NEXT GENERATION HEIGHT REFERENCE FRAME

PART 3/3: TECHNICAL REQUIREMENTS

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Glossary

AHD - Australian Height Datum

AFGN - Australian Fundamental Gravity Network

AGQG – Australian Gravimetric Quasigeoid

ANGD - Australian National Gravity Database

ANLN - Australian National Levelling Network

DEM - Digital Elevation Model

EGM2008 - Earth Geopotential Model 2008

GGM - Global Geopotential Model

GNSS - Global Navigation Satellite System

GPS - Global Positioning System

GSWA - Geological Survey of Western Australia

MDT - Mean Dynamic Topography

MSL - Mean Sea Level

MSS - Mean Sea Surface

NLS - National Levelling Survey

NMC - National Mapping Council

SRTM – Shuttle Radar Topography mission

Notation

C - Geopotential Numbers

g - Gravity

$\bar{g}$ - Integral mean gravity along the plumbline

h - Ellipsoidal height

H - Physical height

m - Metres

mGal – A measure of acceleration ($\frac{10^{-5}m}{s^2}$)

N - Geoid height

ρ – Density of the Earth

s - Seconds

Std - Standard Deviation

U - Gravitational Potential Energy of the Earth Approximating Ellipsoid

W - Gravitational Potential Energy of the Earth

ζ - Quasigeoid Height Anomaly

γ_0 - Normal Gravity on the Ellipsoid Surface

1. Purpose and Scope

The purpose of this study was to assess both the **User Requirements (Part 2/3)** for height determination and the **Technical Options (Part 3/3)** which could be implemented to meet the user requirements.

The only other user requirements study of physical heights in Australia was undertaken in 1988 (Kearsley, 1988) which predates the widespread use of GNSS. Given the technological advances of the past two decades and the modern methods some now use to access physical heights, we felt it necessary to reach out to the user community to assess their needs for physical heights now and into the future.

The results of the user requirements study should be reviewed in conjunction with this technical options report which reviews the height system and height datum options which could be implemented in Australia based on current and future data holdings.

The objective is to identify what requirements users have for height datums in Australia and what can technically be developed to provide users with what they need. The **Executive Summary (Part 1/3)** brings the recommendations of both reports together to describe the preferred option to satisfy the needs of users for physical heights in Australia.

2. Introduction

2.1 Height system and Height datum

A height system is the theoretical definition based on adopted conventions such as the internationally agreed geoid reference potential W_0 value. The physical realisation of a height system is a height datum. Therefore, a height system could have many datums as new theories, computational process and data become available. Generally, each new height datum is a better (more accurate, reliable, robust and fit for purpose) realisation of the height system.

2.2 Introduction to the AHD

The Australian Height Datum (Roelse et al., 1971; AHD) is Australia's first and only national height reference system. It was adopted by the National Mapping Council in 1971 based on a (staged) least squares adjustment of 97,320 km of 'primary' levelling (used in the original adjustment) and approximately 80,000 km of 'supplementary' levelling (included in a subsequent adjustment). Levelling observations ran between junction points (see Figure 1; Filmer et al., 2010) in the Australian National Levelling Network (ANLN), and were known as level sections. These level sections were created by combining levelling observations along level runs (usually following major roads). The interconnected network of level sections and junction points was fixed to mean sea level (MSL) observed for 1966-1968 at 30 mainland tide gauges, with MSL assigned a value of zero AHD at each gauge. The staged least squares adjustment propagated heights above 1966-1968 MSL (or defined AHD zero reference), across the ANLN. The ANLN contains systematic, gross and random errors which have propagated into the AHD as local and regional biases (e.g. Morgan 1992).

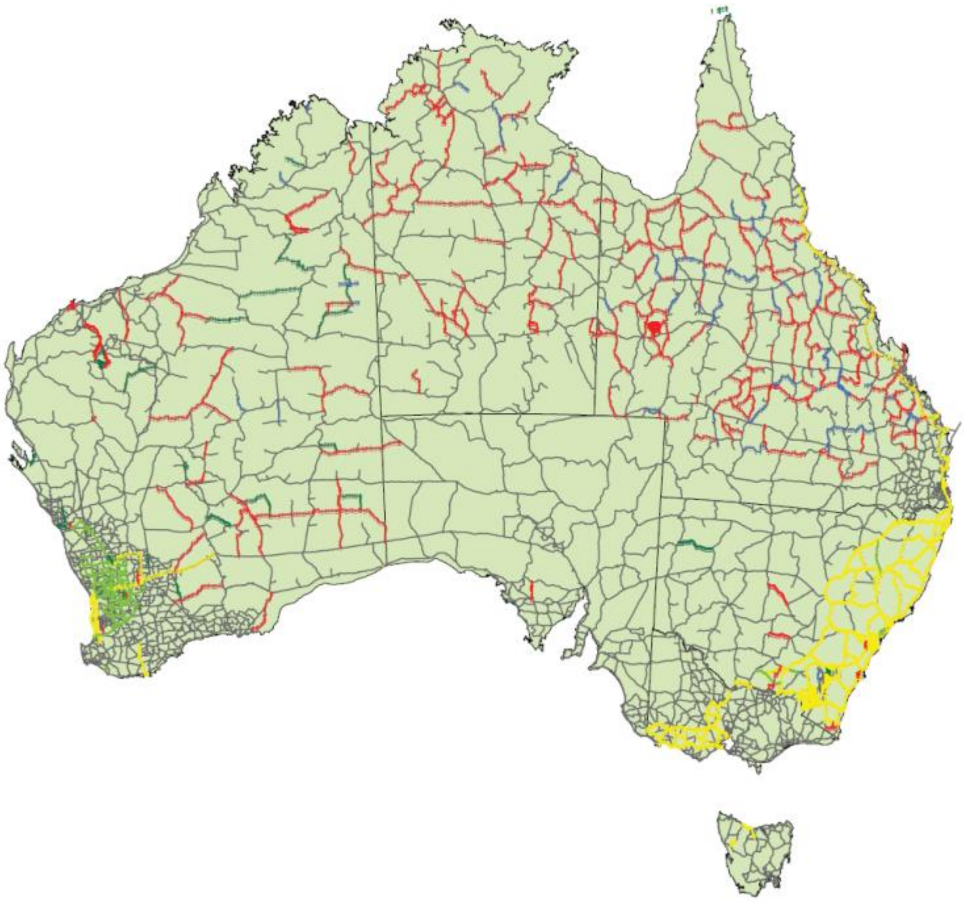


Figure 1: The Australian National Levelling Network (ANLN). First order levelling sections are in yellow, second order sections in light green, third order in fine grey, fourth order in dark green, one way (third order) in red and two-way (order undefined; Steed 2006, pers. comm.) in blue. The junction points are the intersection points of the levelling loops. Lambert projection (figure and description from Filmer et al., 2010).

The Australian Height Datum (Tasmania) 1983 (AHD–TAS83) zero reference is fixed to MSL observed for 1972 only at tides gauges in Hobart and Burnie. It was propagated throughout Tasmania using mostly third order differential levelling over 72 sections between 57 junction points and computed via least-squares adjustment on 17 October 1983. Because AHD (mainland) and AHD (Tas) are referenced to MSL observed at different times and locations, there is a vertical offset between the two datums, which has been estimated (from various methodologies and data) to be ~10 cm (Rizos et al. 1991), between 12 cm and 26 cm (Featherstone, 2000), and ~1 cm (Filmer and Featherstone, 2012a). For the purposes of this document, AHD is used to refer to both the Australian Height Datum 1971 (AHD71; Australian mainland) and Australian Height Datum (Tasmania) 1983 (AHD–TAS83).

2.3 AHD Compatibility with GNSS observations

Global Navigation Satellite System (GNSS) ellipsoidal heights (h) can be converted to physical heights by subtracting the geoid undulation (N) or quasigeoid height anomaly (ζ). This is advantageous since GNSS ellipsoidal heights are relatively cheap and easy to obtain in comparison to large-scale levelling campaigns. AHD heights (H_{AHD}) can be determined by subtracting AUSGeoid model values (ζ_{AHD}) from GNSS ellipsoidal height observations. The AHD height is approximated by Eq. (1).

$$H_{AHD} = h - \zeta_{AHD} \qquad (1)$$

The current AUSGeoid model, AUSGeoid2020 is composed of two components;

1. A gravimetric quasigeoid (cf. Featherstone et al. 2018a) - computed from Global Geopotential Model EGM2008, a 1″x 1″ grid of Faye gravity anomalies onshore (computed from the Australian national gravity database and digital elevation model derived terrain corrections), and satellite altimetry derived gravity anomalies offshore (Sandwell et al 2014).
2. A geometric component (cf. Brown et al. 2018) – computed from 7,634 co-located GPS-levelling points across Australia by least squares prediction to interpolate the geometric offset between the GPS-levelling AHD heights and the gravimetric quasigeoid.

2.4 AHD biases and distortions

AHD has the following biases and distortions which contribute to misalignment with heights determined by Eq. (1).

1. Due to the ocean's time-mean dynamic topography (MDT) and the short tide gauge observation periods, the zero reference of the AHD (MSL at 32 mainland tide gauges) is not coincident with an equipotential surface (e.g. the geoid). The differences largely manifest in a north-south tilt of ~0.7 m in the AHD relative to the geoid across the continent (Featherstone and Filmer 2012a). This tilt can be eliminated by the use of MDT models (e.g., Filmer et al. 2018) at tide gauges to correct for the offset between the sea surface recorded at the tide gauges and the geoid (Filmer et al., 2014).
2. The levelling derived heights contain local and regional distortions due to systematic and gross errors in the Australian National Levelling Network (ANLN) that propagated through the national network adjustments completed in 1971 (Morgan 1992; Filmer and Featherstone 2009; Filmer et al 2014).

These attributes of the current official Australian Height Datum are problematic for many users of the datum. In particular, they have a noticeable impact on users requiring consistent heights over large distances (e.g. greater than 10 km) for engineering purposes (e.g. Snowy Hydro) and scientific studies (e.g. water flow over wide regions). For such cases, an alternative vertical height datum would be valuable presuming there is no strict requirements to align with AHD.

3. Physical Height system options

3.1 What is a physical height?

Physical height systems are based on geopotential numbers $C = W - W_0$ (m^2/s^2), where W is the gravitational potential energy at a point on or above the Earth's surface and W_0 is the gravitational potential on some equipotential (i.e. the gravitational potential is constant) reference surface. Geopotential numbers are converted to meaningful heights H by dividing them by some gravity value g.

$$H = \frac{C}{g} \tag{2}$$

Dimensionally, for the units in metres and seconds, gravity is shown in terms of $\frac{m}{s^2}$ and C is in $\frac{m^2}{s^2}$, therefore H is in terms of metres (Heiskanen and Moritz, 1967, Ch. 4; Jekeli, 2000, Featherstone and Kuhn, 2006).

The equipotential surface which best agrees with mean sea level is called the geoid. The computation of geoid requires that the entire mass of the Earth is enclosed by the geoid. However, in reality the topographic surface is typically outside/above of the geoid. Various computational procedures exist to navigate this dilemma, with some approximations. Molodensky (1945) introduced an alternative theoretical surface called the "quasigeoid" which preserves the position of the topographic surface of the Earth during computation. The quasigeoid agrees exactly with the geoid offshore and approximates to the geoid on-shore (cf. Appendix A for more details).

In relation to equation (2), physical height systems differ from one another in two ways:

1. Choice of zero reference (i.e. geoid or quasigeoid), and
2. How the gravity value, g, is determined.

A description of several different height system is given in Appendix A and is summarised below.

3.2 Types of height systems

In general, height systems can be classified as *dynamic*, *rigorous orthometric* (Tenzer et al. 2005; Santos et al. 2006), *Helmert orthometric* (Helmert 1890), *Mader orthometric* (Mader 1954), *Neithammer orthometric* (Neithammer 1932), *normal* (Molodensky, 1945) and *normal-orthometric* (e.g., Rapp 1961).

(i) Dynamic heights are impractical since they are not geometrically meaningful over long distances (Heiskanen and Moritz, 1967, Ch. 4, and Appendix A)
(ii) Rigorous orthometric heights are extremely difficult to compute, requiring gravity values along the curved, torsioned, plumb line. This computation is dependent on knowledge of precise mass-density variation data through the topography which are currently unavailable in Australia (Appendix A).

For these practical reasons, we discount the dynamic and rigorous orthometric height systems from further consideration.

3.3 Which height systems can we realise in Australia?

In practice, physical heights are typically observed or transferred by:

- levelling,
- converting GNSS observations to physical heights using a model, or
- a combination of both methods.

Below we describe how a user would typically observe or transfer height using these techniques to assist in determining which height system would be practical for use.

3.3.1 Levelling

Levelling is a relative method to determine height differences. Relative height observations are referenced to a "known" height value at, for example, a benchmark or MSL at a tide gauge, which is an approximation of the geoid. Height corrections must be applied to observed height differences to account for the non-parallelism of the equipotential surfaces.

Helmert orthometric, Mader orthometric and **Neithammer orthometric** heights approximate the orthometric height with varying precision and complexity (cf. Appendix A). Their determination requires gravity values along levelling traverses derived from gravity measurements (cf. Appendix A) in order to compute the levelling corrections. Filmer et al. (2010) show that surface gravity values can be obtained from Global Gravity Models (GGM) such as EGM2008 or by interpolating the data from the existing Australian gravity database (cf. Appendix B).

Normal heights similarly require gravity values (normal gravity and surface gravity observations as described in Appendix A). Again, surface gravity data are available using GGM's and/or interpolating values from the Australian National Gravity Database (ANGD) (Filmer and Featherstone 2011; Filmer et al 2010).

Normal-orthometric heights can be determined analytically using a formula which approximates the Earth's shape and subsequent gravity field with that of an ellipsoid (cf. Appendix A).

3.3.2 GNSS

A modern requirement of a height datum is for physical heights to be determined from GNSS observations. Ellipsoidal heights (h) derived from GNSS can be approximately transformed to a physical height (H) by subtracting either the,

(i) modelled geoid value $(\boldsymbol{N})$ as an approximation for Helmert orthometric, Mader orthometric and Neithammer orthometric heights (i.e. $\boldsymbol{H_{orth}} = \boldsymbol{h} - \boldsymbol{N}$) or,

(ii) modelled quasigeoid value $(\boldsymbol{\zeta})$ for normal heights (i.e. $\boldsymbol{H_{norm}} = \boldsymbol{h} - \boldsymbol{\zeta}$).

Normal-orthometric heights are not strictly compatible with the geoid or the quasigeoid since they neglect the influence of local gravity variation. In practice, they are most closely aligned with **normal** heights and the differences are small (Fig. 4.1).

It is possible to model both geoid and the quasigeoid from the existing data holdings over Australia (cf. Appendix B). However, the quasigeoid can be computed exactly whilst the geoid requires approximations for the integral mean of gravity along the plumbline within the topography.

3.4 Summary

Appendix A provides a detailed description and discussion of the variety of height systems and what is required to determine height values from Geopotential numbers. In the following table we have summarised this information. Those sections shown in green indicate a positive response, and those in yellow indicate that extra effort is required, or lower accuracy is achieved. **Dynamic** and **Rigorous orthometric** heights have been included in the table, but greyed out to indicate that we have discounted them.

Table 3.1 Summary of height systems.

Height System	Requires gravity observations for levelling?	Referenced to Geoid or Quasigeoid?	Practical to implement?
Dynamic	No	N/A	No
Rigorous Orthometric	Yes	Geoid	No
Helmert Orthometric	Yes	Geoid	Yes
Mader Orthometric	Yes	Geoid	Yes
Neithammer Orthometric	Yes	Geoid	Yes
Normal	Yes	Quasigeoid	Yes
Normal-orthometric	No	Approx. Quasigeoid	Yes

4. Choice of Height System

4.1 Height system precision evaluation

4.1.1 In theory

The three well-known approximations of orthometric height can be listed in terms of how difficult they are to compute and their rigour with regard to fluid flow (cf. Appendix A).

1. Neithammer Orthometric: Most difficult to compute but most accurate.
2. Mader Orthometric: Second most difficult to compute and second most accurate.
3. Helmert Orthometric: Least difficult to compute but least accurate.

The Neithammer gravity value is the most precise approximation (of those listed here) to the true integral mean gravity along the plumbline, but is more complex to compute than both the Mader and Helmert approximations (Dennis and Featherstone 2002, Appendix A). The gravity value is determined from surface gravity data, the normal gravity gradient, an assumed constant topographic density and terrain corrections values (at the surface of the Earth and at discrete points along the plumbline down to the geoid). All necessary data to compute these heights are available in Australia (Appendix A).

For Mader orthometric heights the gravity value uses the second most precise approximation (of those listed here) to the integral mean gravity (Dennis and Featherstone 2002; Hwang and Hsiao 2003). Although, it is easier to compute than the Neithammer integral mean gravity, being derived from the normal gravity gradient, an assumed constant topographic density, surface gravity data and terrain correction values (at the surface of the Earth and on the geoid) are needed. Helmert orthometric heights apply the most simplistic approximation to the integral mean gravity, however, they are also the least precise (Dennis and Featherstone 2002, and Appendix A).

These orthometric height systems are, in principle, referenced to the geoid. The geoid is possible to compute using Australian data holdings (albeit with some approximations regarding to the topographic density). With a precomputed geoid model, these heights are accessible to GNSS users.

Normal heights similarly require gravity values (normal gravity and surface gravity observations; Appendix A). The surface gravity data are generally available at a high enough precision using GGM's and/or interpolating values from the Australian National Gravity Database (ANGD) (Filmer and Featherstone 2011; Filmer et al 2010). However, these heights are referenced to the quasigeoid. The quasigeoid is an approximation to the geoid, and heights above it [theoretically] do not precisely model how water will flow. For this reason, it is theoretically an inferior reference surface to the geoid. However, it can be modelled without any approximations and has been computed with increasingly improved precision over the Australian continent for the last 20 years (e.g. AUSGeoid98, AGQG09 and AGQG2017).

Normal-orthometric heights are currently used in Australia, being the height system of the AHD. Normal-orthometric heights are the easiest of the height systems to establish, however they are also the most inaccurate with respect to fluid flow. They rely only on the normal gravity field so do not require gravity values in their computation (Appendix A), which is why they have been implemented in numerous national vertical datums where gravity observations were not available. However, they neglect the effect of variations in the Earth's gravity field. This causes theoretical errors for heights established by relative methods (i.e. levelling) along differing paths (non-holonomic) and theoretical inconsistency with GNSS heights converted to physical heights using a quasigeoid model.

4.1.2 In practice

Height determination is limited by the instrumentation and models used. Generally the precision of levelling is defined by $c\sqrt{k}$ where k is the length of the levelling traverse in kilometres and c is in millimetres. High quality levelling is typically precise to $2\sqrt{k}$ mm along a traverse. Physical heights determined by transforming GNSS heights in Australia generally have an absolute accuracy of 5 – 6 cm at best; with ~2 cm of error in the GNSS ellipsoidal heights and ~5 cm of error at best for AGQG2017 (cf. Featherstone et al. 2018a).

Filmer et al. (2010) demonstrate the difference between **Helmert orthometric, normal** and **normal-orthometric** height differences over Australia. The difference between **normal** and **Helmert orthometric** heights are shown in Fig 4.1. Filmer et al. (2010) show the difference between the height systems has a standard deviation of ±1.7 cm (min: -2.9 cm; max: 26.3 cm) across Australia. The standard deviation increases to ±4.3 cm, over alpine regions (min: -0.9 cm; max: 26.3 cm).

For levelling traverses longer than 100 km, first order leveling (precise to $2\sqrt{k}$) will give heights precise to around ±2 cm. This is larger than the typical difference between **normal** and **Helmert orthometric** heights in Australia (i.e. the standard deviation of ±1.7 cm across the whole of Australia). We therefore consider that the difference between the height systems is not significant for the majority of use cases, given the precision to which they can be measured (cf. Filmer and Featherstone 2011).

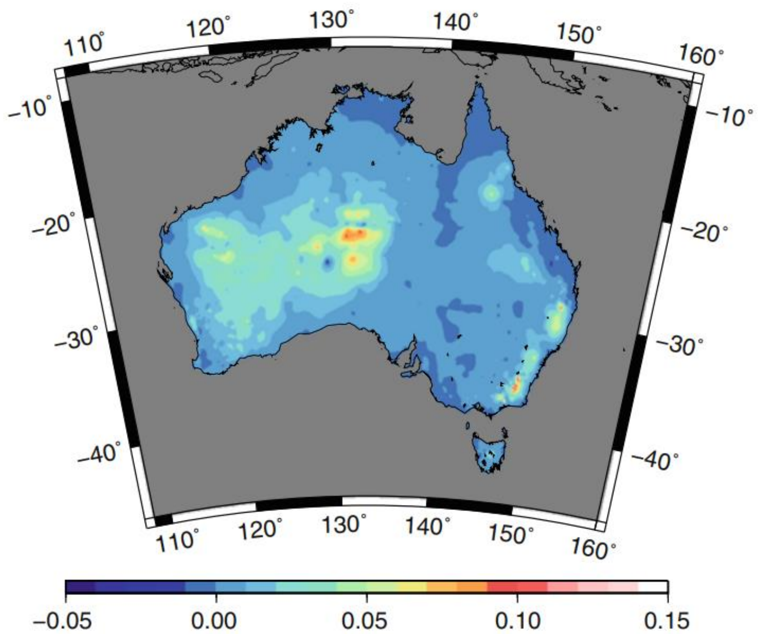


Figure 4.1 Differences between Helmet Orthometric and Normal Heights (in m) over Australia. EGM2008-derived gravity values at benchmarks were used for the levelling corrections (Fig. 6. Filmer et al, 2010).

The difference between normal and normal-orthometric heights are shown in Fig 4.2. Filmer et al. (2010) show the height systems have differences with a nominal standard deviation of ±1.2 cm (min: -2.4 cm; max: 17.7 cm) and a greater standard deviation in alpine regions of ±2.9 cm (min: -2.4 cm; max: 17.7 cm).

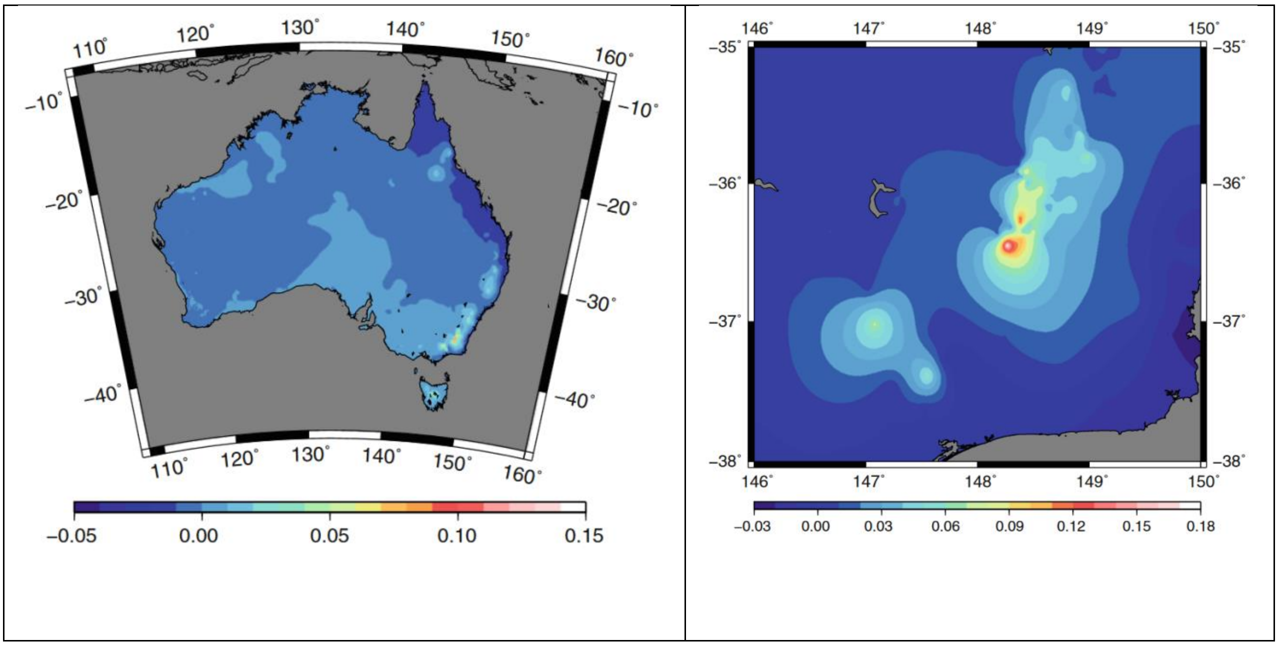


Figure 4.2 (left) Difference between normal and normal-orthometric heights over Australia (from Filmer et al, 2010), and (right) difference between normal and normal-orthometric heights over south east Australia showing maximum differences (from Filmer et al, 2010).

4.2 Evaluation criteria

The following criteria were used to assist in evaluating the strengths and weaknesses of the available height systems:

1. Compatibility with GNSS
2. Ease of access / use
3. Accuracy

We recognise that approximations to the orthometric height (e.g. Neithammer/ Helmert/ Mader) provide a more rigorous and theoretically accurate height system with respect to the direction of fluid flow than normal heights if implemented correctly. For this reason, the geoid may be considered preferable to the quasigeoid; however, modelling the geoid requires approximations of the distribution of the mass density within the topography. The quasigeoid on the other hand can be determined exactly without any approximations. The normal height system, referenced to the quasigeoid is preferable in this practical respect.

The difference between normal and normal-orthometric heights is generally very small over Australia where heights are <1000 m. If a quasigeoid was chosen as the reference surface for the alternative height reference frame, then in theory, the frame would be a realisation of a normal height system. The distinction between normal and normal-orthometric heights would be less than measurement accuracy for most applications of GNSS in Australia, although users should be made aware of the distinction between height systems.

For levelling users operating in a normal height system, Filmer et al. (2010) investigated 1,366 mostly third-order levelling loop closures in Australia to assess the difference in misclosure achieved by applying either normal or normal-orthometric corrections. The results indicate there is negligible benefit to applying normal corrections compared to applying normal-orthometric corrections in Australia. This is most likely attributable to the errors in the third-order levelling at heights <1,000 m being of larger magnitude than the height corrections. This indicates that observing surface gravity values at benchmarks along the levelling traverse to apply normal corrections to levelling in Australia cannot be justified, certainly below heights of 1,000 m.

Table 4.1 Evaluating the strengths and weaknesses of the available height systems.

	Normal	Neithammer orthometric	Normal-orthometric
Compatible with GNSS?	Yes	Yes	Approximately
Ease of Access / Use	Average	Hard	Easy
Accuracy	Average	Highest	Lowest

Based on this analysis, we believe that a normal height system should be adopted for the alternative height reference frame. It is directly compatible with GNSS, moderately easy for users to access with GNSS and there are no practical benefits (at heights <1000 m) to applying normal corrections over normal-orthometric corrections, meaning there is no extra work / effort required by those performing levelling.

5. Recommendations

If an alternative height reference frame was to be established in Australia, a realisation of a normal height system would be the best option. We recommend the Australian Gravimetric Quasigeoid 2017 (AGQG2017) model (Featherstone et. al. 2018a) is used as the initial release of an alternative height reference frame and refined over time as more data becomes available.

AGQG2017 is the gravimetric component of the AUSGeoid2020 model and is based on EGM2008 enhanced with local gravity data (Appendix B). It is aligned with the internationally agreed W_0(=62,636,856.0 m^{2}s^{-2}) value and provides a seamless physical reference surface onshore and offshore.

Given that AGQG2017 is only a model and heights are not provided on benchmarks, levelling users will need GNSS heights to access the alternative height reference frame. They can use the quasigeoid model value to derive normal heights. These can then be used as reference heights / starting points for levelling surveys.

Clear communication on the differences between derived normal heights from AGQG2017 and AHD heights will be critical given that existing AHD benchmarks are not coincident with normal heights derived from AGQG2017 (having differences of up to approximately 0.5 m). Benchmark height values could potentially be provided at permanent GNSS antennas using a quasigeoid model to convert the GNSS heights to normal heights.

Appendix A – Height Systems

Physical height systems are based on geopotential numbers $C = W - W_0$ (m^2/s^2), where W is the gravitational potential energy at a point on or above the Earth's surface and W_0 is the gravitational potential on some equipotential (i.e. the gravitational potential is the same) reference surface. Fluids flow from high geopotential numbers (in absolute value) to low ones. The geopotential numbers are converted to meaningful heights H by dividing them by some gravity value g. $\left(H = \frac{C}{g}\right)$

Dimensionally, for the units in metres and seconds, gravity is shown in terms of $\frac{m}{s^2}$ and C is in $\frac{m^2}{s^2}$, therefore H is in terms of metres (Heiskanen and Moritz, 1967, Ch. 4; Jekeli, 2000, Featherstone and Kuhn, 2006).

Physical height systems differ from one another in two ways:

1. Choice of zero reference (i.e. geoid or quasigeoid), and
2. How the gravity value, g, is determined.

Dynamic heights

Description: Dynamic heights are the most closely related to geopotential numbers. A single constant gravity value $\tilde{\gamma}_0$ is usedto transform the geopotential numbers to units of length. This is typically the gravity value generated by an Earth approximating ellipsoid at some preferred fixed latitude e.g. mean latitude local to the datum or 45 degrees.

***Strengths*:** Dynamic heights are simple to compute. By using a constant gravity value, variations in geopotential numbers are retained in the height values. They precisely map how water will flow and levelling loops will "close" perfectly in theory (Featherstone and Kuhn, 2006).

$$H^D = \frac{C}{\tilde{\gamma}_0} \quad \text{(A1)}$$

***Weaknesses*:** Dynamic height differences are not geometrically meaningful. That is, for points on two different equipotential surfaces, the dynamic height between points on each surface is constant. However, due to gravitational variations, the equipotential surfaces will be geometrically closer or further apart in places. This is negligible over small scales but becomes problematic in terms of consistency over large scales (e.g. over a continent such as Australia).

The correction added to levelling height differences, Δn_{AB}, between two points A and B to obtain dynamic height differences is given by Heiskannen and Moritz (1967) as,

$$DC_{AB} = \int_A^B \frac{g-\gamma_0}{\gamma_0} dn = \sum_A^B \frac{g-\gamma_0}{\gamma_0} dn \quad \text{(A2)}$$

where g are gravity values measured at the Earth's surface (measured independently or interpolated from existing data or a GGM), γ_0 is the normal constant gravity value and dn is the levelled height increment along the route (Heiskanen and Moritz (1967, Ch. 4); Jekeli (2000)).

Orthometric heights

Description: The orthometric height (Fig. A1) is given by the geopotential number divided by the integral mean gravity value along the curved gravitational plumbline between the observation point and the geoid surface $\bar{g}$

(Heiskanen and Moritz, 1967 Ch. 4). The geoid is an equipotential surface which best agrees with mean sea level.

$$H^o = \frac{C}{\bar{g}} \quad \text{(A3)}$$

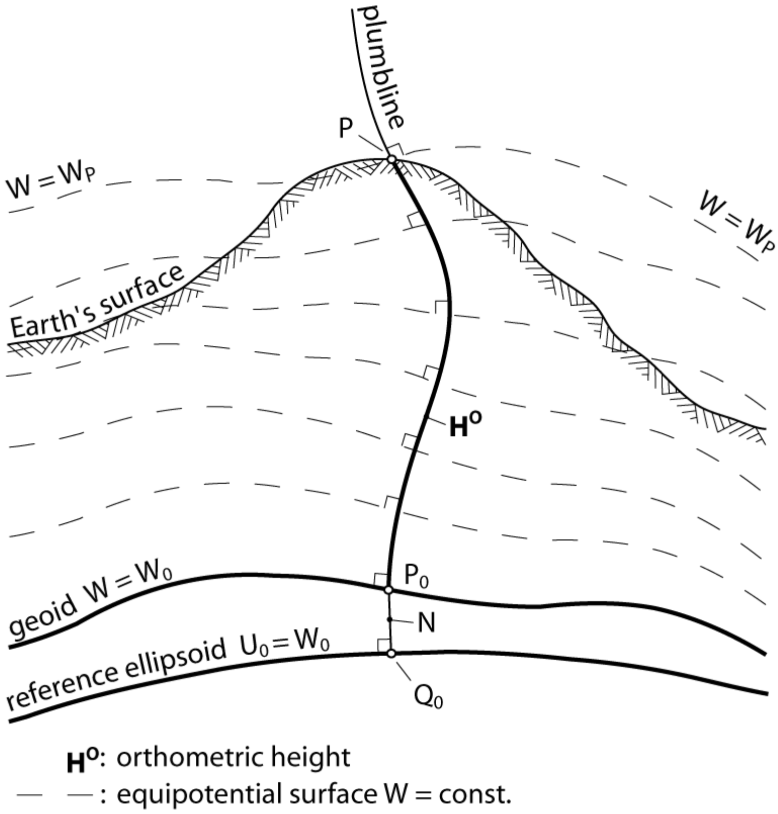


Fig. A1 - Geometry of the orthometric height (Featherstone and Kuhn 2006).

Strengths: This height system is based on the curved and torsioned distance from the equipotential surface which best agrees with MSL (namely, the geoid) to the observation point. Unlike the dynamic height, it is geometrically meaningful. The geoid surface extends everywhere so can be used as a common reference surface in all places and can be modelled with gravity data. These heights are referenced to an equipotential surface so they approximately model how fluids will flow locally. Levelling loops will "close" perfectly in theory.

Weaknesses: The rigorous orthometric correction applied to levelling height differences at points A and B, is given by,

$$OC_{AB} = \sum_A^B \frac{g-\gamma_0}{\gamma_0} dn + \frac{\bar{g}_A-\gamma_0}{\gamma_0} H_A^* - \frac{\bar{g}_B-\gamma_0}{\gamma_0} H_B^* \quad \text{(A4)}$$

This formula requires the integral mean gravity values within the topography, $\bar{g}_A$ and $\bar{g}_B$, and the average of the surface gravity values at the two benchmarks.

In practice, the integral mean gravity values, $\bar{g}_A$ and $\bar{g}_B$, along the curved gravitational plumb line are impossible to obtain without precise knowledge of the mass-density variations through the topography (cf. Tenzer et al. 2005; Strange 1982). This means it is not possible to calculate true orthometric heights. Instead, approximations to obtain the integral mean gravity have been formulated. However, these necessary approximations mean that levelling loops will not "close" perfectly i.e. levelling height differences will accumulate systematic errors (cf. Santos et. al 2006).

Helmert Orthometric Heights

To approximate the integral mean gravity $\bar{g}$ at the benchmarks, the Helmert (1890) approximation uses a gravity observation at the surface g_s, the normal gravity gradient and assumed constant density of 2.67 Mg/m^3 (Mega gram per metre cubed). Formally, $\bar{g}$ is approximated by the Poincare and Prey reduction using a Bouguer plate, (Heiskannen and Moritz, 1967, pg. 163-165)

$$\bar{g} = g_s + \frac{1}{2}\frac{\partial\gamma}{\partial h}H^{orth} - 2\pi GM\rho H^{orth} \quad \text{(A5)}$$

Here, γ is the normal gravity, GM is the Earth gravity constant and ρ is the standard rock density of 2.67 Mg/m^3. $2\pi GM\rho H^{orth}$ corresponds to the effect of an infinite slab of topography. It is possible to compute an approximation of mean gravity, however the gravitational effect of topography is assumed to be modelled adequately by an infinite slab and neglects the effect of topographic height and density variations.

Neithammer Orthometric Heights

To approximate the integral mean gravity at the benchmarks, Neithammer (1932) uses a gravity observation at the surface g_s, the normal gravity gradient and assumed constant density of 2.67 Mg/m^3, and includes the effect of gravitational variations due to the topography differing from an infinite slab. Neithammer orthometric heights $\bar{g}$ are given by the following equation (Neithammer, 1932)

$$\bar{g} = g_s + \frac{1}{2}\frac{\partial\gamma}{\partial h}H^{orth} - 2\pi GM\rho H^{orth} + \delta g^{TC} - \overline{\delta g^{TC}} \quad \text{(A6)}$$

Here δg^{TC} is the terrain correction at the surface of the Earth and $\overline{\delta g^{TC}}$ is the integral mean terrain correction along the plumbline (evaluated in practice using discrete $\overline{\delta g^{TC}}$ along the plumbline). The accurate computation of the terrain correction terms is dependent on adequately sampled topographic height variations.

Mader Orthometric Heights

To approximate the integral mean gravity at the benchmarks, the Mader (1954) orthometric heights use the following approximation to the integral mean gravity anomaly.

$$\bar{g} = g_s + \frac{1}{2}\frac{\partial\gamma}{\partial h}H^{orth} - 2\pi GM\rho H^{orth} + \frac{\delta g^{TC} - \delta g_0^{TC}}{2} \quad \text{(A7)}$$

Here δg^{TC} is the terrain correction at the surface of the Earth. This accounts for variations in topography away from an infinite slab, again assuming constant density. δg_0^{TC} is the terrain correction value at the surface of the geoid. The accurate computation of the terrain correction terms is dependent on adequately sampled topographic height variations.

Normal heights

Definition: Normal heights are reliant upon defining an Earth approximating reference ellipsoid, which has a defined gravity field. At the ellipsoid surface, the normal gravitational potential U_0 agrees with the Earth's gravitational potential at mean sea level W_0. At a point P on the surface of the Earth, a point Q is defined such that the true gravity potential of the actual Earth at the surface W_P is equal to the normal potential at the point Q (i.e. $W_P = U_Q$). The points Q define a theoretical surface known as the telluroid. The normal height of the point P is given as the distance between Q and surface of the ellipsoid along the normal plumbline. The distance between the telluroid and the Earth's surface is termed the height anomaly (Fig. A2) which can also be mapped to difference between the reference ellipsoid and the quasigeoid (Heiskanen and Moritz, 1967, Featherstone and Kuhn, 2006). Heights are determined by dividing the geopotential number by the mean normal gravity along the curved normal plumbline ($\overline{\gamma}$).

$$H^N = \frac{C}{\overline{\gamma}} \qquad \text{(A8)}$$

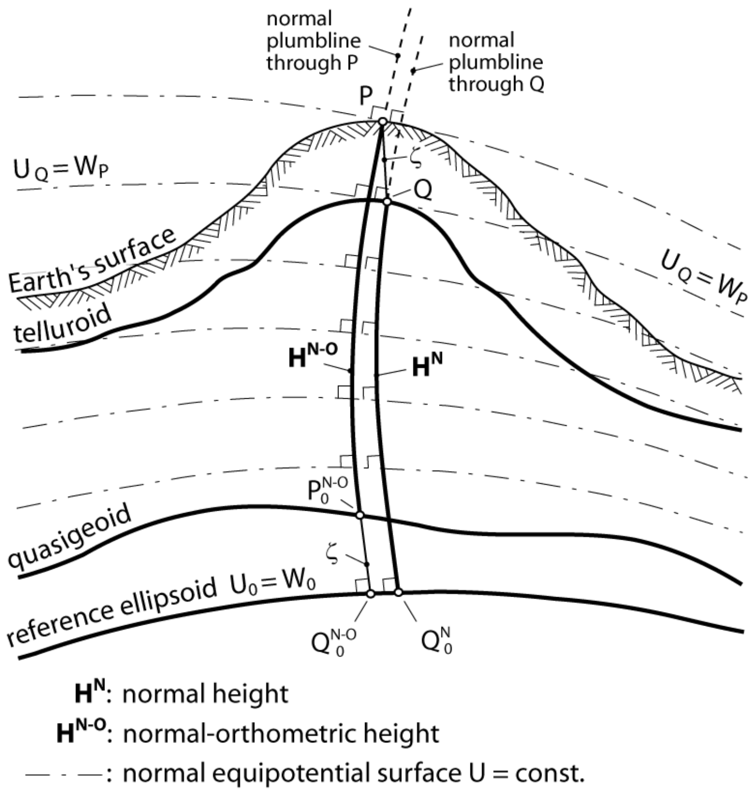


Figure A2 - The geometric interpretation of Normal and Normal-Orthometric heights (Featherstone and Kuhn 2006).

Strengths: This height system avoids the obstacle of computing integral mean gravity anomalies along the plumbline. The mean gravity value is determined from the normal gravity field so it is relatively simple to calculate since it only varies with latitude and height (Featherstone and Kuhn, 2006) i.e. these heights require no approximations of integral mean gravity within the topography**.** The normal correction applied to levelling height differences at points A and B, to convert them to normal heights, is given by,

$$NC_{AB} = \sum_A^B \frac{g-\gamma_0}{\gamma_0} dn + \frac{\overline{\gamma}_A-\gamma_0}{\gamma_0} H_A - \frac{\overline{\gamma}_B-\gamma_0}{\gamma_0} H_B \qquad \text{(A9)}$$

Where g are surface gravity measurements between A and B and $\overline{\gamma}_A$and $\overline{\gamma}_B$ are the average normal gravity along the curved normal plumbline, between the ellipsoid and telluroid.

Weaknesses: As with the discussed orthometric height approximations, normal heights require that gravity observations g must be determined along the levelling traverse (Featherstone and Kuhn, 2006), although these can be interpolated or derived from a GGM. Moreover, they are referenced to the quasigeoid rather than the geoid. The quasigeoid is only an approximation of an equipotential surface so normal heights do not predict how water will flow precisely.

Normal-orthometric heights

Definition: Normal-orthometric heights are purely based on normal gravity. Geopotential numbers are replaced by normal potential differences and divided by the integral mean of the normal gravity between the Earth's surface and the quasigeoid (Fig. A2).

Strengths: This height system only requires the use of the normal gravity field. The mean gravity value is determined from the normal gravity field so there is no need to obtain gravity values along the levelling traverses. Only levelling data and 1D latitude positions are needed to establish the normal-orthometric height. The normal-orthometric correction (Rapp, 1961) (which must be applied to levelling data to get the normal-orthometric height differences between points *a* and *b*) is not dependent on any gravity observations, so these heights are easier to establish with levelling than normal, or orthometric heights.

The full Rapp (1961) normal-orthometric correction (NOC) formula (truncated version implemented in AHD71) is

$$NOC = (AH + BH^2 + CH^3)d\phi \qquad \text{(A10)}$$

Where H is the mean difference between the normal-orthometric height at points *a* and *b*, $d\phi$ is the difference in latitude between the two points in rads,

$$A = 2\sin 2\phi\,\alpha'\left(1 + \cos 2\,\phi\left(\alpha' - \frac{2K}{\alpha'} - 3Kcos^2 2\phi\right)\right)Q \qquad \text{(A11)}$$

$$B = 2\sin 2\phi\,\alpha' t_2\left(t_3 + \frac{t_4}{2\alpha'} + \cos 2\,\phi\left(\frac{3}{2}t_4 + 2\alpha'^{t_3} - \frac{2Kt_3}{\alpha'}\right)\right)Q \qquad \text{(A12)}$$

Where ϕ is the mid latitude between the two points *a* and *b*,

$$\alpha' = \frac{\beta}{2+\beta+2\epsilon} \qquad \text{(A13)}$$

$$K = \frac{-2\epsilon}{2+\beta+2\epsilon} \qquad \text{(A14)}$$

$$t_2 = \frac{2(1+\alpha+c')}{a\left(1+\frac{\beta}{2}+\epsilon\right)} \qquad \text{(A15)}$$

$$t_3 = 1 - \frac{(3\alpha-2.5c')}{2} \qquad \text{(A16)}$$

$$t_4 = 1 - t_3 \qquad \text{(A17)}$$

β and ϵ are the gravity formula constants and α is the spheroid flattening. $c' = \omega^2 a^3/k^2 M$

where ω is the angular velocity of the Earth's rotation, a is the equatorial radius, k^2 is the gravitational constant M is the mass of the Earth and Q is one arc minute in radians.

Weaknesses: These heights do not predict how water will flow precisely**,** and they are not referenced to an equipotential surface, or any defined surface for that matter (Filmer et al 2010). Moreover since these heights are not based on geopotential number differences the levelling loops do not close perfectly.

Appendix B – Data Holdings

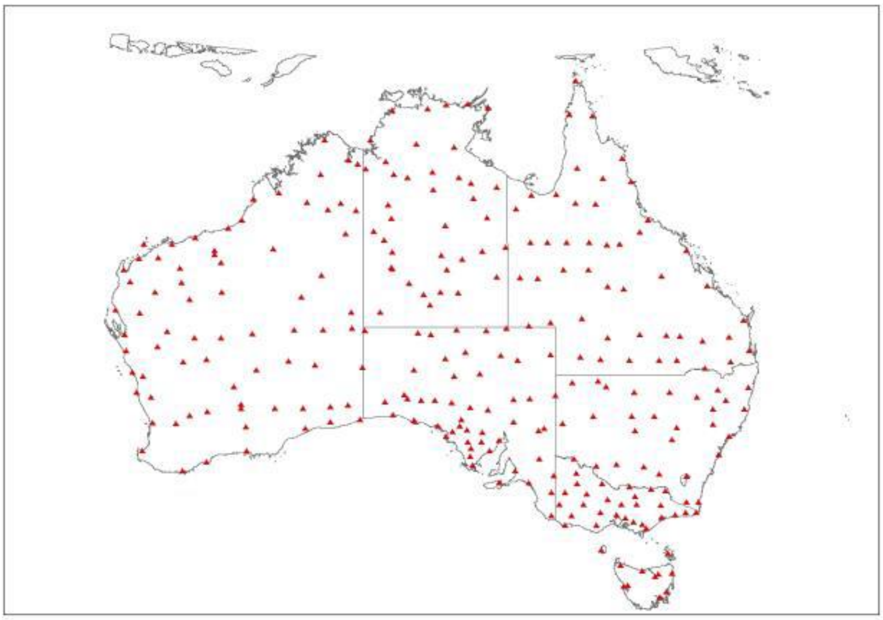
Gravity data

Surface gravity data are needed to determine the height correction value applied to levelling data, transform geopotential number to height values and for the computation of gravimetric geoid/quasigeoid models. This subsection evaluates the gravity data over Australia which are available for these computations.

Absolute gravity

The absolute gravity value refers to the vertical gradient of the Earth's gravitational potential field corrected for the gravitational effect of the Sun, Moon and ocean tides.

The Australian Fundamental Gravity Network (AFGN) was first established in the early 1950s with 59 stations and has grown to consist of around 950 gravity stations at over 250 locations throughout Australia as shown in Figure B1. Wellman et al (1984) suggest an accuracy of ±30 μgals from the 1980 adjustment. In 2001 Geoscience Australia took delivery of an A10 portable absolute gravimeter for the purpose of refurbishing the AFGN. Measurements with the portable absolute gravimeter at 60 AFGN sites confirm 30 microgal accuracy. However the original network demonstrated a bias of ≈ 78 microgals relative to the A10 measurements, and so was subsequently readjusted. The re-adjustment resulted in a new gravity datum, the Australian Absolute Gravity Datum 2007 (Tracy et al. 2007). (cf. http://www.ga.gov.au/afgn/index.jsp)



B1 - Absolute gravity sites (red triangles) included in the AFGN.

Relative gravity

Geoscience Australia maintains the Australian National Gravity Database (ANGD). The data have been collected since the 1930's and by 1974 gravity observations had been made at 11km spacing over most of the continent (Murray 1997). The gravity data comprise AFGN absolute gravity values and relative gravity observations transformed to gravity values (by tying off relative gravity measurements to previously surveyed points or directly to the AFGN absolute gravity values). All of these data points are in terms of the Australian Absolute Gravity Datum 2007. Each gravity data-base entry has corresponding position data, determined by various means, earlier locations were scaled from 1:250,000 photocenter base maps so are no more accurate than ~ 900m, (Fraser et al, 1976). Most contemporary data are positioned with GPS. All position and gravity data channels have a corresponding standard deviation estimate of their precision.

There are 1,767,351 entries in the data base as of 11th December 2018.

Data coverage:

- Estimated surface area of the Australian continent: 7659861 km^2
- Number of gravity observations (on shore) over Australia: 1,767,351
- Average spatial density: 1 gravity observation every 4.334 km^2
- Coverage varies markedly, cf. Fig. B2.

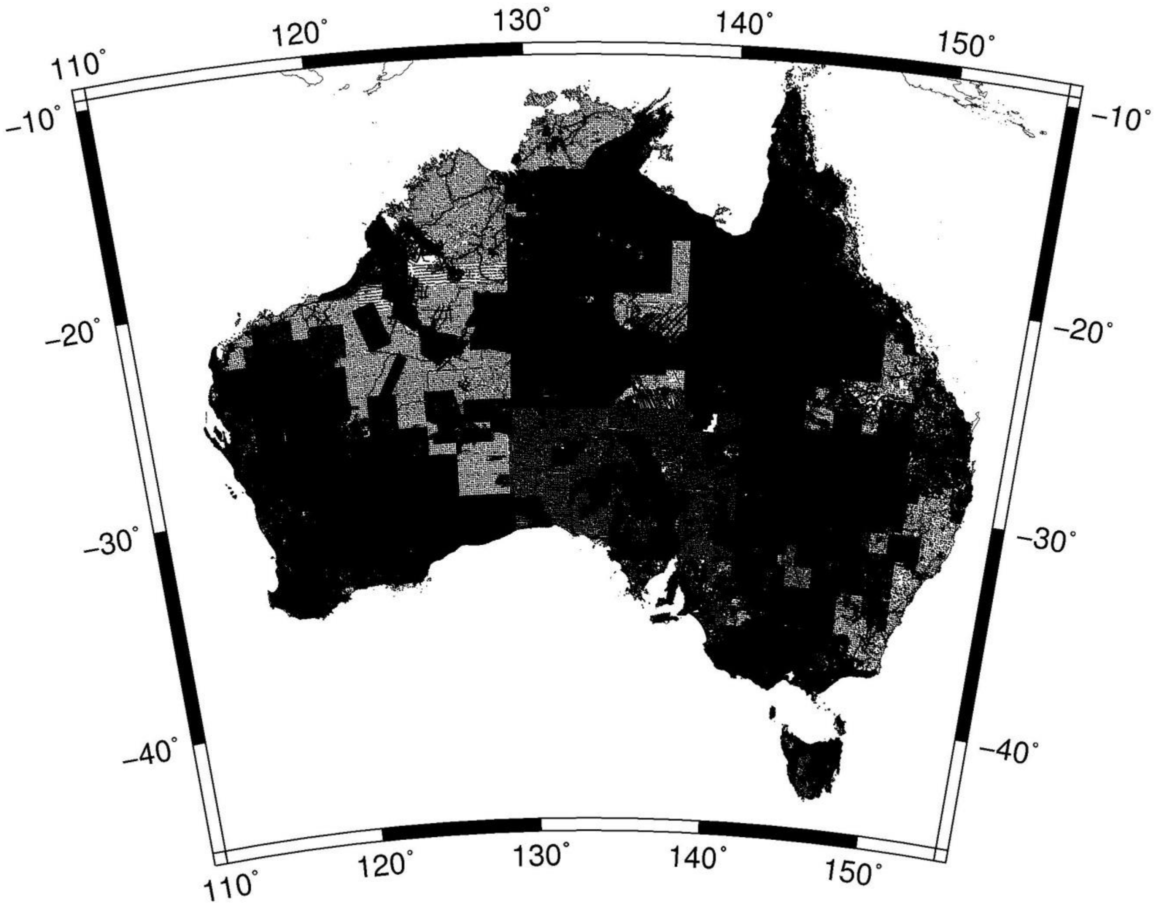


Fig. B2 - Relative gravity data coverage. Average spatial density 1 data point every 4.334 km^2.

Following the methodology of Sproule et al. (2006), the normal-orthometric AHD height of each gravity station have been compared to the 1 second digital elevation model by bi-cubically interpolating the DEM to the location of the gravity sites. The distribution of the differences is shown in Fig. B3. Errors in the heights of the gravity sites will propagate into height correction formulas and gravity anomalies used to compute gravimetric geoids/quasigeoids.

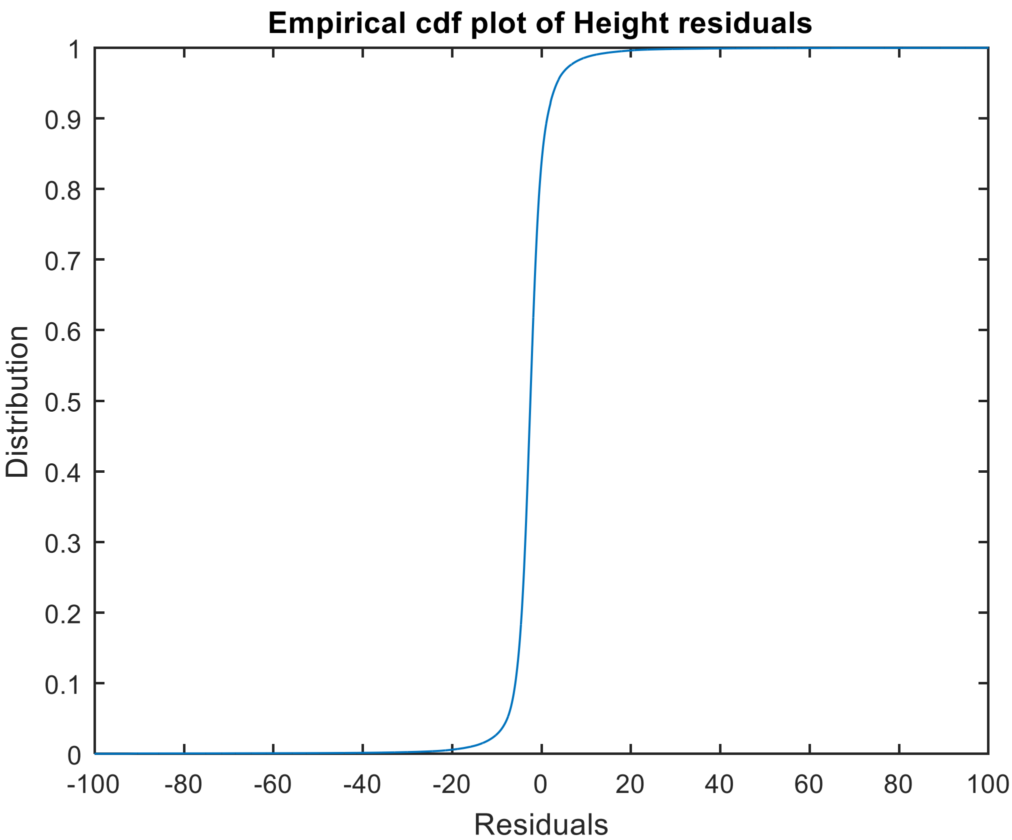


Fig. B3- Empirical cumulative distribution function (cdf) plot of the heights residuals - [min:-652.34, max: 683.92, mean:-0.25, std: 6.08] metres, x-axis in metres

The majority of the older (pre 1990s) gravity observation heights have been determined from aneroid barometers (accurate to around 3-10 m) and horizontal positions were originally scaled from aerial photography which is typically accurate to a few hundred metres (Fraser et al 1976; Murray 1997). In steep topography errors in the marks horizontal positions will result in a larger discrepancy in the difference to the DEM derived height (cf. Filmer et al. 2013). This is less of a problem for more modern sites which will have been located by GPS (Sproule et al, 2006).

A second test has been performed to assess the internal consistency of the simple Bouguer gravity anomalies by cross validation (i.e. each gravity site is left out one at a time; the remaining data are then interpolated to the missing site location and then compared to the missing sites gravity value). The distribution of the differences is shown in figure B4. The standard deviation of the cross validation residuals is 1.024 mGal.

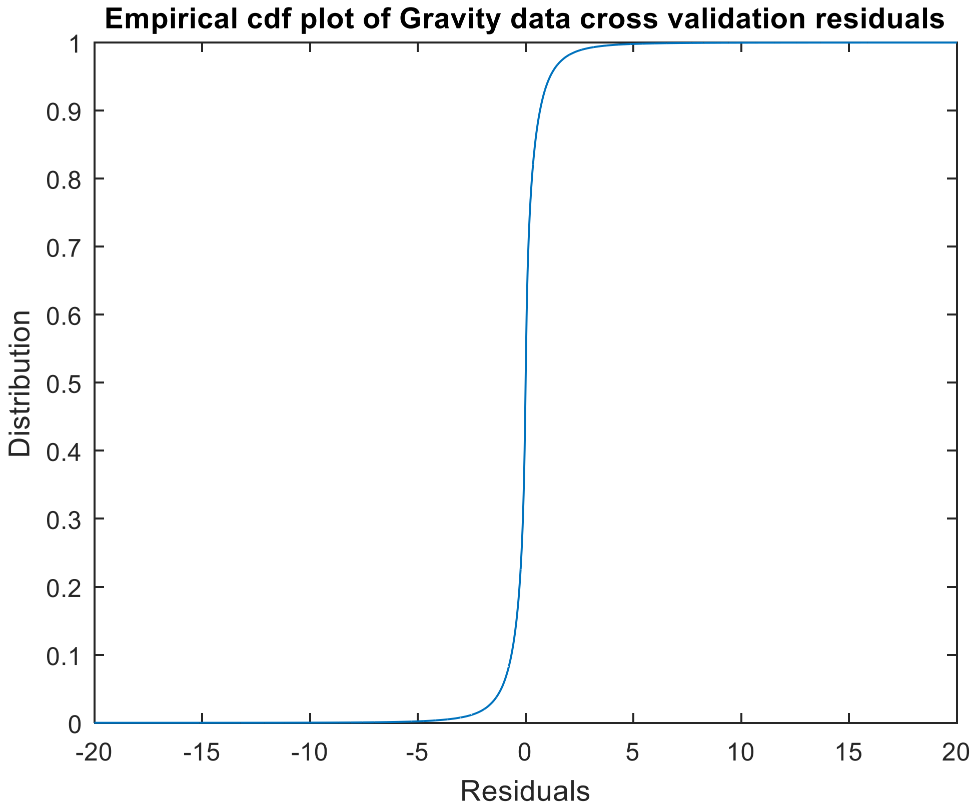


Fig. B4 - Empirical cdf plot of the heights residuals - [min:-95.87, max: 127.82, mean:0.00, std:1.024] mGal, x-axis in mGal

This brief assessment of the gravity database demonstrates that the large majority of the data are reliable although there are long tails on the distributions. The data were plotted but there did not appear to be any obvious spikes. The large residuals on the tails may be due to limitations of this evaluation method (i.e. in steps gravity/topographic gradients).

Tide gauge data

The Bureau of Meteorology (BoM) maintains a database of some tide gauge observations around the Australian coast. The Permanent Service for Mean Sea Level (PSMSL) also holds data for Australian tide gauges. The data base provides monthly mean sea level estimates, based on hourly sea level observations, at 98 tide gauges around the Australian mainland and Tasmania.

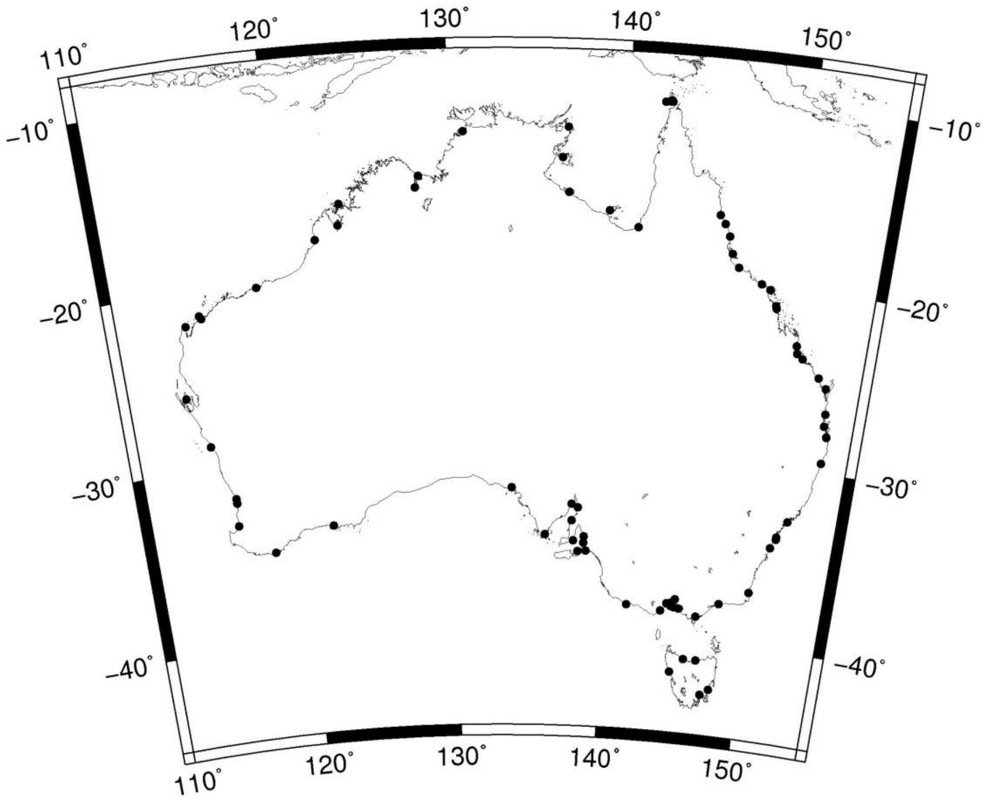


Fig. B5- 98 Tide gauge locations on Australian mainland and on Tasmania

The full database can be downloaded from here.

http://www.bom.gov.au/oceanography/projects/ntc/monthly/index.shtml

MSL is the average of tide gauge sea level observations over a specified time period, and have often been used as the zero reference point for local vertical datums (e.g., AHD) as an approximation of the geoid. However, MSL is dependent upon physical artefacts (such as the ocean temperature or salinity) which are unrelated to Earth's gravity field, and it is now well known that MSL does not coincide with the geoid. Indeed, it has been shown that the AHD has a north-south tilt of ~0.7 m in the AHD relative to the geoid (Featherstone and Filmer 2012).

The difference between the mean sea surface and the geoid is referred to as the mean dynamic topography (MDT). To rigorously constrain a levelling network to multiple tide gauges, the MDT should be corrected. MDT can be estimated at the tide gauge by a number of methods (Woodworth et al. 2012; Filmer et al, 2018). For example, the geodetic approach uses (1) GNSS observations to express the MSL observation as an ellipsoidal height at the tide gauge, or (2) from altimetry-derived MSS at the tide gauge and then subtracting a geoid height derived from a geoid model at the tide gauge from (1) or (2). Alternatively, the ocean approach infers the local MDT from observations of surface currents, temperature and salinity, or from global models based on physics-based dynamical constraints, or a combination of both approaches (cf. Filmer et al, 2018).

Filmer et al (2018) details the results of a number of MDT estimations at 32 tide gauges around Australia, specifically looking at data averaged over the 2003-2007 time period. The results indicate the MDT at the tide gauges can be determined to a precision of around 5 cm.

Australian National Levelling Network (ANLN)

Over the course of many years, differential levelling has been used to establish heights at over 100,000 benchmarks across the Australian continent (Fig. B6). The Australian levelling network was established for mapping geophysical exploration (Granger, 1972) and comprises largely of third order levelling, with first and second order levelling in more densely populated regions. Generally the levelling data are accurate to around 12 mm $\sqrt{km}$ along the levelling lines.

The third order levelling was opted for so that large distances could be covered efficiently by using 90 m sight lengths compared to <50 m for first order standard. It was initially planned for the ANLN to include more first order levelling over time and to be subsequently readjusted (Lambert and Leppert 1975). However this has not been implemented despite some moderate updates and corrections (Morgan, 1992). The ANLN is mostly the same data which were used to establish the AHD.

The ANLN contains many accumulated errors. These errors are particularly problematic in continent wide levelling adjustments since they propagate into the whole network and lead to regional distortions in the AHD. The distortions are particularly prominent in the centre of the continent where the size of the loop misclosures can by larger than 0.5 m (Filmer and Featherstone 2009). Levelling loops in regional areas sometimes exceeded 2000 km in length, so a precision of 12 $\sqrt{km}$, mm permitted misclosures due to random errors of up to 0.5 m. However, this was often due more to gross errors, systematic errors such as refraction, or magnetic effects on early automatic levels (Morgan 1992).

The original levelling connections to the AHD tide gauges are contained within in the ANLN. Connections to later tide gauges (other than QLD) are fewer, and ideally the AHD tide gauge connections should be updated. This is something that can still be (and should be) done.

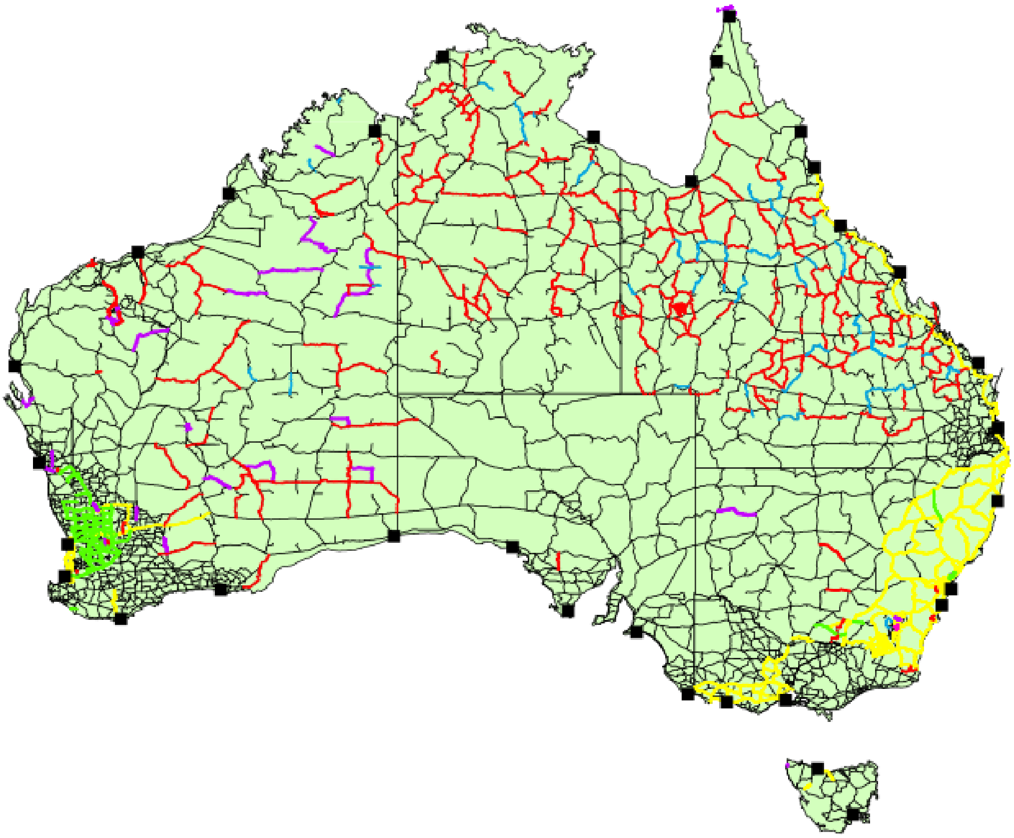


Fig. B6. The Australian National Levelling Network (ANLN). First order levelling sections are in yellow, second order section in light green, third order in fine grey, fourth order in dark green, one way (third order) in red and two-way (order undefined; Steed 2006, pers. comm.) in blue. The junction points are the intersection points of the levelling loops. Lambert projection (figure and description from Filmer, 2010).

GNSS and levelling

Global Navigation Satellite System (GNSS) observations have been made at many of the ANLN benchmarks to obtain ellipsoidal heights. These co-located AHD normal-orthometric heights and GNSS-derived ellipsoidal heights can be used to calculate the geometric separation between the ellipsoid and AHD. They were used for modelling the geometric layer of AUSGeoid2020. As of August 2017, there are a total of 7,635 of these GNSS-levelling data points (Fig B7).

The GNSS data are static dual-frequency occupations of at least six hours' duration. The ellipsoidal heights were computed using Bernese version 5.2 on the Geocentric Datum of Australia 2020 (GDA2020; GRS80 ellipsoid), that is a regional realisation of ITRF2014 (Altamimi *et al.*, 2016), projected to epoch 2020.0 using Australian station velocities. The GDA2020 ellipsoidal heights were output from a least squares adjustment (LSA) along with the associated positional uncertainty (one sigma).

The GNSS-levelling data include some of Australia's offshore territories, which are technically on separate vertical datums, as is the AHD on Tasmania. Though they are all termed AHD heights, they refer to mean sea level observed at a tide gauge on each island. These comprise: Lord Howe Island (1 point), Cocos/Keeling Islands (14 points), Christmas Island (20 points), Tasmania (76 points) and coastal islands close to the Australian mainland (138 points). (Featherstone et al., 2018a)

The data are currently available from: https://github.com/icsm-au

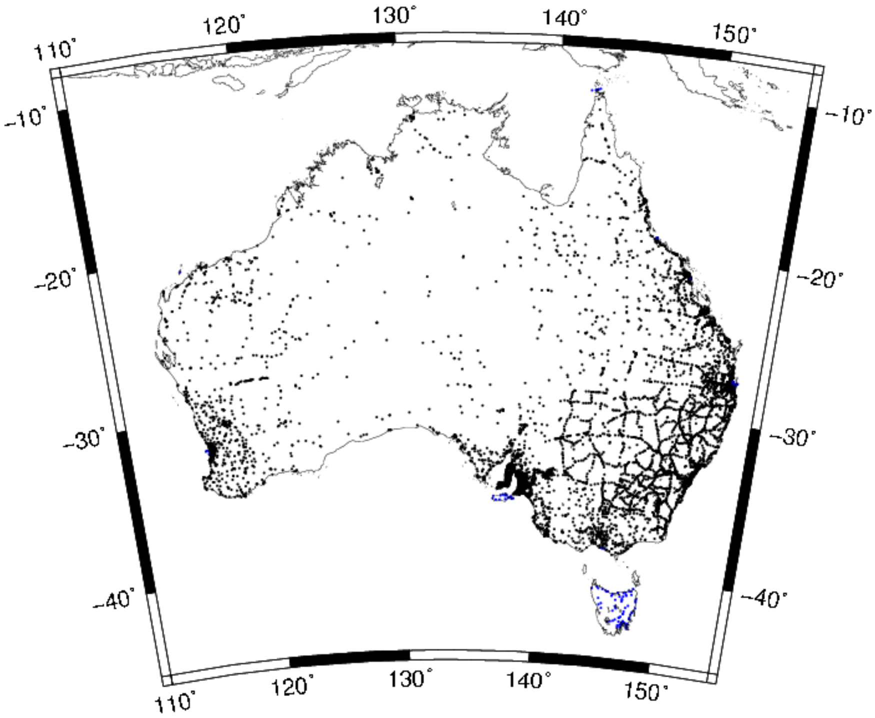


Fig. B7 - Spatial coverage of the May 2017 Australian GNSS-levelling data. Black dots are points on the Australian mainland (a single vertical datum). Blue dots are the separate vertical datums on Tasmania and coastal islands near the mainland. Featherstone et al. (2018b)

These data are predominately isolated to populated lowland areas where the sites are more accessible. The data are sparse in the less populated remote areas of central and north-west Australia. This causes difficulties when modelling the AUSGeoid geometric layer, and evaluating the gravimetric component over the whole continent, in particular where the gravity field is susceptible to high frequency changes (e.g. in rough topography where data is sparse).

Digital elevation model

The gravitational effect of topography is not always well captured by surface gravity data. Digital elevation models (DEM's) can be used to reinforce the gridding/interpolation process of surface gravity anomalies needed for gravimetric geoid/quasigeoid computations and gravity values for levelling height corrections (Featherstone and Kirby, 2000).

Geoscience Australia has produced a 1 second resolution (~ 30 m) DEM-H model (Gallant et al., 2011) (Figure 16). It is derived from SRTM, has vegetation removed to convert the digital surface model to a DEM, adaptively smoothed depending on the roughness of the topography and noise in the SRTM, and hydrological connectivity enforced using the ANUDEM software (Hutchinson, 1989). The 1" DEM-H file size is approximately 100 times larger than the previous 9" GEODATA DEM. (McCubbine et al., 2017)

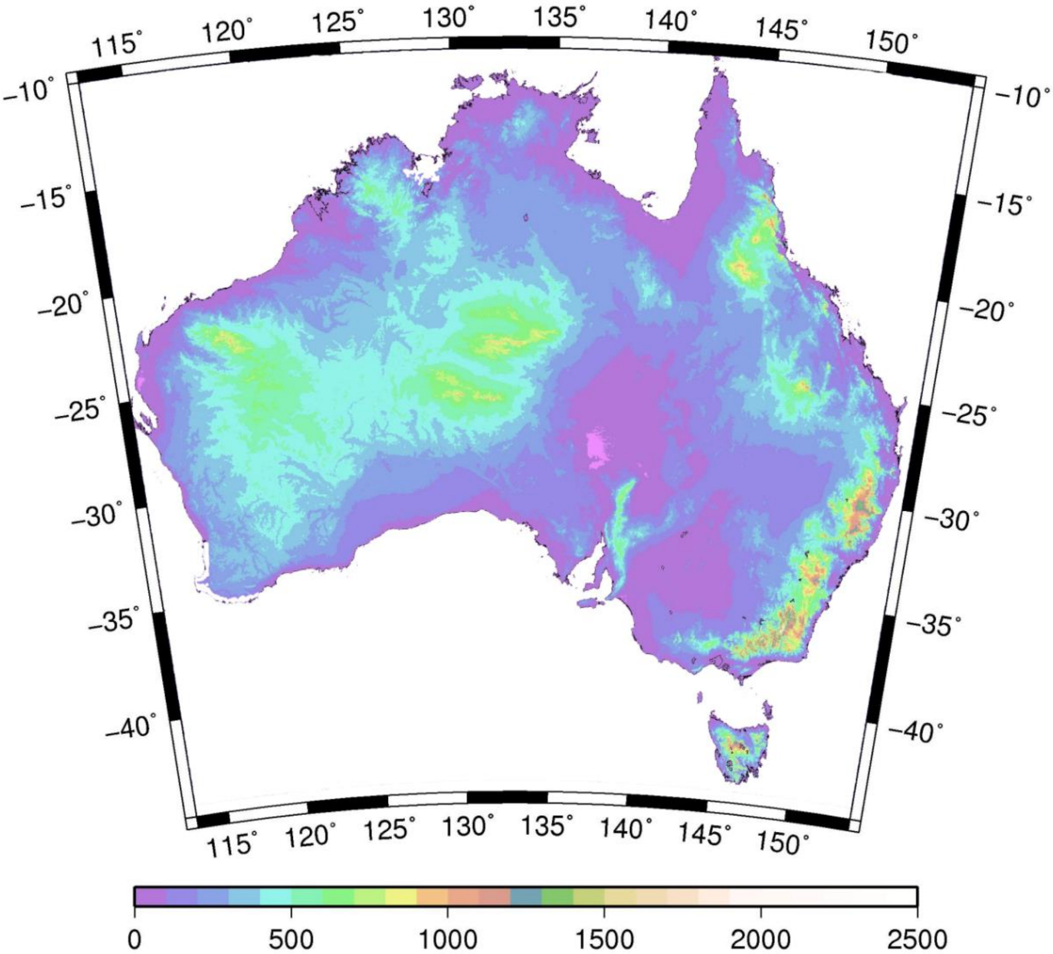


Fig. B8 - 1 second resolution digital elevation model. Scale is in metres

The elevation error in the DEM-H model, assessed by Gallant et al. (2011), indicates that 90% of the DEM elements are within 9.8 m of leveled heights. Although, Gallant et al. (2011) also report that "significant changes to elevation have occurred due to the smoothing and drainage enforcement processes, … errors as large as 200 m occur in some areas".

Gravimetric terrain corrections

The gravimetric quasigeoid computation processing chain for the gravimetric component of AUSGeoid2020 used gridded gravimetric terrain corrections values (at the same resolution of the gridded gravity data). A 1 arc second resolution grid of gravimetric terrain corrections were determined from the 1 second digital elevation model using the fast Fourier transform method (McCubbine et al. 2017). These data were block averaged to 1 arc minute and algebraically added to the gridded free air anomaly to obtain gridded Faye gravity anomalies for AGQG2017. The terrain corrections (block averaged to 1 arc minute) can be seen in Fig. B9.

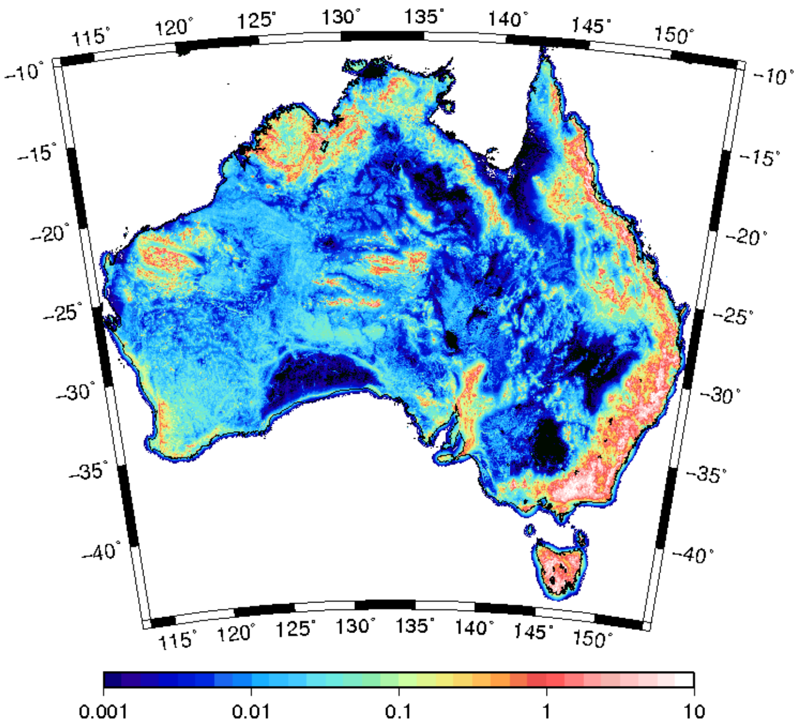


Fig. B9- Gravimetric terrain corrections, block averaged to 1 arc minute. In mGal.

These data are suitable for to be used as the δg^{TC} values in Eqs (A6) and (A7) Neithammer and Mader height correction formulas. However, further computations would have to be performed to obtain the mean terrain correction value $\overline{\delta g^{TC}}$ along the plumbline (for Eq (A7)) and the terrain correction value at the geoid surface δg_0^{TC}. Both of these additional quantities can be obtained using the DEM.

Topographic density models

Stokes integral is used to compute the geoid surface separation from the ellipsoid. The integral is a convolution between Gravity anomalies and Stokes kernel function. These gravity anomalies must be on the surface of the geoid and this requires the complete remove of the gravitational effect of topography outside of the geoid. Standard terrain corrections assume a constant rock density of 2670 $\mathrm{kg/m^3}$ however this can vary from 2.22 $\mathrm{kg/m^3}$ for sandstone through to 2.9 $\mathrm{kg/m^3}$ for dolerite (Washington et al, 1917). This means the standard rock density assumption can cause >10% error in the derived topographic correction values. For precise geoid and levelling height correction computations, a more accurate estimate of rock density should be used.

Kuhn (2003) describes a method to determine a 3D model from seismic reflection data, borehole data and geological maps; although these models are normally only 2D determined from surface geology maps (e.g. Huang et. al. 2001, , Foroughi et. al. 2017). At present, no digital topographic density map currently formally exists for Australia. Nation-wide surface geology maps are available through the GA website, although these alone are not sufficient to produce a 3 dimensional model.

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