PHYSICAL REVIEW D **86,** 016005 (2012)

Looking for meson molecules in B decays

I. Bediaga
Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro, RJ, Brazil

F. S. Navarra and M. Nielsen
Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, SP, Brazil
(Received 7 June 2012; published 13 July 2012)

We discuss the possibility of observing a loosely bound molecular state in a three-body hadronic B decay. In particular, we use the QCD sum rule approach to study a $\eta' - \pi$ molecular current. We consider an isovector-scalar $I^G J^{PC} = 1^- 0^{++}$ molecular current, and we use two- and three-point functions to study the mass and decay width of such a state. We consider the contributions of condensates up to dimension six, and we work at leading order in α_s. We obtain a mass around 1.1 GeV, consistent with a loosely bound state, and a $\eta' - \pi \to K^+K^-$ decay width around 10 MeV.

DOI: 10.1103/PhysRevD.86.016005 PACS numbers: 11.55.Hx, 12.38.Lg, 12.39.−x

I. INTRODUCTION

One of the outstanding open questions in hadron physics is as follows: are there meson-meson bound states? The same mechanism of meson exchange that binds the deuteron could also, in principle, bind two mesons. The interest in this subject was renewed by the discovery of the new charmonium states. Since their first appearance, some of them were considered to be meson molecules. These states have already been discussed in some reviews [1–3].

In this paper we discuss how to look for molecules at the LHCb, taking advantage of the unprecedented high statistics.

We can look for meson molecules in three-body hadronic B decays. Because the phase space is large, we can even try to directly use the Dalitz diagram, which extends up to large values of the variables s_{12} and s_{23}. All the known normal quark-antiquark intermediate resonant states leave an imprint in the Dalitz plot, which is directly related to the quantum numbers of the states and lead to the identification of the state. Examples are the following: a continuous straight line, in the case of scalar states; a line with a hole, in the case of vector states; or a line with two holes, in the case of tensor states.

A sketch of a Dalitz plot for a three-body meson decay is shown in Fig. 1, where for each invariant parameter s_{12} or $s_{23,}$ we show the relative momentum of each one of the two particles 1, 2 or 2, 3. Let us consider the case that particles 1 and 2 are pions coming from the ρ meson decay. Of course, this decay should produce a line parallel to the s_{23} axis at the point $s_{12} = m_\rho^2$. However, since the pions coming from the ρ meson decay must have one unit of angular momentum, they cannot both go in the same direction. Therefore, no pions could be seen in the region where the relative momentum between them is small. From Fig. 1, one can see that this region is just in the middle of the line parallel to the s_{23} axis. Therefore, a line characterizing a vector resonance state must have a hole in the middle, as mentioned above. Now imagine that the resonant state is a loosely bound molecular state of the particles 1, 2. A loosely bound molecular state can only exist when the relative momentum between the two mesons in the molecule is small. In this case, one has exactly the opposite situation of the one discussed before: there will be no signal in the Dalitz plot unless the two mesons in the molecular state go in the same direction. In other words, one can expect a small line parallel to the s_{23} axis in the middle of the Dalitz plot, approximately in the region where there is a hole in the line characterizing a vector resonance.

The final particles observed in three-body B decays are pions and kaons. Therefore, to observe a molecular state in the Dalitz plot for a three-body B decay, this molecular state must decay into pions and/or kaons. Let us consider a $\eta' - \pi$ loosely bound molecule with the quark content $\bar{u}u\bar{s}s$. This resonant state, hereafter called R, is especially interesting because its mass m_R should be approximately given by

$$m_R \sim m_{\eta'} + m_\pi = 958 \text{ MeV} + 138 \text{ MeV} = 1096 \text{ MeV} \tag{1}$$

and, therefore, it is quite visible in the B decay Dalitz plot. Since for an S wave this molecule has $I^G J^{PC} = 1^- 0^{++}$, it cannot decay into $\pi^+\pi^-$, but it will decay into K^+K^-. In particular, there are already data for $B^- \to K^+K^-K^-$, $B^- \to K^+K^-\pi^-$, $B^- \to \pi^+\pi^-\pi^-$, and $B^- \to \pi^+\pi^-K^-$ [4]. The decay could go through the resonant state R only in the first two of these cases, as illustrated in Figs. 2 and 3. For these cases, a small line with $\sqrt{s_{12}} \sim 1.1$ GeV parallel to the s_{23} axis should be seen in the Dalitz plot in the region where the two particles η' and π have a small relative momentum. This signal should be very different from all other established resonant states decaying into K^+K^-, like the a_0 for instance, and should only be seen in the channels $B^- \to K^+K^-K^-$ and $B^- \to K^+K^-\pi^-$. Of course, the figure is very qualitative, and it is not possible

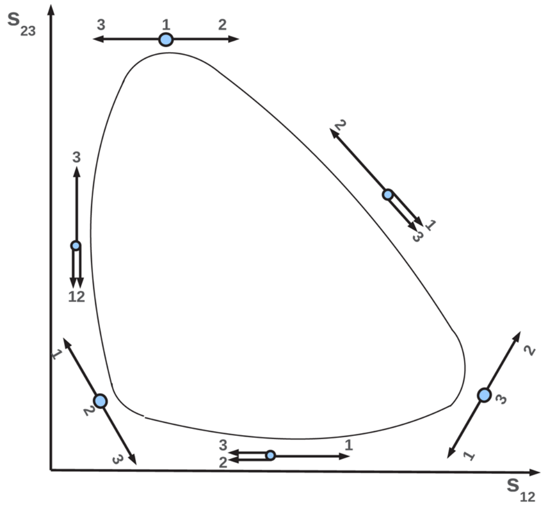

FIG. 1 (color online). Dalitz plot of a three-body B decay. The small drawings illustrate the different kinematical configurations.

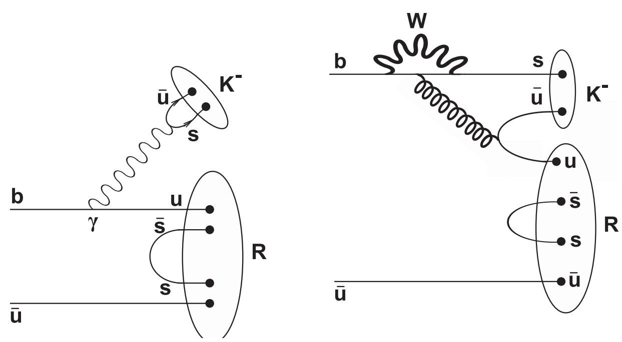

FIG. 2. The two relevant diagrams for $B^- \to K^+K^-K^-$ decay, through the resonance R.

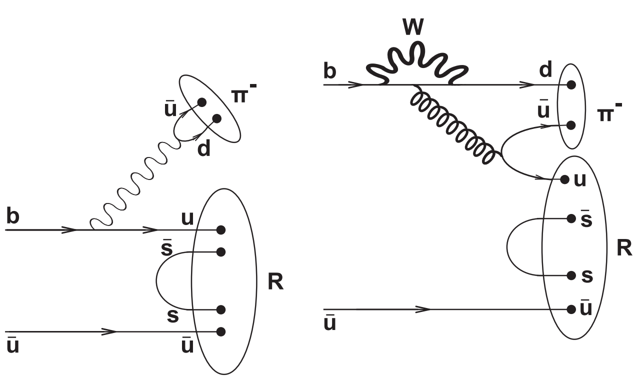

FIG. 3. The two relevant diagrams for $B^- \to K^+K^-\pi^-$ decay, through the resonance R.

to say how large the line segment around the indicated position is. However, the observation of this structure in the Dalitz plot of the two mentioned B decays, and not in the others, would provide strong evidence for the formation of this molecular state. The observation of a line that only appears in a certain part of the s_{23} axis with a fixed value of s_{12}, and only for the decays $B^- \to K^+K^-K^-$ and $B^- \to K^+K^-\pi^-$, could be interpreted as the existence of a weakly bound molecular state.

II. THE $\eta' - \pi$ SCALAR MOLECULE

A. Mass

In a previous work we have investigated the possibility [5] that the light scalar states could be interpreted as tetraquark states. Here, we perform a complementary investigation to understand a possible $\eta'\pi$ meson molecular state in the QCD sum rule (QCDSR) framework [6–8].

The QCDSR approach is based on the correlator of hadronic currents. A generic two-point correlation function is given by

$$\Pi(q) = i\int d^4x e^{iq.x}\langle 0|T[j(x)j^\dagger(0)]|0\rangle, \tag{2}$$

where the local current $j(x)$ contains all the information about the hadron of interest, like quantum numbers, quark content, and so on. A molecular current can be constructed from the mesonic currents that describes the two mesons in the molecule. In the case of the scalar $\eta' - \pi$ state, a possible current is

$$j = \left(\frac{\bar{u}_i\gamma_5 u_i - \bar{d}_i\gamma_5 d_i}{\sqrt{2}}\right)\Big[\sin\theta\left(\frac{\bar{u}_j\gamma_5 u_j + \bar{d}_j\gamma_5 d_j}{\sqrt{2}}\right) + \cos\theta(\bar{s}_j\gamma_5 s_j)\Big], \tag{3}$$

where i, j are color indices, u, d, s are the up down and strange quark fields, respectively, and the mixing angle θ in the η' current is $\theta \sim 40°$ [9–11]. In this work, we use $\theta = 40°$.

In general, there is no one-to-one correspondence between the current and the state since a molecular current can be rewritten in terms of a sum over tetraquark-type currents through a Fierz transformation. However, as shown in [2], if the physical state is a molecular state, it would be better to choose a molecular type of current so that it has a large overlap with the physical state. In any case, it is very important to notice that since the current in Eq. (2) is local, it does not represent an extended object with two mesons separated in space, but rather a very compact object with two singlet quark-antiquark pairs.

The coupling of the scalar resonance R to the scalar current j can be parametrized in terms of a parameter λ as

$$\langle 0|j|R\rangle = \lambda. \tag{4}$$

In the QCD evaluation of the correlator function in Eq. (2), we work at leading order and consider condensates up to dimension six. We deal with the strange quark as a light one and consider the diagrams up to order m_s. We neglect the terms proportional to m_u and m_d. On the phenomenological side, we consider the usual pole plus continuum contribution. Therefore, we introduce the continuum threshold parameter s_0 [12]. In the $SU(2)$ limit, the quarks u and d are degenerate, and we consider the u-quark condensate equal to the d-quark condensate, which we call $\langle\bar{q}q\rangle$. After performing a Borel transform on both sides of the calculation, the sum rule is given by

$$\lambda^2 e^{-m_R^2/M^2} = 3\frac{M^{10}E_4}{2^{13}5\pi^6}(12+\sin^2\theta) - \frac{m_s\langle\bar{s}s\rangle M^6 E_2}{2^7\pi^4}\cos^2\theta + \frac{\langle g^2G^2\rangle M^6 E_2}{2^{13}\pi^6}(4-\sin^2\theta) - \frac{m_s\langle\bar{s}g\sigma.Gs\rangle}{2^7\pi^4} \times M^4 E_1 \cos^2\theta(3.5 - 3\ln(M^2/\Lambda^2_{\rm QCD})) + \frac{M^4 E_1}{2^6\pi^2}(\langle\bar{q}q\rangle^2(1+3\cos^2\theta) + 2\langle\bar{s}s\rangle^2\sin^2\theta), \quad (5)$$

where M is the Borel mass and

$$E_n \equiv 1 - e^{-s_0/M^2}\sum_{k=0}^{n}\left(\frac{s_0}{M^2}\right)^k\frac{1}{k!} \quad (6)$$

accounts for the continuum contribution.

In the numerical analysis of the sum rules, the values used for the quark masses and condensates are [13,14] $m_s = 0.13$ GeV, $\langle\bar{q}q\rangle = -(0.23)^3$ GeV^3, $\langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$, $\langle\bar{q}g\sigma.Gq\rangle = m_0^2\langle\bar{q}q\rangle$ with [8] $m_0^2 = 0.8$ GeV^2 and $\langle g^2G^2\rangle = 0.88$ GeV^4.

In Fig. 4, we show the operator product expansion (OPE) convergence of the sum rule in Eq. (5). From this figure, we see that the convergence is reasonable for $M^2 > 1.2$ GeV^2 and very good for $M^2 > 1.5$ GeV^2. However, as in the case of the light scalars [14], there is no pole dominance for these values of M^2. This result could be interpreted in two different ways: (i) it could indicate that this state does not exist, or (ii) it could indicate that this state is not clearly separated from the continuum. The second interpretation can be applied to very broad states, such as the light scalars σ and κ, because their widths are as large as the difference between their masses and the continuum threshold. In what follows we adhere to the second interpretation.

In order to extract the mass m_R without knowing the value of the constant λ, we take the derivative of Eq. (5) with respect to $1/M^2$ and divide the result by Eq. (5). In Fig. 5, we show the resonance mass as a function of M^2 for different values of $\sqrt{s_0}$. We limit ourselves to the region $M^2 > 1.2$ GeV^2 where the curves are more stable and the OPE convergence is better. Averaging the mass over this entire region we find

$$m_R = (1.15 \pm 0.10)\ \text{GeV}, \quad (7)$$

which is compatible with the experimental threshold in Eq. (1). Knowing the mass, we can also evaluate the value of the parameter λ that gives the coupling between the state and the current. We obtain

$$\lambda = (1.39 \pm 0.27) \times 10^{-3}\ \text{GeV}^5. \quad (8)$$

B. Decay width

In order to study the RK^+K^- vertex associated with the $R \to K^+K^-$ decay, we consider the three-point function

$$T_{\mu\nu}(p, p', q) = \int d^4x d^4y e^{i.p'.x} e^{iq.y}\langle 0|T\{j_{5\mu}^{K^+}(x) j_{5\nu}^{K^-}(y) j^\dagger(0)\}|0\rangle, \quad (9)$$

where $p = p' + q$, j is given in Eq. (3), and we use the axial currents for the kaons:

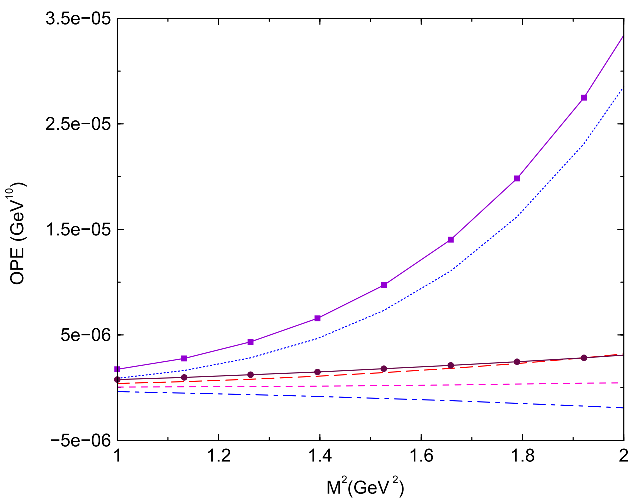

FIG. 4 (color online). The OPE convergence in the region $1.0 \le M^2 \le 2.0$ GeV^2 for $\sqrt{s_0} = 1.5$ GeV. The dotted line, dashed line, long-dashed line, dash-dotted line, solid line with circles, and solid line with squares give the perturbative, quark condensate, gluon condensate, mixed condensate, four-quark condensate, and total contributions, respectively.

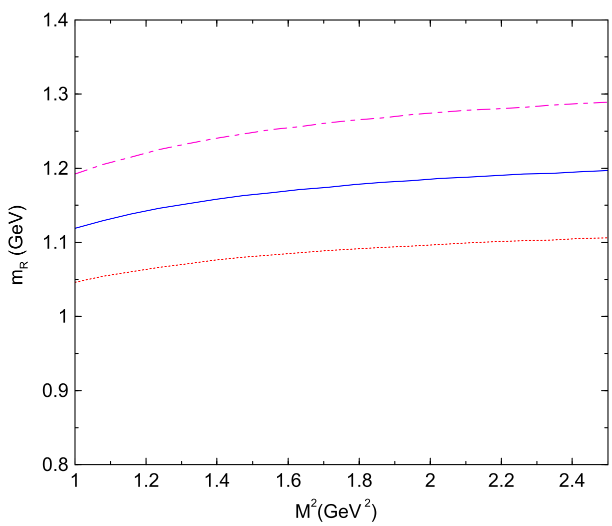

FIG. 5 (color online). The resonance mass as a function of the sum rule parameter (M^2) for different values of the continuum threshold: $\sqrt{s_0} = 1.4$ GeV (dotted line), $\sqrt{s_0} = 1.5$ GeV (solid line), and $\sqrt{s_0} = 1.6$ GeV (dash-dotted line).

$$j_{5\mu}^{K^+} = \bar{s}_a \gamma_\mu \gamma_5 u_a, \qquad j_{5\mu}^{K^-} = \bar{u}_a \gamma_\mu \gamma_5 s_a. \tag{10}$$

To evaluate the phenomenological side, we insert intermediate states for K^+, K^-, and R, and we use the definitions in Eqs. (4) and (11) below:

$$\langle 0 | j_{5\mu}^K | K(p) \rangle = i p_\mu F_K. \tag{11}$$

We obtain the following relation,

$$T_{\mu\nu}^{\text{phen}}(p, p', q) = \frac{F_K^2 \lambda}{(M_R^2 - p^2)(m_K^2 - p'^2)(m_K^2 - q^2)} g_{RKK} p'_\mu q_\nu + \text{higher resonances}, \tag{12}$$

where the coupling constant g_{RKK} is defined by the matrix element:

$$\langle K(p') K(q) | R(p) \rangle = g_{RKK}. \tag{13}$$

Here, we follow Refs. [5,15] and work at the kaon pole as suggested in [7] for the nucleon-pion coupling constant. This method was also applied to nucleon-kaon-hyperon coupling [16,17], $D^* - D - \pi$ coupling [18,19], and to the $J/\psi - \pi$ cross section [20]. It consists in neglecting the kaon mass in the denominator of Eq. (12) in the term $1/(m_K^2 - q^2)$, and working at $q^2 = 0$. On the QCD side, one singles out the leading terms in the operator product expansion of Eq. (9) that match the $1/q^2$ term. Up to dimension six only the diagrams proportional to the quark condensate times m_s and the four-quark condensate contribute. Making a single Borel transform to both $-p^2 = -p'^2 \to M^2$ we get

$$g_{RK^+K^-} \frac{\lambda F_K^2}{m_R^2 - m_K^2} (e^{-m_K^2/M^2} - e^{-m_R^2/M^2}) = \frac{\sqrt{2}\cos\theta}{8} \bigg(\frac{(\langle \bar{q}q \rangle + \langle \bar{s}s \rangle)^2}{3} + \frac{m_s}{8\pi^2} \Big(\langle \bar{q}q \rangle - \frac{\langle \bar{s}s \rangle}{3} \Big) \times M^2 (1 - e^{-s_0^K/M^2}) \bigg), \tag{14}$$

where $s_0^K = (1.0 \pm 0.1)\ \text{GeV}^2$ is the continuum threshold for the kaon.

As discussed in Ref. [21] the problem of performing a single Borel transformation in a three-point function sum rule is the fact that terms associated with the pole-continuum transitions are not suppressed. However, as shown in [21], the pole-continuum transition term has a different behavior as a function of the Borel mass compared with the double pole contribution: it grows with M^2. Therefore, the pole-continuum contribution can be taken into account through the introduction of a parameter A on the phenomenological side of the sum rule in Eq. (14) by making the substitution $g_{RK^+K^-} \to g_{RK^+K^-} + AM^2$ [5,17,18,20].

Using $F_K = 160\ \text{MeV}$, $m_K = 490\ \text{MeV}$, $m_R = 1.15$ GeV, and the parameter λ given by the sum rule in Eq. (5), we show in Fig. 6 the QCDSR results for the vertex coupling constant for different values of s_0 and s_0^K in the interval given above. We see that in the Borel range used for the two-point function, the QCDSR results do have a linear form as a function of the Borel mass. Fitting the QCDSR results by a linear form, $g_{RK^+K^-} + AM^2$ (which is also shown in Fig. 6), the coupling can be obtained by extrapolating the fit to $M^2 = 0$. In the limits of the continuum thresholds mentioned above and taking into account the uncertainties in m_R given in Eq. (7) we obtain

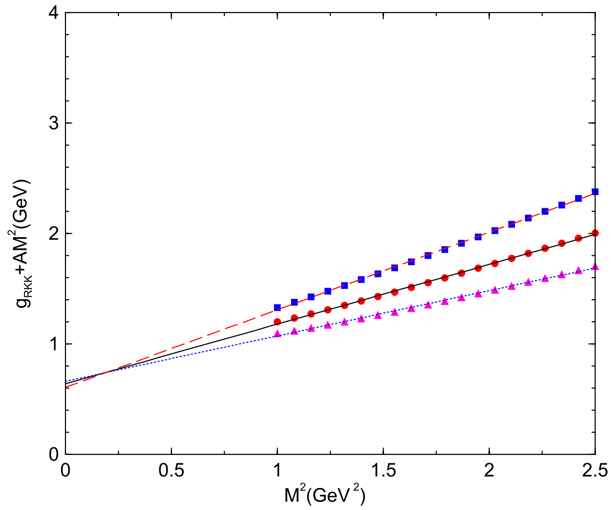

FIG. 6 (color online). The QCDSR result for the coupling constant $g_{RK^+K^-}$ as a function of the sum rule parameter M^2 for different values of s_0 and s_0^K (circles, triangles, and squares for $\sqrt{s_0} = 1.5$ GeV, $s_0^K = 1.0\ \text{GeV}^2$; $\sqrt{s_0} = 1.6$ GeV, $s_0^K = 1.1\ \text{GeV}^2$; and $\sqrt{s_0} = 1.4$ GeV, $s_0^K = 0.9\ \text{GeV}^2$, respectively). The solid, dotted, and dashed lines give linear fits to the QCDSR results.

$$g_{RK^+K^-} = (0.63 \pm 0.06)\ \text{GeV}. \tag{15}$$

The decay width of $R \to K^+K^-$ is given in terms of the hadronic coupling $g_{RK^+K^-}$ as

$$\Gamma(R \to K^+K^-) = \frac{1}{16\pi m_R^3} g_{RK^+K^-}^2 \sqrt{\lambda(m_R^2, m_K^2, m_K^2)}, \tag{16}$$

where $\lambda(m_R^2, m_K^2, m_K^2) = m_R^4 + m_K^4 + m_K^4 - 2m_R^2 m_K^2 - 2m_R^2 m_K^2 - 2m_K^2 m_K^2 = m_R^2(m_R^2 - 4m_K^2)$. Therefore, we get

$$\Gamma(R \to K^+K^-) = (11.4 \pm 2.2)\ \text{MeV}. \tag{17}$$

Of course, this is not the total width of the $\eta'\pi$ molecule because it can also decay into $\eta - \pi$ with a much bigger phase space. However, in the B decays discussed here, only the channel $R \to K^+K^-$ can be observed.

The errors quoted above come directly from the uncertainty in the determination of the continuum threshold parameter, s_0. According to our previous experience, they are the main source of uncertainty in the method. For a detailed analysis of the uncertainty associated with other parameters used in QCDSR we refer the reader to Refs. [2,19].

III. CONCLUSION

We have proposed that a loosely bound molecular state should leave a particular signal in the Dalitz plot. Such a state, of the particles 1, 2, can only exist when the relative momentum between these two particles is small. Therefore, we expect to observe a short line parallel to the s_{23} axis in the middle of the Dalitz plot, approximately in the region where there is a hole in the line, characterizing a vector resonance (see Fig. 1). This signal is different from any signal characterizing the normal quark-antiquark mesons and could be used to identify the existence of loosely bound molecular states.

In the case of three-body B decays, the final particles observed are pions and kaons. Therefore, to observe a molecular state in the Dalitz plot for a three-body B decay, this molecular state must decay into pions and/or kaons. We have considered a $\eta' - \pi$ molecular state. If this state exists as a loosely bound state, its mass should be close to the $\eta' - \pi$ threshold, $\sim$1.1 GeV, which is quite visible in the B decay Dalitz plot. Because for an S wave this molecule has $I^G J^{PC} = 1^- 0^{++}$, it cannot decay into $\pi^+\pi^-$, but it will decay into K^+K^-. Therefore, the observation of a small line with $\sqrt{s_{12}} \sim 1.1$ GeV parallel to the s_{23} axis in the Dalitz plot for the $B^- \to K^+K^-K^-$ and $B^- \to K^+K^-\pi^-$ decays, with negative observation in the Dalitz plot for the $B^- \to \pi^+\pi^-K^-$ and $B^- \to \pi^+\pi^-\pi^-$ decays, would definitively indicate the existence of the $\eta' - \pi$ molecular state.

We have used QCD sum rules to study the mass and the decay width of a $\eta' - \pi$ molecular current using two- and three-point functions, respectively. We considered diagrams up to dimension six in both cases. We found a mass slightly larger than the $\eta' - \pi$ threshold, indicating the possibility of a loosely bound molecular state. We obtained a small width for the $\eta' - \pi \to K^+K^-$ decay around 10 MeV. With this information, it should be possible to experimentally identify this state in the $B^- \to K^+K^-K^-$ and $B^- \to K^+K^-\pi^-$ Dalitz plots if it exists.

The method for identifying resonances (or bound states) discussed here could be applied to other cases. A straightforward extension of our work could be performed for $\eta' - \pi$ with quantum numbers $J^{PC} = 1^{-+}$. The CLEO [22] and COMPASS [23] collaborations have recently searched for this exotic state.

ACKNOWLEDGMENTS

This work was partially financed by the Brazilian funding agencies CNPq and FAPESP.

[1] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011).
[2] M. Nielsen, F. S. Navarra, and S. H. Lee, Phys. Rep. **497**, 41 (2010).
[3] S. L. Olsen, Nucl. Phys. **A827**, 53C (2009).
[4] K. Nakamura *et al.*, J. Phys. G **37**, 075 021 (2010).
[5] T. V. Brito, F. S. Navarra, M. Nielsen, and M. E. Bracco, Phys. Lett. B **608**, 69 (2005).
[6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[7] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[8] S. Narison, *QCD as a Theory of Hadrons*, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. No. 17, (2002), p. 1; *QCD Spectral Sum Rules*, World Sci. Lect. Notes Phys. No. 26 (1989), p. 1; Phys. Rep. **84**, 263 (1982).
[9] V. Anisovich, D. Melikhov, and V. Nikonov, Phys. Rev. D **52**, 5295 (1995); **55**, 2918 (1997).
[10] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998).
[11] I. Balakireva, W. Lucha, and D. Melikhov, Proc. Sci. QFTHEP2011 (2011) 056.
[12] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1981).
[13] S. Narison, Phys. Lett. B **466**, 345 (1999).
[14] R. D. Matheus, F. S. Navarra, M. Nielsen, and R. Rodrigues da Silva, Phys. Rev. D **76**, 056005 (2007).
[15] S. Narison, Phys. Lett. **175B**, 88 (1986).
[16] S. Choe, M. K. Cheoun, and S. H. Lee, Phys. Rev. C **57**, 2061 (1998).
[17] M. E. Bracco, F. S. Navarra, and M. Nielsen, Phys. Lett. B **454**, 346 (1999).
[18] F. S. Navarra, M. Nielsen, M. E. Bracco, M. Chiapparini, and C. L. Schat, Phys. Lett. B **489**, 319 (2000).
[19] M. E. Bracco, M. Chiapparini, F. S. Navarra, and M. Nielsen, arXiv:1104.2864.
[20] F. O. Durães, S. H. Lee, F. S. Navarra, and M. Nielsen, Phys. Lett. B **564**, 97 (2003).
[21] B. L. Ioffe and A. V. Smilga, Nucl. Phys. **B232**, 109 (1984).
[22] G. S. Adams *et al.* (CLEO Collaboration), Phys. Rev. D **84**, 112009 (2011).
[23] T. Schluter (COMPASS Collaboration), eConf C 110613, 83 (2011).