

# P Systems on Hexagonal Picture Languages and Applications

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**ExtendedAbstract** Hexagonal pictures occur in several application areas especially in picture processing and image analysis. Hexagonal Kolam array grammars for generating hexagonal arrays and hexagonal patterns on triangular grids which can be treated as two-dimensional representation of three-dimensional blocks was constructed by Siromoney et al. [19]. We refer to [13,14] for the study of two-dimensional representations of three-dimensional blocks. Recently, the hexagons and the hexagonal tiling have been addressed by a symmetric coordinate frame in [4,12] and possible link of applications in [11].

Contextual grammars were introduced by S.Marcus[8] in 1969 as another model to describe natural languages. A contextual grammar produces a language by starting from a given finite set of strings and adding, iteratively, pairs of strings (called as contexts), associated to sets of words (called selectors) to the string already obtained. Many variants of contextual grammars have been considered in the literature and investigated from a mathematical point of view [3,9,10]. Two special cases of contextual grammars, called internal and external are very natural and have been extensively investigated. An external contextual grammar generates a language starting from a finite set of strings and iteratively adjoining to its contexts. In internal contextual grammars [3], the context are adjoined inside the current string.

In [20], D.G.Thomas et al developed a new method of generating hexagonal arrays based on an extension of contextual grammars called parallel contextual hexagonal array grammars. Their systems yield languages of hexagons using parallel rewriting relations. They make use of 'window movement' on arrow heads to decide whether the languages are generated by array contexts of choice mappings by the applications of array contextual operations parallelly.

The area of membrane computing, was initiated by Paun [15] introducing a new computability model, now called as P system, which is distributed highly parallel theoretical model inspired by the membrane structure and behavior of the living cells. A computation starts from an initial configuration of a system,

defined by a membrane structure with objects and evolution rules in each membrane, and terminates when no further rule can be applied. One uses the Chomsky way of rewriting for computations, in a P system with string objects. In [7] the contextual way of handling string objects in P systems has been considered and that the contextual P systems are found to be more powerful than ordinary string contextual grammars and its variants. James et al [5] have introduced a P systems called external and internal parallel contextual hexagonal P systems to generate pictures using X, Y, and Z directional parallel contextual hexagonal array rules and studies some basic properties of these P systems and proved comparison results in terms of their generative powers. Jayasankar et al [6] a new P system model called parallel contextual hexagonal array insertion-deletion P system (PCHAIDPS) based on X, Y and Z directional contextual rules of parallel contextual hexagonal array grammar and makes use of parallel contextual double window movement along the arrow heads for generating pictures. They studied closure properties of PCHAIDPS and proved PCHAIDPS has more generative power compared to some well-known families of hexagonal languages like HLOC and HREC [1]. In [21], we compared the generative powers of P system models PCHAPS and PCHAIDPS and prove that the family of languages generated by PCHAIDPS properly contains the family of languages generated by the P system PCHAPS. In this paper we review some interesting results done by the authors connecting P Systems and hexagonal picture languages. We discuss some applications of the study in the area of Artificial Intelligence.

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