Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2007, Article ID 10462, 10 pages
doi:10.1155/2007/10462

Research Article

Design and Performance Analysis of an Adaptive Receiver for Multicarrier DS-CDMA

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Received 26 January 2006; Revised 22 January 2007; Accepted 21 May 2007

Recommended by Lee Swindlehurst

An adaptive parallel interference cancelation (APIC) scheme is proposed for the multicarrier direct sequence code division multiple access (MC-DS-CDMA) system. Frequency diversity inherent in the MC system is exploited through maximal ratio combining, and an adaptive least mean square algorithm is used to estimate the multiple access interference. Theoretical analysis on the bit-error rate (BER) of the APIC receiver is presented. Under a unified signal model, the conventional PIC (CPIC) is shown to be a special case of the APIC. Hence the BER derivation for the APIC is also applicable to the CPIC. The performance and the accuracy of the theoretical results are examined via simulations under different design parameters, which show that the APIC outperforms the CPIC receiver provided that the adaptive parameters are properly selected.

1. INTRODUCTION

Future generations of broadband wireless mobile communication systems are expected to support various services over a multitude of channels encountered in indoor, open rural, suburban, and urban environments, while maintaining the required quality of service (QoS) [1, 2]. In order to meet these demands, the signal should be flexibly designed such that it is capable of adapting to these communication conditions. In the existing direct-sequence code-division multiple access (DS-CDMA) systems, the spread spectrum (SS) modulation is exploited in mitigating various problems encountered in different communication media. Recently, the multicarrier (MC) technique has become an important alternative for achieving this goal [3, 4].

A number of MC-CDMA schemes have been proposed in the literature [5–9]. Their performances are analyzed and compared with that of single carrier (SC) DS-CDMA systems in frequency-selective Rayleigh fading channels [9–13]. MC systems are advantageous due to their robustness in combating the frequency selectivity in broadband channels. The underlying reason is the integration of an orthogonal frequency division multiplexing (OFDM) overlay, which can be designed such that each subcarrier undergoes flat fading and hence reduces the severe intersymbol interference (ISI) encountered in SC-DS-CDMA systems.

On the other hand, the critical issue for MC-CDMA systems remains to be the improvement of the system capacity in multiuser communications, in which the multiple access interference (MAI) becomes the major capacity-limiting factor. Much attention has been given to the performance analysis of the MC systems based on the single user detection (SUD) strategy, see [4, 14, 15], for example, where the MAI is simply treated as thermal noise. Significant improvement can be achieved when multiuser detection (MUD) techniques are employed to jointly detect all the users' signals [16, 17]. Most MUD algorithms originally proposed for SC-DS-CDMA are applicable to MC-CDMA systems, where interest in this area has been mainly focused on the performance analysis of these various techniques.

It is well known that the prohibitive complexity of the optimal multiuser detection necessitates suboptimal solutions having lower complexity. A large volume of suboptimal MUD algorithms have been proposed in the literature, and two different approaches emerge, namely, adaptive filtering [18–20] and interference cancelation (IC) [21–23]. Much more research has been dedicated to the latter primarily due to a simpler analysis tractability. There are two main varieties of IC schemes, namely, serial IC (SIC) and parallel IC (PIC). In SIC, MAI is estimated and subtracted from the received signal sequentially. Adaptation of SIC to MC systems can be found in [21, 22], where the system performances are also

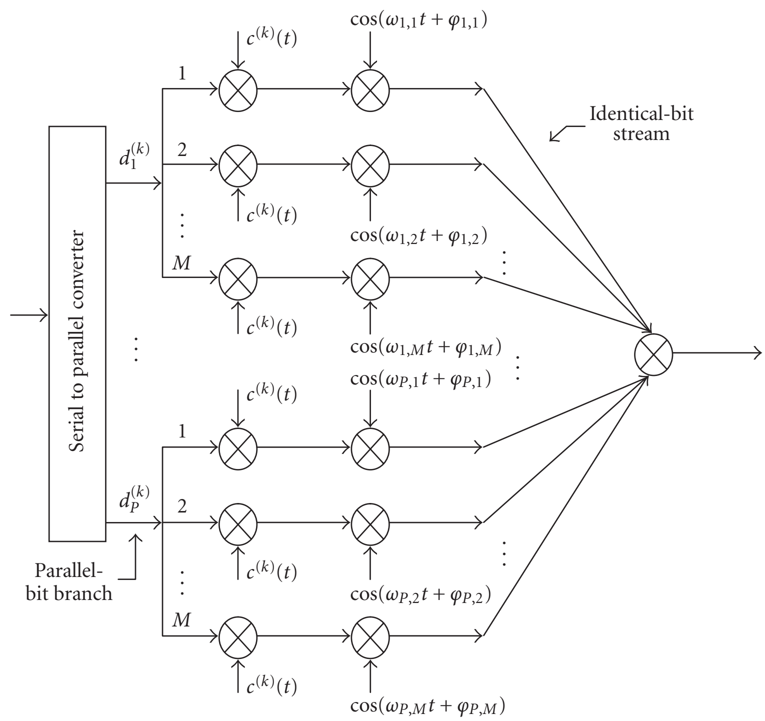

Figure 1: Illustration of signal transmission for user k.

analyzed. PIC appears to be more attractive in the case when high speed detection is preferred, since the cancelation of the interference is performed in parallel. However, the potential gain from PIC depends on the precise estimate of the MAI. A partial PIC is proposed in [24] to mitigate the effect of unreliable MAI estimation. Motivated by [24], a hybrid approach comprising of the PIC and an adaptive technique is proposed for SC-DS-CDMA in [25].

In this paper, an adaptive parallel interference cancelation (APIC) scheme is proposed for the MC-DS-CDMA system, in which the frequency diversity inherent in the MC system is exploited through maximal ratio combining (MRC). The contribution of the paper is twofold. Firstly, the adaptive signal processing as well as the IC technique is designed for the MC system, and the conditions under which the algorithms are able to function properly are investigated. Secondly, instead of simply implementing a heuristic algorithm, we perform a thorough analysis on the system performance and obtain a simple closed form expression for the bit-error rate (BER) of the APIC receiver. Furthermore, under the unified signal model, we show that the theoretical result derived for the APIC is also applicable to the conventional PIC (CPIC), as long as the adaptive step-size is set to zero. The accuracy of the BER derivation is validated by computer simulations. The results showed a significant performance improvement of the APIC over the CPIC receiver.

The organization of this paper is as follows. Section 2 introduces the MC-DS-CDMA system model. Section 3 highlights the structure of the APIC receiver with pre- and post-MRC combining. Section 4 analyzes the performance of the receiver and derives the corresponding closed form BER expression. Numerical results and discussions are presented in Section 5. We conclude the paper in Section 6.

2. SYSTEM MODEL

The structure of the transmitter for user k is shown in Figure 1. A block of P incoming bits is first serial-to-parallel converted into P so-called *parallel-bit* branches. The bit on each parallel-bit branch, denoted as $d_p^{(k)}$, $p = 1, 2, \dots, P$, is then replicated into M streams referred to as *identical-bit* streams, as shown in Figure 1. These $M \times P$ bit streams are spread by the same user-specific pseudorandom spreading sequence $c^{(k)}(t)$, and modulated on subcarriers that are orthogonal to each other. In order to ensure independent fading and hence achieving frequency diversity, the $M \times P$ subcarriers are assigned in such a way that the frequency separation between all identical-bit subcarriers is maximized, as illustrated in Figure 2, where the identical-bit subcarriers $f_{p,m}$, $m = 1, 2, \dots, M$, corresponding to data $d_p^{(k)}$ are separated by a distance of P/T_c for two neighboring subcarriers, for example, $f_{p,m}$ and $f_{p,m+1}$.

In this paper, the channel is assumed to be a slow-varying, frequency-selective Rayleigh fading channel with a delay spread of T_m. Since the spread spectrum system can resolve multipath signals with delay larger than one chip duration, for an SC-DS-CDMA system with a chip duration of T_c', the number of resolvable paths L' is given by

$$L' = \lfloor T_m/T_c' \rfloor + 1, \tag{1}$$

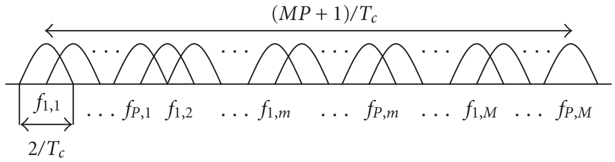

Figure 2: Spectrum of the transmitted signal.

where $\lfloor x \rfloor$ is themaximum integer less than or equal to x. Assuming a passband null-to-null bandwidth, the transmission bandwidth for the SC-DS-CDMA is $2/T_c'$. Maintaining this bandwidth, if the chip duration on each subcarrier of the MC-DS-CDMA system is T_c, then the following condition should be satisfied (refer to Figure 2):

$$\frac{2}{T_c'} = \frac{MP+1}{T_c}. \tag{2}$$

From (1) and (2), the number of resolvable paths L on each subcarrier of the MC-DS-CDMA system is $L = \lfloor T_m/T_c \rfloor + 1 = \lfloor 2(L'-1)/(MP+1) \rfloor + 1$. It is easy to show that when P and M are chosen to satisfy [4]

$$MP \geq 2(L'-1), \tag{3}$$

then $L = 1$ and each subcarrier experiences a flat fading channel. In this case, the complex channel gain for the qth subcarrier of user k can be defined as

$$\zeta_q^{(k)}(t) = \alpha_q^{(k)}(t) \exp\left[j\beta_q^{(k)}(t)\right], \tag{4}$$

where $\alpha_q^{(k)}(t)$ is a Rayleigh-distributed stochastic process with unit second moment and $\beta_q^{(k)}(t)$ is uniformly distributed over 0 and 2π. It is assumed that the channel gain $\zeta_q^{(k)}(t)$ is independent and identically distributed (i.i.d.) for different values of k and q. This is a slight simplification over a real channel which would be correlated in frequency, but typically the difference in performance between a correlated and uncorrelated channel model is small, except that the correlation is noticeable [21, 26]. Furthermore, we investigate synchronous MC-DS-CDMA systems with BPSK modulation to considerably simplify the exposition and analysis. Synchronous systems are becoming more of practical interest since quasisynchronous approach has been proposed for satellite and microcell applications [27]. In [28], an uplink synchronous CDMA system is investigated, where users' signals are assumed aligned at the base station. In this paper, we consider a similar uplink synchronous MC-DS-CDMA system and study its performance.

Assuming that the system consists of K number of users and that all the users employ the same transmitter structure of Figure 1, the received signal at the base station can be written as

$$\begin{aligned} r(t) = \sum_{k=1}^{K}\sum_{p=1}^{P}\sum_{m=1}^{M}\sqrt{\frac{2P_k}{M}}d_p^{(k)} p_{T_b}(t-T_b)c^{(k)}(t) \\ \times \zeta_q^{(k)}(t)\cos\left(\omega_q t + \phi_q\right) + n(t), \end{aligned} \tag{5}$$

where P_k is the power of the kth user, $d_p^{(k)} \in \{-1,+1\}$ is the kth user's pth parallel-bit data, T_b is the transmission interval for each block of data, $p_\tau(t)$ is defined as the rectangular pulse waveform with unit amplitude and duration τ, ω_q and ϕ_q are the frequency and random phase of the qth subcarrier, respectively, and $c^{(k)}(t)$ is the spreading sequence of user k, which is given by

$$c^{(k)}(t) = \sum_{n=-\infty}^{\infty} c^{(k)}(n) p_{T_c}(t-nT_c), \tag{6}$$

where $c^{(k)}(n)$ is the nth chip of the long spreading sequence for user k. Suppose each symbol interval contains N chips, we conduct normalization in each symbol period such that $\sum_{n=lN}^{(l+1)N-1}[c^{(k)}(n)]^2 = 1$, $l \in \mathbb{Z}$. N is called the processing gain. Since we have assumed that the channel is slowly fading, the channel gain $\zeta_q^{(k)}(t)$ will remain constant for one transmission interval. Hence the function of time in $\zeta_q^{(k)}(t)$ will be omitted hereafter. The parameter $q = p + (m-1)P$ is the subcarrier index corresponding to the pth parallel-bit branch and the mth identical-bit stream. The variable $n(t)$ in (5) is the additive white Gaussian noise (AWGN) with zero mean and one-sided power spectral density of N_0.

3. ADAPTIVE RECEIVER STRUCTURE

The structure of the proposed adaptive receiver is illustrated in Figure 3, which can be functionally divided into three parts: the pre- and post-MRC combining, the adaptive MAI estimation, and the parallel IC.

3.1. *Initial stage: MF with MRC (MF-MRC)*

Referring to Figure 3, after the received signal is down converted to its equivalent baseband signal and passed through the fast Fourier transform (FFT) block, the signal can be grouped into P sets, where each set consists of M identical-bit streams. For description simplicity, we only consider the processing of the pth branch in the sequel. For the qth subcarrier where $q = p+(m-1)P$, the coherently detected signal corresponding to the nth chip, $r_q(n)$, is given by

$$\begin{aligned} r_q(n) &= \int_{nT_c}^{(n+1)T_c} r(t)\cos\left(\omega_q t + \phi_q\right)dt \\ &= \sum_{k=1}^{K} \nu_q^{(k)}(n) + \eta_t(n), \quad 0 \leq n \leq N-1, \end{aligned} \tag{7}$$

where

$$\nu_q^{(k)}(n) = \sqrt{\frac{P_k}{2M}} d_p^{(k)} c^{(k)}(n)\zeta_q^{(k)}, \tag{8}$$

$$\eta_t(n) = \int_{nT_c}^{(n+1)T_c} n(t)\cos\left(\omega_q t + \phi_q\right)dt. \tag{9}$$

It is easy to show that $\eta_t(n)$ is a zero mean Gaussian random variable with variance $\sigma_\eta^2 = N_0/2$.

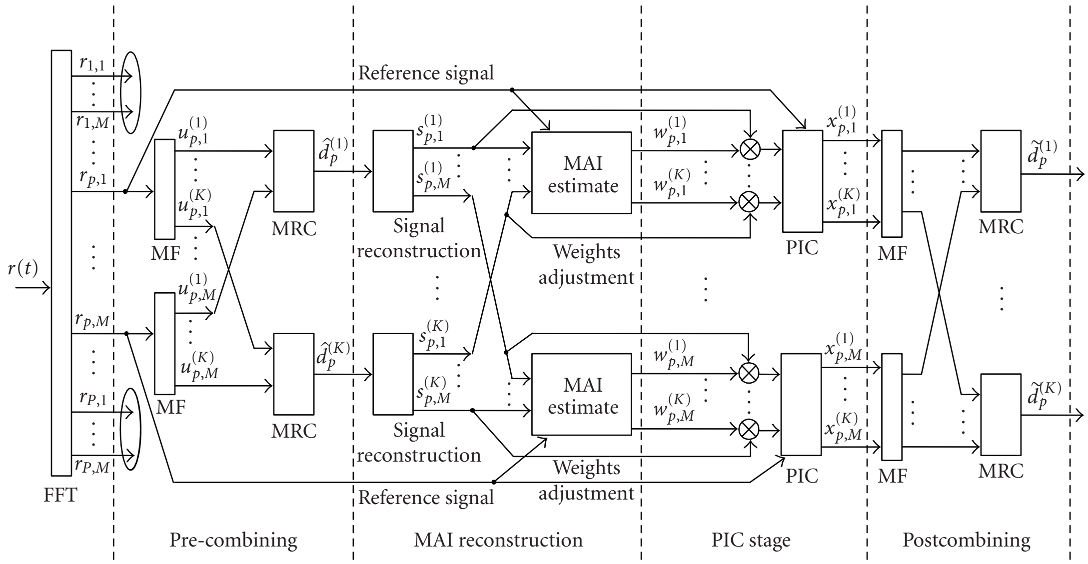

FIGURE 3: Adaptive receiver design.

The signal $r_q(n)$ is then passed to the chip-rate matched filter (MF) bank, as depicted in Figure 3. The output corresponding to the nth chip of the kth user is given by

$$u_q^{(k)}(n) = r_q(n)c^{(k)}(n), \quad 0 \leq n \leq N-1. \tag{10}$$

Assuming perfect channel estimation, the M outputs corresponding to the identical-bit streams are combined together using the MRC coefficients $g_q^{(k)} = [\zeta_q^{(k)}]^*$, where $[x]^*$ is the complex conjugate of x. If the user of interest is user 1, the tentative decision $\widehat{d}_p^{(1)}$ is then given by

$$\widehat{d}_p^{(1)} = \operatorname{sign}\left\{\Re\left[\sum_{n=0}^{N-1}\sum_{m=1}^{M} u_q^{(1)}(n)g_q^{(1)}\right]\right\}. \tag{11}$$

3.2. MAI estimation stage

After the initial decision $\widehat{d}_p^{(k)}$ is obtained, it is replicated into M branches to reconstruct the MAI, as shown in Figure 3. Similar to the transmitter, each of the M copies is spread by the corresponding user's spreading code $c^{(k)}(n)$ and attenuated by the channel gain $\zeta_q^{(k)}$. Hence the regenerated signal of the nth chip corresponding to the qth carrier is given by

$$s_q^{(k)}(n) = \sqrt{\frac{P_k}{2M}}\widehat{d}_p^{(k)}c^{(k)}(n)\zeta_q^{(k)}, \quad 1 \leq m \leq M. \tag{12}$$

The regenerated signals of all the users are multiplied by their corresponding adaptive weights $w_q^{(k)}(n)$ and summed together to produce an estimate $\widehat{r}_q(n)$ of the signal $r_q(n)$, which is written as

$$\widehat{r}_q(n) = \sum_{k=1}^{K} s_q^{(k)}(n)w_q^{(k)}(n), \quad 0 \leq n \leq N-1. \tag{13}$$

The difference between $r_q(n)$ and $\widehat{r}_q(n)$ constitutes the MAI estimation error. Based on this error we define the cost function of the adaptive algorithm as

$$\varepsilon_q = E\left[\left|e_q(n)\right|^2\right] = E\left[\left|r_q(n) - \widehat{r}_q(n)\right|^2\right], \tag{14}$$

where $E[\cdot]$ is the statistical expectation operator and $e_q(n) = r_q(n) - \widehat{r}_q(n)$ is the error of the MAI estimation. In order to minimize the cost, the weights $w_q^{(k)}(n)$ are adjusted at the chip rate according to the normalized LMS algorithm [29]:

$$w_q^{(k)}(n+1) = w_q^{(k)}(n) + \frac{\mu \cdot s_q^{(k)}(n)}{\sum_{i=1}^{K}\left[s_q^{(i)}(n)\right]^2}\left[e_q(n)\right]^*, \tag{15}$$

where μ denotes the step-size. At the end of one transmission interval, the weight $w_q^{(k)}(N-1)$ is determined and it is used by the next stage to assist in the interference cancelation, as depicted in Figure 3.

3.3. Cancelation with MRC stage: PIC-MRC

At this stage, $w_q^{(k)}(N-1)$ is used to weight the input signal $s_q^{(k)}(n)$ over the entire transmission interval. Subtracting the weighted MAI, the "cleaner" signal for user 1 is given by

$$x_q^{(1)}(n) = r_q(n) - \sum_{k=2}^{K}\widehat{v}_q^{(k)}(n), \tag{16}$$

where

$$\widehat{v}_q^{(k)}(n) = s_q^{(k)}(n)w_q^{(k)}(N-1). \tag{17}$$

The signal $x_q^{(1)}(n)$ is then passed to the MF bank and the M identical-bit streams are combined via MRC. The final decision is then obtained according to

$$\widetilde{d}_p^{(1)} = \operatorname{sign}\left\{\Re\left[\sum_{n=0}^{N-1}\sum_{m=1}^{M} x_q^{(1)}(n)c^{(1)}(n)g_q^{(1)}\right]\right\}. \tag{18}$$

A multistage APIC receiver can be realized by repeating the process from (12) to (18).

4. PERFORMANCE ANALYSIS

4.1. BER of the MF-MRC receiver

In Figure 3, if we only consider the first stage, then the structure becomes a conventional MF with MRC combining, which we will refer to as the MF-MRC receiver in the following context. The soft output of the MF-MRC receiver for user 1 is given by

$$Z^{(1)} = D + n_t + I_{\text{MAI}}, \tag{19}$$

where $u_q^{(1)}(n)$ was defined in (10). The desired signal D is given by

$$\begin{aligned} D &= \sum_{m=1}^{M}\sum_{n=0}^{N-1} v_q^{(1)}(n)c^{(1)}(n)\left[\zeta_q^{(1)}\right]^* \\ &= \sqrt{\frac{P_1}{2M}}d_p^{(1)}\sum_{m=1}^{M}\left[\alpha_q^{(1)}\right]^2, \end{aligned} \tag{20}$$

and the noise term n_t is given by

$$n_t = \Re\left\{\sum_{m=1}^{M}\sum_{n=0}^{N-1}\eta_t(n)c^{(1)}(n)\alpha_q^{(1)}\exp\left[-j\beta_q^{(1)}\right]\right\}. \tag{21}$$

n_t is a zero mean Gaussian random variable and its variance is given by

$$\sigma_n^2 = \frac{N_0}{4}\sum_{m=1}^{M}\left[\alpha_q^{(1)}\right]^2. \tag{22}$$

The term I_{MAI} in (19) is the MAI which can be written into two parts:

$$I_{\text{MAI}} = I_{\text{MAI}}^{(s)} + I_{\text{MAI}}^{(d)}, \tag{23}$$

where $I_{\text{MAI}}^{(d)}$ is the interference from the other users on different subcarriers, which simply vanishes in synchronous case [4]. $I_{\text{MAI}}^{(s)}$ is the interference from the other users on the same subcarrier, which is given by

$$\begin{aligned} I_{\text{MAI}}^{(s)} &= \Re\Bigg\{\sum_{m=1}^{M}\int_0^{T_b}\sum_{k=2}^{K}\sqrt{\frac{2P_k}{M}}d_p^{(k)}p_{T_b}(t-T_b)c^{(k)}(t) \\ &\quad\times\zeta_q^{(k)}\cos\left(\omega_q t+\phi_q\right)c^{(1)}(t)\cos\left(\omega_q t+\phi_q\right)\left[\zeta_q^{(1)}\right]^* dt\Bigg\} \\ &= \sum_{m=1}^{M}\sum_{k=2}^{K}\sum_{n=0}^{N-1}\sqrt{\frac{P_k}{2M}}d_p^{(k)}c^{(k)}(n)c^{(1)}(n) \\ &\quad\times\cos\left(\beta_q^{(k)}-\beta_q^{(1)}\right)\alpha_q^{(k)}\alpha_q^{(1)}. \end{aligned} \tag{24}$$

The term $I_{\text{MAI}}^{(s)}$ is commonly approximated as a zero mean Gaussian random variable. Under the assumption that $\alpha_q^{(1)}$ is known at the receiver and the channel is normalized with $E[(\alpha_q^{(k)})^2] = 1$, the variance of $I_{\text{MAI}}^{(s)}$ is given by

$$\text{Var}\left[I_{\text{MAI}}^{(s)}\right] = \sum_{k=2}^{K}\frac{P_k}{4MN}\sum_{m=1}^{M}\left[\alpha_q^{(1)}\right]^2. \tag{25}$$

Let $\gamma = \sum_{m=1}^{M}[\alpha_q^{(1)}]^2$, and assuming that a bit "1" is transmitted, then the error probability conditioned on γ is given by

$$\begin{aligned} P[e \mid \gamma] &= \frac{1}{2}\operatorname{erfc}\left(\frac{E\left[Z^{(1)}\right]}{\sqrt{2\,\text{Var}\left[Z^{(1)}\right]}}\right) \\ &= \frac{1}{2}\operatorname{erfc}\left(\frac{\sqrt{P_1/(2M)}\gamma}{\sqrt{2\left(\text{Var}\left[I_{\text{MAI}}\right]+\sigma_n^2\right)}}\right). \end{aligned} \tag{26}$$

Assuming that any bit can be sent via any of the P parallel branches with equal probability, the final BER of the MF-MRC receiver (the initial stage of the APIC receiver) can be written as

$$P_{\text{ini}}[e] = \frac{1}{P}\sum_{p=1}^{P}\int_0^{\infty}P[e \mid \gamma]p(\gamma)d\gamma, \tag{27}$$

where $p(\gamma)$ is the probability density function of γ and is given by [30]

$$p(\gamma) = \frac{1}{(M-1)}\gamma^{M-1}e^{-\gamma}. \tag{28}$$

4.2. BER of the conventional PIC receiver

In Figure 3, when there is no adaptive process involved, or equivalently by setting the weight $w_q^{(k)}(N-1) = 1$ in the MAI estimation stage, the receiver reduces to a conventional PIC receiver (CPIC). For CPIC, the IC is performed by subtracting the estimated signals of the interfering users from the reference signal $r_q(n)$, which forms a "cleaner" signal $x_q^{(1)}(n)$ as given by

$$x_q^{(1)}(n) = r_q(n) - \sum_{k=2}^{K}s_q^{(k)}(n), \tag{29}$$

where $s_q^{(k)}(n)$ is the regenerated signal defined in (12). The output signal after the MRC combining is given by

$$\begin{aligned} \hat{Z}^{(1)} &= \Re\left\{\sum_{m=1}^{M}\sum_{n=0}^{N-1}x_q^{(1)}(n)c^{(1)}(n)\left[\zeta_q^{(1)}\right]^*\right\} \\ &= D + n_t + I'_{\text{MAI}}. \end{aligned} \tag{30}$$

The desired signal D and the noise term n_t above are identical with the corresponding terms in (20) and (21). The new MAI term is given by

$$\begin{aligned} I'_{\text{MAI}} &= \sum_{m=1}^{M}\sum_{k=2}^{K}\sum_{n=0}^{N-1}\sqrt{\frac{P_k}{2M}}\left(d_p^{(k)}-\hat{d}_p^{(k)}\right)c^{(k)}(n) \\ &\quad\times c^{(1)}(n)\cos\left(\beta_q^{(k)}-\beta_q^{(1)}\right)\alpha_q^{(k)}\alpha_q^{(1)}. \end{aligned} \tag{31}$$

If the BER of the initial stage, $P_{\text{ini}}[e]$, is available, then we have [31]

$$P[\hat{d}_p^{(k)} = -d_p^{(k)} \mid d_p^{(k)}] = P_{\text{ini}}[e], \qquad P[\hat{d}_p^{(k)} = d_p^{(k)} \mid d_p^{(k)}] = 1 - P_{\text{ini}}[e], \tag{32}$$

$$E[(d_p^{(k)} - \hat{d}_p^{(k)})^2] = 4P_{\text{ini}}[e]. \tag{33}$$

From (25) and (33), the variance of I'_{MAI} can be written as

$$\text{Var}[I'_{\text{MAI}}] = 4P_{\text{ini}}[e]\,\text{Var}[I_{\text{MAI}}^{(s)}]. \tag{34}$$

The corresponding BER can then be obtained by using (26) and (27), with $\text{Var}[I_{\text{MAI}}]$ replaced by $\text{Var}[I'_{\text{MAI}}]$. An alternative derivation of the BER of the CPIC receiver can be obtained by regarding the CPIC as a special case of APIC, as shown below.

4.3. BER of the adaptive PIC receiver

Let $\Delta r_q(n)$ be the difference between $r_q(n)$ and the composite estimated signal $\sum_{k=1}^{K} \hat{v}_q^{(k)}(n)$, that is,

$$r_q(n) = \sum_{k=1}^{K} \hat{v}_q^{(k)}(n) + \Delta r_q(n). \tag{35}$$

Comparing with (7), the following relations are satisfied:

$$\sum_{k=1}^{K} \hat{v}_q^{(k)}(n) + \Delta r_q(n) = \sum_{k=1}^{K} v_q^{(k)}(n) + \eta_t(n), \tag{36}$$

$$\Delta r_q(n) = \sum_{k=1}^{K} \Delta v_q^{(k)}(n) + \eta_t(n), \tag{37}$$

where $\Delta v_q^{(k)}(n) = v_q^{(k)}(n) - \hat{v}_q^{(k)}(n)$, by which the term $x_q^{(1)}(n)$ in (29) can be rewritten as

$$\begin{aligned} x_q^{(1)}(n) &= r_q(n) - \sum_{k=1}^{K} \hat{v}_q^{(k)}(n) + \hat{v}_q^{(1)}(n) \\ &= \Delta r_q(n) + \hat{v}_q^{(1)}(n) \\ &= \Delta r_q(n) - \Delta v_q^{(1)}(n) + v_q^{(1)}(n). \end{aligned} \tag{38}$$

From (38), the soft output of the PIC-MRC stage is given by

$$\begin{aligned} \widetilde{Z}^{(1)} &= \Re\left\{ \sum_{m=1}^{M} \sum_{n=0}^{N-1} x_q^{(1)}(n) c^{(1)}(n) [\zeta_q^{(1)}]^* \right\} \\ &= \sum_{m=1}^{M} \sum_{n=0}^{N-1} v_q^{(1)}(n) c^{(1)}(n) [\zeta_q^{(1)}]^* \\ &\quad + \sum_{m=1}^{M} \sum_{n=0}^{N-1} [\Delta r_q(n) - \Delta v_q^{(1)}(n)] c^{(1)}(n) [\zeta_q^{(1)}]^* \\ &= D + I. \end{aligned} \tag{39}$$

The first term D is the desired signal, which is identical to the corresponding term in (20). The second term I is the interference, which is approximated as a zero mean Gaussian random variable.

From (37), with the assumption that $\Delta v_q^{(k)}(n)$ is an i.i.d random variable, we have

$$E\Big[\big(\Delta r_q(n)\big)^2\Big] = K \cdot E\Big[\big(\Delta v_q^{(k)}(n)\big)^2\Big] + \sigma_\eta^2, \tag{40}$$

$$E\Big[\big(\Delta r_q(n) - \Delta v_q^{(1)}(n)\big)^2\Big] = (K-1)E\Big[\big(\Delta v_q^{(k)}(n)\big)^2\Big] + \sigma_\eta^2, \tag{41}$$

where σ_η^2 is the variance of $\eta_t(n)$ in (9).

Substituting (40) into (41) gives

$$E\Big[\big(\Delta r_q(n) - \Delta v_q^{(1)}(n)\big)^2\Big] = \frac{(K-1)E\Big[\big(\Delta r_q(n)\big)^2\Big] + \sigma_\eta^2}{K}. \tag{42}$$

Therefore, the variance of the interference term is given by

$$\text{Var}[I] = \sum_{m=1}^{M} [\alpha_{q,0}^{(1)}]^2 \cdot \frac{(K-1)E\Big[\big(\Delta r_q(n)\big)^2\Big] + \sigma_\eta^2}{2K}, \tag{43}$$

where $E[(\Delta r_q(n))^2]$ is the mean square error (MSE) of the MAI estimation and it can be approximated using the following result.

Proposition 1. *Assume that the source data is i.i.d. with $E[d_p^{(k)} d_p^{(l)}] = \delta_{k,l}$. Furthermore, assume that power control is ideal such that all users' signals have the same power level at the receiver. If the step-size is properly selected such that the misadjustment of the LMS algorithm is less than 10%, then the MSE of the MAI estimation can be approximated as*

$$\text{MSE} \approx \left(1 + \frac{\mu K}{2MN}\right)\left\{\frac{K[1 - (1 - 2P_{\text{ini}}[e])^2]}{2MN} + \sigma_\eta^2\right\}, \tag{44}$$

where μ is the step-size, K, M, N are the number of users in the system, the identical-bit streams and the processing gain, respectively, $P_{\text{ini}}[e]$ is defined in (27) and $\sigma_\eta^2 = N_0/2$.

Proof. Refer to the appendix. □

By approximating $E[(\Delta r_q(n))^2]$ in (43) using the MSE in (44), the corresponding BER of the APIC receiver can then be established by using (26) and (27) with $\text{Var}[Z^{(1)}]$ replaced by $\text{Var}[I]$.

Remark 1. Other than the theoretical approximation, a more accurate value of the MSE can be determined with the aid of computer simulations. It is easy to show that if the adaptive step-size of the algorithm is fixed at 0 and the initial weights are set at 1, the APIC reduces to the CPIC. Under these settings, if the MSE of the CPIC is available through computer simulations, the BER expression originally derived for the APIC can also be applied to the CPIC. The justification of the analysis will be illustrated in the next section.

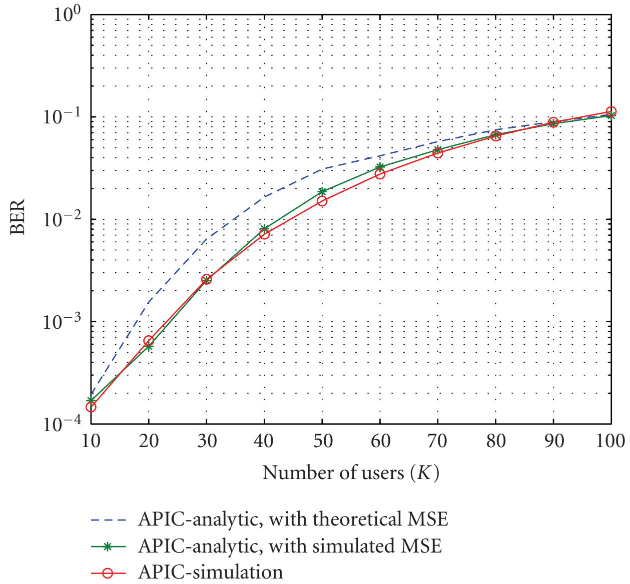

FIGURE 4: Theoretical and simulation results of the one-stage APIC receiver, $N = 32$, SNR $= 20$ dB.

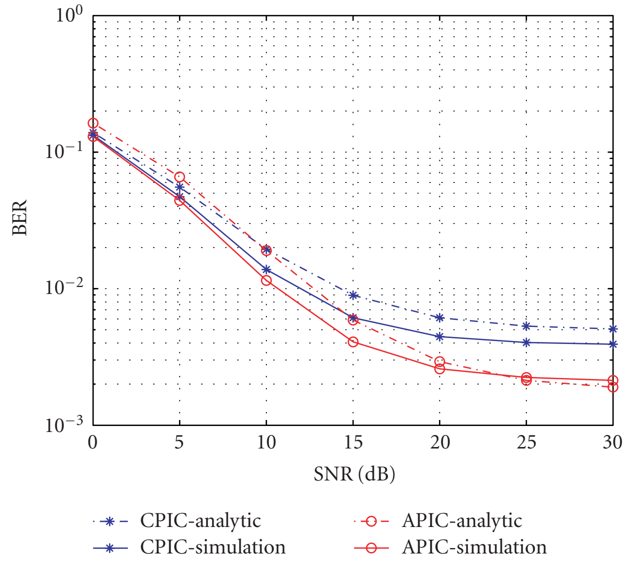

FIGURE 5: Analytical and simulation results of the one-stage CPIC and APIC receivers, $N = 32$, $K = 30$. APIC-analytic uses the simulated MSE.

5. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the performance of the APIC receiver for the MC-DS-CDMA system is studied through numerical results. The channel is frequency-selective with $L' = 3$. The number of identical-bit streams M and parallel-bit branches P should be chosen to satisfy (3) in order to guarantee flat fading on each subcarrier. M is referred to as the *repetition depth* in [2] which bears the tradeoff between the maximum number of users supportable and the achievable frequency diversity, given a fixed number of subcarriers. In the simulations we set $M = 2$ to reflect a certain level of diversity gain. The selection of P, as long as it satisfies (3), does not affect the performance much although it is constrained by the total available bandwidth as well as the system complexity. Considering that the practical cell specific scrambling codes could destroy the orthogonality between users' spreading codes, we utilize random codes as the spreading codes in the simulations.

In Figure 4, the analytical and simulation results of the APIC receiver are presented. The dashed curve is numerically calculated using (26), (27), (28), and (43), where the MSE of the MAI estimation is obtained from the approximation of (44). The step-size and the initial weight for the APIC receiver are $\mu = 0.1$ and $w = 1$, respectively. There is a small discrepancy between the theoretical and the simulation results, due to the approximation of the MSE. However, it can be seen that if the MSE is obtained from the simulations, the resultant BER calculated from the equations is very close to the statistic BER obtained from the Monte Carlo simulations. Under the same settings, Figure 5 illustrates the analytical and simulation results of both the APIC and CPIC receivers in terms of BER versus SNR. The derivations of both receivers are justified through the agreement of the analytical and simulation results.

For the adaptive receiver, the step-size μ plays an important role in system performance. In the sequel, simulations are conducted to investigate the effects of the step-size and the initial weights on the performance of the APIC receiver. It is shown in Figure 6 that the lowest BER is achieved when the initial weight is $w_0 = 1$ at $\mu = 0.3$. Note that when the initial weight is $w_0 = 1$ and the step-size is $\mu = 0$, the APIC reduces to the CPIC. The horizontal line in the figure represents the performance of the CPIC receiver, and we can see that the APIC outperforms the CPIC for $\mu \in (0, 0.5)$.

Under the same simulation settings, the BER performances of the one- and two-stage APIC receiver versus the step-size μ are presented in Figure 7. The initial weight has been fixed at 1. It is shown that the one-stage APIC receiver achieves its best performance at $\mu = 0.3$. However, for the two-stage APIC receiver (with the step-size of the first stage being $\mu_1 = 0.3$), the best performance is at the point where $\mu_2 = 0$. Hence the APIC reduces to the CPIC at the second stage (horizontal line). The underlying reason lies in the fact that, for both the APIC and the CPIC, the MAI estimation has been reliable enough after the first stage processing. Hence the APIC does not have superiority over the CPIC at the second stage. However, when the SNR is low or the system load is heavy such that the first stage cannot perfectly handle the MAI estimation errors, a small nonzero step-size for the second stage guarantees advantage of the APIC over the CPIC. This is verified in Figure 8, where for the APIC, the first stage adopts a step-size of $\mu_1 = 0.3$ and $\mu_2 = 0.05$ for the second stage.

Furthermore, the influence of the choice of the initial weight w_0 is shown in Figure 9, where the MSE performances

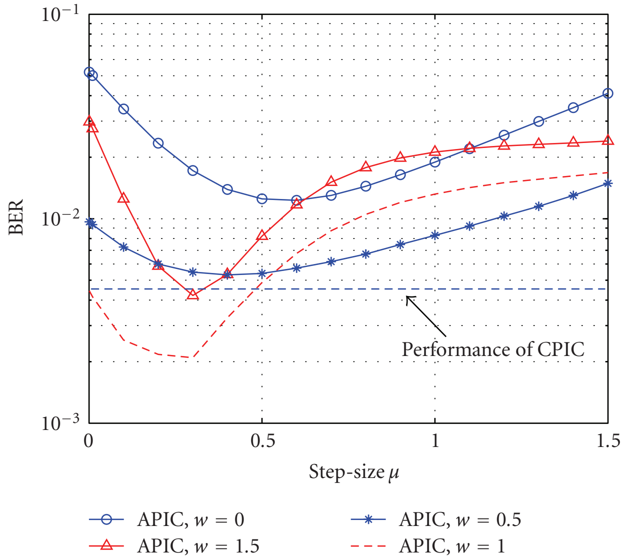

FIGURE 6: BER performance of one stage APIC receiver with different initial weights as a function of the step-size, $N = 32$, SNR = 20 dB, $K = 30$.

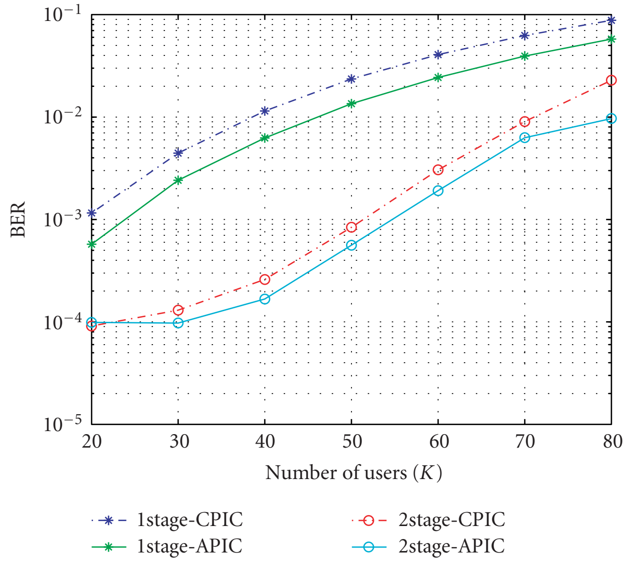

FIGURE 8: BER performances of the CPIC versus the APIC. $N = 32$, SNR = 20 dB. For the APIC, $\mu_1 = 0.3$, $\mu_2 = 0.05$.

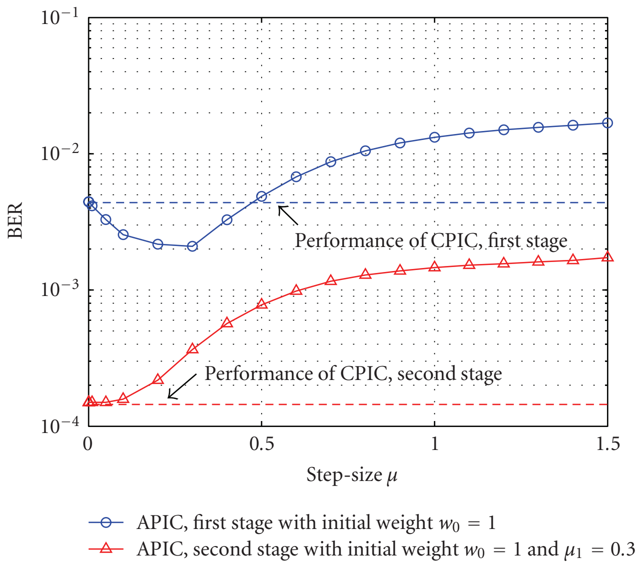

FIGURE 7: BER performance of the first and second stage of the APIC as a function of the step-size.

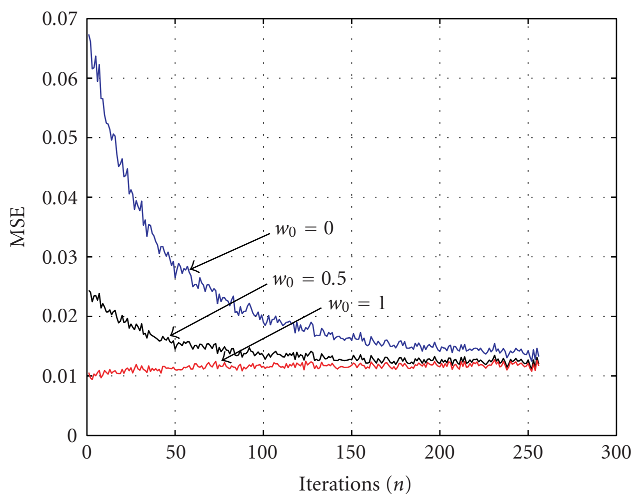

FIGURE 9: Convergence comparison for different initial weights. $N = 32$, $K = 30$, SNR = 20 dB, $\mu = 0.3$.

with different initial weights are presented. It is shown that when the initial weight is randomly chosen, it takes a while for the algorithm to converge. Consequently, when the processing gain of the system is small, the algorithm converges across multiple symbols. A very natural choice of the initial weight for the proposed scheme, however, is $w_0 = 1$. The idea comes with the obvious fact that by choosing $w_0 = 1$, the algorithm starts from the CPIC, which constitutes a stationary starting point.

6. CONCLUSIONS

In this paper, we designed an adaptive receiver for the multicarrier DS-CDMA system over Rayleigh fading channels and evaluated the performance of the system. A closed form expression of the BER is originally derived for the APIC receiver, and it was shown that the derivation of the BER for the CPIC receiver can be unified under the same framework. Simulation results are provided to verify the theoretical derivations. The effect of the design parameters of the APIC receiver, such as the adaptive step-size and the initial weights, are investigated. It is shown that with the appropriate selection of these parameters, the APIC outperforms the CPIC.

APPENDIX

At the MAI estimation stage, the regenerated signal of the nth chip corresponding to the qth carrier given in (12) is rewritten here for convenience of reference:

$$s_q^{(k)}(n) = \sqrt{\frac{P_k}{2M}}\hat{d}_p^{(k)} c^{(k)}(n)\zeta_q^{(k)}, \quad 1 \le m \le M. \tag{A.1}$$

Stack K users' signals in one vector as

$$\mathbf{s}(n) = \left[s_q^{(1)}(n), s_q^{(2)}(n), \ldots, s_q^{(K)}(n)\right]^T, \tag{A.2}$$

such that the $K \times K$ autocorrelation matrix is defined as

$$\mathbf{R} = E\left[\mathbf{s}(n) \quad \mathbf{s}^T(n)\right]. \tag{A.3}$$

Under ideal power control, the channel is statistically identical for all users. The average power received at the base station for users can be assumed to be equal. By normalizing $E[P_k|\zeta_q^{(k)}|^2] = 1$, for all $k = 1, 2, \ldots, K$, and incorporating the i.i.d source data assumption made in Section 4, we then have the following result:

$$\mathbf{R} = \begin{bmatrix} \frac{1}{2MN} & 0 & \cdots & 0 \\ 0 & \frac{1}{2MN} & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{2MN} \end{bmatrix} = \frac{1}{2MN}\mathbf{I}_K, \tag{A.4}$$

where $\mathbf{I}_K$ is the $K \times K$ unit matrix. The inverse of the matrix $\mathbf{R}$ is given by

$$\mathbf{R}^{-1} = 2MN\mathbf{I}_K. \tag{A.5}$$

As in (7), the reference signal is given by

$$r_q(n) = \sum_{k=1}^{K} v_q^{(k)}(n) + \eta_t(n), \quad 0 \le n \le N-1, \tag{A.6}$$

thus we can define the $K \times 1$ cross-correlation vector

$$\mathbf{p} = E\left[\mathbf{s}(n) r_q^*(n)\right]. \tag{A.7}$$

It is then easy to obtain the minimum mean-square error (MMSE) of the MAI estimation as [29]

$$\varepsilon_{\min} = E\left[\left|r_q(n)\right|^2\right] - \mathbf{p}^H \mathbf{R}^{-1}\mathbf{p}. \tag{A.8}$$

The equal power and the i.i.d source data assumptions further lead to the following result:

$$\begin{aligned} E\left[\left|r_q(n)\right|^2\right] &= E\left[\left|\sum_{k=1}^{K} v_q^{(k)}(n) + \eta_t(n)\right|^2\right] \\ &= \sum_{k=1}^{K} E\left[\left|\sqrt{\frac{P_k}{2M}} d_p^{(k)} c^{(k)}(n)\zeta_q^{(k)}\right|^2\right] + \sigma_\eta^2 \\ &= \frac{K}{2MN} + \sigma_\eta^2, \end{aligned} \tag{A.9}$$

where $\sigma_\eta^2 = N_0/2$.

From (A.1), (A.6) and the expression of $v_q^{(k)}(n)$ in (8), we can easily calculate the kth component of $\mathbf{p}$ as

$$E\left[s_q^{(k)}(n) r_q^*(n)\right] = \frac{1}{2MN} E\left[\hat{d}_p^{(k)} d_p^{(k)}\right] = \frac{1 - 2P_{\text{ini}}[e]}{2MN}. \tag{A.10}$$

Hence, the cross-correlation vector $\mathbf{p}$ is given by

$$\mathbf{p} = \frac{1 - 2P_{\text{ini}}[e]}{2MN} \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}}_{K}^T. \tag{A.11}$$

The MMSE in (A.8) can then be written as

$$\varepsilon_{\min} = \frac{K\left[1 - \left(1 - 2P_{\text{ini}}[e]\right)^2\right]}{2MN} + \sigma_\eta^2. \tag{A.12}$$

When the LMS algorithm [29] is utilized for adaptive signal processing, the mean-square error (MSE) of the estimation can be separated into two terms as

$$\text{MSE} = \varepsilon_{\min} + \varepsilon_{\text{excess}}, \tag{A.13}$$

where $\varepsilon_{\text{excess}}$ is the excess MSE which is proportional to $\varepsilon_{\min}$, that is,

$$\varepsilon_{\text{excess}} = \lambda \cdot \varepsilon_{\min}. \tag{A.14}$$

Assuming that the step-size μ is properly selected such that the misadjustment of the LMS algorithm is less than 10%, that is, $\lambda \le 0.1$, we have

$$\lambda = \frac{\mu \cdot \text{tr}[\mathbf{R}]}{1 - \mu \cdot \text{tr}[\mathbf{R}]} \approx \mu \cdot \text{tr}[\mathbf{R}] = \frac{\mu K}{2MN}. \tag{A.15}$$

Finally, the expression of the MSE is given by

$$\begin{aligned} \text{MSE} &= (1 + \lambda) \cdot \varepsilon_{\min} \\ &= \left(1 + \frac{\mu K}{2MN}\right)\left\{\frac{K\left[1 - \left(1 - 2P_{\text{ini}}[e]\right)^2\right]}{2MN} + \sigma_\eta^2\right\}. \end{aligned} \tag{A.16}$$

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their valuable comments.

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