HISTORICAL ORIGIN OF THE DECIMAL SYSTEM

We examine here how the decimal system of numbering was probably invented and how the concepts of addition, subtraction, multiplication and division followed. Clearly the system was based on counting with our ten fingers. If one asks an individual to count off the numbers they will typically start with their thumb on the right hand and call it one , this is followed with the right index finger as two, and so on. Once five is reached one changes to the left hand and calls the thumb as number six and so on ending with the left pinky finger as number ten. The picture being described is as shown-

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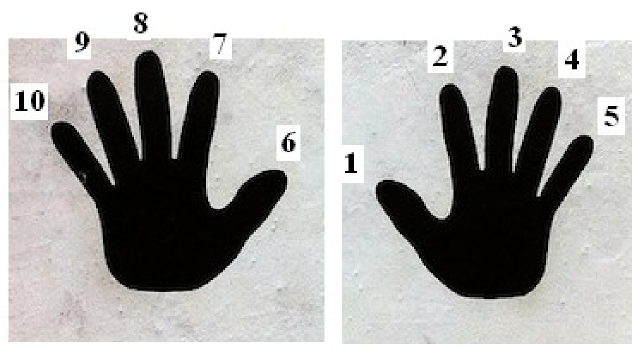

Left Hand **Right Hand**

One thus has the elements of a decimal system S={1,2,3,4,5,6,7,8,9,10}. The operation of addition follows as 6+2=thumb on left hand+index finger on right hand=middle finger on left hand=8. Subtraction goes as 9-5=ring finger on left hand-pinky finger on right hand=ring finger on right hand=4. It must have become obvious that replacing the fingers by abstract number symbols that mathematical operations can be carried out without resorting back to fingers. Indeed, the Chinese as early as 1400BC introduced a bamboo stick decimal numbering system and developed the abacus based on this system. Later these designations were replaced in the west by the Hindu-Arab symbols for one through nine with which we are all familiar today. Also the concept of zero represented by the Chinese as a blank space was replaced by the clearer representation of 0 in the Hindu-Arab system. Europeans became aware of the decimal symbols D={0,1,2,3,4,5,6,7,8,9} shortly after the Moorish invasion of Spain and also due to the efforts of the Italian mathematician Fibonacci in the twelve hundreds .

To express larger numbers such as 3579 in a decimal system, mathematicians quite early looked at this number as-

$$3579 = 3000 + 500 + 70 + 9 = 3x10^3 + 5x10^2 + 7x10^1 + 9x10^0$$

so that the numbers multiplying the nth power of ten is placed at the nth position to the left of the number multiplying 10^0. Therefore one has the following addition-

$$421 + 79 = 4x10^2 + (2+7)x10^1 + (1+9)x10^0 = 4x10^2 + (2+7+1)x10^1 = 5x10^2 = 500$$

and the subtraction-

$$121 - 89 = 1x10^2 + (2-8)x10^1 + (1-9)x10^0 = 1x10^2 - 6x10^1 - 8x10^0 = 100 - 68 = 32$$

To multiply two numbers one has-

$$24x17 = (2x10^1 + 4x10^0)x(1x10^1 + 7x10^0) = (2x1)x10^2 + (2x7 + 4x1)x10^1 + (4x7)x10^0 =$$
$$200 + 180 + 28 = 408$$

and division yields-

$$564 \div 47 = (4x10^2 + 7x10^1 + 9x10^1 + 4x10^0) \div (4x10^1 + 7x10^0) = (10+2) = 12$$

Today one learns these operations by rote and uses hand calculators. This is a pity since it takes away from a true understanding of what these decimal operations really mean. Consider the value of the number 2^{20}=1048576. How would a fifth grader today work out this number if not allowed a calculator? He would most likely try multiplying 2 twenty times which would take him quite awhile. With a true understanding of what this number means , one would write-

$$2^{20} = 2^5 \cdot 2^5 \cdot 2^5 \cdot 2^5 = 32 \cdot 32 \cdot 32 \cdot 32 = 1024 \cdot 1024 = 1048576$$

It should be pointed out that there are other number systems which have been used in the past. These include the base 60 system(sexagesimal) of the Babylonians probably obtained from the approximately 360 days contained in one year. Their angle measure of 360 degrees in a circle survives to this day. The Mayans of the Yucatan used a base 20 number system(vigesimal). This most likely had its origin in counting both their fingers and toes. It is probably a pretty good assumption that advanced space aliens with just three fingers on each hand would have developed a base six number system(heximal) H={0,1,2,3,4,5} so that in their way of counting 3x5=103 and 4+3=11.

The simplest of all number systems is the binary system B={0,1} exclusively used by today's electronic computers. It was first introduced by Leibnitz in 1703 in its present form although earlier Chinese and Indian scholars were already partially aware of it. The system expresses numbers as powers of two. Thus $1=2^0$, $2=2^1$, $3=2^1+2^0$, $4=2^2$, $5=2^2+2^0$, $6=2^2+2^1$,$7=2^2+2^1+2^0$ are written in binary as 1, 10, 11, 100, 101, 110, 111, respectively. Multiplying 32 by 16 in binary is just 100000x10000=1000000000 which yields $2^9=512$ in decimal. The penalty being paid by small number bases is that the number of digits required to represent a number becomes large. The decimal system seems to be a happy medium for human consumption between the binary system used by electronic computers and the sexagesimal system of the Babylonians.