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Slope Angle Estimation Based on Multi-sensor Fusion For a Snake-like Robot

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Abstract—**In this paper, we report on a body state and ground profile estimator for a snake-like robot executing a rolling gait to travel from flat ground to a slope. With the help of the estimator, the snake-like robot can adaptively adjust the body shape and locomotion speed by changing the gait parameters for the purpose of tackling a steep slope. Specifically, we propose a repeating sequence of continuous time dynamical models to fuse kinematic encoder data with on-board Inertial Measurement Unit (IMU) measurements based on extended Kalman filter (EKF). All the sensors are mounted inside each module of the snake-like robot, which measure the joint position, the three-axis acceleration, and the three-axis angular velocity. Further, the robot changes its moving pattern under our policy, judging by the estimated angle of the ground profile. We implement this estimation procedure off-line, using data extracted from repeated runs of the snake-like robot by simulation and evaluate its performance compared to the ground truth.**

Fig. 1. The snake-like robot is rolling forward. This snake-like robot is modular designed. Each joint axis is orthogonal to its neighbors with a rotation range of ±90°. Encoder and IMU sensors are mounted inside each module.

I. Introduction

Snake-like robots are a class of biomorphic hyper-redundant robots [1], designed to imitate the snake-like creatures' biological limbless locomotion with outstanding rapidity, stability, and diversity in wild environment. Typically, these snake-like robots consist of many chain-connected active joint modules, giving them kinematic versatility, like bending, stretching, and crispation. Hirose [2] built up the world's first snake robot controlled by the *serpenoid curve*, which travelled forward on a flat ground. Since then a considerable number of snake-like robots have been implemented in various fields, such as disaster rescuing, factory maintenance, and terrorism surveillance. Those robots are further required to obtain capabilities of moving autonomously and behaving self-adaptively [3], *e.g.*, making decision when, where, and how to move based on different situations of itself and environment.

In order to improve the snake-like robots' autonomy, environment sensing ability is required imperatively, involving robotic vision and multi sensor information fusion technology. For vehicles or aircrafts, autonomous driving [4] or flight [5] have already been achieved under the control based on filtered and fused visual feedback and measurements of inertial sensors. For legged robotics [6, 7], estimating the full pose of the robot under dynamic gaits is significantly important to enhance the adaptation ability for unknown terrain. However, somehow snake-like robots have not take full advantage of these theoretical findings nor hardware sensors for the purpose of autonomous locomotion. The reasons are twofold. First, due to the redundant degrees of freedom, the locomotion of the snake-like robot is difficult to model, especially including the complex interaction between the robot and the environment. Therefore, the estimation model is not accurate and intuitive. Second, the snake-like robot achieves locomotion by twisting its body heavily. Compared with the aforementioned platforms, snake-like robot does not have a relative stable base or body for inertia sensors to output valid and efficient data.

To tackle these challenges, we treat the rolling snake-like robot as a virtual vehicle, based on the theory *virtual chassis* [8]. Then we propose a body state and ground profile estimator for a snake-like robot executing a rolling gait. This estimator gathers and fuses the data of the inertia sensors and encoder in each module based on extended Kalman filter. It can predict the angle of the ground profile quickly when the snake-like robot rolls up a slope from flat ground. Furthermore, we utilize the estimated angle to change the moving pattern to adapt to the environment under our policy. Through simulations, we demonstrate the effectiveness of the ground profile angle estimator and the policy for adapting to critical terrain.

The rest of this paper is organized as follows: Section II presents the related background. The kinematics of the snake-like robot is modelled in Section III. Section IV proposes the algorithm of environment information estimation and Section V comes up with the policy. Section VI gives the experimental analysis. Section VII concludes this paper and presents the future work.

II. Background

A mountain of work about the biological snake locomotion principles [9] have been studied from different angles. These

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are the foundation and prerequisite knowledge, upon which our work has been build.

A. Gait Equation

Gait equation [10] is an intuitive approach to simplify and model the 3D locomotion gaits adopted by snake-like robots. It is extended and developed based on the *serpenoid curve* [11] proposed by Hirose. The basic form of the gait equation is expressed as

$$\begin{cases} \alpha(n,t)_{odd} = C_{odd} + A_{odd} \cdot \sin(\Omega_{odd} \cdot n + \omega_{odd} \cdot t) \\ \alpha(n,t)_{even} = C_{even} + A_{even} \cdot \sin(\Omega_{even} \cdot n + \omega_{even} \cdot t + \delta) \end{cases} \quad (1)$$

where the subscripts *odd* and *even* present the odd and even joint numbers. The other important parameters are the frequency ω and the amplitude A, which effect the speed and the body shape. Full explanations and specific parameters for different gaits can be found in [10]. We will investigate the slope angle prediction problem under *rolling* gait.

B. Virtual Chassis Body Frame

Although gait equation vastly simplifies the modelling of the locomotion for snake-like robots, which is originally complex due to the so many redundant degrees of freedom. A novel modeling method, *Virtual chassis body frame* [12], for a snake-like robot has been proposed, which uses an averaged body frame to present the snake robot's orientation and locomotion. Under the law of SVD (singular value decomposition), the virtual chassis is a body frame that is aligned with the principle components of the robot's overall shape as shown in Fig. 2. Due to the fact that the virtual chassis body frame depends on the snake robot's internal shape changes, rather than the external disturbances in the environment, more accurate, stable and intuitive estimation and observation about the robot's orientation can be reached.

C. Robot State Estimation

Snake robots are unique in both their locomotive capabilities as well as their challenges to estimation and control. Previous work[13, 14] from Biorobotics laboratory in Carnegie Melon University demonstrated estimation of a snake robot's orientation using the robot's proprioceptive sensors and an EKF. For the purpose of autonomous locomotion and based on their work, we build up more precise models to describe the locomotion and predict the environment information not necessarily waiting for a stable pose of the robot.

III. Robot Kinematic Model

This snake-like robot is modular designed. Each joint axis is orthogonal to its neighbors with a rotation range of $\pm 90^\circ$ as shown in Fig 1. Similar to a robotic arm with multi-DoFs, we transfer the coordinate of each module to the world coordinate, with which the coordinate of head module is in accordance. The transfer matrix is derived as

$$T_{k-1}^{k} = TransZ(\frac{L}{2}) \cdot RotateY(\theta) \cdot RotateZ(\frac{\pi}{2}) \cdot TransZ(\frac{L}{2})) \quad (2)$$

where θ is the joint angle position driven as (1) and L depicts the length between the centers of mass of two adjacent modules.

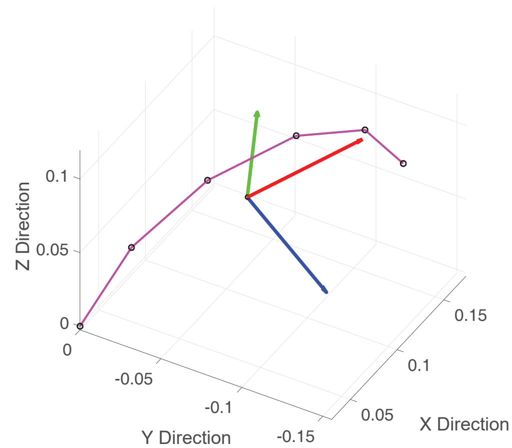


Fig. 2. The virtual chassis illustration. The pink links present the backbone of a six-joint snake-like robot under rolling gait. The coordinate of the virtual chassis of the snake-like robot is represented with three arrows. The red arrow means the direction of the main principal moment of inertia, the green arrow presents the direction of the second principal moment of inertia, and the blue arrow shows the direction of the cross product of the first two vectors.

After attaching all the modules one by one, the posture of the i_{th} module can be presented in the world coordinate by left multiplied the transform matrix T_i^{head} as below.

$$T_i^{head} = T_1^2 T_2^3 \dots T_{i-1}^i \quad (3)$$

The origin coordinate of the virtual body frame is the geometric center of the snake robot and each module geometric center can be described by a matrix P in virtual body frame. According to the *virtual chassis* (VC) theory [8], singular value decomposition (SVD) can extract the 3 principal inertia axes, namely the column vectors of matrix V.

$$P^{body} = USV^T \quad (4)$$

The last step is to transform the pose of each link of the robot to the virtual chassis body frame. If the pose of each link is described by a homogeneous transform, left multiplying each link's transform by V^{-1}, now represents its pose in the virtual chassis body frame.

$$R_k^i = T_i^{body} = \begin{bmatrix} V & \bar{p} \\ 0 & 1 \end{bmatrix}^{-1} \times T_i^{head} \quad (5)$$

where R_k^i is the rotation matrix that describes the orientation of i_{th} module in the body frame at the k_{th} time-step. $\bar{p}$ is the position of the center of mass.

IV. EKF Filter

In order to exploit the orientation state of the snake-like robot, an estimator is designed based on extended Kalman filter. This section starts by defining the state vector of the filter and subsequently continues by formulating the filter models and equations.

A. Filter State Definition

The state vector describes the posture and kinematics of the snake-like robot, from which the robot can be modelled in real-time. Extendeding from Rollison et al.'s work [14], we add the angle of the ground profile as one state and give more accurate equations with less consumptions.

The state vector Γ describes the initial slope angle of the start position. State vector θ_k, $\dot{\theta}_k$ character the joints position and velocity. $a = [a_x, a_y, a_z]$,$q_k = [q_1, q_2, q_3, q_4]$, and $\omega = [\omega_x, \omega_y, \omega_z]$ depict the robot's acceleration, quaternion, and angular velocity in the world coordinate, respectively.

$$x_k = [\Gamma, \theta_k, \dot{\theta}_k, a_k, q_k, \omega_k]^T \tag{6}$$

B. Prediction Model

The estimation equations are used to model the state propagation of the robot from one time-step to the next theoretically. Vector with hat notation depicts the states in the prediction step of the filter, before the update step based on the measurement observations.

The slope angle of the ground profile is predicted in the world coordinate. The value of Γ is integrated by the angular velocity ω over time.

$$\hat{\Gamma}_k = \Gamma_{k-1} + \omega_{k-1} \cdot \Delta_t \tag{7}$$

The joints position and velocity are predicted with (8) and (9). Compared with the constant velocity approximation, we expand the gait equation based on the sinusoid curve.

$$\hat{\theta}_k = \cos(f\,\Delta t)\,\theta_{k-1} - \dot{\theta}_{k-1}\sin(f\,\Delta t)\,\frac{1}{f} \tag{8}$$

$$\hat{\dot{\theta}}_k = \cos(f\,\Delta t)\,\dot{\theta}_{k-1} + \theta_{k-1}\sin(f\,\Delta t) f \tag{9}$$

where f represents the frequency (ω_{odd}, ω_{even}) in (1).

The quaternion is predicted by (10-13). The quaternion representing the orientation of the robot is updated based on the estimated angular velocities at that time-step. We perform a discrete-time update developed by [15].

$$\alpha_\Delta = \left|\overrightarrow{\omega}_k\right| \Delta t \tag{10}$$

$$\overrightarrow{e}_\Delta = \frac{\overrightarrow{\omega}_k}{\left|\overrightarrow{\omega}_k\right|} \tag{11}$$

$$q_\Delta = [cos(\frac{\alpha_\Delta}{2}), \overrightarrow{e}_\Delta sin(\frac{\alpha_\Delta}{2})] \tag{12}$$

$$\hat{q}_k = q_\Delta \cdot q_{k-1} \tag{13}$$

where α_Δ is the rotation angle in Δt, q_k is the quaternion at the k_{th} time-step.

C. Observation Model

The snake-like robot is equipped with a single-axis joint angle encoder, a 3-axis accelerometer and a 3-axis gyro located in each module. Therefore, the measurement vector in this filter has $7n$ dimensions, where n is the number of the modules of the snake-like robot.

$$z_k = [\phi_k,\ \alpha_k,\ \gamma_k]^T \tag{14}$$

where, z_k domains all the measurements at k_{th} time-step. ϕ_k measures the robot's joint angle, α_k measures the 3-axis acceleration, and γ_k is the gyroscope measurements.

Expected joint angle measurements are predicted directly from the estimated angles in the state vector (6).

$$\hat{\phi}_k = \hat{\theta}_k \tag{15}$$

The acceleration measurements consist of the gravity, the internal and external motion, as shown in (16). As we all know, the internal motion of each module is rotating along the axis connected to the previous module. This consistent motion drives the robot to accelerate or decelerate, which can be derived into $a_{interal}$ and $a_{external}$, i.g., the acceleration existing inside the robot and the acceleration exhibiting outside the robot.

$$\hat{\alpha}_k = \hat{a}^i_{gravity} + \hat{a}^i_{internal} + \hat{a}^i_{external} \tag{16}$$

The former acceleration $a_{internal}$ can be further divided into a_{radial} and $a_{tangential}$, along the radial and tangential directions of each module as shown below.

$$\hat{a}^i_{radial} = (\dot{\theta}^i_k)^2\ R \tag{17}$$

$$\theta^i_k - \theta^i_{k-1} = \dot{\theta}^i_{k-1}\ \Delta t + \frac{\omega_{acceleration}}{2}\ \Delta t^2 \tag{18}$$

$$\omega_{acceleration} = \frac{2\ (\theta^i_k - \theta^i_{k-1}) - 2\ \dot{\theta}^i_{k-1}\ t}{\Delta t^2} \tag{19}$$

$$a_{tangential} = \omega_{acceleration} \cdot R \tag{20}$$

Therefore, the $\hat{a}^i_{internal}$ can be calculated as

$$\hat{a}^i_{internal} = (\dot{\theta}^i_k)^2\ R + 2R\ (\theta^i_k - \theta^i_{k-1} - \dot{\theta}^i_{k-1}\Delta t)\ /\Delta t^2 \tag{21}$$

The external acceleration consists of two parts, i.e., the gravitational acceleration and motion acceleration. In our case, *rolling* is considered as an uniform linear motion with constant velocity, therefore the motion acceleration $\hat{a}^i_{external}$ is regard as zero. Transforming the estimated gravity vector g into the frame of each module, we can obtain

$$\hat{a}^i_{gravity} = {R^i_k}^{-1}\ \hat{V}_k\ g \tag{22}$$

where V_k is the estimate of the rotation matrix representing the quaternion pose q in the state vector.

The predicted gyro measurements for each module are generated by differentiating the orientation of the robot at two nearby time-steps. If $\hat{R}^i_k$ and $\hat{R}^i_{k-1}$ are rotation matrices that describe the orientations of module i in the body frame at two timesteps, then gyro measurements [16] due to the robot's motion in the body frame at two time-steps, k and $k-1$ can be approximated by

$$\dot{R}(t) = S(\omega) \cdot R(t) \tag{23}$$

where $S(\omega)$ is the antisymmetric matrix of the angular velocity ω.

$$S(\omega) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{24}$$

$R(t)$ can be differentiate as

$$\dot{R}(t) \approx \frac{R(t+\Delta t) - R(t)}{\Delta t} \tag{25}$$

$$R(t+\Delta t) \approx (\Delta t S(\omega) + I_{3\times 3}) \cdot R(t) \tag{26}$$

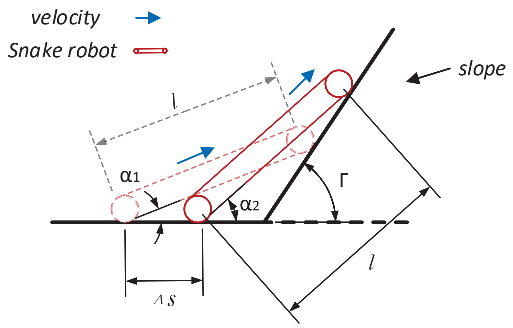


Fig. 3. Side view to describe two contiguous states of the snake-like robot when it is rolling up to a slope.

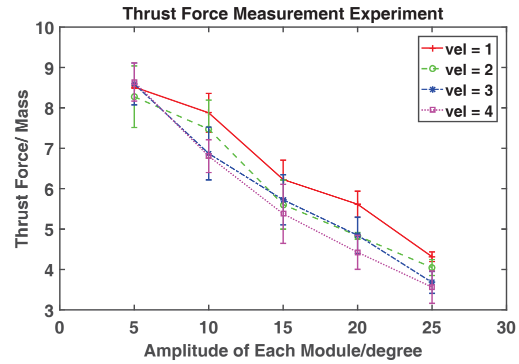


Fig. 4. The amplitude selection range is referred as [17]. 60 trials are performed in order to ensure the statistical sufficiency.

Therefore, the rotation matrix corresponding to angular velocity ω is as

$$\begin{bmatrix} 1 & -\hat{\omega}_z^i & \hat{\omega}_y^i \\ \hat{\omega}_z^i & 1 & -\hat{\omega}_x^i \\ -\hat{\omega}_y^i & -\hat{\omega}_x^i & 1 \end{bmatrix} \Delta t \approx \frac{R_k^i}{R_{k-1}^i} \tag{27}$$

where R_k^i describes the orientation of module i in the body frame at the time step k.

$$\hat{\gamma}_k^i = \hat{\omega}^i + (R_k^i)^{-1}\hat{\omega}_k \tag{28}$$

V. Slope Angle Estimation

To achieve autonomous locomotion, the robot should have the capacities of obtaining useful environment information via sensors. Then the robot can behave felicitously by adjusting the moving pattern, e.g, the body geometry, the velocity, or even stop the locomotion.

A. Real Time Slope Angle Estimation

After feeding the raw sensory data to the aforementioned estimator, the pitch angle of plane where the robot stands on, can be estimated. Then we propose a method to predict the forthcoming slope angle based on two contiguous time-steps. The scenario that the snake-like robot rolls to a slope in two contiguous steps is intuitively presented in Fig. 3 from the side view. At the previous step, the angle between the cross section of the robot and the ground is α_1. After Δt at the current step, the angle is α_2 and the travel distance in this period is ΔS. Then we can predict the slope angle Γ by using

$$tan\Gamma = \frac{l(sin\alpha_2 - sin\alpha_1)}{l(cos\alpha_2 - cos\alpha_1) + \Delta s} \tag{29}$$

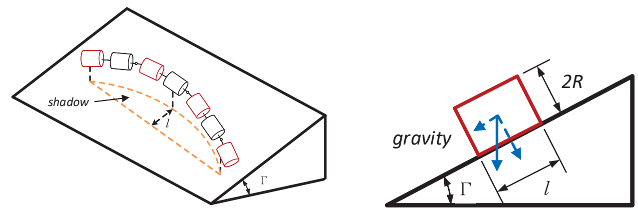
$$\Delta s \approx vel \cdot \frac{cos\alpha_2 + cos\alpha_1}{2} \cdot \Delta t \tag{30}$$

where l and vel represent the length and the forward velocity of the snake-like robot from the side view, respectively. Both of these parameters are constant value under the same gait parameters. Therefore, θ can be calculated as

$$\theta \approx arctan\frac{2l(sin\alpha_2 - sin\alpha_1)}{2l(cos\alpha_2 - cos\alpha_1) + vel\Delta t(cos\alpha_2 + cos\alpha_1)} \tag{31}$$

As a result, we can approximatively calculate the predicted angle of the ground profile during the climbing process, instead of waiting until it is fully on the slope.

B. Policy for Motion Adjustment



(a) Isometric view. (b) Side view.

Fig. 5. The illustration for the stability analysis of the snake-like robot. The robot is rolling on a slope. Then angle of the profile is presented by Γ. In figure 5(a), the supporting area is marked with dotted lines. The sagitta of the body arc is l. In figure 5(b), the gravity is divided into two components along the direction of the ground and the direction perpendicular to the ground.

The purpose of estimating the slope angle for the snake robot is to adjust itself to adapt to different terrain. Specifically, there is a trade-off between the stability and the agility of the snake-like robot. If the body forms into an arc shape with high curvature, the snake-like robot will be more stable, since it has a larger space to support itself. The other way round, the robot will drag itself and slow down the forward speed.

For the stability analysis as shown in Fig. 5, the whole robot body can be regarded as a triangle box on a slope. If the slope is so steep that the gravity vector is out of the supporting section, the robot will be topped over. Therefore, the l should meet the condition as below.

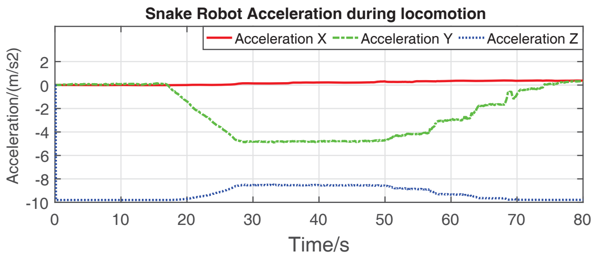
$$\tan\Gamma \leq \frac{l}{2} \div R \tag{32}$$

where l can be calculated according to the geometry in Fig. 5(a).

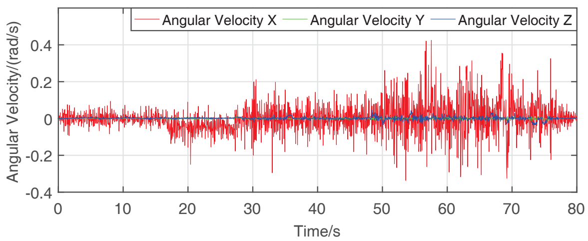
$$l = \frac{L}{A} \cdot (1 - cos(\frac{nA}{2})) \tag{33}$$

where A represents the amplitude (A_{odd}, A_{even}) in (1). As we can see, the larger is the amplitude, the larger supporting section will the robot have. Therefore, we can calculate a minimum value to ensure the stability of the robot on the slope.

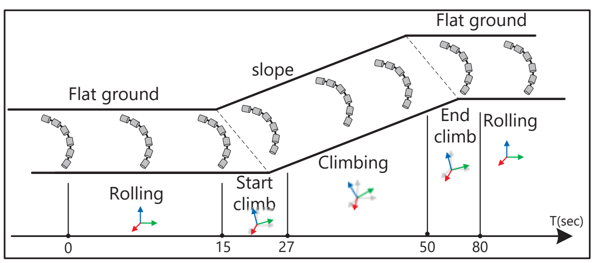
For the speed analysis, the propulsion of each module can be decomposed along the moving direction and the tangential



(a) The acceleration of the virtual chassis of the snake-like robot in the world coordinate.



(b) The angular velocity of the virtual chassis of the snake-like robot in the world coordinate.



(c) Illustration of the rolling process from the flat ground to slop for the snake-like robot.

Fig. 6. Estimation results of the travelling process. The angle of the slope is 30-degree slope.

direction. As we know, the propulsion is generated by the rotation of the body. If the robot body forms an arc with higher curvature, the forward propulsion will be smaller. For the extreme case that the body is rolling into a straight line, it will reach the fastest speed under the same frequency. Then we can conclude that the larger the amplitude, the slower the robot is. To testify this conclusion, we conduct a set of simulations to measure the thrust force of the rolling gait under different velocities and amplitude values. The results are shown in Fig. 4. With the same velocity, for instance, $vel = 1$, the thrust is decreasing from $28N$ to $14N$ with the amplitude changing from 5° to 25°. With the same amplitude value, for instance, 15°, the thrust is decreasing from $20N$ to $16N$ with the velocity changing from $1m/s$ to $4m/s$.

Here, we adopt a linear fitting equation to approximately describe the relationship between the thrust force and the amplitude angle.

$$\frac{F_{thrust}}{m} = p_1 \times A + p_2 \tag{34}$$

where the fitting parameters are $p_1 = -0.2507$, $p_2 = 9.523$.

According to the conclusion, we should decrease the velocity and the amplitude of the snake-like robot to generate larger

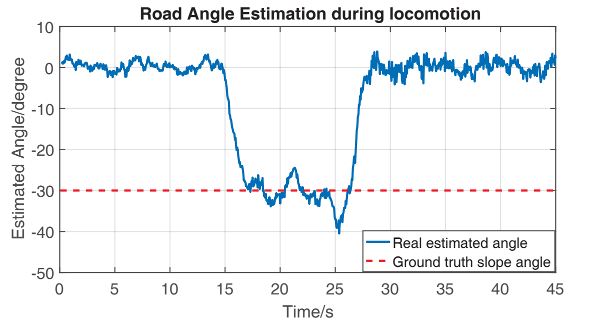


Fig. 7. Real time estimation of the profile angle during the climbing process. The ground truth is 30 degree.

thrust force to overcome the component of the gravity force along the slope. Therefore, we have a maximum value for the amplitude value for the rolling gait as

$$\frac{F_{thrust}}{m} \geq g \sin \Gamma \tag{35}$$

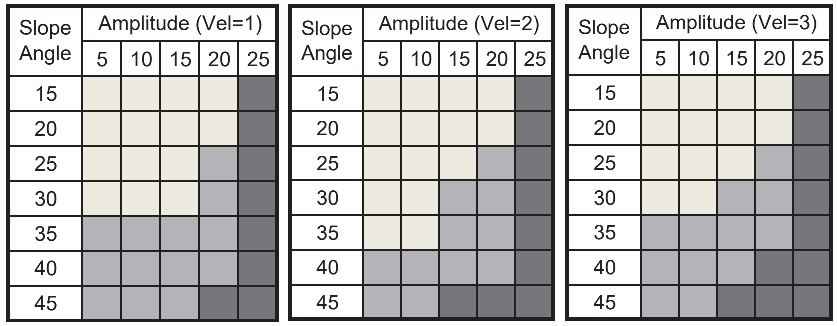
Overall speaking, a pari of boundaries for the amplitude should be satisfied for a critical steep slope. The maximum approximate value can be limited as (34, 35) and the minimum value is bounded as (32, 33).

VI. Experiment Evaluation

For further verification, a scenario that the snake-like robot rolls from flat ground to a slope is simulated using Virtual Robot Experimentation Platform (V-REP EDU) and all the sensor data are off-line filtered.

The estimated results are presented as shown in Fig. 6. When the robot starts to roll till $t = 15s$, the robot is travelling on the flat ground, therefore the virtual chassis is aligned with the world coordinate. The acceleration of x and y axes are zero and the acceleration of z direction is the gravity (9.8 m/s^2). The angular velocities of all 3 axes are zero, since the virtual chassis of the robot is just shifting forward. Afterwards, the snake-like robot starts to roll up the a 30° slope from $t = 15s$ to $t = 27s$. We can find out that the x acceleration is zero, the accelerations of y and z change with time during this period. Meanwhile, the x angular velocity is around 0.1 rad/s and the other two angular velocity remain zero. When the snake robot steadily travels on the slope, the acceleration and the angular velocity of the virtual chassis remain stable, from $t = 27s$ to $t = 50s$. The y acceleration is around $-4.9m/s^2$ and the z acceleration decreases to $-8.49m/s^2$. As we predict, the y acceleration is about $g \sin 30^\circ$ (g is gravity), meanwhile the z acceleration is about $g \cos 30^\circ$. Then the robot climbs up to another flat ground before $t = 80s$ with changing accelerations. Finally, the robot reaches the top of the slope and restores to the initial status which is aligned with the world coordinate.

In order to examine our slope angle prediction algorithm, we further process the filtered acceleration data as (31). With this method, we can deduce the angle of the slope even the robot has not fully moved up to the slope. The results are shown in Fig. 7. The predicted angle is calculated every 20 time-step which is one second. When the robot starts to roll at $t = 15s$, the algorithm quickly work out the forthcoming slope angle



(a) Velocity is $1m/s$. (b) Velocity is $2m/s$. (c) Velocity is $3m/s$.

Fig. 8. Slope rolling experiments with different amplitudes and velocities. Light color means successful rolling. Gray color means that the robot can slip on the slope but unable to climb up. Black color means that the robot is unable to climb to the slope.

in two seconds. The result presents that the angle is around 30°, which is very close to the true value. For the purpose of changing moving pattern adaptively and examining our policy, we also conducted a set of slope rolling experiments with different angles and different gait parameters. The results are shown in Fig. 8. Light color means successful slope rolling, gray color means that the robot can climb up to the slope however can not rolling on the slope surface. The black color means the robot can not fully reach the slope. From observing the color tendency, we can conclude that smaller amplitude, the steeper slope the robot can roll up.

VII. Conclusion and Future Work

Adaptive locomotion for snake-like robot based on estimating the environment information via sensors is not easy. We tried to tackle this problem by proposing a multi-sensors fusion model and an adaptive algorithm to adjust the locomotion appropriately. Specifically, the proposed model can predict the angle of the ground profile by integrating the data of the encoder with on-board IMU. Based on the predicted angle of the ground profile, the adaptive algorithm changes the body shape to vary the stability and the velocity of the snake-like robot. Through simulation experiments, we illustrated that the posture of the virtual chassis of the robot was accurately estimated. Then the angle of the profile was predicted during the process even the snake robot had not fully climbed up the slope.

For future work, we plan to examine our theory by implementing it with the prototype under several practical gaits, like slithering.

VIII. Acknowledgement

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