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ORIGINAL ARTICLE

Improved transmission of vector quantized data over noisy channels

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Abstract The conventional channel-optimized vector quantization (COVQ) is very powerful in the protection of vector quantization (VQ) data over noisy channels. However, it suffers from the time consuming training process. A soft decoding self-organizing map (SOM) approach for VQ over noisy channels is presented. Compared with the COVQ approach, it does not require a long training time. For AWGN and fading channels, the distortion of the proposed approach is comparable to that of COVQ. Simulation confirmed that our proposed approach is a fast and practical method for VQ over noisy channels.

Keywords Vector quantization · Self-organizing map

1 Introduction

Vector quantization (VQ) is a widely used data compression method [1–6]. Traditionally, the codebook of the VQ process alone is optimized for the data source only. Obviously, data encoded by this VQ approach is not effectively transmitted over noisy channels [7]. Hence, the channel noise results in significant degradations in the system performance. Although channel coding techniques [8] can be used to protect the data, in a heavy noise environment channel coding cannot correct all transmission errors and then the VQ process still faces an equivalent noisy channel. To reduce the degradation due to channel noises, joint source-channel coding (JSCC) is usually considered [7– 12]. There are two common JSCC approaches.

In the first approach, namely robust VQ (RVQ) [7, 9–11], a codebook is first trained for a noiseless channel. Afterwards, an index assignment procedure is carried out. It assigns each codevector to a signal of the signal constellation. The issue of sensitivity to channel noise for VQ is formulated as an assignment problem. As the problem is NP-hard, finding the optimum solution is impractical for a large codebook. Hence, several heuristics were proposed [7, 9–11]. One feature common to the heuristics is the need to calculate all the pairwise distances between two distinct codevectors. Hence, they involve very large overhead when the size of the codebook is large. Also, the time-consuming assignment procedure must be carried again when a new codebook is used.

In the second approach, namely channel-optimized VQ (COVQ) [12, 13], a codebook is trained for a specified channel. Simulation results showed that the performance of this approach is better than that of some RVQ techniques. However, this approach requires a reliable feedback channel for COVQ training. Also, to obtain a good codebook, the number of required training epoches is very large. If we further add a time-consuming decoding channel code, such as turbo code [13], on the top the COVQ, the training time will become impractically long.

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An alternative form of RVQ is the self-organizing map (SOM) approach originally proposed in [14] for hard-decoding. The SOM approach provides channel robustness by preserving a neighborhood structure. It avoids the undesirable time-consuming index assignment process. However, the key problem in the SOM approach [14] is that the VQ decoder uses the hard decision method. Hence, the VQ decoder is not a minimum mean-square error (MMSE) estimator [15].

This paper investigates the soft decoding SOM approach over noisy channels. Our approach does not need a time-consuming index assignment process when a new codebook is used or the noise power of the channel changes. Also, it does not require a long training time and a reliable feedback channel for training. Simulation results show that under the similar number of training epoches our approach is better than the conventional COVQ approach. To further improve the performance, a SOM-based COVQ approach is also discussed.

The rest of this paper is organized as follows. In Sect. 2, the background information is presented. Section 3 describes the proposed SOM based approaches. Simulation results are presented in Sect. 4. Section 5 concludes the paper.

2 Background

In VQ, the codebook Y partitions the data space $\Re^k$ into M regions $\Omega = \{\Omega_1, \ldots, \Omega_M\}$. Given an input vector $\vec{x}$, the output is an index i^* whose corresponding codevector $\vec{c}_{i*}$ is the closest codevector to $\vec{x}$. To transmit the index, the modulator puts the corresponding channel signal $\vec{s}_{i*} \in S = \{\vec{s}_1, \ldots, \vec{s}_M\}$, where S is the signal constellation, to the channel. Two common modulation methods, shown in Fig. 1, namely quadrature phase shift keying (QPSK) and quadrature amplitude modulation (QAM), are usually employed. [tb]

The channel is assumed to be an additive Gaussian noise channel (AWGN) or an independent Rayleigh

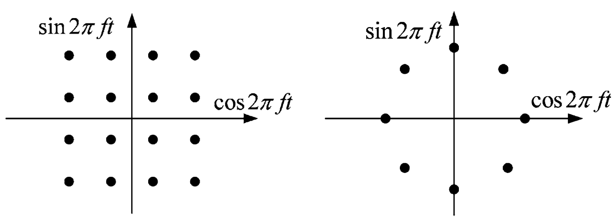


Fig. 1 Two modulation methods. **a** QPSK and **b** QAM

fading channel [16]. The received channel signal is given by

$$\vec{r} = a\vec{s}_{i*} + \vec{n}, \tag{1}$$

where a is the distortion with the Rayleigh distribution and the noise $\vec{n}$ is a Gaussian random vector, with variance σ^2 in each dimension. In the receiver side, the symbol detector provides the conditional probability density values (likelihood values) $p(\vec{r}|\vec{s}_i)'$s, for $i = 1, \ldots, M$. In the conventional hard decoding rule, the output $\hat{\vec{x}}$ is given by

$$\hat{\vec{x}} = \vec{c}_{i'}, \tag{2}$$

where $p(\vec{r}|\vec{s}_{i'}) > p(\vec{r}|\vec{s}_i)$ for all $i \neq i'$.

When the estimated codevector $\vec{c}_{i'}$ is not equal to the transmitted codevector $\vec{c}_{i*}$, a symbol error occurs. The distortion from $\vec{c}_{i'}$ to $\vec{c}_{i*}$ depends on the association between the codebook $Y = \{\vec{c}_1, \cdots, \vec{c}_M\}$ and $S = \{\vec{s}_1, \cdots, \vec{s}_M\}$. If the association is not created in a proper manner, the distortion in the received data is very large.

To reduce the effect from symbol errors, in the RVQ approach [7, 9–11], a codebook is first trained for a noiseless channel. An index assignment procedure is then carried out. The objective function of the assignment procedure is to minimize the cost function, given by

$$D = \sum_{i=1}^{M}\sum_{j=1}^{M} \|\vec{c}_j - \vec{c}_i\|^2 P(\vec{s}_j|\vec{s}_i)P(\vec{c}_i), \tag{3}$$

where $P(\vec{c}_i)$ is the probability that the codevector $\vec{c}_i$ is transmitted and $P(\vec{s}_j|\vec{s}_i)$ is the conditional symbol error probability. However, the above assignment problem is NP-hard, finding a good solution is very time-consuming [17].

In the COVQ approach [12, 13], a codebook is trained for minimizing the objective function, given by

$$D(Y, \Omega) = \sum_{i=1}^{M} \int_{\Omega_i} E\left\{ p(\vec{r}|\vec{s}_i)\|\vec{x} - \hat{\vec{x}}(\vec{r})\|^2 \right\} p(\vec{x})\, \mathrm{d}\vec{x}, \tag{4}$$

where the expectation $E\{\cdot\}$ is operated on the channel noise. The main drawback of this approach is that the training time is very long. Also, training cannot be performed unless we have an additional reliable channel for training. Note that it is not possible for the transmitter to know the receiver's situation because the channel is noisy.

3 SOM approaches

3.1 SOM training

In the SOM learning scheme [3, 6, 14, 18–20], a neighborhood structure, represented by a graph $G = \{V, E\}$, is imposed on a codebook, where $V = \{ v_1, \ldots, v_M \}$ is a set of vertices and E is the set of edges in this graph. In this representation, a vertex v_i is associated with a codevector $\vec{c}_i$. If the codevector $\vec{c}_i$ is defined to be a neighbor of $\vec{c}_j$, two corresponding vertices v_i and v_j are joined by an edge with weighted value equal to 1. The neighborhood distance between $\vec{c}_i$ and $\vec{c}_j$ is the length of the shortest path between v_i and v_j in the graph G. A codevector $\vec{c}_i$ is a level-u neighbor of $\vec{c}_j$ if the neighborhood distance between the two codevectors is less than or equal to u. The collection of level-u neighbors of a codevector $\vec{c}_i$ is denoted as $N_i(u)$. The order u^G of a topological order is the longest neighborhood distance in G.

Figure 2 shows some commonly used structures. Given an neighborhood structure, the learning algorithm is summarized as follows:

1. Given the tth training vector $\vec{x}(t)$, calculate the distances $d_i = \|\vec{x}(t) - \vec{c}_i(t)\|'$s from $\vec{x}(t)$ to all training codevectors.
2. Find the closest codevector $\vec{c}_{i*}(t)$, where $d_{i*} < d_i \forall i \neq i^*$.
3. Update the codebook as follows:

$$\vec{c}_i(t+1) = \vec{c}_i(t) + \alpha_t(\vec{x}(t) - \vec{c}_i(t)) \quad \forall \vec{c}_i(t) \in N_{i*}(u_t), \tag{5}$$

otherwise

$$\vec{c}_i(t+1) = \vec{c}_i(t). \tag{6}$$

The parameter u_t controls which codevectors should be updated at the training iteration t. In our experience, the initial value u_0 is equal to $u^G/4$. The learning rate α_t is a gain that controls the percentage of the update on codevectors.

A trained SOM usually has the ordering-preservation property [6, 21]. That is, when two codevectors are neighbors to each other in the graph, after training, their Euclidean distance in the data space is usually small. Figure 3 shows a typical run of a SOM. Initially, the codevectors do not form a good ordering in the data space. After training, the codevectors form some ordering, i.e., two codevectors are very close in the data space if they are neighbors to each other.

3.2 SOM mapping

According the ordering-preservation property, we can simply use the neighborhood structure to create the index assignment between the codebook and signal constellation. Given a signal constellation, we use its neighborhood structure as the neighborhood structure of the SOM.

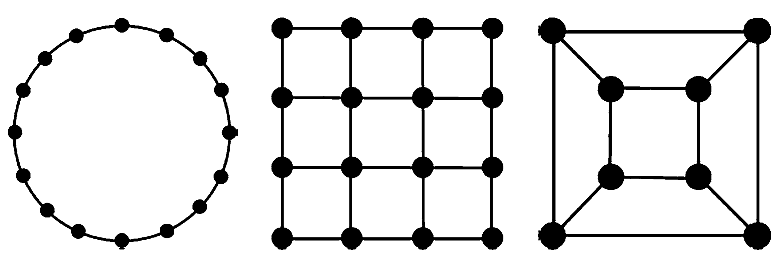
For example, if the signal constellation is 16 QPSK, we can use the one-dimensional (1-D) circular structure as the neighborhood structure of the codebook. If the signal constellation is 16 QAM, we can use the two-dimensional (2-D) grid structure shown in Fig. 2 as the neighborhood structure of the codebook. After training, the association, shown in Fig. 4, between the signal constellation and codebook is automatically created. With this approach, an error event in the receiver only causes a small distortion in the VQ. This SOM approach avoids the undesirable time-consuming index assignment process and the training process over noisy channels.

3.3 Soft decoding SOM

In the hard decoding SOM approach, the output $\hat{\vec{x}}$ is given by

$$\hat{\vec{x}} = \vec{c}_{i'}, \tag{7}$$

where $p(\vec{r}|\vec{s}_{i'}) > p(\vec{r}|\vec{s}_i)$ for all $i \neq i'$ and $p(\vec{r}|\vec{s}_{i'})$s are conditional likelihood values. However, the key problem in this hard decoding SOM approach [14] is that the VQ decoder uses the hard decision method. It does



(a) circular (b) regular grid (c) hypercube

Fig. 2 Three neighborhood structures. **a** Circular, **b** regular grid and **c** hypercube. Since the graphs are used to describe the neighborhood structure of SOM. Note that they do not reflect any actual geometric information of codebooks

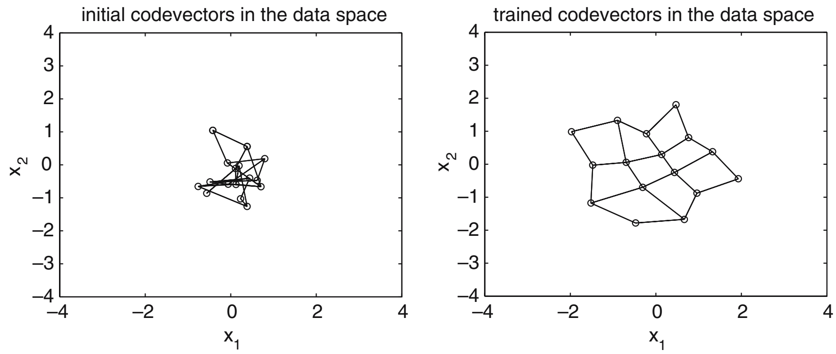


Fig. 3 A typical run of a SOM. The figure shows the codevectors in the data space. To indicate the neighbor structure of the SOM, we use an edge to join two codevectors if they are defined as neighbors to each other

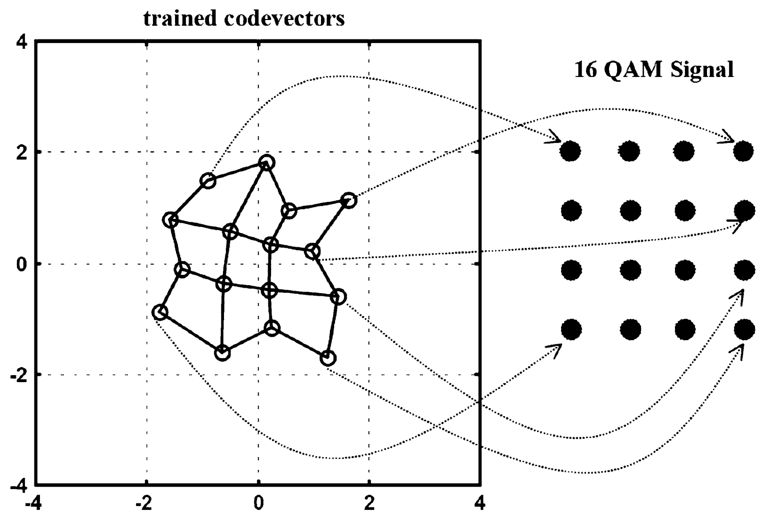


Fig. 4 Association between codevectors and signal constellation

not utilize all the conditional likelihood values provided by the channel. Hence, it is not a MMSE estimator [15].

To further improve the SOM approach, we should utilize all the likelihood values $p(\vec{r}|\vec{s}_j)'$s. In this case, the output is given by

$$\hat{\vec{x}} = \sum_{i=1}^{M} P(\vec{s}_i|\vec{r})\vec{c}_i, \tag{8}$$

$$\hat{\vec{x}} = \frac{\sum_{i=1}^{M} p(\vec{r}|\vec{s}_i)P(\vec{s}_i)\vec{c}_i}{\sum_{i=1}^{M} p(\vec{r}|\vec{s}_i)P(\vec{s}_i)}, \tag{9}$$

where $P(\vec{s}_i|\vec{r})$ is the conditional probability that the transmitted signal (codevector) is $\vec{s}_i$ ($\vec{c}_i$) given the received signal, and $P(\vec{s}_i)$ is the a priori probability that $\vec{s}_i$ is transmitted. In soft decoding, the decision rule utilizes all likelihood values provided by the channel. Hence, the decision is the optimal in the statistical sense.

3.4 SOM–COVQ approach

Since the SOM training method can produces a codebook with a good a neighborhood structure, it is also interested to investigate the hybrid model of the SOM and COVQ approaches. In this model, we use the SOM training method to train a good codebook Y_{som} which has strong resistance to channel noise. Afterwards, we use the trained codebook Y_{som} as the initial codebook of the COVQ algorithm to get a better codebook. We call the hybrid approach as SOM–COVQ.

3.5 Training time

In SOM, data samples are sequentially and repeatedly presented. Equation (5) is used for updating the codevectors. Hence, the training complexity for each training example in a training epoch is equal to $O(kM)$. In COVQ and SOM–COVQ, the cobebook is trained over the noisy channel. Data samples are first quantized and then the soft decoding outputs are collected at the receiver. Afterwards, we update the codebook. According to the soft decoding rule (9), the complexity for each training example in a training epoch is equal to $O(kM)$. As the training complexity of SOM is the same as that of COVQ, the number of required training epoches for convergence determine the training efficient.

4 Simulation

The performance of various data protection schemes, COVQ, SOM, and SOM–COVQ, are investigated. Also, the performance without any data protection (LBG trained codebook) is presented. Two analog sources: Gaussian source and image data are used. Two channel models are considered. They are AWGN and independent Rayleigh fading channel. In the fading channel, the estimated fading factor is assumed to be available at the receiver.

4.1 Comparing with sufficient trained COVQ

In this section, two Gaussian data sources are considered. Each source contains 1,024 k-dimensional samples, where $k = 2$ or 3. The number of codevectors are 16. The signal constellation is 16 QAM. The number of training epoches for SOM is 10 only. In COVQ and SOM–COVQ, when we set the number of training epoches to 40 such that the codebooks of COVQ and SOM–COVQ are well trained.

The performance, in terms of signal-to-reconstruction error ratio (SRER) versus SNR in the channel, is summarized in Figs. 5 and 6. The performance of those data protection schemes is better than that of the simple LBG algorithm. Compared with the LBG, the hard coding SOM can improve the distortion. When the soft decoding SOM is used, the performance is further improved. The performance of the two soft decoding SOM approaches (SOM and SOM–COVQ) is better than that of the COVQ approach. Also, the performance of SOM–COVQ is a bit better than that of SOM for high SNR values. That means, a good initial codebook for COVQ is important.

For a fixed SRER in the reconstruction data, the two soft decoding SOM approaches can achieve about 1–4 dB channel gains. For example, in the 2-D Gaussian data and fading channel case (Fig. 7a), to achieve 7 dB in the SRER, the SNR's of the two soft decoding SOM approaches should be around 9 dB. In the COVQ case, to achieve the same SRER, the SNR in the channel should be around 14 dB.

For a fixed SNR in the channel, the two soft decoding SOM approaches can achieve about 1–2 dB in SRER gains. For example, in Fig. 7a, when SNR in the channel is equal to 10 dB, the SRERs of the two soft decoding SOM approaches are around 7 dB while the SRER of the COVQ approach is around 5.5 dB only.

4.2 Image data: convergence

We use the image, Lena, as the data source to compare the performance of the four soft decoding data protection schemes: LBG, SOM, COVQ and SOM–COVQ. The modulation scheme is 16 QPSK. The image is divided into a number of 4×2 blocks. Each block is regarded as an 8-D input vector. The codebook size is equal to 256. For the SOM codebook, the Cartesian product of two 1-D circular graphs is used as the neighborhood structure. In SOM, the number of training epoches is equal to 10 only and their trained codebooks are used for all channel SNR values.

For COVQ and SOM–COVQ, we vary the number of training epoches. In COVQ and SOM–COVQ for a large codebook, we find that if we only present each sample one time in a training epoch, the convergence,

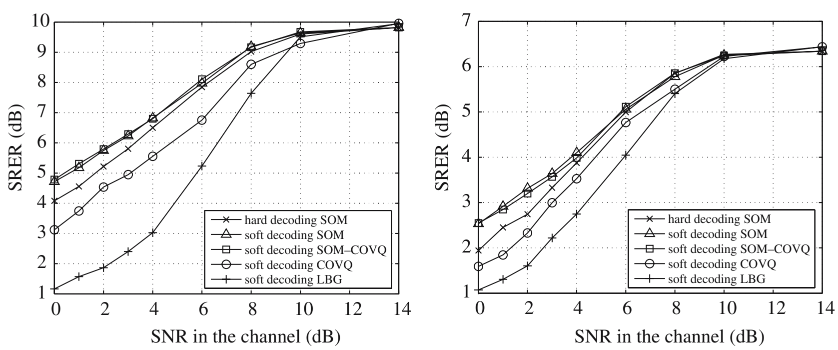
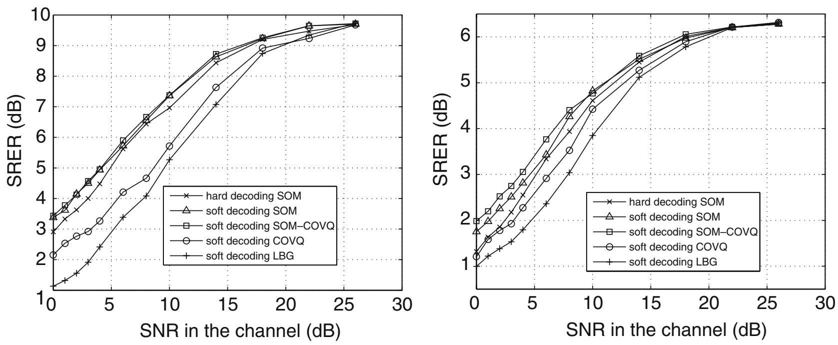


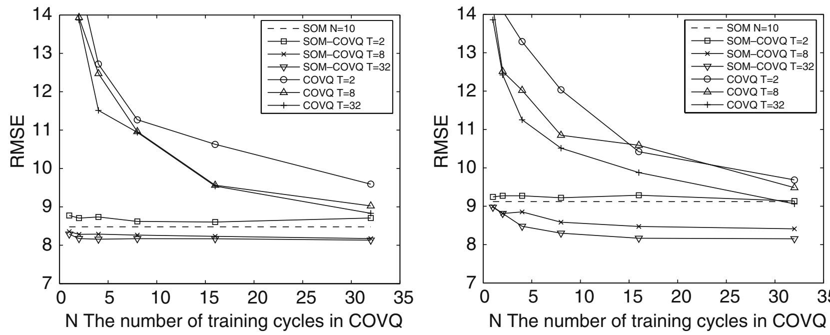
Fig. 5 The signal-to-reconstruction error ratio (*SRER*) in dB. The channel is an AWGN channel

(a) 2-D Gaussian Source **(b)** 3-D Gaussian Source



(a) 2-D Gaussian Source **(b)** 3-D Gaussian Source

Fig. 6 The signal-to-reconstruction error ratio (*SRER*) in dB. The channel is a fading channel



(a) AWGN channel, SNR = 14 dB **(b)** Fading channel, SNR = 29 dB

Fig. 7 The effect of insufficient training. The image, Lena, is used. In the SOM approach, the number of training cycles is equal to 10. In COVQ or SOM–COVQ, the effective number of training epoches is equal to $N \times T$, where T is the number of retransmission in each epoch. For the LBG case, the number of training cycles is equal to 20. For the AWGN case, the RMSE of LBG is equal to 23.34. For the fading channel, the RMSE of LBG is equal to 15.29

in terms of training epoches and reconstruction error, is very poor. This is because the COVQ training may not be able to capture enough noise statistics of the channel. So, we consider the re-transmissions of the training samples in each epoch. Therefore, in COVQ and SOM–COVQ, there are two training parameters: one is the number N of training epoches, and the other is the number T of re-transmissions of the training examples in each epoch. Of course, in terms of distortion, it is desirable to have N and T sufficiently large so that distortion is small and does not further decrease significantly. Note that the additional bandwidth/power/delay requirement associated with the COVQ is proportional to the value of $N \times T$. In practice, the minimum sufficient values of $N \times T$ should be used such that the distortion does not further decrease significantly. *In the LBG and SOM approaches, there is no re-transmission at each training epoch*.

Figure 7 shows the root-mean-square-error (RMSE) performance for the two channel models. For AWGN, the channel SNR value is equal to 14 dB. For the fading channel, the channel SNR value is equal to 29 dB. Other values of SNR have similar results. In terms of the training convergence, the SOM approach is much better than the COVQ approach. For COVQ, with insufficient value of T (Fig. 7, $T = 2$), the COVQ training process may not converge well even after performing a large number of training cycles. With the sufficient value of T, the convergence becomes better for large value of T.

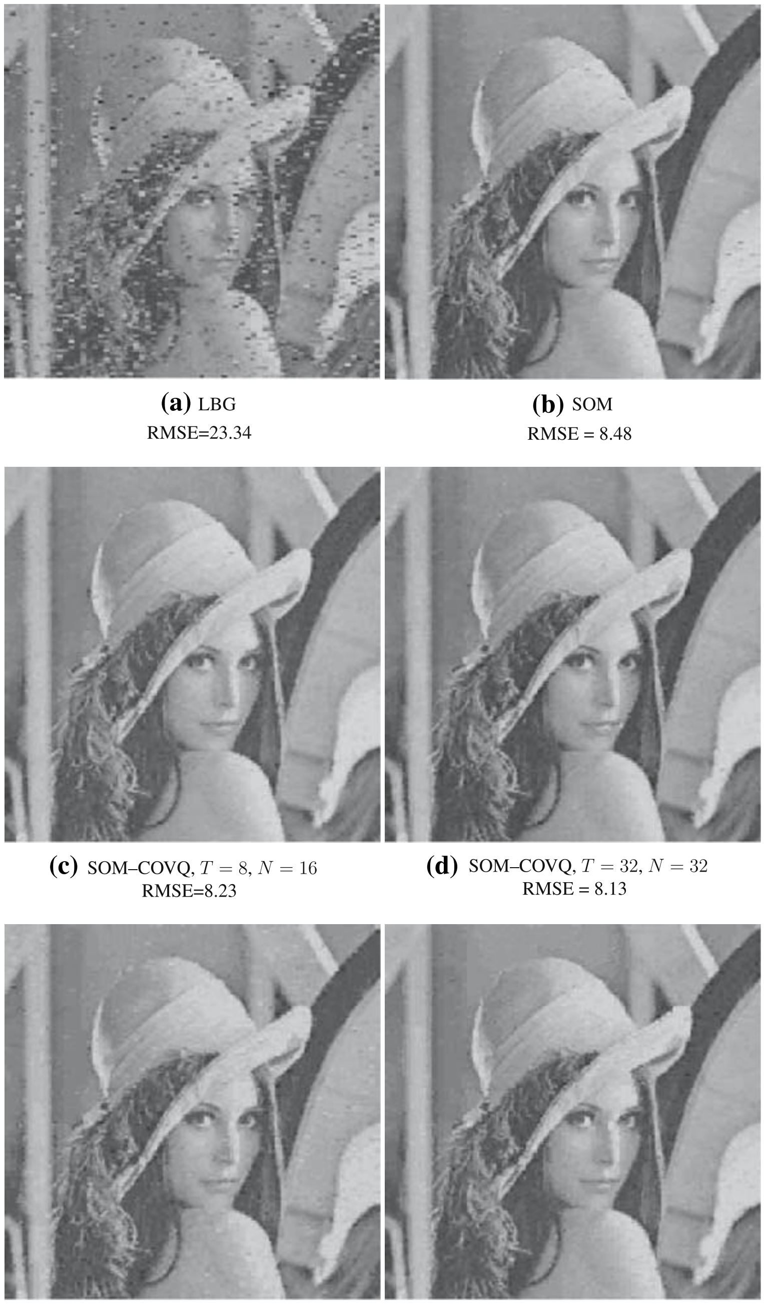
For the SOM–COVQ approach, a sufficient value of T is also very important. With very small value T, even we increase the number of training epoches, the SOM–COVQ cannot achieve a performance better than the SOM approach. However, with a large value of T, the SOM–COVQ is better than the SOM approach. That confirms that we can further improve the performance of the SOM codebook if we use the codebook of SOM as the initial codebook of COVQ. Compared with COVQ, with a large value of T, the convergence of SOM–COVQ is much better.

Those observations suggest that the COVQ approach is beneficial, only if sufficiently large values of T and N (hence, possibly long delay, large bandwidth and high power/channel SNR required for training) are affordable; otherwise, the performance may actually degrade. This justifies that the SOM as an excellent RVQ. Note that in COVQ and SOM–COVQ, a different codebook for a different channel SNR is required.

Finally, reconstructed versions of the image "Lena" at SNR = 14 dB for the AWGN are shown in Fig. 8. Compared with the conventional LBG without enhanced channel robustness, the SOM approach significantly improves the reconstruction quality of images. Also, the visual performance of the SOM is comparable to that of the sufficiently trained COVQ and SOM–COVQ.

4.3 Image data: sufficiently trained COVQ

We use six images, shown in Fig. 9, as data source to compare the performance of SOM with a sufficiently trained COVQ. The three out of the six images, Lena,



(a) LBG
RMSE=23.34

(b) SOM
RMSE = 8.48

(c) SOM–COVQ, $T = 8$, $N = 16$
RMSE=8.23

(d) SOM–COVQ, $T = 32$, $N = 32$
RMSE = 8.13

(e) COVQ, $T = 8$, $N = 16$
RMSE=9.56

(f) COVQ, $T = 32$, $N = 32$
RMSE = 8.83

Fig. 8 The reconstructed images at SNR = 14 dB for the AWGN channel

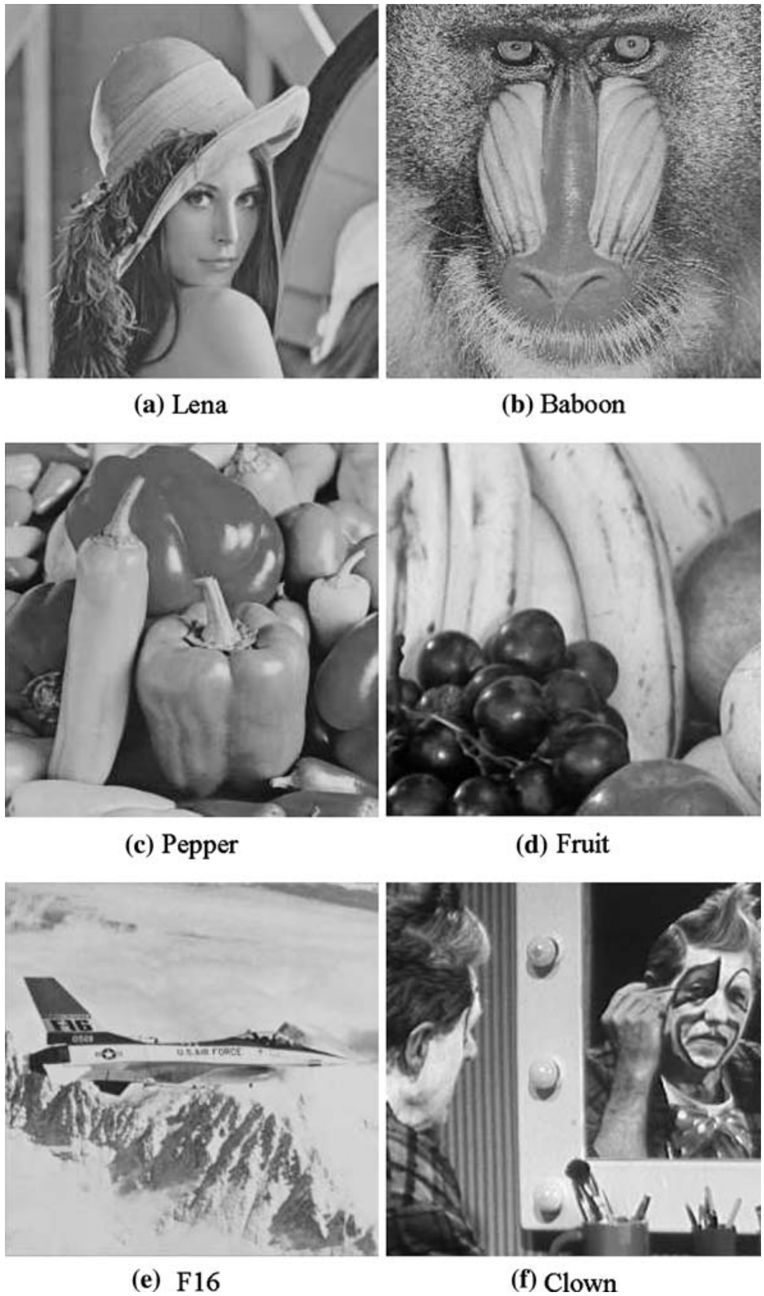


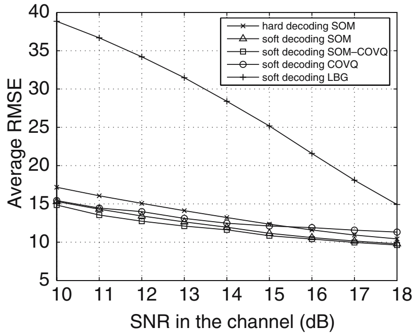
Fig. 9 The six images used in the simulations

Pepper and Baboon, are applied as training set and the whole six images are testing set. The modulation scheme is 16 QPSK. Other settings are same as those of Sect. 4.2. In the COVQ and SOM–COVQ, $T = 32$ and $N = 32$. This means that the effective number of training epoches in COVQ and SOM–COVQ is equal to 1,024. While for SOM, the number of training epoches is equal to 10 only.

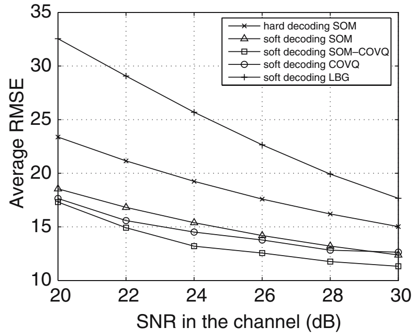
The average RMSE of the received images are shown in Fig. 10. We can observe that SOM, COVQ and SOM–COVQ are much better than LBG. Also, soft decoding SOM is better hard decoding SOM. The result of soft decoding SOM is comparable with that of COVQ. However, the soft decoding SOM requires less training epoches. Also, the proposed approach SOM–COVQ can provide further improvement.

5 Conclusion

This paper presents a soft decoding SOM approach that can effectively suppress the effect of channel noisy on VQ data. In our comparative study, the soft decoding SOM approach is shown to be an excellent



(a) AWGN channel



(b) Fading channel

Fig. 10 The average RMSE of reconstruction images over all six images when sufficient training is applied on COVQ and SOM–COVQ. In the SOM approach, the number of training cycles is equal to 10. For the LBG case, the number of training cycles is equal to 20. In the COVQ and SOM–COVQ, T = 32 and N = 32. This means that the effective number of training epoches in COVQ and SOM–COVQ is equal to 1,024

RVQ with performance comparable with that of the COVQ. Besides, as a RVQ, the soft decoding SOM approach avoids the time-consuming index assignment process of traditional RVQs and unlike COVQ, it does not require a reliable feedback channel and the additional bandwidth/power/delay requirement associated with the noisy data training. We also propose the SOM–COVQ approach to further improve the performance.

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