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# OUTPATHS OF ARCS IN REGULAR 3-PARTITE TOURNAMENTS

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**Abstract**

Guo [*Outpaths in semicomplete multipartite digraphs*, Discrete Appl. Math. 95 (1999) 273–277] proposed the concept of the outpath in digraphs. An outpath of a vertex $x$ (an arc $xy$, respectively) in a digraph is a directed path starting at $x$ (an arc $xy$, respectively) such that $x$ does not dominate the end vertex of this directed path. A $k$-outpath is an outpath of length $k$. The outpath is a generalization of the directed cycle. A $c$-partite tournament is an orientation of a complete $c$-partite graph.

In this paper, we investigate outpaths of arcs in regular 3-partite tournaments. We prove that every arc of an $r$-regular 3-partite tournament has 2- (when $r \geq 1$), 3- (when $r \geq 2$), and 5-, 6-outpaths (when $r \geq 3$). We also give the structure of an $r$-regular 3-partite tournament $D$ with $r \geq 2$ that contains arcs which have no 4-outpaths. Based on these results, we conjecture that for all $k \in \{1, 2, \ldots, r-1\}$, every arc of $r$-regular 3-partite tournaments with $r \geq 2$ has $(3k-1)$- and $3k$-outpaths, and it has a $(3k+1)$-outpath except an $r$-regular 3-partite tournament.

**Keywords:** multipartite tournament, regular 3-partite tournament, outpaths.

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