Korteveg-de Vries solitons in a cold quark-gluon plasma

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The relativistic heavy ion program developed at RHIC and now at LHC motivated a deeper study of the properties of the quark-gluon plasma (QGP) and, in particular, the study of perturbations in this kind of plasma. We are interested on the time evolution of perturbations in the baryon and energy densities. If a localized pulse in baryon density could propagate throughout the QGP for long distances preserving its shape and without loosing localization, this could have interesting consequences for relativistic heavy ion physics and for astrophysics. A mathematical way to prove that this can happen is to derive (under certain conditions) from the hydrodynamical equations of the QGP a Korteveg-de Vries (KdV) equation. The solution of this equation describes the propagation of a KdV soliton. The derivation of the KdV equation depends crucially on the equation of state (EOS) of the QGP. The use of the simple MIT bag model EOS does not lead to KdV solitons. Recently we have developed an EOS for the QGP which includes both perturbative and nonperturbative corrections to the MIT one and is still simple enough to allow for analytical manipulations. With this EOS we were able to derive a KdV equation for the cold QGP.

I. INTRODUCTION

Korteweg-de Vries (KdV) solitons are very interesting nonlinear waves, which may exist in many types of fluids from ordinary water to astrophysical plasmas [1]. In recent years we have started to produce a new kind of fluid in laboratory: the quark-gluon plasma (QGP). This is a state where quarks and gluons, usually confined in the interior of baryons (such as the proton) and mesons, are free to travel longer distances. With the beginning of the LHC era, we have means to study larger and longer living samples of QGP and even the propagation of perturbations in this new medium. In this context a natural question is can we have KdV solitons in the QCD plasma? In this work answer this question.

Before the QGP there were other fluids made of strongly interacting hadronic matter and the existence of KdV solitons in these fluids had already been investigated. The first works on the subject were published in [2], where the authors considered the propagation of baryon density pulses in proton-nucleus collisions at intermediate energies. In this scenario the incoming proton would be absorbed by the nuclear fluid generating a KdV soliton, which, traversing the whole nucleus without distortion, would escape from the target as a proton and would simulate an unexpected transparency. In [2] the existence of the KdV soliton relied solely on the equation of state (EOS), which had without substantive justification. In [3] we reconsidered the problem, introducing an equation of state derived from relativistic mean field models of nuclear matter. We concluded that the homogeneous meson field approximation was too strong and would exclude the existence of KdV solitons. We could also trace back the derivative terms in the energy density to derivative couplings between the

nucleon and the vector meson. In [4] we extended our analysis to relativistic hydrodynamics and in [5] to spherical and cylindrical geometries. In [6] we considered hadronic matter at finite temperature and studied the effects of temperature on the KdV soliton. In [7] we started the study of perturbations in the QGP at zero and finite temperature. The conclusion found in that work was that the existence of KdV solitons in a QGP depends on details of the EOS and with a simple MIT bag model EOS there is no KdV soliton! A further study of the equation of state, carried out in [8], showed that if nonperturbative effects are included in the EOS through gluon condensates, then new terms appear in the expression of the energy density and pressure. In [8] we focused on the simpler case of a cold ($T = 0$) plasma. The obtained EOS, because of nonperturbative effects and because of the inclusion of hard gluons, is able to generate more pressure and thus can explain the puzzling massive neutron stars recently observed [9].

In the present work we show how the new derivative terms lead to a KdV equation, after the proper treatment of the hydrodynamical equations. This new EOS can be applied to the study of the propagation of perturbations in the core of dense stars.

In the next section we briefly review the equations of one-dimensional relativistic fluid dynamics. In section III we introduce the equation of state, in Sec. IV we derive the KdV equation and in Sec. V we present a numerical analysis of the obtained equation.

II. RELATIVISTIC FLUID DYNAMICS

Relativistic hydrodynamics is well presented in the textbooks [10,11]. The relativistic version of the Euler equation [7,10,11] is given by

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{(\varepsilon + p)\gamma^2}\left(\vec{\nabla}p + \vec{v}\frac{\partial p}{\partial t}\right), \quad (1)$$

where $\vec{v}$, ε, p, and γ are the velocity, energy density, pressure, and the Lorentz factor, respectively. We employ the natural units $c = 1$ and $\hbar = 1$. Space and time coordinates will be in fm(1 fm $= 10^{-15}m$). The relativistic version of the continuity equation for the baryon density is [10]

$$\partial_\nu j_B{}^\nu = 0 \quad (2)$$

Since $j_B{}^\nu = u^\nu \rho_B$ the above equation can be rewritten as [7]

$$\frac{\partial \rho_B}{\partial t} + \gamma^2 v \rho_B\left(\frac{\partial v}{\partial t} + \vec{v} \cdot \vec{\nabla}v\right) + \vec{\nabla} \cdot (\rho_B \vec{v}) = 0, \quad (3)$$

where ρ_B is the baryon density.

Equations (1) and (3) may be studied in the two-dimensional case which is relevant for heavy ion collisions, but in this work we treat only the one-dimensional case for simplicity and also because it may be realistic in the case of radial, spherically symmetric perturbations (relevant for stars). The present work is in Cartesian coordinates and as mentioned before, we have already developed this formalism for the case of a spherical geometry [5].

In the one-dimensional Cartesian relativistic fluid dynamics the velocity field is written as $\vec{v} = v(x, t)\hat{x}$ where $\hat{x}$ is the unit vector in the x direction. Equations (1) and (3) can be rewritten in the simple form:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{(v^2 - 1)}{(\varepsilon + p)}\left(\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial t}\right) \quad (4)$$

and

$$v\rho_B\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) + (1 - v^2)\left(\frac{\partial \rho_B}{\partial t} + \rho_B\frac{\partial v}{\partial x} + v\frac{\partial \rho_B}{\partial x}\right) = 0. \quad (5)$$

III. THE QGP EQUATION OF STATE

In what follows we present the mean field treatment of QCD developed in [8] (for previous works on the subject see [12,13]) and go beyond the homogeneous field approximation, including the terms with gradients.

The Lagrangian density of QCD is given by

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{q=1}^{N_f}\bar{\psi}^q_i[i\gamma^\mu(\delta_{ij}\partial_\mu - igT^a_{ij}G^a_\mu) - \delta_{ij}m_q]\psi^q_j, \quad (6)$$

where

$$F^{a\mu\nu} = \partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} + gf^{abc}G^{b\mu}G^{c\nu}. \quad (7)$$

The summation on q runs over all quark flavors, m_q is the mass of the quark of flavor q, i and j are the color indices of the quarks, T^a are the SU(3) generators and f^{abc} are the

SU(3) antisymmetric structure constants. For simplicity we will consider massless quarks, i.e. $m_q = 0$. Moreover, we will drop the summation and consider only one flavor. At the end of our calculation the number of flavors will be recovered. Following [12,13], we shall write the gluon field as

$$G^{a\mu} = A^{a\mu} + \alpha^{a\mu}, \quad (8)$$

where $A^{a\mu}$ and $\alpha^{a\mu}$ are the low ("soft") and high ("hard") momentum components of the gluon field, respectively. We will assume that $A^{a\mu}$ represents the soft modes which populate the vacuum and the terms containing $A^{a\mu}$ will be replaced by their expectation values $\langle A^{a\mu}\rangle$, $\langle A^{a\mu}A^a_\mu\rangle$, etc. in the plasma. $\alpha^{a\mu}$ represents the modes for which the running coupling constant is small.

In a cold quark-gluon plasma the density is much larger than the ordinary nuclear matter density. These high densities imply a very large number of sources of the gluon field. Assuming that the coupling constant is not very small, the existence of intense sources implies that the bosonic fields tend to have large occupation numbers at all energy levels, and therefore they can be treated as classical fields. This is the famous approximation for bosonic fields used in relativistic mean field models of nuclear matter [14]. It has been applied to QCD in the past and amounts to assume that the hard gluon field, α^a_μ, is simply a function of the coordinates:

$$\alpha^a_\mu(\vec{x}, t) = \delta_{\mu 0}\alpha^a_0(\vec{x}, t) \quad (9)$$

with $\partial_\nu\alpha^a_\mu \neq 0$. This space and time dependence goes beyond the standard mean field approximation [14], where α^a_μ is constant in space and time and consequently $\partial_\nu\alpha^a_\mu = 0$. We keep assuming, as in [8], that the soft gluon field $A^{a\mu}$ is independent of position and time and thus $\partial^\nu A^{a\mu} = 0$. Following the same steps introduced in [8] we obtain the following effective Lagrangian:

$$\mathcal{L}_0 = -\frac{1}{2}\alpha^a_0(\vec{\nabla}^2\alpha^a_0) + \frac{m_G{}^2}{2}\alpha^a_0\alpha^a_0 - \mathcal{B}_{QCD} + \bar{\psi}_i(i\delta_{ij}\gamma^\mu\partial_\mu + g\gamma^0T^a_{ij}\alpha^a_0)\psi_j, \quad (10)$$

where m_G is the dynamical mass of the hard gluon α generated by its interaction with the soft gluons $A^{a\mu}$ from the vacuum and it is related to the dimension-two $\langle A^2\rangle$ gluon condensate. The constant $\mathcal{B}_{QCD}$ is related to the dimension four gluon condensate $\langle F^2\rangle$ (see [8] for details).

The effective Lagrangian (10) is quite similar to the one obtained in [8] and the only difference is the first term, which is new and comes from the gradients. The equations of motion [6] are given by

$$\frac{\partial\mathcal{L}}{\partial\eta_i} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\eta_i)} + \partial_\nu\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\partial_\nu\eta_i)}\right] = 0. \quad (11)$$

Inserting (10) into (11) with $\eta_1 = \alpha^a_0(\vec{x}, t)$ and $\eta_2 = \bar{\psi}(\vec{x}, t)$ we find

$$-\vec{\nabla}^2 \alpha_0^a + m_G{}^2 \alpha_0^a = -g\rho^a \tag{12}$$

$$(i\gamma^\mu \partial_\mu + g\gamma^0 T^a \alpha_0^a)\psi = 0, \tag{13}$$

where ρ^a is the temporal component of the color vector current given by $j^{a\nu} = \bar{\psi}_i \gamma^\nu T_{ij}^a \psi_j$. In the end of the calculation the color index a will be averaged and the energy density and pressure will be color neutral as they should be. The quark densities will have their color indices contracted and final quantities appearing in the equation of state will be number densities (or baryon number densities).

The energy-momentum tensor reads [6]

$$T^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu \eta_i)}(\partial^\nu \eta_i) - g^{\mu\nu}\mathcal{L} - \left[\partial_\beta \frac{\partial\mathcal{L}}{\partial(\partial_\mu \partial_\beta \eta_i)}\right](\partial^\nu \eta_i)$$
$$+ \frac{\partial\mathcal{L}}{\partial(\partial_\mu \partial_\beta \eta_i)}(\partial_\beta \partial^\nu \eta_i). \tag{14}$$

From the above expression we can obtain the energy density ($\varepsilon = <T_{00}>$) which turns out to be [8]

$$\varepsilon = \frac{1}{2}\alpha_0^a(\vec{\nabla}^2 \alpha_0^a) - \frac{m_G{}^2}{2}\alpha_0^a \alpha_0^a + \mathcal{B}_{\text{QCD}} - g\rho^a \alpha_0^a$$
$$+ 3\frac{\gamma_Q}{2\pi^2}\int_0^{k_F} dk k^2 \sqrt{\vec{k}^2 + m^2}, \tag{15}$$

where γ_Q is the quark degeneracy factor $\gamma_Q = 2(\text{spin}) \times 3(\text{flavor})$. The sum over all the color states was already performed and resulted in the prefactor 3 in the expression above. k_F is the Fermi momentum defined by the quark number density ρ:

$$\rho = \langle N|\psi_i^\dagger \psi_i|N\rangle = \frac{3}{V}\sum_{\vec{k},\lambda}\langle N|N\rangle = 3\frac{\gamma_Q}{(2\pi)^3}\int d^3k$$
$$= 3\frac{\gamma_Q}{2\pi^2}\int_0^{k_F} dk k^2 = \frac{\gamma_Q}{2\pi^2}k_F{}^3. \tag{16}$$

In the above expression $|N\rangle$ denotes a state with N quarks. In a first approximation the field α_0^a may be estimated from (12). Neglecting the derivative term $\vec{\nabla}^2 \alpha_0^a$ of (12) we have [6]:

$$\alpha_0^a \cong -\frac{g}{m_G{}^2}\rho^a. \tag{17}$$

Inserting (17) in the first term of (12) and then solving it for α_0^a we find

$$\alpha_0^a = -\frac{g}{m_G{}^2}\rho^a - \frac{g}{m_G{}^4}\vec{\nabla}^2 \rho^a. \tag{18}$$

We can write the color charge density ρ^a in terms of the quark number density ρ through

$$\rho^a \rho^a = 3\rho^2. \tag{19}$$

Analogously we have

$$\rho^a \vec{\nabla}^2 \rho^a = 3\rho \vec{\nabla}^2 \rho, \qquad \rho^a \vec{\nabla}^2(\vec{\nabla}^2 \rho^a) = 3\rho \vec{\nabla}^2(\vec{\nabla}^2 \rho). \tag{20}$$

Inserting (18)–(20) into (15), performing the momentum integral and using the baryon density, which is $\rho_B = \frac{1}{3}\rho$, we arrive at the final expression for the energy density in one spatial dimension:

$$\varepsilon = \left(\frac{27g^2}{2m_G{}^2}\right)\rho_B{}^2 + \left(\frac{27g^2}{2m_G{}^4}\right)\rho_B\frac{\partial^2\rho_B}{\partial x^2} + \left(\frac{27g^2}{2m_G{}^6}\right)\rho_B\frac{\partial^4\rho_B}{\partial x^4}$$
$$+ \left(\frac{27g^2}{2m_G{}^8}\right)\frac{\partial^2\rho_B}{\partial x^2}\frac{\partial^4\rho_B}{\partial x^4} + \mathcal{B}_{\text{QCD}} + 3\frac{\gamma_Q}{2\pi^2}\frac{k_F{}^4}{4}. \tag{21}$$

The pressure is given by $p = \frac{1}{3}<T_{ii}>$. Repeating the same steps mentioned before we arrive at

$$p = \left(\frac{27g^2}{2m_G{}^2}\right)\rho_B{}^2 + \left(\frac{18g^2}{m_G{}^4}\right)\rho_B\frac{\partial^2\rho_B}{\partial x^2} - \left(\frac{9g^2}{m_G{}^6}\right)\rho_B\frac{\partial^4\rho_B}{\partial x^4}$$
$$- \left(\frac{9g^2}{2m_G{}^4}\right)\frac{\partial\rho_B}{\partial x}\frac{\partial\rho_B}{\partial x} + \left(\frac{9g^2}{2m_G{}^6}\right)\frac{\partial^2\rho_B}{\partial x^2}\frac{\partial^2\rho_B}{\partial x^2}$$
$$- \left(\frac{9g^2}{m_G{}^8}\right)\frac{\partial^2\rho_B}{\partial x^2}\frac{\partial^4\rho_B}{\partial x^4} - \left(\frac{9g^2}{2m_G{}^8}\right)\frac{\partial^3\rho_B}{\partial x^3}\frac{\partial^3\rho_B}{\partial x^3}$$
$$- \left(\frac{9g^2}{m_G{}^6}\right)\frac{\partial\rho_B}{\partial x}\frac{\partial^3\rho_B}{\partial x^3} - \mathcal{B}_{\text{QCD}} + \frac{\gamma_Q}{2\pi^2}, \tag{22}$$

where now k_F defined by (16) is given by $\rho_B = k_F{}^3/\pi^2$.

IV. THE KDV EQUATION

We now combine Eqs. (4) and (5) to obtain the KdV equation which governs the space-time evolution of the perturbation in the baryon density. We first write (4) and (5) in terms of the dimensionless variables:

$$\hat{\rho} = \frac{\rho_B}{\rho_0}, \qquad \hat{v} = \frac{v}{c_s}, \tag{23}$$

where ρ_0 is an equilibrium (or reference) density, upon which perturbations may be generated, and c_s is the speed of sound. Next, we introduce the ξ and τ "stretched" coordinates [2,15]:

$$\xi = \sigma^{1/2}\frac{(x - c_s t)}{R}, \qquad \tau = \sigma^{3/2}\frac{c_s t}{R}, \tag{24}$$

where σ is a small expansion parameter, R is a typical size scale of the problem. After this change of variables we expand (23) as

$$\hat{\rho} = 1 + \sigma\rho_1 + \sigma^2\rho_2 + \dots \tag{25}$$

$$\hat{v} = \sigma v_1 + \sigma^2 v_2 + \dots. \tag{26}$$

Having rewritten (4) and (5) in the $\xi - \tau$ space and having expanded them in powers of σ up to σ^2 we organize the two equations as series in powers of σ. After these steps (4) and (5) become

$$\sigma\left\{-\left[\left(\frac{27g^2\rho_0^2}{m_G^2}\right)c_s^2 + 3\pi^{2/3}\rho_0^{4/3}c_s^2\right]\frac{\partial v_1}{\partial\xi} + \left[\left(\frac{27g^2\rho_0^2}{m_G^2}\right) + \pi^{2/3}\rho_0^{4/3}\right]\frac{\partial\rho_1}{\partial\xi}\right\}$$

$$+ \sigma^2\left\{\left[\left(\frac{27g^2\rho_0^2}{m_{G^2}}\right) + \pi^{2/3}\rho_0^{4/3}\right]\frac{\partial\rho_2}{\partial\xi} - \left[\left(\frac{27g^2\rho_0^2}{m_G^2}\right)c_s^2 + 3\pi^{2/3}\rho_0^{4/3}c_s^2\right]\frac{\partial v_2}{\partial\xi}\right.$$

$$+ \left[\left(\frac{27g^2\rho_0^2}{m_G^2}\right)c_s^2 + 3\pi^{2/3}\rho_0^{4/3}c_s^2\right]\left(\frac{\partial v_1}{\partial\tau} + v_1\frac{\partial v_1}{\partial\xi}\right) + \left(\frac{27g^2\rho_0^2}{m_G^2}\right)\rho_1\frac{\partial\rho_1}{\partial\xi} + \pi^{2/3}\rho_0^{4/3}\frac{\rho_1}{3}\frac{\partial\rho_1}{\partial\xi}$$

$$\left. - \left[\left(\frac{27g^2\rho_0^2}{m_G^2}\right)2c_s^2 + 4\pi^{2/3}\rho_0^{4/3}c_s^2\right]\rho_1\frac{\partial v_1}{\partial\xi} - \left[\left(\frac{27g^2\rho_0^2}{m_G^2}\right)c_s^2 + \pi^{2/3}\rho_0^{4/3}c_s^2\right]v_1\frac{\partial\rho_1}{\partial\xi} + \left(\frac{18g^2\rho_0^2}{m_G^4R^2}\right)\frac{\partial^3\rho_1}{\partial\xi^3}\right\} = 0 \quad (27)$$

and

$$\sigma\left\{\frac{\partial v_1}{\partial\xi} - \frac{\partial\rho_1}{\partial\xi}\right\} + \sigma^2\left\{\frac{\partial v_2}{\partial\xi} - \frac{\partial\rho_2}{\partial\xi} + \frac{\partial\rho_1}{\partial\tau} + \rho_1\frac{\partial v_1}{\partial\xi}\right.$$

$$\left. + v_1\frac{\partial\rho_1}{\partial\xi} - c_s^2 v_1\frac{\partial v_1}{\partial\xi}\right\} = 0, \quad (28)$$

respectively. In the last two equations each bracket must vanish independently and so $\{\ldots\} = 0$. From the first term of (28) we obtain $\rho_1 = v_1$. Using this identity in the first term of (27) we obtain an equation, which solved for c_s yields

$$c_s^2 = \frac{\left(\frac{27g^2\rho_0^2}{m_G^2}\right) + \pi^{2/3}\rho_0^{4/3}}{\left(\frac{27g^2\rho_0^2}{m_G^2}\right) + 3\pi^{2/3}\rho_0^{4/3}}. \quad (29)$$

Inserting these results into the terms proportional to σ^2, we find, after some algebra the KdV equation:

$$\frac{\partial\rho_1}{\partial\tau} + \left[\frac{(2 - c_s^2)}{2} - \left(\frac{27g^2\rho_0^2}{m_G^2}\right)\frac{(2c_s^2 - 1)}{2A}\right.$$

$$\left. - \frac{\pi^{2/3}\rho_0^{4/3}}{A}\left(c_s^2 - \frac{1}{6}\right)\right]\rho_1\frac{\partial\rho_1}{\partial\xi} + \left[\frac{9g^2\rho_0^2}{m_G^4R^2A}\right]\frac{\partial^3\rho_1}{\partial\xi^3} = 0, \quad (30)$$

where

$$A = \left(\frac{27g^2\rho_0^2}{m_G^2}\right)c_s^2 + 3\pi^{2/3}\rho_0^{4/3}c_s^2$$

$$= \left(\frac{27g^2\rho_0^2}{m_G^2}\right) + \pi^{2/3}\rho_0^{4/3}. \quad (31)$$

Returning to the $x - t$ space we obtain

$$\frac{\partial\hat\rho_1}{\partial t} + c_s\frac{\partial\hat\rho_1}{\partial x} + \alpha c_s\hat\rho_1\frac{\partial\hat\rho_1}{\partial x} + \beta\frac{\partial^3\hat\rho_1}{\partial x^3} = 0, \quad (32)$$

where

$$\alpha \equiv \left[\frac{(2 - c_s^2)}{2} - \left(\frac{27g^2\rho_0^2}{m_G^2}\right)\frac{(2c_s^2 - 1)}{2A}\right.$$

$$\left. - \frac{\pi^{2/3}\rho_0^{4/3}}{A}\left(c_s^2 - \frac{1}{6}\right)\right] \quad (33)$$

and

$$\beta = \left[\frac{9g^2\rho_0^2 c_s}{m_G^4 A}\right], \quad (34)$$

which is the KdV equation at zero temperature for the small perturbation in the baryon density $\hat\rho_1 \equiv \sigma\rho_1$. If we neglect the space and time derivatives in (21) and (22) and repeat the derivation sketched above we arrive at

$$\frac{\partial\hat\rho_1}{\partial t} + c_s\frac{\partial\hat\rho_1}{\partial x} + \alpha c_s\hat\rho_1\frac{\partial\hat\rho_1}{\partial x} = 0, \quad (35)$$

which is a breaking wave (BW) equation for $\hat\rho_1$. We close this section emphasizing that Eq. (32) is the main result of this work. It shows that for suitable choices of the parameters α and β we can have KdV solitons in a quark-gluon plasma.

V. NUMERICAL ANALYSIS

The KdV equation (32) has an analytical soliton solution given by [1]

$$\hat\rho_1(x, t) = \frac{3(u - c_s)}{\alpha c_s}\,\text{sech}^2\left[\sqrt{\frac{(u - c_s)}{4\beta}}(x - ut)\right], \quad (36)$$

where u is an arbitrary supersonic velocity. A soliton is a localized pulse which propagates without change in shape and in this case it has the width λ defined by

$$\lambda = \sqrt{\frac{4\beta}{(u - c_s)}} = \sqrt{\frac{36g^2\rho_0^2 c_s}{(u - c_s)m_G^4 A}}. \quad (37)$$

For the numerical estimates we shall use the following values of the parameters: $\mathcal{B}_{\text{QCD}} = 0.0006$ GeV^4, $g = 0.35$, $m_G = 290$ MeV and $\rho_0 = 2$ fm^{-3} which, when substituted in (29) yield $c_s^2 = 0.5$ ($c_s = 0.7$). We choose $u = 0.8$. With the help of (37) we find $\lambda = 1.7$ fm for the soliton width and 0.6 for its amplitude. Even though this work is essentially qualitative the chosen numbers are well appropriate to study a realistic situation of a perturbation traversing the QGP.

In Fig. 1 we show the numerical solution of (32) with initial condition $\hat\rho_1(x, t = 0)$ and $\hat\rho_1(x, t)$ given by (36). We can observe the time evolution of the initial Gaussian-like pulse as a well-defined soliton, keeping its shape and form.
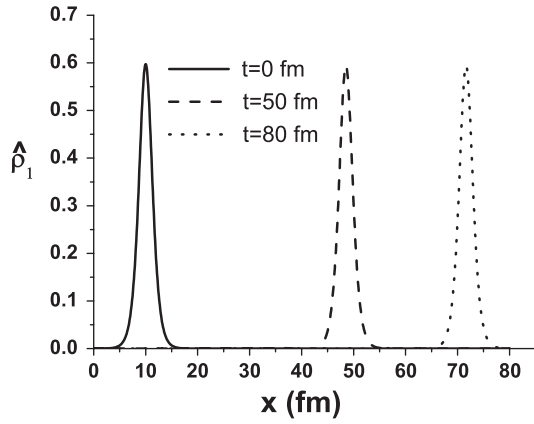
FIG. 1.   Numerical solution of (32) with (36) as initial condition calculated at different times.
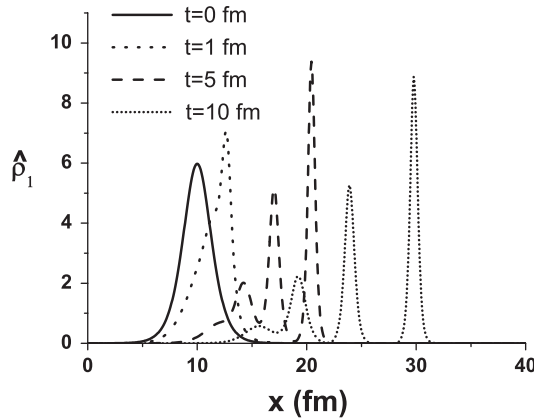

FIG. 2.   The same as Fig. 1 with an initial amplitude 10 times larger.

This solution shows the behavior expected from the analytic solution. We see that using the correct input for the amplitude and width we obtain a pulse which propagates without distortion. This initial condition is a very special
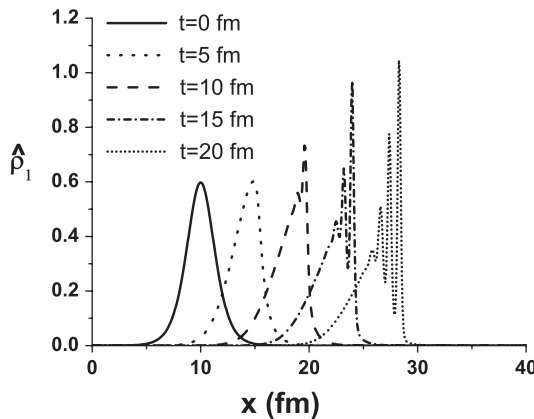

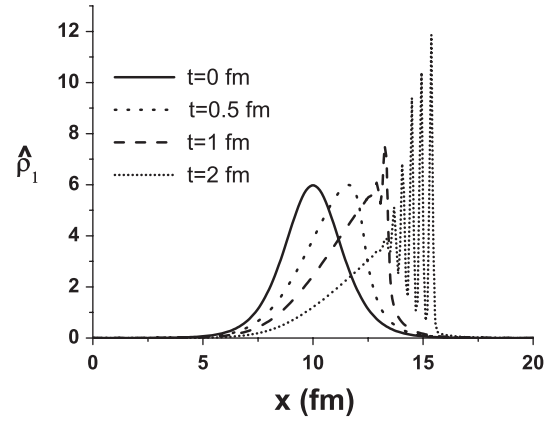FIG. 3.   Numerical solution of (35) with (36) as initial condition.

FIG. 4.   The same as Fig. 3 with an initial amplitude 10 times larger.

case of little practical interest. A real perturbation produced in the QGP will most likely have the ''wrong'' amplitude and wrong width. For arbitrary amplitudes the solution must be numerically calculated.

In Fig. 2 we show again the numerical solution of (32) with the initial condition given by (36) multiplied by a factor 10. Now we observe that the initial pulse starts to develop secondary peaks, which are called ''radiation'' in the literature. Further time evolution will increase the strength of these peaks until the complete loss of localization.

In Fig. 3 we show the numerical solution of (35) with the initial condition given by (36). We observe the gradual formation of a ''wall'' followed by the dispersion of the initial pulse. In Fig. 4 the amplitude of the initial pulse (36) is multiplied by a factor 10 and used as initial condition for (35). As expected the dispersion takes place much earlier than in Fig. 3.

VI. SUMMARY

The main conclusion of this work is that it is indeed possible to have KdV solitons in QCD, provided that two conditions are satisfied. The first condition is that the gluon field have a dynamical mass. In this case the equation of motion (12) can be solved in the weak inhomogeneity approximation yielding (18). The existence of a dynamical gluon mass has been intensely discussed in the literature during the last years and seems to be well established (for details see the references given in [8]). For a massless gluon field we can only have a breaking wave equation, as it was found in our previous work [7]. The second condition is the existence of second order derivative terms in the energy density and pressure. These terms appear naturally from the formalism, as we can see in (15). However it is necessary to keep these derivative terms. The use of uniform field approximations prevents us from finding KdV solitons. If we neglect the derivatives we arrive at the breaking wave Eq. (35). The practical

difference between perturbations governed by the KdV and BW equations is that the former propagate much longer keeping its localization whereas the latter loose localization and may generate unstable "walls". The numerical analysis of some cases confirms the anticipated qualitative expectation. The application of the formalism developed in this work to problems in the theory of compact stars is in progress.

ACKNOWLEDGMENTS

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