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Momentum-dependent flow fluctuations as a hydrodynamic response to initial geometry

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Abstract

We propose a redefinition of the principal component analysis (PCA) of anisotropic flow that makes it more directly connected to fluctuations of the initial geometry of the system. Then, using state-of-the-art hydrodynamic simulations, we make an explicit connection between flow fluctuations and a cumulant expansion of the initial transverse geometry. In particular, we show that the second principal component of elliptic flow is generated by higher-order cumulants, and therefore probes smaller length scales of the initial state. With this information, it will be possible to put new constraints on properties of the early-time dynamics of a heavy-ion collision, including small-scale structure, as well as properties of the quark-gluon plasma.

Keywords: heavy-ion collisions, collective dynamics, anisotropic flow, principal component analysis

1. Introduction

In ultrarelativistic heavy-ion collisions, momentum-space anisotropies in the final distribution of particles are understood to originate from pressure gradients in the early stages of the quark-gluon plasma (QGP). Here, we study how the small-scale structure of the initial state affects momentum-dependent correlations of the anisotropic flow, which can be efficiently visualized by employing a principal component analysis (PCA) [1, 2]. In order to connect this analysis with features of the initial state, we redefine the PCA observables and investigate how cumulants of the initial geometry correlate with flow harmonics in the final state, in an event-by-event basis [3, 4, 5].

2. Principal Components of Anisotropic Flow

Principal component analysis is a statistical method that allows one to find, among a set of correlated variables, linearly independent combinations of maximum variance. Here, we study the PCA of the flow

harmonics $V_n(p_T)$, defined as Fourier coefficients of the azimuthal distribution of particles at transverse momentum p_T:

$$\frac{dN}{p_T dp_T \, d\varphi} = \frac{1}{2\pi} N(p_T) \sum_{n=-\infty}^{\infty} V_n(p_T) \, e^{-in\varphi} \,. \tag{1}$$

The principal components $V_n^{(\alpha)}(p_T)$ are found by diagonalizing the flow covariance matrix, such that:

$$V_{n\Delta}(p_{T1}, p_{T2}) = \langle V_n(p_{T1}) V_n^*(p_{T2}) \rangle \simeq \sum_{\alpha=1}^{\alpha_{\max}} V_n^{(\alpha)}(p_{T1}) V_n^{(\alpha)}(p_{T2}) \,, \tag{2}$$

where the components are put in descending order: $\|V_n^{(\alpha)}\| \geq \|V_n^{(\alpha+1)}\|$. Since a large hierarchy is found among the different components, the RHS of Eq. (2) might be truncated, say, at $\alpha_{\max} = 2$ [1].

In [2], we have shown that the original flow PCA observables proposed by Bhalerao *et al.* in [1], and measured by the CMS Collaboration [6], are sensitive not only to fluctuations of anisotropic flow $V_n(p_T)$, but also to fluctuations of the number of particles in each bin, $N(p_T)$. The reason behind this is that they were defined from the covariance matrix $V_{n\Delta}^N(p_{T1}, p_{T2}) = \langle N(p_{T1}) N(p_{T2}) V_n(p_{T1}) V_n^*(p_{T2}) \rangle$. Since the anisotropic flow per particle V_n is better correlated with the initial geometry, the sensitivity to fluctuations of $N(p_T)$ is undesirable for our purposes.[1] To solve this issue, we divide the covariance matrix $V_{n\Delta}^N$ by $\langle N(p_T^a) N(p_T^b) \rangle$, so as to approximately cancel out particle-number fluctuations [2]:

$$V_{n\Delta}^R(p_{T1}, p_{T2}) = \frac{\langle N(p_{T1}) N(p_{T2}) V_n(p_{T1}) V_n^*(p_{T2}) \rangle}{\langle N(p_{T1}) N(p_{T2}) \rangle} \simeq \langle V_n(p_{T1}) V_n^*(p_{T2}) \rangle \,. \tag{3}$$

To elucidate how the different PCA observables respond to particle-number and anisotropic flow fluctuations, we study them in a controlled toy model. We sample the distribution of particles in p_T with an azimuthal distribution with constant $|V_2(p_T)|$ and a simple exponential spectra $N(p_T) \propto e^{-p_T/\overline{p_T}}$, with a stochastic value of the total charged multiplicity. We test the PCA observables within the toy model in four different scenarios, with

1. only global particle-number fluctuations, with no transverse-momentum dependence;
2. fluctuations of the event plane, with some profile $\Psi_{EP}(p_T)$, as a simplified model of subleading flow;
3. fluctuations of the spectrum, with $\overline{p_T} \rightarrow \overline{p_T} + \delta\overline{p_T}$;
4. fluctuations of both the event plane $\Psi_{EP}(p_T)$ and the mean transverse momentum $\overline{p_T}$.

Results are shown in Fig. 1. In this figure, we see that, unlike the original PCA observables proposed by Bhalerao *et al.*, the new PCA observables are insensitive to particle-number fluctuations, thus more directly probing fluctuations of the initial geometry.

For more details on the redefinition of the PCA of anisotropic flow, see Ref. [2].

3. Mapping Hydrodynamic Response

In order to investigate the physical content of the principal components of anisotropic flow [8, 9], we apply the mapping of hydrodynamic response to initial geometry from [3, 4, 5]. This mapping provides an extension of the usual eccentricity scaling of elliptic flow through a systematic expansion, starting from the assumption that the posterior evolution of the system and the final one-particle distribution are fully determined by the transverse energy-density profile $\rho(\tau_0, \vec{x})$ at an initial proper time $\tau = \tau_0$. The initial transverse profile of the system, in turn, can be fully characterized by a (infinite) set of cumulants $W_{n,m}$ [5]. For instance,

$$W_{0,2} = \frac{i^2}{4} \left[\{z^*z\} - \{z^*\}\{z\} \right], \qquad W_{2,2} = \frac{i^2}{8} \left[\{z^2\} - \{z\}^2 \right], \qquad \{(\cdots)\} \equiv \frac{\int d^2x \, \rho(\vec{x}) \, (\cdots)}{\int d^2x \, \rho(\vec{x})} \,, \tag{4}$$

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[1] While fluctuations of particle number can be of interest, they can be characterized separately by the covariance matrix $\langle \Delta N(p_{T1}) \Delta N(p_{T2}) \rangle$ [1, 7].
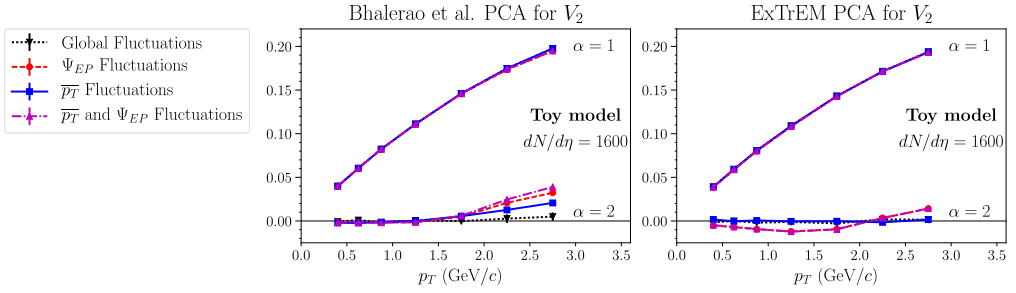
Fig. 1. Toy model results for the PCA of elliptic flow for the original definition in Bhalerao *et al.* [1] (left panel) and the new improved definition proposed in [2] (right panel). Scenario 1 (black dotted curve) includes only global fluctuations; scenario 2 (red dashed curve) also includes subleading flow in the form of event plane fluctuations; scenario 3 (solid blue curve) includes fluctuations of the mean transverse momentum; scenario 4 (dot-dashed magenta curve) includes all the three sources of fluctuations. Both definitions of the PCA yield a subleading mode in the presence of subleading flow, and neither of them are sensitive to global multiplicity fluctuations. However, the Bhalerao *et al.* PCA [1] is sensitive to fluctuations of the mean transverse momentum, while the new, improved PCA is sensitive to subleading flow alone [2].

where $z \equiv x + iy$ and we have defined the "spatial average" $\{(\cdots)\}$. From them, we can define generalized eccentricities

$$\epsilon_{n,m} \equiv -W_{n,m}/(W_{0,2})^{m/2}, \tag{5}$$

where the first index n specifies the symmetry under rotations, while m is related to the typical length scales being probed, with higher values of m corresponding to increasingly finer details of the initial conditions.

Since we assume that the full evolution of the system is determined from $\rho(\tau_0, \vec{x})$, the flow harmonics V_n can be predicted by the set of all eccentricities and, if they are not too large, can be approximated by a Taylor series in $\epsilon_{n,m}$. Also assuming that large scales dominate the final flow pattern, we can truncate the expansion of $V_n[\{\epsilon_{n,m}\}]$ at a finite value of m. The elliptic flow V_2, for instance, is approximated by

$$
\begin{aligned}
V_2(p_T) &\simeq \kappa_0(p_T)\,\epsilon_{2,2} + \kappa_1(p_T)\,\epsilon_{2,4} + \kappa_2(p_T)\,\epsilon_{2,6} + \kappa_3(p_T)\,\epsilon_{2,8} + O(m = 10) \\
&+ \kappa_4(p_T)\,|\epsilon_{2,2}|^2\,\epsilon_{2,2} + \kappa_5(p_T)\,\epsilon_4\epsilon_2^* + \kappa_6(p_T)\,\epsilon_{1,3}^2 + \ldots + O(\epsilon^3),
\end{aligned}
\tag{6}
$$

where linear terms are placed on the first line and only terms with the correct symmetry under rotations are included. In hydrodynamic simulations, the coefficients $\kappa_i(p_T)$ can be independently fixed for each p_T bin, by maximizing the correlation between the final $V_2(p_T)$ and the predictor in Eq. (6).

4. Results and Conclusions

To investigate the connection between the PCA of the flow per particle and the initial geometry, we employ a state-of-the-art event-by-event hydrodynamic model (TrENTO+MUSIC+iSS+UrQMD), with parameters optimized to reproduce LHC data [2, 10, 11, 12, 13, 14, 15, 16]. In particular, we test if the PCA of the flow per particle can be reproduced by the approximation in Eq. (6), and study which terms in this expression are the most important for each principal component. Results are shown in Fig. 2, where we check that the description of flow fluctuations is visibly improved by considering higher-order eccentricities of the initial geometry. In particular, we find that the second principal component of V_2 is largely sensitive to the subleading eccentricity $\epsilon_{2,4}$, which is sensitive to the smaller-scale structure of the initial geometry. In this figure, the PCA is defined directly from $\langle V_n(p_{T1})V_n^*(p_{T2})\rangle$, which is approximately equivalent to the matrix in Eq. (3) [2].

We conclude that the redefined subleading principal component of elliptic flow is a valuable probe of fluctuations of the initial stages at smaller length scales. As such, it will provide fresh insight into the formation and properties of the QGP, establishing new constraints for models of high-energy heavy-ion collisions.
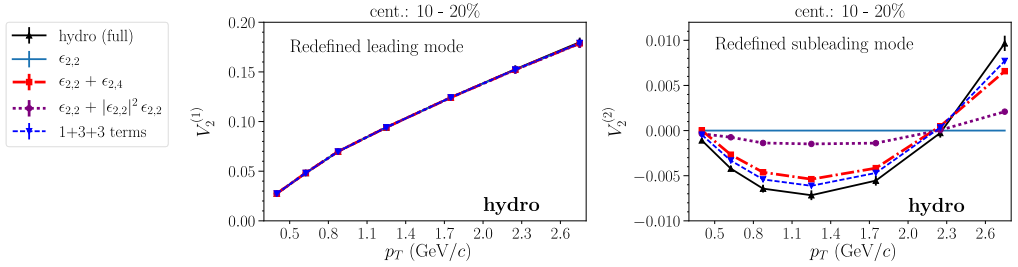
Fig. 2. Comparison between the principal components of the elliptic flow per particle in a full hydrodynamic simulation (solid black curve) and the ones predicted from Eq. (6) (dashed blue curve). Also shown are the predictions from Eq. (6) when only the first subleading linear term $\epsilon_{2,4}$ (dash-dotted red curve) or the leading nonlinear term $|\epsilon_{2,2}|^2\epsilon_{2,2}$ (magenta dotted line) are considered. While the leading principal component (left panel) is correctly predicted as long as the leading eccentricity $\epsilon_{2,2}$ is included, the subleading principal component (right panel) is only reproduced if the linear response to $\epsilon_{2,4}$ is included.

Acknowledgments

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