow and the uniform velocity, see Ref. [5]. The agreement between theory and experiment shows that both effects are in fact of secondary importance in the context of the present work.

4. Wave-current interaction for irregular waves

Suppose here an ocean current defined by the velocity

$\mathbf{U} = -U\mathbf{i}$ in the vicinity of the floating body and an irregular wave being propagated in a direction that makes an angle β with the $\mathbf{i}$-direction. Let also $S(\omega)$ be the wave spectrum measured in a reference system where the medium is stationary. In a practical situation $S(\omega)$ may be identified with the wave spectrum existing before the presence of the current or else, in the reference system moving with the current, with the spectrum generated by the wavemaker in the sketch of Fig. 1. For a given regular wave with frequency ω and unit amplitude, let $\mathbf{D}_0(\omega,\beta)$ be the standard $(U = 0)$ generalized second order steady wave force vector in the horizontal plane; the second order steady wave force in the body, excited by the irregular wave and influenced by the ocean current, is then given by (see Eqs. (2a)–(2c))

$$\hat{\mathrm{D}}_U(\beta) = 2\int_0^\infty \left(1 - 4\frac{U\omega}{g}\right) S(\omega)\cdot\mathbf{D}_0(\omega_e, \beta_1)\,\mathrm{d}\omega, \tag{5a}$$

$$\beta_1 = \beta + 2\frac{U\omega}{g}\sin\beta; \qquad \omega_e = \omega\left(1 - \frac{U\omega}{g}\cos\beta\right).$$

Suppose now that $S_U(\omega_e)$ is the *actual* wave spectrum measured in vicinity of the body in the presence of the ocean current; from energy conservation $S_U(\omega_e)\,\mathrm{d}\omega_e = S(\omega)\,\mathrm{d}\omega$, see Refs. [14,15], and disregarding again terms of order U^2 one then obtains

$$\hat{\mathrm{D}}_U(\beta) = 2\int_0^\infty \left(1 - 4\frac{U\omega_e}{g}\right) S_U(\omega_e)\cdot\mathbf{D}_0(\omega_e, \beta_1)\,\mathrm{d}\omega_e, \tag{5b}$$

$$\beta_1 = \beta + 2\frac{U\omega_e}{g}\sin\beta.$$

In the *design analysis* of an ocean system both the wave spectrum and the ocean current are, in general, defined independently and then, in this case, Eq. (5a) should be used; in the other hand, if the actual spectrum $S_U(\omega_e)$ is known, either by a direct measurement of the wave elevation or else by the use of the refraction theory when the current field is given in a geographical scale, then Eq. (5b) must be used instead.

As a last remark it should be observed that the relevance of the wave-current interaction for a turret configuration is not restricted to the academic conditions analyzed in the present work. In Campos Basin, for example, the major ocean current goes south (current of Brazil), with estimated deccenary value of 1.58 m/s, and the centenary wave goes north, with significant wave height of 7.6 m and zero up-crossing period of 9.2 s. If ψ is the yaw angle with respect to the north–south line and the sea state is represented by the Pierson–Moskowitz spectrum, see Eq. (5a), the following values are theoretically predicted for the same VLCC analyzed in this paper, with a turret placed at $0.2L$: $\psi = 74.1°$ when the wave-current interaction is ignored while $\psi = 28.8°$ when this effect is incorporated in the analysis, a discrepancy similar to the ones observed before in this paper. The experimental yaw angle determined at IPT wave tank is $\psi = 28.0°$, in a very close agreement with the theoretical value when the wave-current interaction is accounted for.

5. Conclusions

The wave drift damping formula (2c), obtained in Refs. [6,7], is exact in the context of the pertinent theory, where the flow is assumed potential and terms of order U^2 are disregarded (notice that some algebraic mistakes in the demonstration given in Ref. [7] have been already corrected; see Ref. [16]). In the high frequency limit $(\omega \to \infty)$ and for low velocity $(U\omega/g < 1)$, expression (2c) predicts that $\mathbf{D}_U(\infty, \pi) = (1 + 4U\omega/g)\cdot\mathbf{D}_0(\infty, \pi)$, a result apparently obtained a long time ago by Bessho, see Ref. [12], and recovered by Sierevogel and Hermans [11] in their numerical computation of the ray theory for a half immersed sphere and a circular cylinder.

Besides this analytical background, two experimental results, obtained in distinct contexts and in different laboratories, reaffirm not only the adequacy of the wave drift damping formula to describe the wave-current interaction but, even more important than this, the adequacy of the basic model behind this formula (namely, conservation of the wave action in the far field) to describe this complex interaction. The first of these experiments, done at MARIN and analyzed by Trassoudaine and Naciri [8], is the original work by Wichers dealing with the free decay test of a large tanker; the second one, presented in this work, deals with the equilibrium configuration of a tanker with a turret. In these two cases, and in spite of the essential difference in the experimental setup, the agreement between the measurements and the results predicted from the wave drift damping formula was very good in the whole range of frequencies $0 < \mathrm{KL} < 15$, that includes the resonant regime of the tanker. The alleged discrepancy, for a ship like structure, between the direct numerical computation and the wave drift damping formula in the frequency range $\mathrm{KL} > 5$, as obtained by Finne and Grue and Malenica [9,10], points not to an inadequacy of the formula but to an expected numerical ill behavior in high frequency.

From a more practical point of view, the results obtained in the present work show the importance that the wave-current interaction may have in the design of a floating production system. It not only influences markedly, as seen in this work, the equilibrium position of a turret system but also, as analyzed in Ref. [15], it may have an important impact in the slow drift phenomenon, once it affects, sometimes in a strong way, the spectrum of the low frequency exciting forces. In this perspective, it is a happy circumstance that some practically important nonlinear phenomenon can be well described by a simple formula, that depends exclusively on hydrodynamic coefficients determined from a linear frequency domain model.

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