ASSESSMENT OF EGM2008 OVER BRITAIN USING VERTICAL DEFLECTIONS, AND THE PROBLEMS WITH HISTORICAL DATA

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ABSTRACT

Vertical deflections synthesised from the Earth Gravitational Model 2008 (EGM2008) agree with astrogeodetic vertical deflections observed over mainland Britain to within ~1.2" RMS (north-south) and ~1.4" RMS (east-west), which is commensurate with values reported for North America, Australia and parts of continental Europe. For this assessment in Britain, there has been the additional need to transform the observed relative vertical deflections to absolute ones. Not applying horizontal datum transformations led to spurious results, so absolute vertical deflections must always be used to assess EGMs. Three datum transformations were trialled (three-parameter, seven-parameter and OSTN02), which show similar results when considering the estimated ~0.3" precision of these historical (1950 to 1976) astrogeodetic observations. Several other problems were encountered because of the historical nature of the data, comprising destruction of survey pillars, ambiguous station names, and a mixture of horizontal geodetic datums available in Britain.

KEYWORDS: EGM2008, vertical deflections, British geodetic datums

INTRODUCTION

The Earth Gravitational Model 2008 (EGM2008) [26, 27] has been assessed globally and regionally using a variety of control data sets, such as satellite orbits, GPS-levelling, terrestrial (land, marine and airborne) gravity anomalies and/or disturbances, and vertical deflections (e.g. [22]). Vertical deflections (aka vertical deviations) provide more powerful tests of the higher degree coefficients of an EGM [18], and can also be used to test (e.g., [34] and [14]) or to refine [7] regional geoid models.

For readers unfamiliar with vertical deflections, [18] gives a concise explanation of their subtly different definitions. Since EGM2008 is a geocentric model (cf. [19]), absolute vertical deflections are needed for its assessment. However, historical astrogeodetic vertical deflections usually refer to local horizontal geodetic datums and their associated reference ellipsoids. These are termed relative vertical deflections (cf. [18]) and were used for astrogeodetic levelling to determine the separation between the geoid and the local reference ellipsoid (e.g., [1]). These local geoid models were needed for the rigorous reduction of geodetic data to the local ellipsoid (cf. [8]). Therefore, relative deflections have to be transformed to absolute deflections if they are to be used to assess EGMs.

Vertical deflections are arguably preferable for testing EGMs in Britain because the ODN [Ordnance Datum Newlyn] vertical datum contains a significant slope (e.g. [33]), with [3] advising that the ODN should not be used for scientific purposes. [28] have isolated this to systematic errors in the levelling that generate a south-north slope of around –(20-25) mm/degree of latitude as well as regional distortions in ODN. As such, GPS-levelling should not really be used to test geoid models in Britain, though this has been done by, e.g., [35] and [6].

Absolute vertical deflections have previously been used to assess EGM2008 in other regions. Continental European vertical deflections were observed with digital zenith cameras that used GPS to determine geocentric coordinates and thus give absolute deflections [34; 12; 14]. Australian and Canadian vertical deflections are absolute because geocentric coordinates were used to re-compute them [7; 4; 16]. The United States vertical deflections are also absolute by applying axial rotations

and origin shifts **[18; 26]**, but no scale change was applied, which will be discussed later in the context of the British deflections.

This paper will examine some methods available to transform relative deflections to absolute deflections for the purpose of assessing EGM2008 in Britain. However, this has been complicated by having to deal with historical data, such as ambiguous ground-station identification, destruction or relocation of survey pillars, and a mixture of horizontal geodetic datums in Britain, some of which have not been related to a geocentric datum. Nevertheless, the results are commensurate with those reported in North America, Australia and parts of continental Europe.

DEFLECTION DATA

The astrogeodetic vertical deflections used herein were collected by **[29; 30; 31]** between 1950 and 1954 and the Ordnance Survey (OS) in 1953 and between 1969 and 1976 **[5]**. Robbins's deflections referred to an unknown geodetic network available at that time, with Appendix I of **[31]** noting "these deviations have been computed with current geodetic co-ordinates not adjusted for the known scale and azimuth errors of the triangulation." Therefore, we did not use his tabulated values, instead relying on the 192 relative deflections listed in Appendix B of **[5]**, which are all relative to triangulation stations on the OSGB70(SN) [Ordnance Survey of Great Britain 1970 (Scientific Network)] horizontal geodetic datum (cf. **[20]**). Most of these deflections were used by **[5]** and **[24]** to generate astrogeodetic geoid models of Britain. Searching the archives at http://benchmarks.ordnancesurvey.co.uk/pls/htmldb/f?p=111:13:6255072357299077241::NO:13 indicates that around half of the astrogeodetic stations have since been destroyed, but this search was complicated by ambiguous station names.
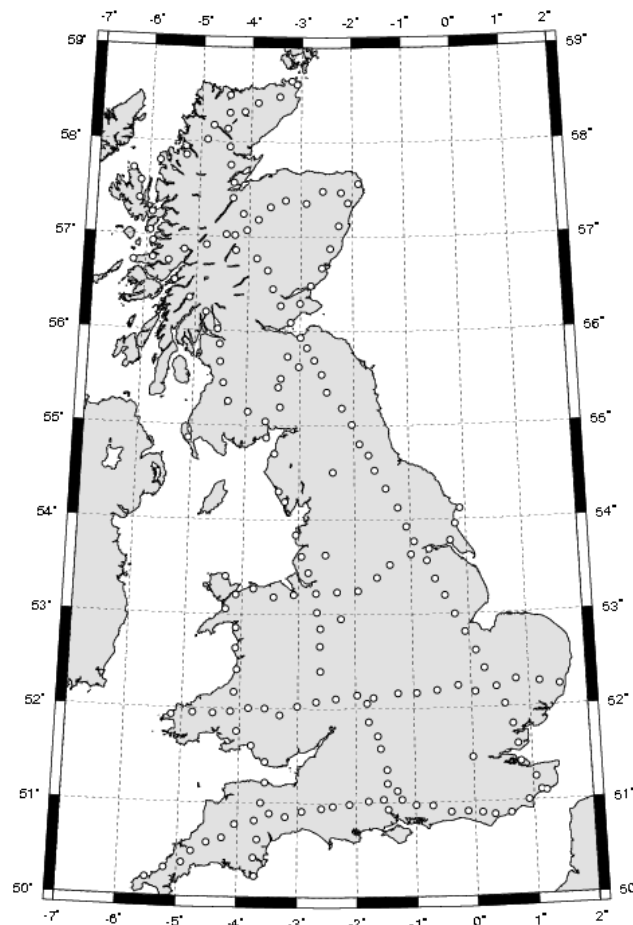

Figure 1. Coverage of the 191 astrogeodetic vertical deflections in Britain (Lambert projection)

One additional vertical deflection was found in an archive at Oxford University, which appears to have been measured at or near Greenwich Observatory. This deflection is different to that listed in **[31]**, suggesting that it is referred to OSGB70(SN). It was also found that **[24]** had omitted one of the vertical deflections listed in **[5]** (3135, Royal Oak). The vertical deflections at Oxford and Herstmonceux listed in **[31]** were - for unknown reasons - not included in **[5]**. As we do not have OSGB70(SN) or any other geodetic coordinates for them, these stations cannot be used with any confidence. In order to provide a fairer comparison with **[24]**, station 3135 has been omitted and the Greenwich deflections added. Another station (3051, GS05/NM7479) presented as a consistent outlier in all comparisons, so was removed to leave 191 vertical deflections (Fig 1).

A principal problem with using these British deflections to test EGM2008 is that the horizontal geodetic coordinates refer to OSGB70(SN), whereas the transformation methods described below relate to OSGB36 [Ordnance Survey of Great Britain 1936], even though this datum contains horizontal scale errors. Therefore, we examined the difference between OSGB70(SN) and OSGB36 coordinates, but this is fraught with uncertainty as follows. The astrogeodetic station occupied was not uniquely identified, nor was information given if a reference mark was occupied (e.g. a cairn or other monument may cover the primary ground mark). Therefore, there is additional uncertainty that the correct coordinates are being transformed. This will be tested in the next section.

ODN heights of some of the astrogeodetic stations are listed in the archive at Oxford University, excepting point 3135 and the 4000-series of stations observed by Robbins (cf. **[5]**). It is unclear whether these are spirit levelled or determined from trigonometric heighting. From the number of decimal places quoted and the large heights of many of the stations, we infer that most are determined from trigonometric heighting, which is logical because heights of triangulation stations in Britain were rarely levelled. The ODN heights of the remaining stations were estimated using the "Google heights" application embedded in http://www.mapmate.co.uk/userzone/p/locate/s/position, which claims a precision of about 10 m.

METHODS

As stated, relative vertical deflections have to be converted to absolute vertical deflections if they are to be used to assess geocentric EGMs. This can be done using the techniques of **[18]** or **[24]**, but an arguably more direct approach is to use the geocentric geodetic coordinates of the astrogeodetic stations, as follows.

Vertical deflections, where ξ is the north-south component and η is the east-west component, are computed from co-located astrogeodetic (Φ, Λ) and geodetic (φ, λ) coordinates using (e.g., **[29; 18]**):

$$\xi = \Phi - \varphi + \frac{\eta^2}{2}\tan\varphi \qquad (1)$$

$$\eta = (\Lambda - \lambda)\cos\varphi \qquad (2)$$

When local geodetic coordinates are used in equations (1) and (2), they yield relative deflections, while when geocentric geodetic coordinates are used, they yield absolute deflections. Therefore, local coordinates must be transformed to geocentric coordinates in order to calculate absolute deflections. We shall experiment with the following options available in Britain. Beforehand, it is important to reiterate that the deflections were originally observed with respect to OSGB70(SN), whereas the transformations below refer to OSGB36.

Three-parameter transformation
Conceptually, a three-parameter horizontal transformation (aka Molodensky transformation) is the least accurate of the methods tested here as it only applies an origin shift, making no account for scale, rotations or regional distortions between the datums. Since **[25]** does not provide Molodensky transformation parameters, the OSGB36 to geocentric WGS84 [World Geodetic System 1984]

parameters were taken from **[23]** as ΔX = +(375±10) m, ΔY = –(111±10) m, ΔZ = +(431±15) m. The large uncertainty of these parameters reflects the lower accuracy of this method.

Seven-parameter transformation

The seven-parameter transformation advances on the three-parameter model by accounting for axial rotations and a single scale factor, but it does not attempt to model regional distortions between datums. The parameters from OSGB36 to the geocentric ETRS89 [European Terrestrial Reference System 1989] were computed by **[25]** as ΔX = 446.448 m, ΔY = –125.157 m, ΔZ = 542.060 m, rx = 0.1502", ry = 0.2470", rz = 0.8421", ds = 20.4894 ppm. No uncertainties are provided for these parameters, but **[25]** gives a blanket estimate of 5 m accuracy for this transformation. When applying the seven-parameter transformation, care is also needed to apply the axial rotations with the correct sign convention **[32]**. The large scale parameter of ~20 mm/km reflects the distortion in OSGB36 (cf. **[20; 30]**).

OSTN02 transformation

This is the conceptually most accurate method because it also attempts to model regional distortions between datums. **[25]** and **[10]** describe the OSTN02 [Ordnance Survey National Grid Transformation 2002] model that transforms coordinates from OSGB36 to ETRS89. It requires British-National-Grid-projected OSGB36 coordinates and applies northing and easting shifts interpolated from a 1-km-resolution grid. We used the GridInQuest software (version 7.0.0 build 4737) to transform OSGB70(SN) eastings and northings to ETRS89 geodetic longitudes and latitudes. The GridInQuest software help pages indicate a transformation accuracy (one sigma) of 0.1 m, which appears to be derived from **[10]**.

Transformation comparisons

In an attempt to quantify the relative performance of the above transformations, as well as to determine the additional uncertainty introduced by using OSGB70(SN) instead of OSGB36, we made the following comparisons. We searched for 'GPS-observed' ETRS89 coordinates of the passive stations of the OS's GPS network (http://gps.ordnancesurvey.co.uk/passive.asp) for seemingly co-located stations. This only delivered 21 of 192 stations, but also highlighted the undesirable ambiguity regarding station names (e.g., 1011 Rottington and 3011 Malvern), reducing the number of reliably co-located stations to 14. Such a small sample has to be used with caution.

Table 1. *Differences between transformed OSGB70(SN) and 'GPS-observed' ETRS89 coordinates at 14 reliably co-located astrogeodetic stations.*

Latitude difference (")	Mean	RMS
Three-parameter transformation	0.22	±0.23
Seven-parameter transformation	0.03	±0.09
OSTN02 transformation	0.03	±0.07

Longitude difference (")	Mean	RMS
Three-parameter transformation	-0.02	±0.03
Seven-parameter transformation	-0.06	±0.12
OSTN02 transformation	-0.06	±0.09

Linear difference (m)	Mean	RMS
Three-parameter transformation	7.02	±7.41
Seven-parameter transformation	5.46	±6.50
OSTN02 transformation	4.21	±4.93

The mean and RMS coordinate differences in Table 1 are presented in terms of angular measure of latitude difference and longitude difference scaled by meridional convergence (cf. equation 2) so as to allow for easier comparison with the deflections. To convert to a linear offset in metres on the ground, these have been multiplied by 31 m and combined with Pythagoras. Still acknowledging the small sample size, Table 1 shows that the relative accuracy of the transformations concurs with expectation. However, there remains a bias of several metres. The most plausible explanation for this is the scale error in the OSGB36 with respect to OSGB70(SN), which will be elaborated upon at the end of the section on "Results and Discussion".

EGM2008 syntheses

Absolute Helmert vertical deflections (i.e., geocentric and at the Earth's surface) were synthesised from the tide-free EGM2008 coefficients using the transformed geocentric coordinates from the three methods described above and ellipsoidal heights of the astrogeodetic stations by adding OSGM02 [Ordnance Survey Geoid Model 2002] to the ODN heights. OSGM02 has been fitted to the ODN using GPS at benchmarks and least squares collocation [9; 17], so accounts for the systematic errors in the ODN [28], thus giving a more accurate transformation of ODN heights to ellipsoidal heights. Ellipsoidal heights are necessary to synthesise EGM2008 vertical deflections on the Earth's surface as shown in Table 5 of [15] and to apply an additional correction for the curvature of the normal plumbline, which will be described in the following section. EGM2008 values were calculated with respect to the GRS80 [Geodetic Reference System 1980] reference ellipsoid [21], which is used with ETRS89.

Other corrections

[18] describes a series of small, but some of which are systematic, conversion terms that have to be applied to make vertical deflections synthesised from an EGM compatible with Helmert astrogeodetic deflections at the Earth's surface. Jekeli's terms for the origin translations and axial rotations are not needed here because these have already been accounted for by the above datum transformations. However, Jekeli omits a correction term for the scale parameter in the seven-parameter transformation, which will be addressed at the end of the section on "Results and Discussion".

Plumbline curvature: The largest conversion term accounts for the north-south curvature of the normal plumbline, and is given by [18]

$$\delta\xi_{norm.curv} \approx 0.17\, h_{km} \sin 2\varphi \quad [\text{arc sec}] \qquad \delta\eta_{norm.curv} = 0 \qquad (3)$$

As we have the [albeit transformed] ellipsoidal heights of the astrogeodetic stations, this term can be applied directly.

Permanent tides: Although not documented in [5], it is assumed that the Helmert astrogeodetic vertical deflections are in the mean tide system (cf. [18]). Applying the Love and Shida numbers for the elastic response of the Earth to the permanent tides, the conversion from mean tide to tide-free vertical deflections (and thus compatible with the tide-free release of EGM2008) is [18]

$$\delta\xi_{perm.tide} = 0.012 \sin 2(\varphi - v) \quad [\text{arc sec}] \qquad \delta\eta_{perm.tide} = 0 \qquad (4)$$

where

$$v \approx f(1 - f/2)\sin 2\varphi \quad [\text{rad}] \qquad (5)$$

and $f = 1/298.257222101$ is the flattening of the GRS80 reference ellipsoid [21].

Approximation corrections: Sections 3 and 5 of [18] derive two other conversion terms for the approximations made in the spherical harmonic synthesis of vertical deflections: one is for an approximation of gravity; the other is for the misalignment between the ellipsoidal surface normal and the geocentric radial. These conversions are respectively

$$\delta\xi_g \approx \xi \frac{\Delta g_P}{\gamma_Q} \quad [\text{arc sec}] \qquad\qquad \delta\eta_g \approx \eta \frac{\Delta g_P}{\gamma_Q} \quad [\text{arc sec}] \qquad\qquad (6)$$

$$\delta\xi_v = v \frac{\delta g_P}{\gamma_Q} \quad [\text{rad}] \qquad\qquad \delta\eta_v = 0 \qquad\qquad (7)$$

where Δg_P is the Molodensky free-air gravity anomaly on the Earth's surface and δg_P is the gravity disturbance, also evaluated on the Earth's surface, and γ_Q is normal gravity on the ellipsoid's surface, computed by [21]

$$\gamma_Q = \gamma_a \frac{1 + k\sin^2\varphi}{\sqrt{1 - e^2\sin^2\varphi}} \quad [\text{ms}^{-2}] \qquad\qquad (8)$$

where, for GRS80, $k = 0.001\,931\,851\,353$ is the normal gravity constant, $\gamma_a = 9.780\,326\,7715$ ms^{-2} is normal gravity on the equator, and $e^2 = 0.006\,694\,380\,02290$ is the square of the first numerical eccentricity. To evaluate these correction terms, the gravity disturbance and Molodensky free-air gravity anomaly were synthesised from EGM2008 at the 3D geocentric geodetic coordinates of the astrogeodetic stations.

Table 2. *Descriptive statistics (") of Jekeli's conversion terms* [18]
when applied to the 191 astrogeodetic stations in Britain.

Correction term	Max	Min	Mean	STD	RMS
$\delta\xi_{norm.curv}$	0.142	0.008	0.032	±0.022	±0.038
$\delta\xi_{perm.tide}$	0.012	-0.012	0.000	±0.009	±0.009
$\delta\xi_g$	<0.001	<0.001	<0.001	<0.001	<0.001
$\delta\xi_v$	0.020	-0.022	0.002	±0.008	±0.008
$\delta\eta_g$	0.001	<0.001	<0.001	<0.001	<0.001

Table 2 shows that the above conversion terms are generally small, but the normal curvature and permanent tide contributions are systematic in a north-south direction, acting to reduce the mean difference of the north-south deflection component [18].

RESULTS AND DISCUSSION

The first row of Table 3 shows the descriptive statistics of the differences when the EGM2008 deflections are synthesised at the OSGB70(SN) station locations and compared to the relative Helmert astrogeodetic deflections. Conceptually this is incorrect, but is presented only to show the effect of not using compatible deflections. The large bias and RMS values demonstrate that relative deflections must never be used to assess EGMs. The mean deflection differences in Table 3 indicate the mean slopes of the geoid relative to the Airy 1830 spheroid compared to the geocentric geoid modelled by EGM2008 with respect to GRS80. Therefore, deflection corrections are needed if reducing terrestrial geodetic survey data to ETRS89 in Britain (cf. [8]).

After application of the conversion terms in Table 2, the EGM2008 deflections were subtracted from the absolute Helmert astrogeodetic deflections that were recomputed (equations 1 and 2) using geocentric geodetic coordinates from: (i) three-parameter-transformed OSGB70(SN) to WGS84; (iii) seven-parameter-transformed OSGB70(SN) to ETRS89; and (iv) OSTN02-transformed OSGB70(SN) to ETRS89. The results of [24] are also included in Table 3, which are similar to the three-parameter transformation, but applied at the origin of the network. The conversion terms in Table 2 were not applied to the results of [24].

No statistical outlier rejection was applied, apart from the one station (3051, GS05/NM7479) that was consistently out by ~4" (~125 m on the ground) in each deflection component for all transformation methods tested. This suggests that a different ground mark may have been occupied. The RMS differences for the east-west deflections (η) are slightly worse than the north-south deflections (ξ) in Table 3, which fits with expectation because of timing uncertainties, especially for astrogeodetic observations made in the 1950s. The same feature occurs for astrogeodetic deflections used to assess EGM2008 in the United States and Australia (Table 9 of [26]) and Canada (Table 4 of [16]). The omission error in EGM2008 is included implicitly in the statistics in Table 3 (cf. [18; 15; 16]). Another smaller contributor is the accuracy of the FK4 star catalogue used to calculate the astrogeodetic deflections [5], which may contain biases of 0.05-0.10" [2].

Table 3. *Descriptive statistics (") of 191 Helmert astrogeodetic deflections minus EGM2008 in Britain after application of the correction terms in Table 2.*

ξ **minus EGM2008 (north-south)**	**Max**	**Min**	**Mean**	**STD**	**RMS**
Relative deflections	7.74	-14.93	-4.58	±3.87	±6.01
Three-parameter transformation	3.92	-5.00	0.26	±1.13	±1.16
Seven-parameter transformation	4.24	-4.95	0.27	±1.16	±1.19
OSTN02 transformation	4.32	-4.99	0.28	±1.18	±1.21
Olliver [1992]	3.68	-5.11	0.10	±1.15	±1.15

η **minus EGM2008 (east-west)**	**Max**	**Min**	**Mean**	**STD**	**RMS**
Relative deflections	14.47	-10.14	2.22	±4.14	±4.70
Three-parameter transformation	2.96	-4.13	-0.03	±1.30	±1.30
Seven-parameter transformation	3.04	-4.25	-0.06	±1.33	±1.33
OSTN02 transformation	2.98	-4.25	-0.08	±1.34	±1.35
Olliver [1992]	3.02	-6.47	-0.03	±1.38	±1.38

Of the transformation methods used, OSTN02 should conceptually be the superior assessor of EGM2008 (cf. Table 1), but the use of OSGB70(SN) instead of OSGB36 coordinates appears to be clouding the comparisons. Nevertheless, the RMS differences for each transformation method in Table 3 are very similar. Moreover, the differences among them are statistically insignificant given the estimated ~0.3" precision of the astrogeodetic observations [5].

Lastly, we turn to effect that the ~20 mm/km scale error (taken from the seven parameter transformation) assumed in the OSGB36 network (cf. [31; 20]) could bias the results, particularly for the mean differences in the north-south deflection component in Table 3. Applying this scale parameter over the north-south extent of the British astrogeodetic network (~8.5 degrees) gives an approximate mean coordinate difference of ~9 m. Knowing that ~31 m on the ground equates to ~1", this could remove ~0.3" from the mean differences for ξ in Table 3. However, without the exact differences between OSGB70(SN) and OSGB36, this can only remain as speculation, but the similarity of the values does render it a prime suspect.

CONCLUSION AND RECOMMENDATION

In summary, 191 absolute Helmert astrogeodetic deflections have been used to assess EGM2008 in Britain, showing RMS agreements to within ~1.2" (north-south component) and ~1.4" (east-west component), which is commensurate with results reported for North America [26; 16], Australia [26; 4] and parts of continental Europe [13].

This assessment required transformation of the relative deflections to absolute deflections and the consideration of small, yet some systematic, conversion terms identified by [18]. The spurious

results achieved when a transformation was not applied demonstrate that absolute deflections must only be used to assess EGMs. As per the recommendations of **[3]** and **[28]**, GPS-ODN should not be used to test EGM2008 because of the south-north systematic levelling error of –(20-25) mm/degree latitude and regional distortions.

The three datum transformation methods tested (three-parameter, seven-parameter and OSTN02) delivered similar results, and which are insignificantly different when considering the ~0.3" precision of these historical (1950 to 1976) astrogeodetic observations. However, the vagaries of the British geodetic datums and the use of historical data (e.g. destruction or relocation of ground marks and ambiguous station names) may also contribute a bias. In particular, the ~20 mm/km scale error in the OSGB36 could account for the mean differences seen for the north-south deflection component (Table 3), but this remains somewhat speculative.

Since modern Helmert astrogeodetic vertical deflections have proven their utility elsewhere (e.g. **[12]**), the OS may wish consider to conduct a new or re-observation programme (e.g. at the active and passive stations of the GPS network) using modern astrogeodetic cameras (e.g. **[13; 11]**).

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