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Shock wave formation in hot nuclear matter

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Abstract : Assuming that nuclear matter can be treated as a perfect fluid, we study the propagation of perturbations in the baryon density at high temperature. The equation of state is derived from the non-linear Walecka model. The expansion of the Euler and continuity equations of relativistic hydrodynamics around equilibrium configurations lead to the breaking wave equation for the density perturbation. We solve it numerically for this perturbation and follow the propagation of the initial pulses.

Keywords : Mach cone, quark gluon plasma, jet quenching

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1. Introduction

Hydrodynamics of strongly interacting systems has been applied to low and high energy nuclear reactions and to phenomenon taking place in dense stars. Recently hydrodynamical models became more sophysticated and received more support from experimental data, in particular from the measurement of elliptic flow at RHIC [1]. Moreover, there is compelling evidence that we have seen “the perfect fluid” at RHIC. This evidence might be significantly reinforced by the observation of waves. Waves in a hadronic medium are produced in many physical situations. In fact, there are already some indications that these waves have been formed. In relativistic heavy ion collisions we may have hard parton - parton collisions in which the outcoming partons have to traverse the surrounding fluid to escape and form jets. Their passage may form Mach shock waves, which will affect the transverse momentum distribution of the observed final particles. These “Mach cones” have been observed at RHIC [1].

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From the theoretical point of view one has tried to describe this process with linearized hydrodynamics of a perfect fluid of quarks and gluons. It has been shown that this approach leads to Mach cone formation and to a possible explanation of the observed experimental data. However, given the complexity of the problem, all calculations presented so far still contain strong simplifications and approximations. In order to improve them one has to take into account several effects. One of the aspects that must be considered is the following: after their formation inside the quark gluon plasma these shock waves propagate outwards. The fluid where they live is, in most of its lifetime, in a mixed phase with both QGP and a hot hadronic gas. Therefore we have to worry about what happens to the shock waves when they have to traverse hot hadronic matter. In this contribution we extend some previous studies to finite temperature and present a numerical study of the propagation of a pulse in a hot hadronic medium.

2. Hydrodynamics

In this section we briefly review the main formulas and procedures that take us from the equations of hydrodynamics to a differential equation describing the propagation of a perturbation in the baryon density. In previous works, we have studied wave propagation in cold nuclear matter and the formation of Kortweg-de Vries solitons in non-relativistic [2] and relativistic [3, 4] hydrodynamics. In [5] this study was extended to spherical geometry and finally, in [6], we have considered the problem at finite temperature. All the details and approximations involved in the derivation of the formulas presented below can be found in the mentioned works.

In relativistic hydrodynamics energy-momentum conservation is ensured by:

$$\partial_\nu T_\mu^\nu = 0 \tag{1}$$

where the energy momentum tensor is, as usual:

$$T_{\mu\nu} = (\varepsilon + p) u_\mu u_\nu - p g_{\mu\nu} \tag{2}$$

where ε and p are the energy density and pressure respectively ($g_{00} = -g_{ii} = 1$ and $g_{\mu\nu} = 0$ if $\mu \neq \nu$). u_μ is the four-velocity of the fluid. The continuity equation for the baryon number is:

$$\partial_\nu j_B{}^\nu = 0 \tag{3}$$

where $j_B{}^\nu = u^\nu \rho_B$ and ρ_B is the baryon density. The equation of state is derived from the non-linear Walecka model [7] :

$$\mathcal{L}_{QHD} = \bar{\psi}\left[\gamma_\mu\left(i\partial^\mu - g_V V^\mu\right) - \left(M - g_S\phi\right)\right]\psi + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_S{}^2\phi^2\right) +$$

$$-\frac{b\phi^3}{3} - \frac{c\phi^4}{4} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_V{}^2 V_\mu V^\mu$$

where $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. As usual, the degrees of freedom are the baryon field ψ, the neutral scalar meson field ϕ and the neutral vector meson field V_μ, with the respective couplings and masses. From the Lagrangian density we can obtain, in the mean field approximation, the quantities ε and p, which appear in Eq. (2) and Eq. (1). Dividing the density by its equilibrium value, ρ_0, and the velocity by the sound velocity, c_s, we construct the following dimensionless variables:

$$\hat{\rho} = \frac{\rho_B}{\rho_0}, \quad \hat{v} = \frac{v}{c_s}. \tag{4}$$

Now we may expand the above expressions around their equilibrium values:

$$\hat{\rho} = 1 + \sigma\rho_1 + \sigma^2\rho_2 + ... = 1 + \hat{\rho}_1 + \hat{\rho}_2 + ... \tag{5}$$

$$\hat{v} = \sigma v_1 + \sigma^2 v_2 + ... \tag{6}$$

where $0 < \sigma < 1$ is a (small) expansion parameter. Inserting Eq. (5) and Eq. (6) into Eq. (1) and Eq. (3), keeping only the first terms up to the order σ^2 and combining the resulting equations, we arrive at:

$$\frac{\partial \hat{\rho}_1}{\partial t} + c_s \frac{\partial \hat{\rho}_1}{\partial x} + \left(3 - c_s^2\right) c_s \hat{\rho}_1 \frac{\partial \hat{\rho}_1}{\partial x} = 0 \tag{7}$$

where x is the one-dimensional Cartesian coordinate. Similar equations may be obtained for spherical coordinates.

3. Numerical results and discussion

Equation (7) is a "shock wave equation". Solving it numerically we find out that, starting from smooth initial perturbations this equation creates shock waves. We can see this process in one dimensional Cartesian coordinates in Fig. 1. In the Figure, we fix one initial gaussain-like profile and study its time evolution for three different temperatures. Figs. 1 (a), (b) and (c) show the development of a shock wave at T = 20 MeV, 70 MeV and 120 MeV respectively. As it can be seen (see also [6]), with increasing temperatures the pulse moves faster and the shock formation and the subsequent dispersive breaking occurs sooner. In all cases we observe a steepening of the profile until the formation of the shock, followed by the dispersion of the wave. In [6] we could also observe that the higher is the initial amplitude, the sooner the wave breaking and dispersion occurs.

As mentioned in the introduction, there is, so far, no complete simulation, which includes the passage of the shock wave through a hadronic phase. In spite of its limitations, our results strongly suggest that the hot hadronic medium is relatively "transparent" to pulses formed in the QGP phase. These pulses can easily traverse 20 fm remaining localized and without "breaking". So if Mach cones are formed in QGP, they may survive the final part of the nuclear collision.

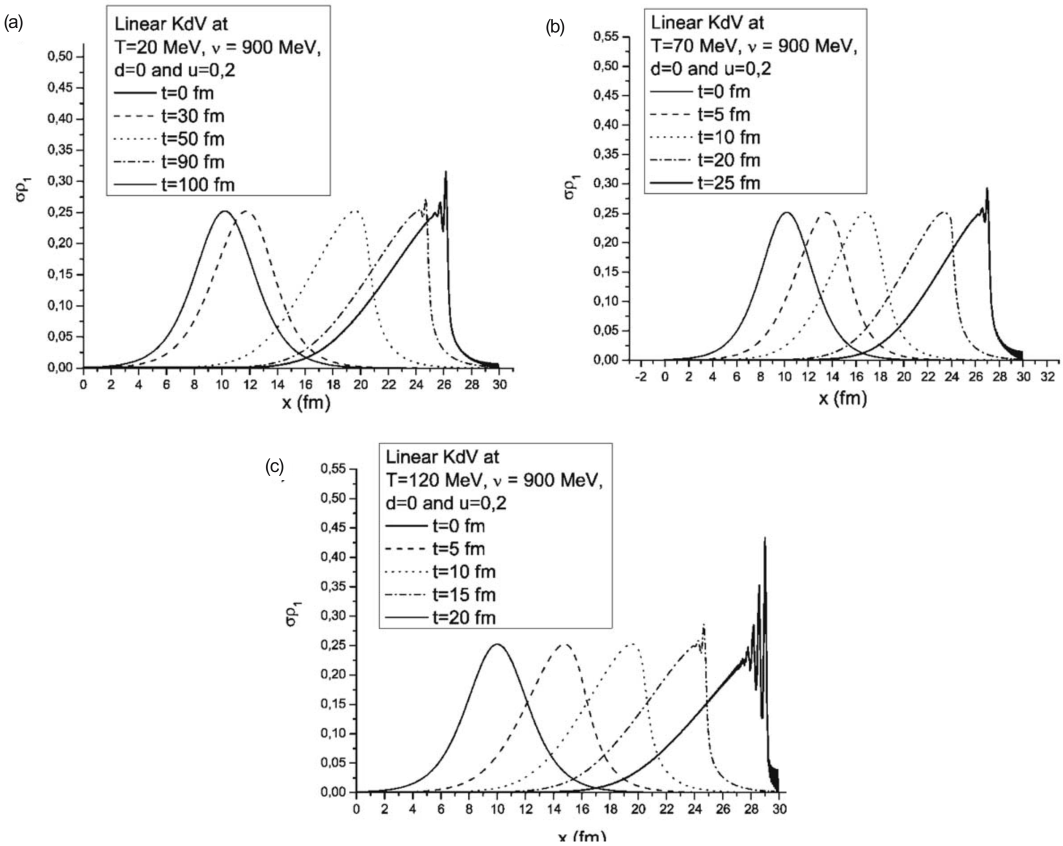

Figure 1. Shock wave formation in one dimensional Cartesian coordinates for different temperatures. In the legend, KdV is Eq (7) and γ = 900 MeV is chemical potential. In (a) T = 20 MeV, (b) T = 70 MeV and (c) T = 120 MeV with d = 0 is Eq. (7) and ν = 900 MeV is the chemical potential.

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