Pasting Systems with Picture Tiles Yielding Interesting Patterns

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Extended Abstract

Formal language theory initially dealt with string generating grammars. Later on Giammarresi and Restivo (Giammarresi and Restivo 1997) extended to two dimensional languages. A two dimensional language is a set of pictures. The study of picture languages was initially motivated by the problems of multidimensional pattern recognition, image processing and scene analysis. Motivated by problems in tiling. Nivat et al. (Nirat et al. 1991) proposed a class of grammars called puzzle grammars. Kalyani et al. (Kalyani, Dare, and Thomas 2007) introduced iso array grammars generating iso picture languages.

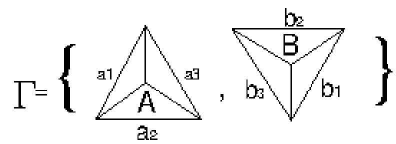
The art of tiling has played a vital role in the field of architecture and pattern generation very early in human civilization (Grunbaum and Shephard 1987). Kolam is a traditional art of decorating floors and court yards in South India and is still followed (Robinson 2007). Over the ages intricate tiling patterns have been used to decorate and cover floors and walls.

A two-dimensional parallel generating model called pasting system was introduced to generate tiling pattern. This model allows two square tiles to be pasted together on their sides based on the rules given in the system (Robinson, Dare, and Subramanian 1999). Motivated by this, triangular pasting systems, that paste edge-to-edge of isosceles right angled triangles in the two-dimensional plane are introduced (Kalyani et al. 2006). Many variants of pasting system like extended pasting system, tabled pasting system and parametric pasting system plays a vital role in picture generation (Kalyani, Dare, and Thomas 2007; Robinson, Dare, and Subramanian 1999).

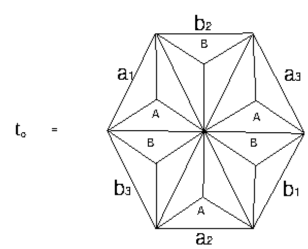
Motivated by the study of array grammar system, tetrahedral array grammar system generating tetrahedral picture languages was proposed in (Raman, Kalyani, and Thomas 2020). Regular tetrahedral array grammar, context-free tetrahedral array grammar and basic puzzle tetrahedral array grammar systems were introduced and compared their properties both respect to generative capacity. In this paper we give a survey of tetrahedral picture languages generated by tetrahedral array grammar. System and 3D-patterns generated by tetrahedral pasting system. We also extend the study of pasting system by proposing controlled labled tetrahedral tile pasting system.

K-tabled tetrahedral tile pasting system, which generates digitized three-dimensional tiling patterns was introduced in (Kalyani, Raman, and Thomas 2020). Some interesting patterns are generated by this system.

Example 1. *A one-tabled Tetrahedral Tile Pasting System, generating a sequence of three-dimensional patterns whose boundaries are hexagons and stars alternatively is given below:*
$M = (\Gamma, E, P, t_0)$ *where*



$E = \{a_1, a_2, a_3, b_1, b_2, b_3\}$;
$P = \{T_1\}$
$T_1 = \{(a_1, b_1), (a_2, b_2), (a_3, b_3), (b_1, a_1), (b_2, a_2), (b_3, a_3)\}$



The first three members of T(M) are shown in Figure 1.

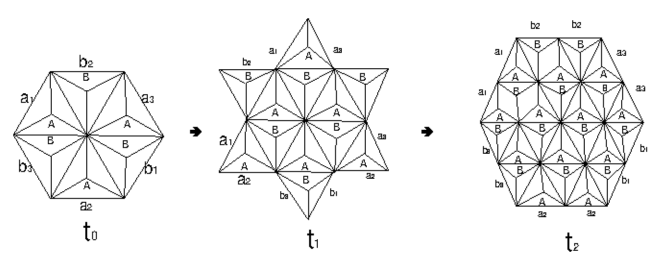
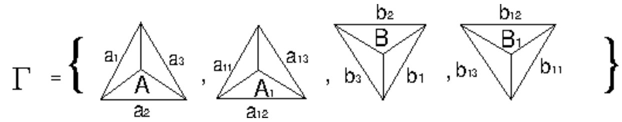


Figure 1: Hexagon and Star polyhedral

Example 2. *A two - tabled Tetrahedral tile pasting system, generating a sequence of three-dimensional mango-leaf patterns is given below.*
$M = (\Gamma, E, P, t_0)$
where



$E = \{a_1, a_2, a_3, a_{11}, a_{12}, a_{13}, b_1, b_2, b_3, b_{11}, b_{12}, b_{13}\}$
$P = \{T_1, T_2\}$
$T_1 = \{(b_2, a_2), (a_3, b_3), (b_1, a_1), (a_2, b_2), (b_3, a_3), (a_1, b_1),$
$(b_2, a_{12}), (a_3, b_{13}), (b_1, a_{11}),$
$(a_2, b_{12}), (b_3, a_{13}), (a_1, b_{11})\}$
$T_2 = \{(a_{13}, b_3), (b_{11}, a_1), (a_{12}, b_2), (b_{13}, a_3), (a_{11}, b_1),$
$(b_{12}, a_2), (a_{11}, b_1), (b_{12}, a_2),$
$(a_{13}, b_3), (b_{11}, a_1), (a_{12}, b_2), (b_{13}, a_3)\}$.

The first three members of T(M) are shown in Figure 2.

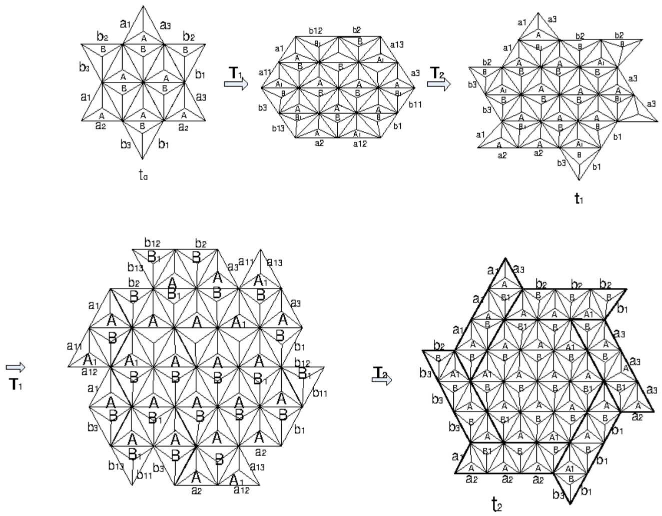


Figure 2: Mango-leaf polyhedrals

In this paper controlled tabled tetrahedral tile pasting system is proposed and it is compared with K-TTTPS and we prove that CTTTPS have more generative capacit tha K-TTTPS.

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