Physics Letters B 703 (2011) 274–280

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Relation between $T_{cc,bb}$ and $X_{c,b}$ from QCD

J.M. Dias [a], S. Narison [b], F.S. Navarra [a], M. Nielsen [a,*], J.-M. Richard [c]

[a] Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil
[b] Laboratoire Particules et Univers de Montpellier, CNRS-IN2P3, Case 070, Place Eugène Bataillon, 34095, Montpellier Cedex 05, France
[c] Université de Lyon et Institut de Physique Nucléaire de Lyon, IN2P3-CNRS-UCBL, 4, rue Enrico Fermi, F-69622 Villeurbanne, France

ARTICLE INFO

Article history:
Received 8 June 2011
Received in revised form 22 July 2011
Accepted 28 July 2011
Available online 4 August 2011
Editor: W. Haxton

Keywords:
QCD spectral sum rules
Non-perturbative methods
Exotic multiquark states
Heavy quarkonia

ABSTRACT

We have studied, using double ratio of QCD (spectral) sum rules, the ratio between the masses of T_{cc} and $X(3872)$ assuming that they are respectively described by the $D-D^*$ and $D-\bar{D}^*$ molecular currents. We found (within our approximation) that the masses of these two states are almost degenerate. Since the pion exchange interaction between these mesons is exactly the same, we conclude that if the observed $X(3872)$ meson is a $D\bar{D}^*+c.c.$ molecule, then the DD^* molecule should also exist with approximately the same mass. An extension of the analysis to the b-quark case leads to the same conclusion. We also study the SU(3) breakings for the T^s_{QQ}/T_{QQ} mass ratios. Motivated by the recent Belle observation of two Z_b states, we revise our determination of X_b by combining results from exponential and FESR sum rules.

© 2011 Elsevier B.V. All rights reserved.

1. Introduction

The existence of exotic hadrons is a long-standing problem. By exotic we mean a state whose quantum numbers and main properties cannot be explained by a simple quark–antiquark or three-quark configuration.

The $X(3872)$ resonance (assumed to be an 1^{++} axial vector meson) has, indeed, stimulated *many activities in the physics of hadrons*. It was discovered by BELLE in B-decays [1], and confirmed by BaBar [2], CDF [3] and D0 [4]. It is rather narrow, with a width $\leqslant 2.3$ MeV. Its most popular picture which consists of a molecular configuration, $D\bar{D}^*+\bar{D}D^*$, with $J^{PC}=1^{++}$, has been attributed to the narrow ($\leqslant 2.3$ MeV width) $X(3872)$.[1]

The case of the four-quark state $(QQ\bar{u}\bar{d})$ with quantum numbers $I=0$, $J=1$ and $P=+1$ which, following Ref. [6], we call T_{QQ}, is especially interesting. As already noted previously [6,7], the T_{bb} or T_{cc} states with $J^P=1^+$ cannot split into a pair of two $\bar{B}$ or two D mesons which is restricted to $J^P=0^+,2^+,\ldots$. If their masses are below the $\bar{B}\bar{B}^*$ or $DD\pi$ thresholds, these decays are also forbidden. As a result, T_{QQ} becomes stable with respect to strong interaction, and must decay radiatively, or even weakly if the mass becomes lower than the threshold made of two pseudoscalar mesons.

2. T_{QQ} from potential models

In constituent models with a flavor-independent central potential, the stability of $(QQ\bar{q}\bar{q})$ configurations comes from a favorable effect when the charge-conjugation symmetry is broken, as noted many years ago [8]. This is the same mechanism by which, in QED, the loosely bound positronium molecule evolves into the very stable hydrogen molecule.

It is worth noting that in the large m_Q limit, the light degrees of freedom cannot resolve the closely bound QQ system. This results in bound states similar to the $\bar{\Lambda}_Q$ states, with QQ playing the role of the heavy antiquark [9].

The $(QQ\bar{q}\bar{q})$ states have been studied using a variety of simple or elaborated potential models [7,8,10–12]. The corresponding four-body problem is very delicate. For instance, an expansion on harmonic-oscillator states was used in [11]. It is efficient for deep binding but converges very slowly for weak binding. If truncated, this expansion may fail to demonstrate stability with potentials that do bind, because it lacks explicit $(Q\bar{q})-(Q\bar{q})$ components, which are important near threshold [7], and are included in the Gaussian expansion sketched in [12] and systematically developed in [6]. See, also, Ref. [13] for a discussion about the four-quark problem. All authors agree that such states become bound when the quark over the antiquark mass ratio becomes sufficiently large. Detailed four-body calculations, using a pairwise central potential supplemented by a chromomagnetic interaction, indicate that T_{bb}

* Corresponding author.
E-mail addresses: jdias@if.usp.br (J.M. Dias), snarison@yahoo.fr (S. Narison), navarra@if.usp.br (F.S. Navarra), mnielsen@if.usp.br (M. Nielsen), j-m.richard@ipnl.in2p3.fr (J.-M. Richard).

[1] For references about some other possible interpretations of the $X(3872)$ see, e.g., [5].

0370-2693/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2011.07.082

is rather well bound, and T_{cc} possibly bound by a few MeV below DD^*. For instance, the prediction of Ref. [6] is, in units of MeV:

$$M_{T_{cc}} = 3876\text{–}3905, \qquad M_{T_{bb}} = 10519\text{–}10651. \tag{1}$$

A non-pairwise confinement has also been considered [14], inspired by the large coupling regime of QCD, where it is shown [15] that it is more favorable to build stable tetraquarks. In this improved quark model, as well as in conventional quark models, it is found that $(QQ\bar{q}\bar{q})$ has an energy lower than $(Q\bar{Q}q\bar{q})$.

Another variant was considered in [16], with a chiral potential model, which includes meson-exchange forces between quarks, instead of the chromomagnetic interaction.

The existence of a $D\bar{D}^* + \bar{D}D^*$ molecule was predicted in Ref. [17] on the basis of the pion-exchange dynamics.[2] Here, the pion is exchanged between the hadrons, as in the Yukawa theory of nuclear forces. The $DD^*\pi$ and $\bar{D}\bar{D}^*\pi$ vertices are identical, as well as the $D^*D^*\pi$ and $\bar{D}^*\bar{D}^*\pi$ ones. There is only an overall change of sign, due to the G-parity of the pion. Therefore, if the pion-exchange dynamics[3] is able to bind the $D\bar{D}^* + \bar{D}D^*$ molecule, the same is true for the DD^* molecule. The difference between these two states can only come from the short-range part of the interaction.

3. T_{QQ} from QCD (spectral) sum rules

The first study of tetraquarks with two heavy quarks within QCD (spectral) sum rules (QCDSR) was done in [19] by using diquark–antidiquark current. This study is revisited and improved in the present Letter. Our aim is also to compare in detail the $(QQ\bar{q}\bar{q})$ and $(Q\bar{Q}q\bar{q})$ configurations. Such a comparison is attempted in Ref. [20], where the authors study heavy tetraquarks using a crude color-magnetic interaction, with flavor symmetry breaking corrections. They assume that the Belle resonance, $X(3872)$, is a $cq\bar{c}\bar{q}$ tetraquark, and use its mass as input to determine the mass of other tetraquark states. They obtain, in units of MeV:

$$M_{T_{cc}} \simeq 3966, \qquad M_{T_{bb}} \simeq 10372, \tag{2}$$

in agreement with the previous results in Eq. (1) and the ones from QCD (spectral) sum rule, in units of GeV [19]:

$$M_{T_{cc}} = 4.2 \pm 0.2, \qquad M_{T_{bb}} = 10.2 \pm 0.3. \tag{3}$$

The short-range part of the interaction can be tested by the QCD (spectral) sum rules approach [21–23]. Therefore, in this work, we study the ratio of the masses of the T_{cc} and $X(3872)$ states, by using the double ratios of sum rules (DRSR) introduced in [24], which is widely applied for accurate determinations of the ratios of couplings and masses [25–31] and form factors [32]. This accuracy is reached due to partial cancellations of the systematics of the method and of the QCD corrections in the DRSR. More recently, the DRSR was used to study different possible currents for the $X(3872)$ [27]. It was found that (within the accuracy of the method) the different structures ($\bar{3}-3$ and $\bar{6}-6$ tetraquarks and $D\bar{D}^* + \bar{D}D^*$ molecule) lead to the same prediction for the mass. This result could indicate that the short-range part of the interaction alone may not be sufficient to reveal the nature of the $X(3872)$.

3.1. Two-point functions and forms of the sum rules

The two-point functions of the $X(3872)$ (assumed to be an 1^{++} axial vector meson) and the T_{cc} (assumed to be a $J^P = 1^+$ state) is defined as:

$$\begin{aligned}\Pi_i^{\mu\nu}(q) &\equiv i\int d^4x\, e^{iq.x}\langle 0|T\left[j_i^\mu(x) j_i^{\nu\dagger}(0)\right]|0\rangle \\ &= -\Pi_{1i}(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \Pi_{0i}(q^2)\frac{q^\mu q^\nu}{q^2},\end{aligned} \tag{4}$$

where $i = X, T_{cc}$. The two invariants, Π_1 and Π_0, appearing in Eq. (4) are independent and have respectively the quantum numbers of the spin 1 and 0 mesons.

We assume that the $X(3872)$ and T_{cc} states are described by the molecular currents:

$$\begin{aligned}j_X^\mu(x) = \left(\frac{g}{\Lambda}\right)^2_{\text{eff}} \frac{1}{\sqrt{2}}\big[&\big(\bar{q}_a(x)\gamma_5 c_a(x)\bar{c}_b(x)\gamma^\mu q_b(x)\big) \\ &- \big(\bar{q}_a(x)\gamma^\mu c_a(x)\bar{c}_b(x)\gamma_5 q_b(x)\big)\big]\end{aligned} \tag{5}$$

and

$$j_{T_{cc}}^\mu(x) = \left(\frac{g'}{\Lambda}\right)^2_{\text{eff}} \big(\bar{q}_a(x)\gamma_5 c_a(x)\bar{q}_b(x)\gamma^\mu c_b(x)\big), \tag{6}$$

where a and b are color indices.

In the molecule assignment, it is assumed that there is an effective local current and the meson pairs are weakly bound by a van der Vaals force in a Fermi-like theory with a strength $(g/\Lambda)^2_{\text{eff}}$ which has nothing to do with the quarks and gluons inside each meson.

Due to its analyticity, the correlation function, Π_{1i} in Eq. (4), can be written in terms of a dispersion relation:

$$\Pi_{1i}(q^2) = \int_{4m_c^2}^{\infty} ds\, \frac{\rho_i(s)}{s - q^2} + \cdots, \tag{7}$$

where $\pi\rho_i(s) \equiv \mathrm{Im}[\Pi_{1i}(s)]$ is the spectral function.

The sum rule is obtained by evaluating the correlation function in Eq. (4) in two ways: using the operator product expansion (OPE) and using the information from hadronic phenomenology. In the OPE side we work at leading order of perturbation theory in α_s, and we consider the contributions from condensates up to dimension six. In the phenomenological side, the correlation function is estimated by inserting intermediate states for the X and T_{cc} states via their couplings λ_i to the molecular currents:

$$\langle 0|j_i^\mu|M_i\rangle = \lambda_i \epsilon^\mu, \tag{8}$$

where $M_i \equiv X,\ T_{cc},\ j_i^\mu$ are the currents in Eqs. (5) and (6).

Using the ansatz: "one resonance" $\oplus$ "QCD continuum", where the QCD continuum comes from the discontinuity of the QCD diagrams from a continuum threshold t_c, the phenomenological side of Eq. (4) can be written as:

$$\Pi_{\mu\nu}^{phen}(q^2) = \frac{\lambda_i^2}{M_i^2 - q^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_i^2}\right) + \cdots, \tag{9}$$

where the Lorentz structure $g_{\mu\nu}$ projects out the 1^+ state. The dots denote higher axial-vector resonance contributions that will be parametrized, as usual, by the QCD continuum. After making an inverse-Laplace (or Borel) transform on both sides, and transferring the continuum contribution to the QCD side, the moment sum rule and its ratio read:

[2] For further references on this approach, see e.g. [18].

[3] Usually, the G parity rule transforms an attractive potential into a repulsive one. Here, however, it only changes the sign of the transition potential $D\bar{D}^* \to D^*\bar{D}$, and thus just a phase in the two-component bound state wave function.

$$\mathcal{F}_i(\tau) \equiv \lambda_i^2 e^{-M_i^2\tau} = \int_{4m_c^2}^{t_c} ds\, e^{-s\tau} \rho_i(s),$$

$$\mathcal{R}_i(\tau) \equiv -\frac{d}{d\tau} \log \mathcal{F}_i(\tau) \simeq M_i^2 \tag{10}$$

where $\tau \equiv 1/M^2$ is the sum rule variable with M being the inverse-Laplace (or Borel) mass. In the following, we shall work with the DRSR [24]:

$$r_{T_{cc}/X} \equiv \sqrt{\frac{\mathcal{R}_{T_{cc}}}{\mathcal{R}_X}} \simeq \frac{M_{T_{cc}}}{M_X}. \tag{11}$$

3.2. QCD expression of the spectral functions

The QCD expressions of the spectral densities of the two-point correlator associated to the current in Eq. (5) are given in Ref. [33]. Up to dimension-six condensates the expressions associated to the current in Eq. (6), in the structure $g_{\mu\nu}$ are:

$$\rho(s) = \rho^{pert}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle G^2\rangle}(s) + \rho^{mix}(s) + \rho^{\langle\bar{q}q\rangle^2}(s), \tag{12}$$

with

$$\begin{aligned}
\rho^{pert}(s) &= \frac{1}{3.2^{13}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)\big[(\alpha+\beta)m_c^2 - \alpha\beta s\big]^3 \\
&\quad \times \big[2m_c^2\big(13\alpha^2 + 13\alpha\beta + 7\alpha + 5\big) - 15\alpha\beta s(1+\alpha+\beta)\big], \\
\rho^{\langle\bar{q}q\rangle}(s) &= -\frac{m_c\langle\bar{q}q\rangle}{2^6\pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2}(1+\alpha+\beta) \\
&\quad \times \big[(\alpha+\beta)m_c^2 - \alpha\beta s\big]^2, \\
\rho^{\langle G^2\rangle}(s) &= \frac{\langle g^2G^2\rangle}{2^{13}.3^2\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} \big\{12\alpha\beta(5\alpha+5\beta-3) \\
&\quad \times \big[(\alpha+\beta)m_c^2 - \alpha\beta s\big]^2 + m_c^4(1-\alpha-\beta)^2\alpha^2 \\
&\quad \times (5+\alpha+\beta) + m_c^2(1-\alpha-\beta)\big[9\alpha^2(2+3\alpha+4\beta) \\
&\quad + \alpha\beta(2+2\alpha+11\beta) + 15\alpha - 2\beta\big] \\
&\quad \times \big[(\alpha+\beta)m_c^2 - \alpha\beta s\big]\big\}, \\
\rho^{mix}(s) &= -\frac{m_c\langle\bar{q}g\sigma.Gq\rangle}{3.2^8\pi^4} \Bigg\{7\int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha(1-\alpha)}\big[m_c^2 - \alpha(1-\alpha)s\big] \\
&\quad - \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2}\big[12\alpha^2 + 17\alpha\beta - \beta\big] \\
&\quad \times \big[(\alpha+\beta)m_c^2 - \alpha\beta s\big]\Bigg\}, \\
\rho^{\langle\bar{q}q\rangle^2}(s) &= \frac{\rho\langle\bar{q}q\rangle^2}{3.2^6\pi^2} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha\big[13m_c^2 - 5\alpha(1-\alpha)s\big],
\end{aligned} \tag{13}$$

where: m_c, $\langle g^2G^2\rangle$, $\langle\bar{q}q\rangle$, $\langle\bar{q}g\sigma.Gq\rangle$ are respectively the charm quark mass, gluon condensate, light quark and mixed condensates; ρ indicates the violation of the four-quark vacuum saturation. The integration limits are given by:

$$\alpha_{min} = \frac{1}{2}(1-v), \qquad \alpha_{max} = \frac{1}{2}(1+v),$$

$$\beta_{min} = \alpha m_c^2/\big(s\alpha - m_c^2\big) \tag{14}$$

where v is the c-quark velocity:

$$v \equiv \sqrt{1 - 4m_c^2/s}. \tag{15}$$

Table 1
QCD input parameters. For the heavy quark masses, we use the range spanned by the running $\overline{MS}$ mass $\overline{m}_Q(M_Q)$ and the on-shell mass from QCD (spectral) sum rules compiled in pages 602, 603 of the book in [23]. The values of Λ and $\hat{\mu}_q$ have been obtained from $\alpha_s(M_\tau) = 0.325(8)$ [38] and from the running masses: $(\overline{m}_u + \overline{m}_d)(2) = 7.9(3)$ MeV [39]. The original errors have been multiplied by 2 for a conservative estimate of the errors.

Parameters	Values	Ref.
$\Lambda(n_f = 4)$	(324 ± 15) MeV	[38,40]
$\hat{\mu}_q$	(263 ± 7) MeV	[23,39]
$\hat{m}_s$	(0.114 ± 0.021) GeV	[23,39,40]
m_c	$(1.23 \sim 1.47)$ GeV	[23,39–43]
m_b	$(4.2 \sim 4.7)$ GeV	[23,39–42]
m_0^2	(0.8 ± 0.2) GeV^2	[28,44,45]
$\langle\alpha_s G^2\rangle$	$(6 \pm 2) \times 10^{-2}$ GeV^4	[25,38,46–51]
$\rho\alpha_s\langle\bar{d}d\rangle^2$	$(4.5 \pm 0.3) \times 10^{-4}$ GeV^6	[38,44,46]

3.3. T_{cc}/X ratio of masses

In the following, we shall extract the mass ratio T_{cc}/X using the DRSR in Eq. (11). For the numerical analysis we shall introduce the renormalization group invariant quantities $\hat{m}_s$ and $\hat{\mu}_q$ [34,35]:

$$\begin{aligned}
\bar{m}_s(\tau) &= \frac{\hat{m}_s}{(-\log\sqrt{\tau}\Lambda)^{-2/\beta_1}}, \\
\langle\bar{q}q\rangle(\tau) &= -\hat{\mu}_q^3(-\log\sqrt{\tau}\Lambda)^{-2/\beta_1}, \\
\langle\bar{q}g\sigma.Gq\rangle(\tau) &= -m_0^2\hat{\mu}_q^3(-\log\sqrt{\tau}\Lambda)^{-1/3\beta_1},
\end{aligned} \tag{16}$$

where $\beta_1 = -(1/2)(11 - 2n/3)$ is the first coefficient of the β function for n flavors. We have used the quark mass and condensate anomalous dimensions reported in [23]. We shall use the QCD parameters in Table 1. At the scale where we shall work, and using the parameters in Table 1, we deduce: $\rho = 2.1 \pm 0.2$, which controls the deviation from the factorization of the four-quark condensates. We shall not include the $1/q^2$ term discussed in [36,37], which is consistent with the LO approximation used here as the latter has been motivated by a phenomenological parametrization of the larger order terms of the QCD series.

Using QCD (spectral) sum rules, one can usually estimate the mass of the X-meson, from the ratio $\mathcal{R}_X$ in Eq. (10), which is related to the spectral densities obtained from the current (5). A tetraquark current for the $X(3872)$ was used in Ref. [26]. At the sum rule stability point and using a slightly different (though consistent) set of QCD parameters than in Table 1, one obtains, with a good accuracy, for $m_c = 1.26$ GeV [26][4]:

$$M_X \simeq \sqrt{\mathcal{R}_X} = (3925 \pm 127)\ \text{MeV}, \tag{17}$$

and the correlated continuum threshold value fixed simultaneously by the Laplace and finite energy sum rules (FESR) sum rules:

$$\sqrt{t_c} \simeq (4.15 \pm 0.03)\ \text{GeV}. \tag{18}$$

[4] The use of $m_c = 1.47$ GeV increases the central value by about (160–200) MeV.

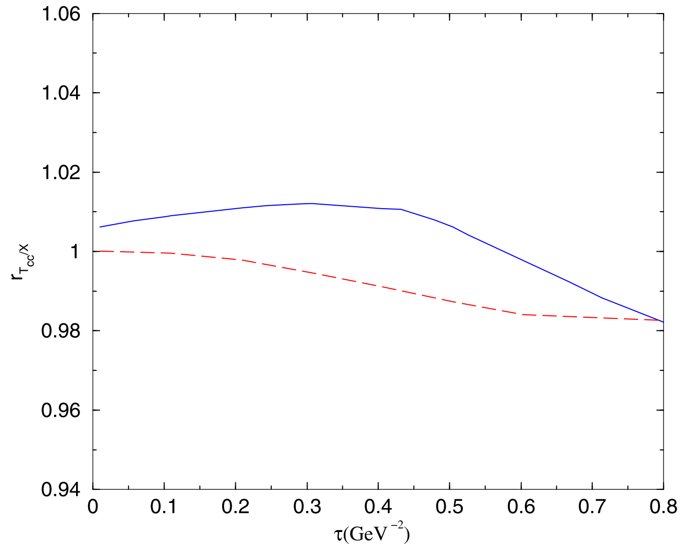

Fig. 1. The double ratio $r_{T_{cc}/X}$ defined in Eq. (11) as a function of τ for $\sqrt{t_c} = 4.15$ GeV and for two values of $m_c = 1.23$ (solid line) and 1.47 GeV (dashed line).

In Ref. [27] it was obtained that the DRSR for the tetraquark current and for the molecular current is:

$$r_{mol/3} = \sqrt{\frac{\mathcal{R}_{mol}}{\mathcal{R}_3}} \simeq 1.00, \tag{19}$$

with a negligible error. Therefore, the result in Eq. (18) is the same for the current in Eq. (5). Although the uncertainty in Eq. (17) is still large, considering the fact that this result was obtained in a Borel region where there is pole dominance and OPE convergence, one can say that the QCD sum rules supports the existence of such a state and that the value obtained for M_X is in reasonable agreement with the experimental candidate [40]:

$$M_X|_{exp} = (3872.2 \pm 0.8)\ \text{MeV}. \tag{20}$$

We now study the DRSR of the T_{cc}/X defined in Eq. (11). In Fig. 1, we show the τ-dependence of the ratio for $\sqrt{t_c} = 4.15$ GeV and for two values of $m_c = 1.23$ GeV and 1.47 GeV. From Fig. 1 one can see that there is a τ-stability around $\tau \simeq 0.4\ \text{GeV}^{-2}$ and for this value of τ, we get:

$$r_{T_{cc}/X} = 1.00 \pm 0.01. \tag{21}$$

In Fig. 2, we show the t_c-dependence of the ratio for $\tau = 0.4\ \text{GeV}^{-2}$ and for two values of $m_c = 1.23$ GeV and 1.47 GeV. From this figure one can see that the ratio increases with t_c. However, considering the large range of t_c presented in the figure, the ratio does not differ more than 3% from 1.

Our analysis has shown that the $D\bar{D}^* + c.c.$ and DD^* currents lead to the same mass predictions within the accuracy of the approach. The accuracy of the DRSR is bigger than the normal QCDSR because the DRSR are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions. As mentioned before, this accuracy is reached due to partial cancellations of the systematics of the method and of the QCD corrections in the DRSR. Therefore, if the observed $X(3872)$ is a molecular $D\bar{D}^* + c.c.$ state its molecular cousin DD^* should also be a bound state. Its mass can be obtained by using the experimental mass for the $X(3872)$ in Eq. (21):

$$M_{T_{cc}} = (3872.2 \pm 39.5)\ \text{MeV}. \tag{22}$$

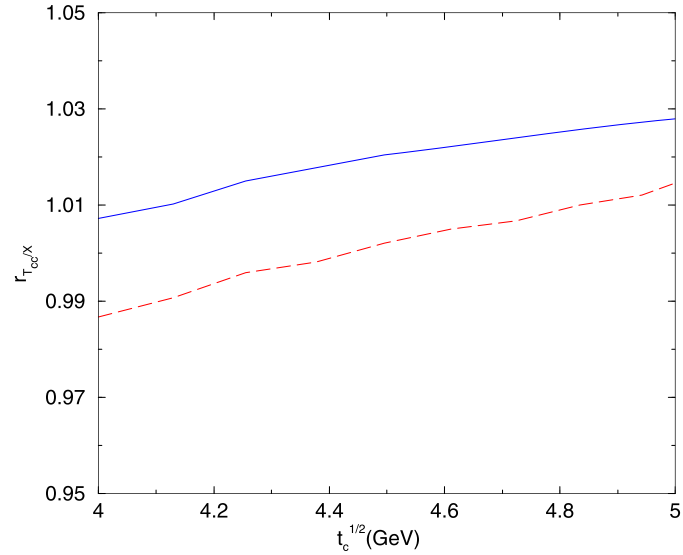

Fig. 2. The double ratio $r_{T_{cc}/X}$ as a function of t_c for $\tau = 0.4\ \text{GeV}^{-2}$ and for two values of $m_c = 1.23$ (solid line) and 1.47 GeV (dashed line).

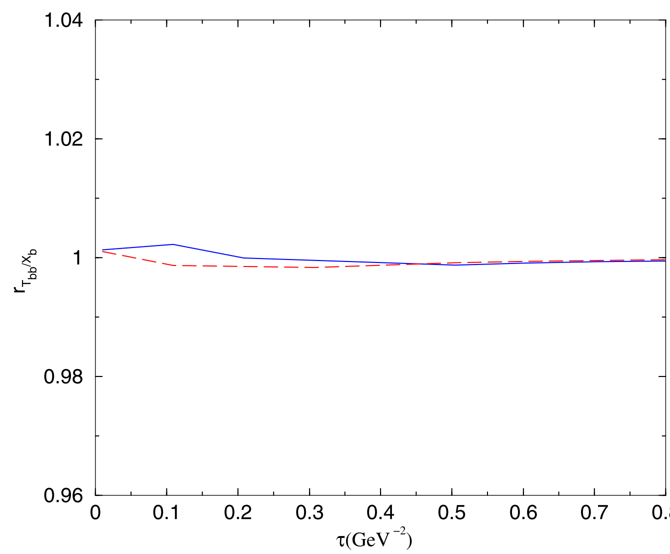

Fig. 3. Same as Fig. 1 for r_{T_{bb}/X_b} for $\sqrt{t_c} = 10.5$ GeV and for two values of $m_b = 4.2$ (solid line) and 4.7 GeV (dashed line).

3.4. T_{bb}/X_b ratio of masses

Using the same interpolating field in Eqs. (5) and (6) with the charm quark replaced by the bottom one, we can analyse the DRSR:

$$r_{T_{bb}/X_b} \equiv \sqrt{\frac{\mathcal{R}_{T_{bb}}}{\mathcal{R}_{X_b}}} \simeq \frac{M_{T_{bb}}}{M_{X_b}}. \tag{23}$$

In Fig. 3, we show the τ-dependence of the ratio in Eq. (23) for $\sqrt{t_c} = 10.5$ GeV and for two values of m_b. From this figure one can see the ratio is very stable. The same happens for the dependence of this ratio with t_c, as can be seen by Fig. 4. We get:

$$r_{T_{bb}/X_b} = 1.00. \tag{24}$$

Therefore, we can predict the degeneracy between the masses of the T_{bb} and of the X_b given in Eq. (25).

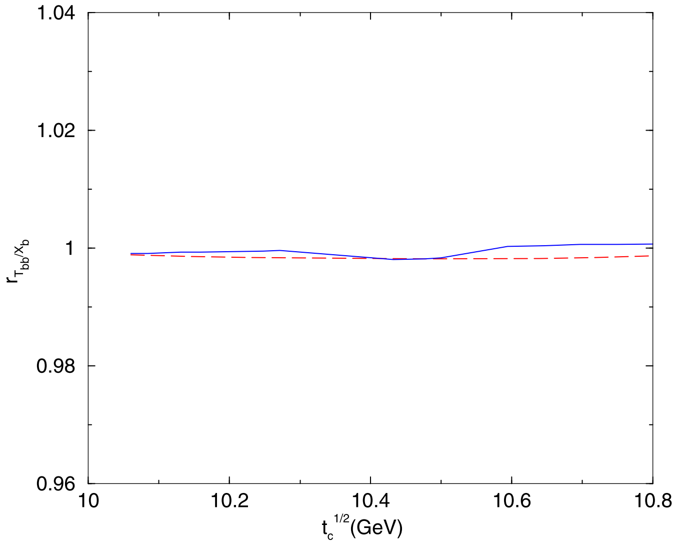

Fig. 4. Same as Fig. 2 for r_{T_{bb}/X_b} for $\tau = 0.2\ \text{GeV}^{-2}$ and for two values of $m_b = 4.2$ (solid line) and 4.7 GeV (dashed line).

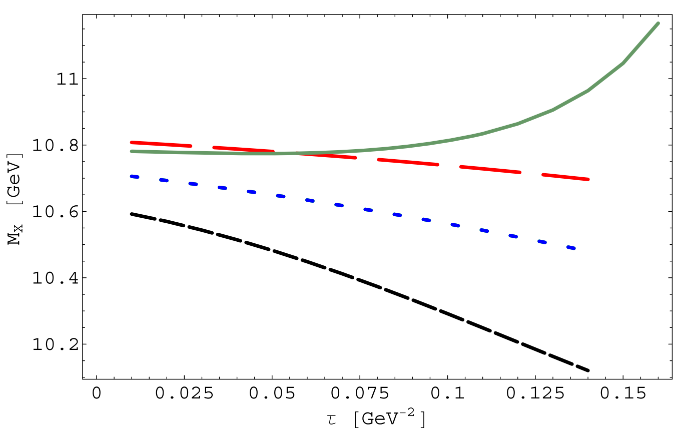

Fig. 5. M_{X_b} in GeV as a function of τ in GeV^{-2} from Laplace sum rule for $m_b = 4.7$ GeV and $\sqrt{t_c} = 11$ GeV: long dashed (red): (1) = perturbative (Pert) contribution; small dashed (blue): (2) = Pert + $\langle\bar{d}d\rangle + \langle\alpha_s G^2\rangle$ contributions (the one of the gluon condensate is relatively negligible); continuous (olive): (3) = (2) + mixed condensate $\langle g\bar{d}Gd\rangle$; medium dashed (black): (4) = (3) + $\langle\bar{d}d\rangle^2$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

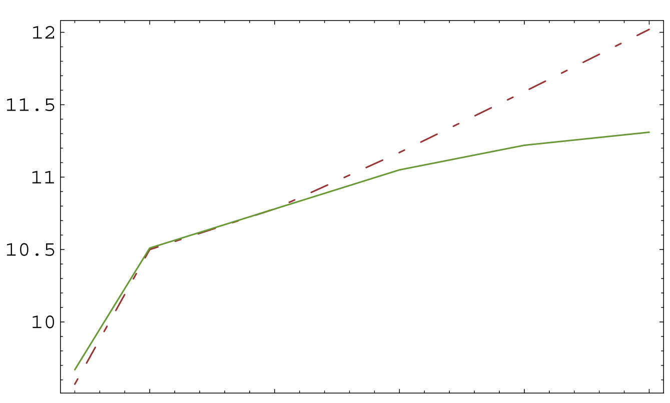

Fig. 6. M_{X_b} in GeV as a function of $\sqrt{t_c}$ in GeV from FESR (dashed curve) and the value at the τ-stability from Laplace sum rule for $m_b = 4.7$ GeV. The OPE has been truncated at $D = 5$.

4. Revisiting the determination of the X_b mass

The X_b was studied in Ref. [26]. At the sum rule stability point and using the perturbative $\overline{MS}$-mass $m_b(m_b) = 4.24$ GeV, they get:

$$10.06\ \text{GeV} \leqslant M_{X_b} \leqslant 10.50\ \text{GeV}, \tag{25}$$

for $10.2\ \text{GeV} \leqslant \sqrt{t_c} \leqslant 10.8$ GeV, while combining the Laplace sum rule with FESR, they obtain a slightly lower but more precise value:

$$M_{X_b} = (10.14 \pm 0.10)\ \text{GeV}. \tag{26}$$

We complete the previous analysis by using here the value of the on-shell mass $m_b = 4.7$ GeV due to the ambiguous definition of the quark mass used as we work to leading order of radiative corrections. For a close comparison with the analysis in [26], we shall work with the two-point function associated to the four-quark current.[5] We notice that the contribution of the $D = 6$ condensate $\langle\bar{q}q\rangle^2$ destabilizes the result [medium dashed (black) curve],[6] which is restored if one adds the $D = 8$ condensate given in [26]. However, we refrain to add such a term due to the eventual uncertainties for controlling the high-dimension condensate contributions (violation of factorization, complete $D = 8$ contributions) and find safer to limit the analysis to the $D = 5$ contribution like in Ref. [26]. In this way, the ratio of sum rules present τ-stability at about 0.1 GeV^{-2} (continuous curve in Fig. 5). We show in Fig. 6 the t_c-behavior of M_{X_b} versus the continuum threshold t_c, where a common solution is obtained in units of GeV:

$$M_{X_b} = 10.50 \sim 10.78 \quad \text{for } \sqrt{t_c} = 10.5 \sim 11.0, \tag{27}$$

which combined with the result in Eq. (26) leads to the conservative range of values:

$$M_{X_b} = 10.14 \sim 10.78 \quad \text{for } \sqrt{t_c} \approx M_{X_b'} = 10.5 \sim 11.0, \tag{28}$$

where one can notice the relatively small mass-difference between $\sqrt{t_c}$ and M_{X_b} eventually signaling the nearby location of the radial excitations as mentioned in [26].

Very recently the Belle Collaboration studied the $\Upsilon(5S) \to \Upsilon(nS)\pi^\pm$ and $\Upsilon(5S) \to h_b(mP)\pi^\pm$ ($n = 1, 2, 3$ and $m = 1, 2$) decay processes looking for resonant substructures. They found two narrow states in units of MeV:

$$M_{Z_b} = 10610 \quad \text{and} \quad M_{Z_b'} = 10650, \tag{29}$$

with a hadronic width in units of MeV:

$$\Gamma_{Z_b} = 15.6 \pm 2.5 \quad \text{and} \quad \Gamma_{Z_b'} = 14.4 \pm 3.2, \tag{30}$$

respectively [52]. The analysis of the Z_b states decay in the channel $Z_b^+ \to \Upsilon(2S)\pi^+$ favors the $J^P = 1^+$ assignment, which is the same as the one of the X_b, although X_b has positive charge conjugation and the neutral partner of Z_b should have negative charge conjugation.

Considering the errors in Eq. (28) and the small mass difference between X_b and $\sqrt{t_c}$, it is difficult to identify the two observed Z_b states with the X_b.

5. SU(3) mass-splittings

We extend the previous analysis to study the ratio between the strange T_{cc}^s ($cc\bar{s}\bar{s}$) and non-strange T_{cc} states.

[5] The result using the $D\bar{D}^*$ molecule current would be the same as we have shown in [27] that the masses obtained from the four-quark and molecule currents are degenerate.

[6] We have neglected the contribution of the triple gluon condensate $\langle g^3 f_{abc} G^3\rangle$ due to the $(1/16\pi^2)^2$ loop factor suppression compared to $\langle\bar{q}q\rangle^2$.

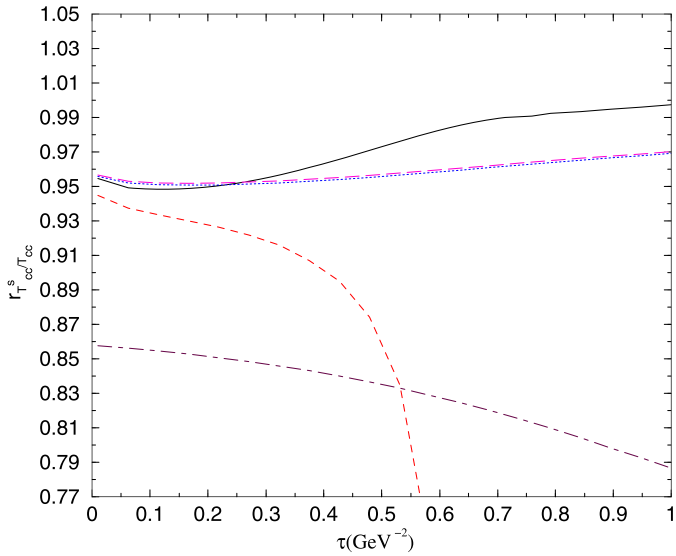

Fig. 7. Different QCD contributions to the DRSR $r_{T^s_{cc}/T_{cc}}$ defined in Eq. (32) for $\sqrt{t_c} = 4.15$ GeV and $m_c = 1.23$ GeV: (1) = Pert + m_s: dot-dashed (maroon); (2) = (1) + $\langle\bar{q}q\rangle$: long-dashed (magenta); (3) = (2) + $\langle\alpha_s G^2\rangle$: dotted (blue); (4) = (3) + $\langle g\bar{q}Gq\rangle$: dashed (red); (5) = (4) + $\langle\bar{q}q\rangle^2$: continuous (black). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

The QCD expression for the spectral function proportional to m_s for the current in Eq. (6) is:

$$\begin{aligned}
\rho^{m_s}(s) = {} & \frac{m_s}{2^8\pi^4}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha}\Bigg\{\langle\bar{s}s\rangle\frac{7[m_c^2-\alpha(1-\alpha)s]^2}{(1-\alpha)} \\
& + \int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}\big[(\alpha+\beta)m_c^2-\alpha\beta s\big] \\
& \times\Bigg[\langle\bar{s}s\rangle\big(m_c^2(21+\alpha+\beta)-6\big[(\alpha+\beta)m_c^2-\alpha\beta s\big]\big) \\
& - \frac{m_c}{2\pi^2\alpha\beta^2}(3+\alpha+\beta)(1-\alpha-\beta) \\
& \times\big[(\alpha+\beta)m_c^2-\alpha\beta s\big]^2\Bigg]\Bigg\}. \qquad (31)
\end{aligned}$$

We start by studying the DRSR:

$$r_{T^s_{cc}/T_{cc}} \equiv \sqrt{\frac{\mathcal{R}_{T^s_{cc}}}{\mathcal{R}_{T_{cc}}}} \simeq \frac{M_{T^s_{cc}}}{M_{T_{cc}}}. \qquad (32)$$

We study the convergence of the DRSR versus τ in Fig. 7, where the show the strength of the different contributions in the OPE. We note that we have τ-stability around $\tau = 0.2$ GeV^{-2}, while the OPE breaks down for $\tau \geqslant 0.5$ GeV^{-2}.

In Fig. 8, we show the τ-dependence of the ratio in Eq. (32), for $\sqrt{t_c} = 4.15$ GeV and for two values of m_c. From this figure one can deduce around the τ-stability:

$$r_{T^s_{cc}/T_{cc}} = 0.95 \sim 0.98, \qquad (33)$$

which gives a smaller mass for T^s_{cc} than for T_{cc}. This result is similar to the result obtained for X^s in Ref. [26]. However, in this case, the decrease in the mass is even bigger than the obtained for X^s: $r_{X^s/X} = 0.984 \pm 0.009$. This result is consistent with what is obtained from the DRSR:

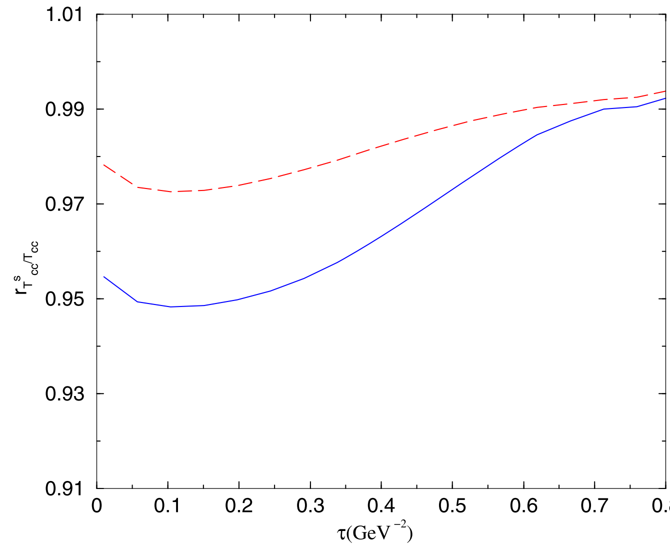

Fig. 8. Same as Fig. 1 for $r_{T^s_{cc}/T_{cc}}$ defined in Eq. (32) for $\sqrt{t_c} = 4.15$ GeV and for two values of $m_c = 1.23$ (solid line) and 1.47 GeV (dashed line).

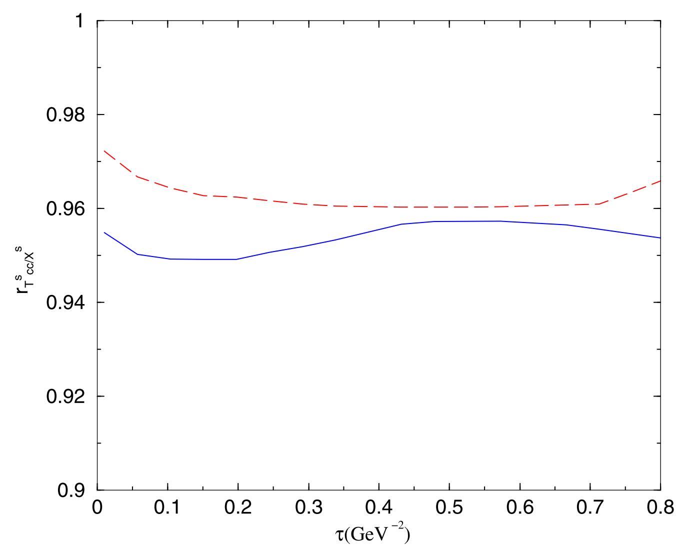

Fig. 9. Same as Fig. 1 for $r_{T^s_{cc}/X^s}$. The solid and dashed lines are for $m_c = 1.23$ and 1.47 GeV respectively.

$$r_{T^s_{cc}/X^s} \equiv \sqrt{\frac{\mathcal{R}_{T^s_{cc}}}{\mathcal{R}_{X^s}}} \simeq \frac{M_{T^s_{cc}}}{M_{X^s}}, \qquad (34)$$

as can be seen in Fig. 9.

In Fig. 9 we have used $t_c = 4.15$ GeV. However, the result is very stable as a function of t_c, as can be seen in Fig. 10.

Since for the T^s_{cc} state, considered as a $D_s D^*_s$ molecule, there is no allowed pion exchange, one cannot conclude, from the analysis above, that the T^s_{cc} should be more deeply bound than the T_{cc}. On the contrary, if the pion exchange is important for binding the two mesons, the T^s_{cc} may not be bound.

6. Conclusion

In conclusion, we have studied the mass of the T_{cc} using double ratios of sum rules (DRSR), which are more accurate than the usual simple ratios used in the literature. We found that the molecular currents $D\bar{D}^* + c.c.$ and DD^* lead to (almost) the same mass predictions within the accuracy of the method. Since the pion

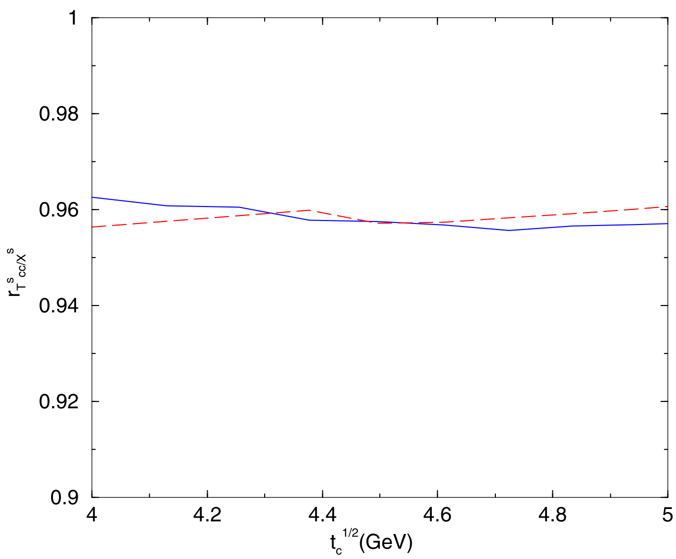

Fig. 10. Same as Fig. 2 for $r_{T^s_{cc}/X^s}$. The solid and dashed lines are for $m_c = 1.23$ and 1.47 GeV respectively.

exchange interaction between these mesons is exactly the same, we conclude that if the observed $X(3872)$ meson is a $D\bar{D}^* + c.c.$ molecule, then the DD^* molecule should also exist with approximately the same mass. A recent estimate of the production rate indicates that these states could be seen in LHC experiments [53].

We have also studied the double ratio r_{T_{bb}/X_b} using molecular currents $\bar{B}\bar{B}^*$ and $B\bar{B}^* + c.c.$ for T_{bb} and X_b respectively. In this case the degeneracy between the two masses is even better than in the charm case. Therefore, we also conclude for the bottom case that if a molecular state $B\bar{B}^* + c.c.$ exist, then the $\bar{B}\bar{B}^*$ molecule should also exist with the same mass.

Acknowledgements

This work has been partly supported by the CNRS-FAPESP program, by CNPq-Brazil and by the CNRS-IN2P3 within the project Non-perturbative QCD and Hadron Physics. We thank R.M. Albuquerque for checking some of the QCD expressions of the two-point correlator.

References

[1] S.-K. Choi, et al., Belle Collaboration, Phys. Rev. Lett. 91 (2003) 262001.
[2] B. Aubert, et al., BaBar Collaboration, Phys. Rev. D 71 (2005) 071103.
[3] D. Acosta, et al., CDF II Collaboration, Phys. Rev. Lett. 93 (2004) 072001.
[4] V.M. Abazov, et al., D0 Collaboration, Phys. Rev. Lett. 93 (2004) 162002.
[5] M. Nielsen, F.S. Navarra, S.H. Lee, Phys. Rept. 497 (2010) 41.
[6] D. Janc, M. Rosina, Few Body Sust. 35 (2004) 175.
[7] S. Zouzou, B. Silvestre-Brac, C. Gignoux, J.M. Richard, Z. Phys. C 30 (1986) 457.
[8] J.P. Ader, J.M. Richard, P. Taxil, Phys. Rev. D 25 (1982) 2370;
L. Heller, J.A. Tjon, Phys. Rev. D 35 (1987) 969.
[9] H.J. Lipkin, Phys. Lett. B 172 (1986) 242;
C.-K. Chow, Phys. Rev. D 51 (1995) 3541.
[10] J. Vijande, A. Valcarce, K. Tsushima, Phys. Rev. D 74 (2006) 054018.
[11] B. Silvestre-Brac, C. Semay, Z. Phys. C 57 (1993) 273;
B. Silvestre-Brac, C. Semay, Z. Phys. C 59 (1993) 457;
C. Semay, B. Silvestre-Brac, Z. Phys. C 61 (1994) 271.
[12] D.M. Brink, Fl. Stancu, Phys. Rev. D 57 (1998) 6778.
[13] J. Vijande, A. Valcarce, N. Barnea, Phys. Rev. D 79 (2009) 074010, arXiv:0903.2949 [hep-ph].
[14] J. Carlson, V.R. Pandharipande, Phys. Rev. D 43 (1991) 1652.
[15] J. Vijande, A. Valcarce, J.M. Richard, Phys. Rev. D 76 (2007) 114013.
[16] S. Pepin, Fl. Stancu, M. Genovese, J.M. Richard, Phys. Lett. B 393 (1997) 119.
[17] N.A. Törnqvist, Z. Phys. C 61 (1994) 525.
[18] E.S. Swanson, Phys. Rept. 429 (2006) 243;
J.M. Richard, talk given at QCD 05 (Montpellier, 4–8th July 2005), Nucl. Phys. B (Proc. Suppl.) 164 (2007) 131, hep-ph/0601043.
[19] F.S. Navarra, M. Nielsen, S.H. Lee, Phys. Lett. B 649 (2007) 166.
[20] Y. Cui, X.-L. Chen, W.-Z. Deng, S.-L. Zhu, High Energy Phys. Nucl. Phys. 31 (2007) 7.
[21] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.
[22] L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rept. 127 (1985) 1.
[23] For a review and references to original works, see e.g., S. Narison, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. 17 (2002) 1, arXiv:hep-ph/0205006;
S. Narison, World Sci. Lect. Notes Phys. 26 (1989) 1;
S. Narison, Acta Phys. Pol. B 26 (1995) 687;
S. Narison, Riv. Nuov. Cim. 10N2 (1987) 1;
S. Narison, Phys. Rept. 84 (1982) 263.
[24] S. Narison, Phys. Lett. B 210 (1988) 238.
[25] S. Narison, Phys. Lett. B 387 (1996) 162.
[26] R.D. Matheus, S. Narison, M. Nielsen, J.M. Richard, Phys. Rev. D 75 (2007) 014005.
[27] S. Narison, F.S. Navarra, M. Nielsen, Phys. Rev. D 83 (2011) 016004.
[28] S. Narison, Phys. Lett. B 605 (2005) 319.
[29] S. Narison, Phys. Lett. B 322 (1994) 327.
[30] S. Narison, Phys. Rev. D 74 (2006) 034013;
S. Narison, Phys. Lett. B 358 (1995) 113;
S. Narison, Phys. Lett. B 466 (1999) 345.
[31] R.M. Albuquerque, S. Narison, M. Nielsen, Phys. Lett. B 684 (2010) 236;
R.M. Albuquerque, S. Narison, Phys. Lett. B 694 (2010) 217.
[32] S. Narison, Phys. Lett. B 337 (1994) 166;
S. Narison, Phys. Lett. B 668 (2008) 308.
[33] R.D. Matheus, F.S. Navarra, M. Nielsen, C.M. Zanetti, Phys. Rev. D 80 (2009) 056002.
[34] E.G. Floratos, S. Narison, E. de Rafael, Nucl. Phys. B 155 (1979) 155.
[35] S. Narison, R. Tarrach, Phys. Lett. B 125 (1983) 217.
[36] K. Chetyrkin, S. Narison, V.I. Zakharov, Nucl. Phys. B 550 (1999) 353;
S. Narison, V.I. Zakharov, Phys. Lett. B 522 (2001) 266;
S. Narison, V.I. Zakharov, Phys. Lett. B 679 (2009) 355.
[37] For reviews, see V.I. Zakharov, Nucl. Phys. B (Proc. Suppl.) 164 (2007) 240;
S. Narison, Nucl. Phys. B (Proc. Suppl.) 164 (2007) 225.
[38] S. Narison, Phys. Lett. B 673 (2009) 30.
[39] For reviews, see S. Narison, Phys. Rev. D 74 (2006) 034013, arXiv:hep-ph/0202200;
S. Narison, Phys. Lett. B 216 (1989) 191;
S. Narison, H.G. Dosch, Phys. Lett. B 417 (1998) 173;
S. Narison, N. Paver, E. de Rafael, D. Treleani, Nucl. Phys. B 212 (1983) 365;
S. Narison, E. de Rafael, Phys. Lett. B 103 (1981) 57;
C. Becchi, S. Narison, E. de Rafael, F.J. Yndurain, Z. Phys. C 8 (1981) 335.
[40] C. Amsler, et al., Phys. Lett. B 667 (2008) 1.
[41] S. Narison, Phys. Lett. B 693 (2010) 559, and references therein;
S. Narison, arXiv:1105.2922 [hep-ph];
S. Narison, arXiv:1105.5070 [hep-ph].
[42] S. Narison, Phys. Lett. B 197 (1987) 405;
S. Narison, Phys. Lett. B 341 (1994) 73;
S. Narison, Phys. Lett. B 520 (2001) 115.
[43] B.L. Ioffe, K.N. Zyablyuk, Eur. Phys. J. C 27 (2003) 229;
B.L. Ioffe, Prog. Part. Nucl. Phys. 56 (2006) 232.
[44] Y. Chung, et al., Z. Phys. C 25 (1984) 151;
H.G. Dosch, Non-Perturbative Methods, Montpellier, 1985;
H.G. Dosch, M. Jamin, S. Narison, Phys. Lett. B 220 (1989) 251.
[45] B.L. Ioffe, Nucl. Phys. B 188 (1981) 317;
B.L. Ioffe, Nucl. Phys. B 191 (1981) 591;
A.A. Ovchinnikov, A.A. Pivovarov, Yad. Fiz. 48 (1988) 1135.
[46] G. Launer, S. Narison, R. Tarrach, Z. Phys. C 26 (1984) 433.
[47] S. Narison, Phys. Lett. B 300 (1993) 293.
[48] R.A. Bertlmann, G. Launer, E. de Rafael, Nucl. Phys. B 250 (1985) 61;
R.A. Bertlmann, et al., Z. Phys. C 39 (1988) 231.
[49] F.J. Yndurain, Phys. Rept. 320 (1999) 287, arXiv:hep-ph/9903457.
[50] J.S. Bell, R.A. Bertlmann, Nucl. Phys. B 227 (1983) 435;
R.A. Bertlmann, Acta Phys. Austriaca 53 (1981) 305;
R.A. Bertlmann, H. Neufeld, Z. Phys. C 27 (1985) 437.
[51] S. Narison, Phys. Lett. B 361 (1995) 121;
S. Narison, Phys. Lett. B 624 (2005) 223.
[52] I. Adachi, et al., Belle Collaboration, arXiv:1105.4583.
[53] Yu-qi Chen, Su-zhi Wu, arXiv:1101.4568 [hep-ph].