Reliability ratio based weighted bit-flipping decoding for low-density parity-check codes

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A novel reliability ratio based weighted bit-flipping decoding scheme is proposed for low-density parity-check codes. A coding gain of 1 dB is achieved in comparison to the weighted bit-flipping scheme, when communicating over an AWGN channel, while maintaining the same decoding complexity.

Introduction: The family of low-density parity-check (LDPC) codes proposed by Gallager [1] has attracted substantial research interest in the information theory community. LDPC codes can be decoded using various decoding schemes [1–4] such as hard-decisions, soft-decisions and hybrid decoding schemes. The high-complexity sum-product algorithm (SPA) was shown to achieve a near-capacity performance [4]. However, the weighted bit-flipping (WBF) algorithm [2] strikes a good trade-off between the associated decoding complexity and the achievable performance. The attractive property of the WBF algorithm is that during each iteration the weighted sum of the same values is computed, resulting in a significantly lower decoding in comparison to the SPA. An improved WBF (I-WBF) algorithm was proposed by Zhang and Fossorier [3]. In this Letter, the following problems of the WBF and the I-WBF algorithm are addressed. 1. Both the WBF and I-WBF algorithms attribute the violation of a particular parity check to *only* the least reliable bit. 2. If the weighting factor α utilised in the I-WBF algorithm [3] is not optimum, the BER performance may be significantly degraded. Thus in this contribution, the BER performance of the I-WBF algorithm is further improved by using more sophisticated bit-flipping, while avoiding any pre-processing such as finding the optimal weighting factor α of the I-WBF algorithm.

An (N, K, j) LDPC code can be uniquely represented by an $M \times N$ parity-check matrix (PCM), where $M = N - K$ and each column of the PCM has an average weight of j. By representing the PCM using the Tanner graph [5], each column of the PCM corresponds to a message node in the Tanner graph and each row of the PCM is associated with a check node. We will use the notation $\mathbf{H}$ for representing the PCM of the LDPC code, and H_{mn} denotes the binary entry in the mth row and the nth column. We denote the set of bits participating in the mth check by $\mathcal{N}(m) = \{n: H_{mn} = 1\}$. The term $\{n: H_{mn} = 1\}$ indicates the specific set of values for the column index n, where the value of the PCM entry H_{mn} at the mth row and nth column is one. Similarly, the set of checks in which the nth bit participates is denoted as $\mathcal{M}(n) = \{m: H_{mn} = 1\}$. When an information block of size K is encoded by an LDPC encoder, a codeword $\mathbf{c}$ of length N will be produced, and the coded bits will be mapped using BPSK modulation onto the corresponding constellation point $\mathbf{x}$. When the Gaussian noise is added to the transmitted signal, a noise-contaminated received sequence $\mathbf{y}$ will be obtained. Based on the sequence $\mathbf{y}$, an initial hard decision can be made and we arrive at a binary sequence $\mathbf{z}$ of length N.

Weighted bit-flipping algorithm: The standard WBF algorithm [2] initially finds the most unreliable message node participating in each individual check. Since the magnitude of the received soft value y_i determines the reliability of the hard decision z_i, the least reliable message node's magnitude for each individual check during the algorithm's initialisation step is given by:

$$y_m^{\min} = \min_{n: n \in \mathcal{N}(m)} |y_n| \tag{1}$$

where $|y_n|$ denotes the absolute value, i.e. the magnitude, of the nth message node's soft value, while $y_m^{\min}$ is the lowest magnitude of all message nodes participating in the mth check. The iterative WBF process is then implemented as follows. (1) The bit sequence $\mathbf{z}$ obtained by hard decision is multiplied with the transpose $\mathbf{H}^T$ of the PCM, and the resultant syndrome vector $\mathbf{s} = (s_1, s_2, \ldots, s_i, \ldots, s_M)$ is derived. (2) For each message node at position n, the WBF algorithm computes

$$E_n = \sum_{m \in \mathcal{M}(n)} (2s_m - 1) y_m^{\min} \tag{2}$$

The error-term E_n is used to quantify the probability that the bit at position n would be flipped. When the mth parity check in which the nth message node participates is violated, i.e. when we have $s_m = 1$, then the term $(2s_m - 1)$ in (2) will contribute $+1$ otherwise -1 to the error term E_n being calculated. Thus in the next step of the WBF algorithm, the bit having the highest error term E_n will be deemed the least reliable bit and hence flipped. (3) Flip the specific bit in $\mathbf{z}$, which has the highest error-term E_n. The foregoing three steps are repeated, until an all-zero syndrome vector $\mathbf{s}$ is obtained, or the maximum affordable number of iterations has been reached.

Improved weighted bit-flipping algorithm: As seen in (2), the WBF algorithm proposed by Kou *et al.* [2] only considers the check-node based information during the evaluation of the error-term E_n. By contrast, the I-WBF algorithm proposed by Zhang and Fossorier [3] enhanced the performance of the WBF algorithm, since it considered both the check-node based and the message-node based information during the evaluation of E_n. As seen from (2), when the error-term E_n is high, the corresponding bit is likely to be an erroneous bit and hence ought to be flipped. However, when the soft-value $|y_n|$ of a certain bit is high, the message node itself is demonstrating some confidence that the corresponding bit should not be flipped. Hence (2) was modified in [3] as follows:

$$E_n = \sum_{m \in \mathcal{M}(n)} (2s_m - 1) y_m^{\min} - \alpha \cdot |y_n| \tag{3}$$

Equation 3 considers the extra information provided by the message node itself, thus a message node having a higher soft-value magnitude has a lower chance of being flipped, despite having a high error term E_n owing to encountering unreliable parity checks. We note however that for LDPC codes having different column weights, or operating at different SNRs, we should weight the effect of the soft-value $|y_n|$ differently [3]. Thus, when (3) is used for decoding a particular LDPC code, an optimum α value should be found experimentally. This requirement is eliminated by the algorithm proposed below.

Reliability-ratio based weighted bit-flipping algorithm: A drawback of the I-WBF algorithm is that the optimum α value has to be found specifically for each particular column weight and its value should be optimised for each individual SNR [3]. Furthermore, both the WBF and the I-WBF algorithms consider only the specific check-node based information, which relies on the message node having the lowest soft-value $|y_n|$. However, all message nodes participating in the mth parity check are contributing, i.e. all message nodes might be liable to change, if the check they participate in is violated. However, for two different message nodes participating in the same violated parity check, the probability that the check is violated owing to the message node having a high soft magnitude is lower than that associated with the message node having a low soft magnitude. Hence, hereby we would like to introduce a new quantity termed as the *Reliability Ratio* (RR) defined as follows:

$$R_{mn} = \beta \frac{|y_n|}{|y_m^{\max}|} \tag{4}$$

where the notation $|y_m^{\max}|$ is used to denote the highest soft magnitude of all the message nodes participating in the mth check. The variable β is a normalisation factor introduced for ensuring that we have $\sum_{n: n \in \mathcal{N}(m)} R_{mn} = 1$. Hence, instead of calculating the error-term E_n as in (2) using $y_m^{\min}$, we propose the employment of the following formula:

$$E_n = \sum_{m \in \mathcal{M}(n)} \frac{2s_m - 1}{R_{mn}} \tag{5}$$

The rest of the RR-WBF algorithm is the same as the standard WBF algorithm and the iterations will be terminated when the resultant syndrome vector becomes an all-zero vector or when the maximum affordable complexity has been exhausted.

Simulation results: Below we characterise the achievable performance of the proposed RR-WBF algorithm. The scheme will be benchmarked against both the standard WBF algorithm [2] and the I-WBF algorithm using the optimal weighting factor of $\alpha = 0.4$ [3]. The three schemes will be invoked for decoding a (200,100,3) and a

(1000,500,3) regular LDPC code using a BPSK modulation scheme communicating over an AWGN channel. Since two LDPC codes having different blocklengths are utilised in this experiment and because the bit-flipping algorithm only changes a single bit during each iteration, for the sake of fair comparison we allow a maximum number of 20 and 100 iterations for the (200,100,3) and (1000,500,3) LDPC codes, respectively. Hence for both codes having different length, 20% of the coded bits have the chance of being corrected. It can be observed in Fig. 1 that the RR-WBF algorithm outperforms both benchmark schemes. At the BER of 10^{-4}, a coding gain of 0.75 and 0.25 dB is achieved for the short blocklength of 200 bits in comparison to the standard WBF and I-WBF algorithm, respectively, while invoking 20 iterations. For the longer blocklength of 1000 bits, a coding gain of 1 and 0.75 dB is achieved in comparison to the two benchmark schemes.

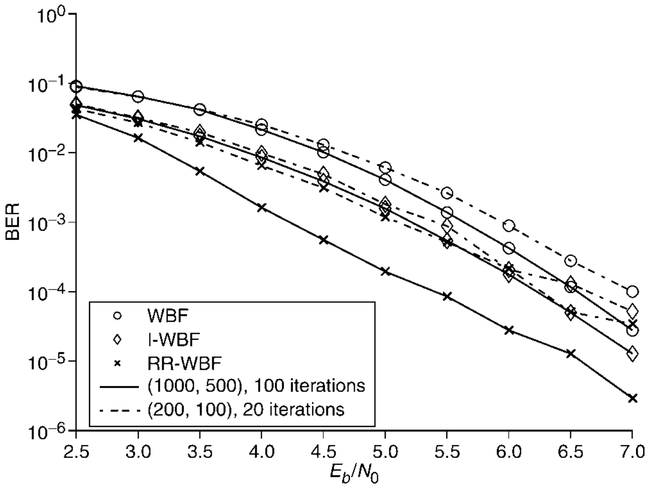

Fig. 1 *BER performance of (200,100,3) and (1000,500,3) regular LDPC codes decoded by WBF, I-WBF and RR-WBF algorithms, respectively*

BPSK modulation is used when communicating over an AWGN channel. Maximum number of iterations are 20 and 100 for (200,100,3) and (1000,500,3) codes, respectively. Optimum α for I-WBF decoder is 0.4

Conclusion: A novel reliability ratio based weighted bit-flipping algorithm is proposed. This scheme uses a similar error-term evaluation method to that of (2), hence imposing no complexity increase during the iterative decoding process. Furthermore, the RR-WBF algorithm utilises all the message-node-based information to produce the check-node-based information, thus an improved BER performance is observed in Fig. 1. In contrast to the I-WBF scheme, the RR-WBF does not have to have any *a priori* knowledge concerning the weighting factor α, which is a further advantage.

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