Simultaneous Network Line Planning and Traffic Assignment

Karl Nachtigall and Karl Jerosch

Technical University Dresden, Faculty of Traffic Sciences,
Institute for Logistics and Aviation
karl.nachtigall@tu-dresden.de
karl-friedemann.jerosch@siemens.com

Abstract. One of the basic problems in strategic planning of public and rail transport is the line planning problem to find a system of lines and its associated frequencies. The objectives of this planning process are usually manifold and often contradicting. The transport operator wants to minimize cost, whereas passengers want to have travel time shortest routes without any or only few changings between different lines. The travel quality of a passenger route depends on the travel time and on the number of necessary changings between lines and is usually measured by a disutility or impedance function. In practice the disutility strongly depends on the line plan, which is not known, but should be calculated. The presented model combines line planning models and traffic assignment model to overcome this dilemma. Results with data of Berlin's city public transportion network are reported.

Key words: line planning problem, integer programming

1 Related literature

In the last years a lot of mathematical integer programming models have been proposed (see e.g. [1–3, 6, 10]). The line plan model presented by Borndörfer, Grötschel and Pfetsch [1, 2] minimizes the combination of line costs and system travel time disregarding transfers between lines and waiting times. To make the model feasible for real instances of local traffic systems, the line frequency variables are not forced to be integer, however knowing that the fractual solution can be quite far from the integer optimum. Because in general the minimum system travel time (called Beckmann-User-Equality) do not really reflect the passenger's "selfish" behavior, the use of the system travel time seems to be disadvantageous.

Both facts are respected in the model of Schöbel and Scholl [11]. This line plan model minimizes the passengers inconvenience under the restriction of a fixed budget for all line costs, whereas the passengers inconvenience is the sum of the travel time and the time needed for transfers concerning one origin-destination relation. The model in [11] assumes that the given passenger travel demand must be satisfied by the line plan. For all origin-destination pairs the passengers will

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travel on a time shortest path. In the model presented here not all passenger demand must be covered. This is controlled by a limited budget for the operational cost of the line plan. For larger networks the model of Schöbel and Scholl suffers from large memory requirements for solving the shortest path problem in the *change& go network*, which contains for each line and each pair of consecutive served stations a special travel edge. Large number of lines and stations requires a big amount of memory. We try to overcome this problem by the use of a column generation scheme. The resulting pricing problem for those passenger flow variables is a shortest path problem.

2 The Basic Combined Model

2.1 General notations

The proposed model is restricted to the case to find a line plan for one homogeneous transport carrier. Each of the edges $e \in E$ of the underlying network (V, E) is assigned with a homogeneous travel time t_e. The nodes or vertices $n \in V$ of the network represent stations. Terminal stations are nodes, at which a line is allowed to start or to terminate. By a potential line L we will understand a path between two terminal nodes, which running time is in maximum the product of the detour factor ρ and the shortest travel time between the two terminal nodes. For simplicity we use the notation $e \in L$ to indicate, that the line L uses edge e. By $L_{|(A,B)}$ we denote the set of all edges, which are used by line L running from station A to station B. This set might be empty, if A and B are not served by L.

2.2 Traffic Assignment

The traffic assignment problem is modelled by some kind of multi-commodity flow problem, where we consider for each travel demand pair (O, D) and lines $L \in \mathcal{L}$ partial passenger routes $p^{O,D}_{(A,B),L}$. The variables

$$\varphi^{O,D}_{(A,B),L}$$

define the traffic flow of all OD-passengers using line L between station A and station B.

A total passenger route p from the origin node O to the destination node D is simply the concatenation of partial routes (see figure 1). The disutility of impedance of a passenger route is a measure for the inconvenience for the trip by this route. In traffic assignment theory this impedance is modelled in dependence of the travel time, the number of changings and the frequency of the service. In practice, there are used rather complex and nonlinear models for this function (see e.g. [4, 7]). Here, we simplify the model by using a linear approximation: For a path p denote the travel time by $t(p)$ and define the number of line changings by $c(p)$, which are penalized by the parameter β. Then $imp(p) := t(p) + \beta c(p)$ is a reasonable measure for the disutility of using the trip route p.

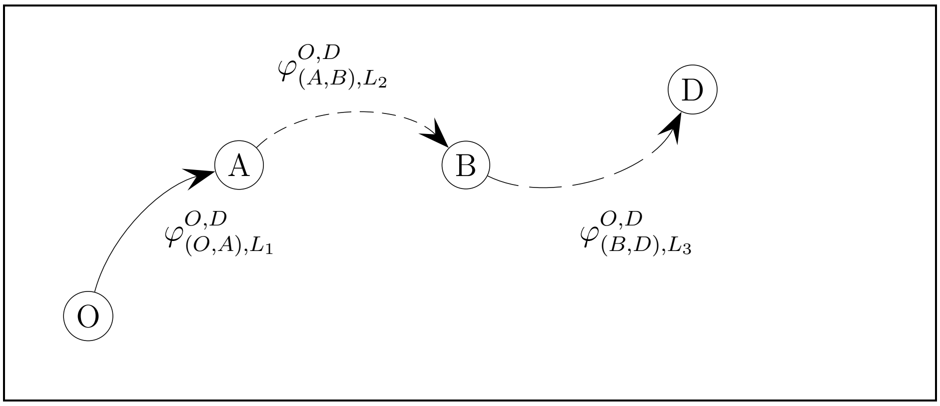

Fig. 1. Partial passenger route flow variables

In order to keep the resulting model linear, we do **not** use the common approach ([4]) to measure quality or utility by a function $f(p) \sim \frac{1}{imp(p)}$, but define the *travel quality* by

$$\omega(p) = \rho \cdot t^*_{OD} - t(p) - \beta c(p) \tag{1}$$

t^*_{OD} denotes the minimum possible travel time from origin O to destination D. The detour factor $\rho \geq 1$ should be defined in such a way, that passengers will accept all those routes for which the travel time added with the change penalty is at most a $\rho-$ multiple of the minimum travel time. The quality measure $\omega(p)$ can be interpreted as that time, what a passenger can save by using a connection p compared to the maximal accepted travel time for that OD-relation.

The best quality is given by $t^*_{OD} \cdot (\rho - 1)$. If the travel time or/and the number of necessary changings becomes too large, the quality turns out to be negative. If there exists no alternative, more favourable path with positive quality, we will assume to 'loose' those passengers. Figure 2 illustrates this approach.

In order to split the total travel time onto the line parts, we define $t_{L;(A,B)}$ to be the travel time of line L from station A to station B. The quality weights

$$\omega^{O,D}_{(A,B),L} := \begin{cases} -t_{L;(A,B)}, & if A = O \text{ and } B \neq D \\ -t_{L;(A,B)} - \beta, & if A \neq O \text{ and } B \neq D \\ \rho t^*_{OD} - t_{L;(A,B)} - \beta, & if A \neq O \text{ and } B = D \\ \rho t^*_{OD} - t_{L;(A,B)}, & if A = O \text{ and } B = D \end{cases}$$

obviously have the property, that its sum on each total path from origin node O to destination node D equals with (1) (see Figure 3).

Combining flow conservation laws and travel demand leads to the traffic assignment model:

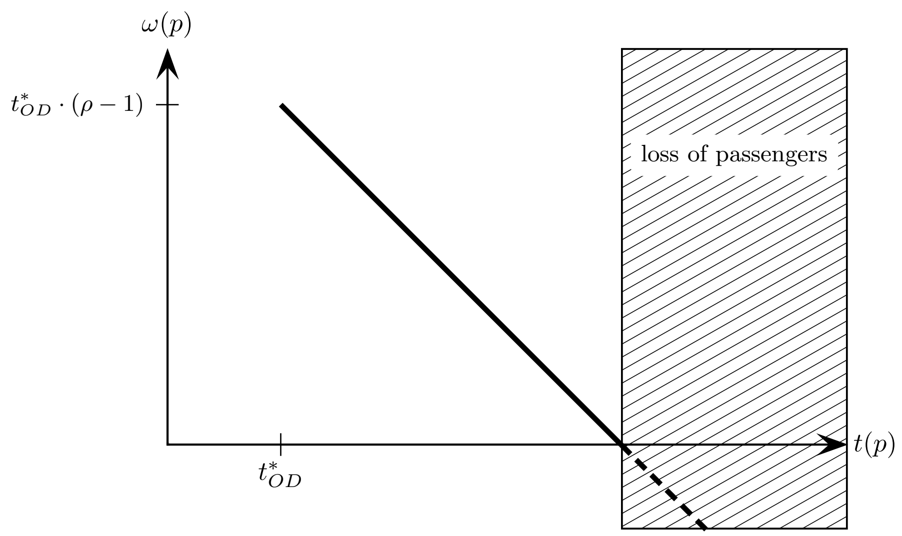

Fig. 2. Quality measure for passenger routes.

Traffic Assignment Model

$$\sum_{O,D,A,B,L} \omega^{O,D}_{(A,B),L} \varphi^{O,D}_{(A,B),L} \rightarrow max$$

$$\sum_{L,A} \varphi^{O,D}_{(A,D),L} \leq v_{OD}$$

$$\sum_{L,A} \varphi^{O,D}_{(A,X),L} - \sum_{L,B} \varphi^{O,D}_{(X,B),L} = 0 \quad \text{forall } X \notin \{O, D\}$$

$$\varphi^{O,D}_{(A,B),L} \in \mathbb{R}^+$$

Note that this model **maximizes travel quality**.

2.3 Line planning model

For the line planning part of our model we use the basic linear programming approach of M. Bussieck ([3]), who defines for each line L a frequency variable $x_L \in \mathbb{Z}^+$, which gives the service frequency of the line per time unit, say one hour. $x_L = 0$ implies, that L is not considered in the line plan. Values $x_L = 1, 2, ...$ indicate, that this line is served once, twice,... per time period. Since we consider a homogeneous line system, we may assume that the passenger capacity of all

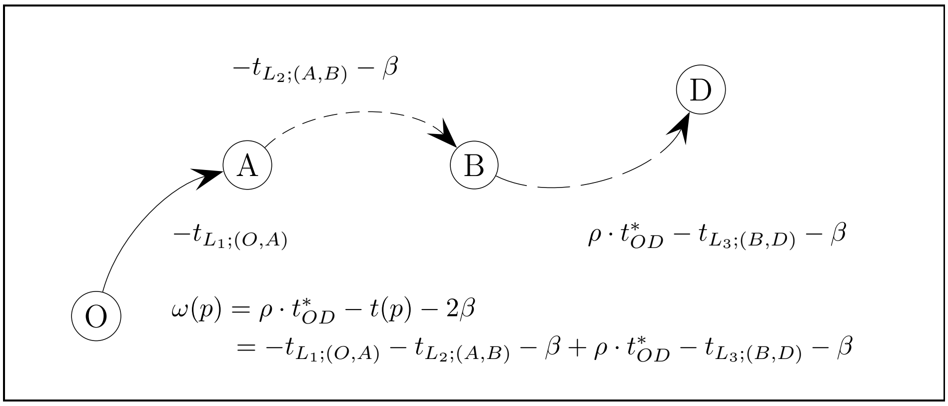

Fig. 3. Splitting route quality to the partial passenger route flows.

single trains is defined by a constant value C. Infrastructural capacity constraints for edges and nodes can be simply modelled by the restrictions

$$\sum_{e \in L} x_L \leq \overline{f}(e) \qquad (\mu_e)$$

$$\sum_{L \text{ serves node } n} x_L \leq \overline{f}(n) \qquad (\nu_n)$$

To model operational cost, we use a simple, linear approach and define the costs for serving line L with frequency x_L, by $c_L x_L$. Operational costs are discussed in more detail by K.-F. Jerosch (see [5]).

2.4 Combining traffic assignment and line planning

Traffic assignment and line planning are combined by the capacity constraints

$$\sum_{O,D,A,B: e \in L_{|(A,B)}} \varphi^{O,D}_{(A,B),L} \leq C x_L$$

which guarantees for each edge e that the line plan provides enough capacity for the passengers. The right side of this inequality is the amount of available capacity provided by the line L with frequency x_L. The left hand summarizes the passengers from all origin-destination pairs (O, D) and all route parts (A, B), which are using the line L on edge e. The modelling approach allows two different possibilities to treat operational cost:

1. Operational cost may be placed into the objective

$$\max \sum_{O,D,A,B,L} \omega^{O,D}_{(A,B),L} \varphi^{O,D}_{(A,B),L} - \alpha \left(\sum_L c_L x_L \right)$$

2. Operational may be restricted by a given budget, which leads to the constraint

$$\sum_L c_L x_L \le c^{max}$$

In summary, we obtain the full model

Full model

$$\begin{aligned}
\sum_{O,D,A,B,L} \omega^{O,D}_{(A,B),L} \varphi^{O,D}_{(A,B),L} &\to max \\
\sum_{L,A} \varphi^{O,D}_{(A,D),L} &\le v_{OD} \\
\forall X \notin \{O,D\} : \sum_{L,A} \varphi^{O,D}_{(A,X),L} - \sum_{L,B} \varphi^{O,D}_{(X,B),L} &= 0 \\
\forall e \in E, L \in \mathcal{L} : \sum_{O,D,A,B: e \in L_{|(A,B)}} \varphi^{O,D}_{(A,B),L} &\le C x_L \\
\sum_L c_L x_L &\le c^{max} \\
\sum_{e \in L} x_L &\le \overline{f}(e) \\
\sum_{L \text{ serves node } n} x_L &\le \overline{f}(n) \\
\varphi^{O,D}_{(A,B),L} &\in \mathbb{R}^+ \\
x_L &\in \mathbb{N}
\end{aligned}$$

2.5 Model strengthening by cutting planes

Using standard techniques from integer programming (see e.g. [8]), it is easy to see that

$$\varphi - \left(b - C\left\lfloor \frac{b}{C} \right\rfloor\right) x \le \left[C - \left(b - C\left\lfloor \frac{b}{C} \right\rfloor\right)\right] \left\lfloor \frac{b}{C} \right\rfloor \tag{2}$$

is a valid inequality of the polyhedron

$$P = \{(\varphi, x) \in \mathbb{R} \times \mathbb{Z} \mid \varphi - Kx \le 0;\ \varphi \le b\}$$

Due to limited travel demand, the passenger flows

$$\varphi^{max}_e := \sum_{O,D,A,B: e \in L_{|(A,B)}} \varphi^{O,D}_{(A,B),L}$$

are bounded from above. Reasonable bounds can be calculated from the origin-destination matrix and the detour factor ρ. Such bounds are discussed by M. Bussieck ([3]) and can be calculated in polynomial time. Define 'reduced' capacity by

$$\tilde{C}_e := \left(\varphi_e^{max} - C\left\lfloor\frac{\varphi_e^{max}}{C}\right\rfloor\right)$$

and the residual capacity by

$$C_e^* := \left[C - \left(\varphi_e^{max} - C\left\lfloor\frac{\varphi_e^{max}}{C}\right\rfloor\right)\right]\left\lfloor\frac{\varphi_e^{max}}{C}\right\rfloor,$$

then by (2) the valid inequality

$$\sum_e \sum_{O,D,A,B:e\in L_{|(A,B)}} \varphi_{(A,B),L}^{O,D} - C\left(\sum_{L:e\in L} x_L\right) \leq 0 \tag{3}$$

implies the cutting plane

$$\sum_e \sum_{O,D,A,B:e\in L_{|(A,B)}} \varphi_{(A,B),L}^{O,D} - \tilde{C}_e\left(\sum_{L:e\in L} x_L\right) \leq C_e^* \tag{4}$$

For the case, that $\varphi_e^{max} < C$, we have $C_e^* = 0$ and $\tilde{C}_e \leq C$. Then we may replace the inequality (3) by the more tighten inequality (4).

3 Solution Methods

3.1 Overview Solution Approach

In real world examples, there will arise a huge amount of possible passenger flow parts. Hence, those variables **can** only be handled by the use of a column generation scheme. For the potential lines (i.e. all frequency variables are contained in the model) the resulting pricing problem is quite straightforward and leads to a shortest path problem with non-negative arc weights. If the potential line frequency variables are not known a priori, the approach becomes much more complicated, since x_L and passenger route variables must be generated simultaneously.

The following algorithm summarizes the discussed method for the case that we use a fixed line pool. A more detailed description of the algorithm steps is given in the next sections.

Algorithm 1

1. *Calculate the pool of potential lines.*
2. *Calculate an initial line plan, which is feasible with the budget constraint.*

3. Solve the relaxation of the initial linear model.

4. Pricing iteration:

(a) solve the pricing problem for the passenger flow variables and add those with negative reduced cost to the formulation

(b) resolve relaxation. Otherwise stop, the optimal solution has been found.

(c) solve the knapsack problem (3.3) and the flow problem (3.3)[1] *to calculate an upper bound of the problem.*

(d) in case of fractional variables $\tilde{x}_L$ *apply the primal heuristic to find a possibly improved solution.*

3.2 Column Generation

In the following we consider the normalized model with associated dual variables given in brackets on the right side.

$$
\begin{aligned}
\sum_{O,D,A,B,L} \omega^{O,D}_{(A,B),L} \varphi^{O,D}_{(A,B),L} &\rightarrow max \\
\sum_{L,A} \varphi^{O,D}_{(A,D),L} &\leq v_{OD} && (\pi^{OD}_D) \\
\forall X \notin \{O,D\} : \sum_{L,A} \varphi^{O,D}_{(A,X),L} - \sum_{L,B} \varphi^{O,D}_{(X,B),L} &= 0 && (\pi^{OD}_X) \\
\forall e \in E, L \in \mathcal{L} : \sum_{O,D,A,B: e \in L_{|(A,B)}} \varphi^{O,D}_{(A,B),L} - C x_L &\leq 0 && (\xi_{e,L}) \\
\sum_L c_L x_L &\leq c^{max} && (\gamma) \\
\sum_{e \in L} x_L &\leq \overline{f}(e) && (\mu_e) \\
\sum_{L \text{ serves node } n} x_L &\leq \overline{f}(n) && (\nu_n) \\
\varphi^{O,D}_{(A,B),L} &\in \mathbb{R}^+ \\
x_L &\in \mathbb{N}
\end{aligned}
$$

The dual *potential* variables π^{OD}_X of the underlying passenger route flow problem are only defined for $X \neq O$. For technical reasons we define $\pi^{OD}_O = 0$ [2]

[1] The solution requires no extra effort, because this problem is already solved during the pricing problem for the passenger flow variables.

[2] This allows to model the dual shortest path problem by regarding $\pi^{OD}_Y - \pi^{OD}_X$ to be a shortest path length from node X to node Y.

Pricing of partial travel routes In dependence on the nodes A, B define

$$s_{(A,B)} := \begin{cases} 0, & if A = O \text{ and } B \neq D \\ -\beta, & if A \neq O \text{ and } B \neq D \\ \rho t^*_{OD} - \beta, & if A \neq O \text{ and } B = D \\ \rho t^*_{OD}, & if A = O \text{ and } B = D \end{cases}$$

Then, the reduced cost of a partial travel route flow variable $\varphi^{O,D}_{(A,B),L}$ are given by

$$\begin{aligned} r\omega^{O,D}_{(A,B),L}) &= -\omega^{O,D}_{(A,B),L} + \sum_{e \in L_{|(A,B)}} \xi_{e,L} + \pi^{OD}_B - \pi^{OD}_A \\ &= -s_{(A,B)} + \sum_{e \in L_{|(A,B)}} t_e + \sum_{e \in L_{|(A,B)}} \xi_{e,L} + \pi^{OD}_B - \pi^{OD}_A \\ &= -s_{(A,B)} + \sum_{e \in L_{|(A,B)}} (\xi_{e,L} + t_e) + \pi^{OD}_B - \pi^{OD}_A \end{aligned}$$

In order to find flow variables with negative reduced cost, we have to minimize the generalized path length $\alpha^* := \min_L \sum_{e \in L_{|(A,B)}} (\xi_{e,L} + t_e)$ and compare this shortest path length by

$$r < 0 \iff \alpha^* < s_{(A,B)} + \pi^{OD}_B - \pi^{OD}_A$$

Note, that the right side $s_{(A,B)} + \pi^{OD}_B - \pi^{OD}_A$ is independently from line L.

In summary, pricing out partial passenger routes can be solved by a shortest path problem from node A to node B with respect to non-negative edge cost $\xi_{e,L} + t_e \geq 0$.

Pricing of line variables The reduced cost of a frequency line variable x_L is given by

$$r(x_L) = \gamma c_L - C \sum_{e \in L} \xi_{e,L} + \sum_{e \in L} \mu_e + \sum_{L \text{ serves node } n} \nu_n$$

Setting $\nu_e = \nu_n$ for $e = (n, x)$ and using a linear approximation

$$c_L = \sum_{e \in L} c_{e,L}$$

for the operational costs, we receive reduced cost as path length

$$\begin{aligned} r(x_L) &= \gamma c_L - C \sum_{e \in L} \xi_{e,L} + \sum_{e \in L} \mu_e + \sum_{L \text{ serves node } n} \nu_n \\ &= \sum_{e \in L} \left(\underbrace{\gamma c_{e,L} + \mu_e + \nu_e}_{\text{shadow cost}} - \underbrace{C\xi_{e,L}}_{\text{shadow profit}} \right) \end{aligned}$$

The dual variables of the arc cost split into

– a cost part $\gamma c_{e,L} + \mu_e + \nu_e$ which are shadow cost with respect to infrastructural capacity limits (μ and ν) and budget restriction $\gamma c_{e,L}$ and
– a negative signed gain part $C\xi_{e,L}$ which may be interpreted as some kind of income in the context of the active line system. Large positive values of $\xi_{e,L}$ indicate mismatched supply and demand (large demand or small supply).

Hence, the negative value $-r(x_l) = \sum_{e\in L}(C\xi_{e,L}) - \sum_{e\in L}(\gamma c_{e,L} + \mu_e + \nu_e)$ can be interpreted to be some kind of profit = income - expenditures, which should be maximized during the pricing step.

Theorem Line pricing
Line pricing is a longest path problem, i.e. to find an elementary (cycle free) path with maximum 'profit'

$$\delta_e := \underbrace{C\xi_{e,L}}_{\text{shadow profit}} \quad - \underbrace{\gamma c_{e,L} + \mu_e + \nu_e}_{\text{shadow cost}}$$

3.3 Lagrange Relaxation

Lagrangian relaxation is a standard technique which moves hard constraints into the objective. Traffic assignment and line planning model are only coupled by the constraints

$$\sum_{O,D,A,B:e\in L_{|(A,B)}} \varphi^{O,D}_{(A,B),L} \le Cx_L \iff \sum_{O,D,A,B:e\in L_{|(A,B)}} \varphi^{O,D}_{(A,B),L} - Cx_L \le 0$$

By using the associated dual variables $\xi_{e,L}$, the movement of those constraints leads to the Lagrangian objective

$$\begin{aligned} &\sum_{O,D,A,B,L} \omega^{O,D}_{(A,B),L}\varphi^{O,D}_{(A,B),L} + \sum_{e,L}\xi_{e,L}\left[Cx_L - \sum_{O,D,A,B,L:e\in L}\varphi^{Q,Z}_{(A,B),L}\right] \\ = &\underbrace{\sum_{Q,Z,A,B,L}\left(\omega^{O,D}_{(A,B),L} - \sum_{L:e\in L}\xi_{e,L}\right)\varphi^{O,D}_{(A,B),L}}_{\text{flow part}} + \underbrace{C\sum_{e,L}\xi_{e,L}x_L}_{\text{line plan part}}\ , \end{aligned}$$

which separates into the sum of two linear terms depending either on the passenger flow variables or the line plan variables. Hence, solving the problems

FlowProblem

$$\varphi^* := \sum_{O,D,A,B,L} \left(\omega^{O,D}_{(A,B),L} - \sum_{O,D,A,B,L:e\in L} \xi_{e,L} \right) \varphi^{O,D}_{(A,B),L} \to \max$$

$$\sum_{L,A} \varphi^{O,D}_{(A,D),L} \leq v_{OD}$$

$$\forall X \notin \{O,D\} : \sum_{L,A} \varphi^{O,D}_{(A,X),L} - \sum_{L,B} \varphi^{O,D}_{(X,B),L} = 0$$

$$\varphi^{Q,Z}_{(A,B),L} \in \mathbb{R}^+$$

Generalized Knapsack Problem

$$\xi^* := C \sum_{e,L} \xi_{e,L} x_L \to \max$$

$$\sum_{L} c_L x_L \leq c^{max}$$

$$\sum_{e\in L} x_L \leq \overline{f}(e)$$

$$\sum_{L \text{ serves node } n} x_L \leq \overline{f}(n)$$

$$x_L \in \mathbb{N}$$

leads to the upper bound $\varphi^* + \xi^*$.

Any integer solution $(\tilde{x}_L)_{L\in\mathcal{L}}$ of the generalized knapsack problem leads to feasible line plan of the origin problem and can therefore be used to generate a primal feasible solution by solving the traffic assignment problem.

Primal Heuristic Traffic Assignment

$$\sum_{O,D,A,B,L} \omega^{O,D}_{(A,B),L} \varphi^{O,D}_{(A,B),L} \rightarrow max$$

$$\sum_{L,A} \varphi^{O,D}_{(A,D),L} \leq v_{OD}$$

$$\sum_{L,A} \varphi^{O,D}_{(A,X),L} - \sum_{L,B} \varphi^{O,D}_{(X,B),L} = 0 \quad \text{forall } X \notin \{O,D\}$$

$$\forall e \in E, L \in \mathcal{L}: \sum_{O,D,A,B:e \in L_{|(A,B)}} \varphi^{O,D}_{(A,B),L} \leq C\tilde{x}_L$$

$$\varphi^{O,D}_{(A,B),L} \in \mathbb{R}^+$$

4 Computational Results

4.1 The Traffic Sample

The software tool LINOP, developed by the Technical University Dresden, Faculty of Traffic Sciences, Institute for Logistics and Aviation, includes the presented mathematical model. Linear programs are solved by using the COIN-BCP-Solver.

We applied our method to the bus network of Berlin city. By kindly support of the Berlin city authority of city development and transportation we used origin destination data, which are not allowed to be published. For this reason the computational results do not report the total passenger demand. Instead, the results are given in percentage of optimal travel quality. 100% percent means, that for each origin destination pair a direct connection could be provided by the line plan.

Using an origin-destination matrix of passenger demands including all stops of Berlin city, it was necessary to add tramway, subway and the local train network to the existing bus infrastructure. A system split (see [9]) splits the total origin destination matrix into separate matrices for the tramway, subway, local train, express bus and bus.

4.2 Instance 1: Bus network of Berlin City

The modeled infrastructure consists of 2.590 nodes and 3.200 edges represents the Berlin city public bus network of July 2005. Bus stops are defined as nodes and edges describe possible connections beetween two bus stops. There are three different kinds of bus services operating on the Berlin infrastructure network, called express bus, metro bus and local bus. The bus services are different considering operation time, frequency or travel time. For instance the expressbus

Fig. 4. Resulting bus line plan

stops not as often as a local bus and consequently the expressbus has a reduced travel time. Due to large differences of the travel times we defined the expressbus as independent transportation product in our model. However the metro bus and the local bus are very similar and consequently these bus services are combined in the model as one transport carrier.

Table 1. *characteristics of one bus line plan*

parameter	value
minimum length of a line:	10 minutes travel time
budget:	7755 minutes operation time
changing penalty:	1 min/changing
detour factor:	1.1
number of lines:	119 lines
lines with frequency of 10 minutes:	33 lines
lines with frequency of 20 minutes:	86 lines
optimization objective value (travel quality):	914.354

For the most challenging instance, what is the expressbus and bus network, we performed for different budget parameter up to 3 pricing iterations, (see table 1), each of which took approximately 40 minutes. The instance was computed by an UNIX-PC with 2x3,2 Ghz and 8 gigabyte RAM. Due to the huge amount of memory needed (Out of memory!), for the bus network only a few number of iterations could be performed.

A typical solution is reported by the Figure 4 and Table 1.

4.3 Instance 2: Tram network of Berlin city

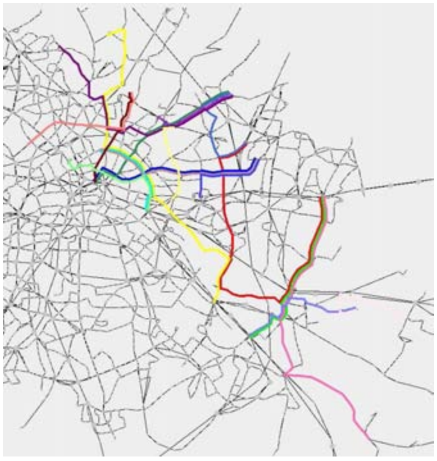

Fig. 5. tramway infrastructure

The second instance is the tram network of Berlin city. Which is of much smaller size compared to the bus network. The initial heuristic solution has an objective of 66.20, which could be improved during the iterations up to a travel quality of 71.00. By Langrange Relaxation we obtained the best upper bound by 89.45.The characteristics are presented in table 2 and in figure 6.

Table 2. *characteristics of one tramway line plan*

parameter	value
nodes	382
edges	435
minimum length of a line:	20 minutes travel time
budget:	1905 minutes operation time
changing penalty:	5 min/changing
detour factor:	1.2
number of lines:	18 lines
lines with frequency of 10 minutes:	18 lines
optimization objective value (travel quality):	71.000

Figure 6 illustrates the performance of the algorithm. Please note that the first line plan represents the start heuristic solution. Initialisation and preprocessing took approximately 10 minutes. The best solution with our hardware-performance for the tram network is found after 12 minutes. The tram line plan was solved with the same hardware as the bus network. In contrast to the bus network for this smaller instance the route node of the branch and bound tree was priced out rather quickly. But after a branch and bound depth of 120 the computer run out of memory again.

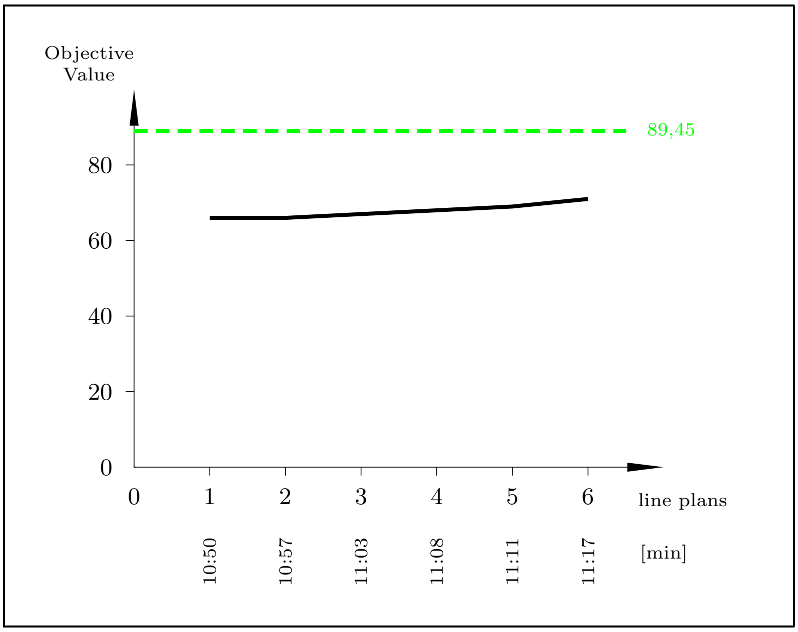

Fig. 6. *objective value (example tram line plan of Berlin city)*

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