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EXTREMAL MATCHING ENERGY OF COMPLEMENTS OF TREES

Tingzeng Wu[1]

School of Mathematics and Statistics
Qinghai Nationalities University
Xining, Qinghai 810007, P.R. China

e-mail: mathtzwu@163.com

Weigen Yan

School of Sciences, Jimei University
Xiamen 361021, P.R. China

e-mail: weigenyan@263.net

AND

Heping Zhang

School of Mathematics and Statistics, Lanzhou University
Lanzhou, Gansu 730000, P.R. China

e-mail: zhanghp@lzu.edu.cn

Abstract

Gutman and Wagner proposed the concept of the matching energy which is defined as the sum of the absolute values of the zeros of the matching polynomial of a graph. And they pointed out that the chemical applications of matching energy go back to the 1970s. Let T be a tree with n vertices. In this paper, we characterize the trees whose complements have the maximal, second-maximal and minimal matching energy. Furthermore, we determine the trees with edge-independence number p whose complements have the minimum matching energy for $p = 1, 2, \ldots, \lfloor \frac{n}{2} \rfloor$. When we restrict our consideration to all trees with a perfect matching, we determine the trees whose complements have the second-maximal matching energy.

Keywords: matching polynomial, matching energy, Hosoya index, energy.

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[1]Corresponding author.

References

[1] J. Aihara, *A new definition of Dewar-type resonance energies*, J. Amer. Chem. Soc. **98** (1976) 2750–2758.
doi:10.1021/ja00426a013

[2] L. Chen and Y. Shi, *The maximal matching energy of tricyclic graphs*, MATCH Commun. Math. Comput. Chem. **73** (2015) 105–119.

[3] L. Chen, J. Liu and Y. Shi, *Matching energy of unicyclic and bicyclic graphs with a given diameter*, Complexity **21** (2015) 224–238.
doi:10.1002/cplx.21599

[4] D. Cvetković, M. Doob, I. Gutman and A. Torgašev, Recent Results in the Theory of Graph Spectra (North-Holland, Amsterdam, 1988).

[5] M.J.S. Dewar, The Molecular Orbital Theory of Organic Chemistry (McGraw-Hill, New York, 1969).

[6] E.J. Farrell, *An introduction to matching polynomials*, J. Combin. Theory Ser. B **27** (1979) 75–86.
doi:10.1016/0095-8956(79)90070-4

[7] C.D. Godsil, Algebraic Combinatorics (Chapman and Hall, New York, 1993).

[8] C.D. Godsil and I. Gutman, *On the theory of the matching polynomial*, J. Graph Theory **5** (1981) 137–144.
doi:10.1002/jgt.3190050203

[9] I. Gutman, *The matching polynomial*, MATCH Commun. Math. Comput. Chem. **6** (1979) 75–91.

[10] I. Gutman and S. Wagner, *The matching energy of a graph*, Discrete Appl. Math. **160** (2012) 2177–2187.
doi:10.1016/j.dam.2012.06.001

[11] I. Gutman, *The energy of a graph: old and new results*, in: Algebraic Combinatorics and Applications, A. Betten, A. Kohnert, R. Laue, A. Wassermann (Ed(s)), (Springer-Verlag, Berlin, 2001) 196–211.
doi:10.1007/978-3-642-59448-9_13

[12] I. Gutman, X. Li and J. Zhang, *Graph energy*, in: Analysis of Complex Networks From Biology to Linguistics, M. Dehmer, F. Emmert-Streib (Ed(s)), (Wiley-VCH, Weinheim, 2009) 145–174.
doi:10.1002/9783527627981.ch7

[13] S. Ji, X. Li and Y. Shi, *Extremal matching energy of bicyclic graphs*, MATCH Commun. Math. Comput. Chem. **70** (2013) 697–706.

[14] H. Li, Y. Zhou and L. Su, *Graphs with extremal matching energies and prescribed parameters*, MATCH Commun. Math. Comput. Chem. **72** (2014) 239–248.

[15] S. Li and W. Yan, *The matching energy of graphs with given parameters*, Discrete Appl. Math. **162** (2014) 415–420.
doi:10.1016/j.dam.2013.09.014

[16] X. Li, Y. Shi and I. Gutman, Graph Energy (Springer, New York, 2012). doi:10.1007/978-1-4614-4220-2

[17] L. Lovász, Combinatorial Problems and Exercises, Second Edition (Budapest, Akadémiai Kiadó, 1993).

[18] D.B. West, Introduction to Graph Theory, Second Edition (Pearson Education, Singapore, 2001).

[19] T. Wu, *On the maximal matching energy of graphs*, J. East China Norm. Univ. **1** (2015) 136–141.

[20] K. Xu, Z. Zheng and K.C. Das, *Extremal t-apex trees with respect to matching energy*, Complexity (2015), in press. doi:10.1002/cplx.21651

[21] K. Xu, K.C. Das and Z. Zheng, *The minimum matching energy of* (n, m)*-graphs with a given matching number*, MATCH Commun. Math. Comput. Chem. **73** (2015) 93–104.

[22] W. Yan, Y. Yeh and F. Zhang, *Ordering the complements of trees by the number of maximum matchings*, J. Quan. Chem. **1055** (2005) 131–141. doi:10.1002/qua.20688

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