The Cooperation and Competition Between an Added Value MVNO and an MNO Allowing Secondary Access

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***Abstract*—Mobile Virtual Network Operators (MVNOs) are an increasingly growing segment of the market for wireless services. MVNOs do not own their own network infrastructure and so must cooperate with existing Mobile Network Operators (MNOs) to gain access to the network infrastructure needed to enter this market. Cooperating with an MVNO is a non-trivial decision for an MNO in part because the MVNO may then become a potential competitor for customers. One motive for entering into such an arrangement is that the MVNO receives an added value from serving customers beyond what it earns from charging them for wireless service. We study a game theoretic model for the cooperation and competition between an MNO and such an added value MVNO based on models for price competition with congestible resources. Our model captures two different dimensions of how an MNO may cooperate. The first dimension is the payment scheme between the MNO and the MVNO. The second dimension is the access priority that the MNO chooses to offer to the MVNO's customers. We characterize the pros and cons of different cooperation modes and analyze the optimal cooperation mode under different conditions.**

I. Introduction

In recent years, the global MVNO market has been growing fast. According to [1], [2], the size of the global MVNO market has been increasing from 2012 to 2017 and is projected to continue increasing. Compared to traditional MNOs, MVNOs can have advantages of providing a product mix or having access to special types of customers[1] that incumbent mobile operators cannot match [3]. These advantages have been well studied in [3], [12]. However, in addition to these potential advantages, in many cases these MVNOs might have additional "added value" beyond the money they collect from wireless service subscriptions. For example, some *brand-oriented* MVNOs (e.g., Virgin Mobile) might benefit from advertising their brands to their mobile users or might be able to offer mobile users incentives to utilize other products within this brand. These MVNOs get added value from their own customers beyond the wireless service. Their added value depends on the number of their own mobile subscribers. Comparatively, some other *access* MVNOs can benefit from letting more users have wireless access, regardless of whether these users subscribe to their own service or that of the MNO. An example of this is a company like Google, who benefits from any user having wireless access and thus being more likely to use the company's other services. Finally, other *hybrid* MVNOs can have both types of benefits, gaining some profit from expanding access and some additional profit from its own subscribers. In this paper, we focus on these "added value" MVNOs.

Forging a win-win agreement with an MNO is a key part in any MVNO's success. Depending on how the cooperation between the MVNO and MNO is structured, the MNO may be able to expand their market shares, differentiate the customers and services and bring new revenue streams outside the wireless service market by cooperating with the MVNO. However, there are also challenges for the MVNOs. Chief among these is convincing the MNO that an MVNO is not a threat to its business.[2] Concerns of increased competition from an MVNO can be offset when the MVNO has added value. By designing an appropriate revenue sharing scheme, the MNO could also benefit from this added value. Studying this for added value MVNOs is one of the key goals of this paper. We consider two different revenue sharing schemes: a *flat-rate payment* and a *usage-based payment*. In a *flat-rate payment* scheme, the MVNO only needs to pay a flat fee to get access to the MNO's network, while in a *usage-based payment* scheme, the fee paid by the MVNO depends on the traffic it puts on the MNO's network.

Another aspect that can impact the cooperation with an MVNO is the policy used by the MNO for sharing its resources, which we refer to as the *resource sharing policy*. We note that this sharing can refer to sharing both spectrum and infrastructure as is the normal case today. The sharing can also refer to settings where an MVNO shares the MNO's spectrum but has its own infrastructure. We consider two resource sharing policies. The first, which we refer to as *primary access mode*, is where the MNO treats the MVNO's traffic and its own traffic equally. The second, referred to as *secondary access mode*, is where the MNO gives priority access to its own customers, similar to the primary-secondary model for resource sharing [17].

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[1]For example, Boost (an MVNO in the U.S.) has music streaming service and hence can attract the customers who like music streaming.

[2]Indeed, the first MVNO, Sense Communication, was not able to reach commercial agreements with MNOs and ended in bankruptcy [5].

A. Related Work

There has been prior work, e.g., [12]–[15], that studied the economics of MNOs and MVNOs in the wireless market. For example, [13], [14] focused on the impact of the user type on the cooperation and competition of MNOs and MVNOs. Our paper models the strategic interaction between the MVNO and MNO, which was assumed to be given in [13] and [14], and focuses on the impact of the MNO's band resource, the MVNO's added value, and the cooperation mode. Reference [12] considered a strategic model of price competition between the MNO and MVNO. However, it focused on an MVNO that attracts a different market segment of users compared to the MNO, instead of an added value MVNO as in our work. Futhermore, [12] assumed that the MNO charges the MVNO a usage-based payment with predetermined per source access fee and [15] assumed that MNO offers the MVNO secondary access with a flat-rate payment. However, in our paper, we model the payment between the MVNO and MNO as a decision of the MNO, and consider different resource sharing policies between the MNO and MVNO.

There has also been related work studying the competition among MNOs under different resource sharing policies but did not include MVNOs, e.g., [8]–[10], [16]–[18], [21], [22]. In this paper, we adopt the competition model in [8], [9], and model secondary access in a similar approach as in [10].

B. Contributions

In this paper, we build a game theoretical model to study the cooperation and competition between an MNO and an added value MVNO. We consider the impact of the resource sharing policy and the payment scheme on their cooperation. In addition to this, we consider the option of *commitment*, which means that the MNO has the option to commit not to compete with the MVNO after the MVNO enters. If the MVNO and MNO agree to cooperate without any commitment, we also model the competition between them for customers. The main conclusions are as follows:

- Offering secondary access with the proper choice of payment scheme and commitment always provides the MNO with the largest possible profit compared to offering primary access.
- When the MVNO's added value from its own customers is much larger than that from the MNO's customers, the MNO should choose the flat-rate payment scheme.
- When the MVNO's added value from its own customers and the MNO's customers are small and close, the payment scheme should be chosen based on the band resource. If the band resource is limited, the MNO should choose the flat-rate payment scheme. If the band resource is abundant, the MNO should choose the usage-based payment scheme.

II. Model

In this paper, we focus on a case where there is one monopolist MNO, denoted as SP1, and one potential MVNO, denoted as SP2, having access to a common pool of customers. We build a sequential game model to study the cooperation and competition between SP1 and SP2. We introduce the timing of the game as follows:

- Stage I: If SP1 could profit from cooperating with SP2, SP1 will decide a contract to offer to SP2 including (i) the *resource sharing policy*, (ii) *revenue sharing scheme* and (iii) whether to commit not entering at later stages. We consider two possible *resource sharing policies* (i.e., the primary access mode and secondary access mode) and two commonly used *revenue sharing schemes* (i.e., usage-based payment and flat-rate payment).
- Stage II: SP2 decides whether to sign the contract and become an MVNO.
- Stage III: If SP2 refuses to sign, SP1 will keep serving customers as a monopolist. If SP2 signs and SP1 has made a commitment not to compete with SP2, SP2 will set a price to maximize its profit as a monopolist. If SP2 signs and SP1 remains free to compete, they will set their prices for customers and compete for a common pool of customers.

A. Price Competition Model

We first model the price competition between the SPs when SP2 enters as an MVNO. As in [8], [9], we assume that the SPs in the market compete for a common pool of customers to maximize their revenue. The customers are modeled as non-atomic users with a total mass of 1. Each customer can choose an SP considering the SPs *delivered price*, given by the sum of its service price and a congestion cost. Here, the congestion cost captures the customer dissatisfaction caused by the network delay. This term characterizes the SP's Quality of Service (QoS). As in [10], we assume that the congestion cost experienced by a customer depends on the number of customers served at each priority tier (where if primary access mode is used, all customers are at the same primary tier). For a user at the primary tier, its congestion cost is given by $g(x^P)$, where x^P denotes the mass of customers in the primary tier, and $g(\cdot)$ is a given increasing function. For a user at the secondary tier, its congestion cost is given by $g(x^T)$, where $x^T = x^P + x^S$ is the total mass of customers being served, and x^S is the mass of secondary users. This models the characteristic that the primary users do not experience any degradation due to the existence of the secondary users, while the secondary users do experience degradation due to the existence of the primary users. To simplify our analysis, as in [10], [11], we focus on the case where $g(\cdot)$ is a linear function, i.e., $g(x) = \frac{x}{B}$, where B is band resource, or spectrum utilization efficiency, increasing with the available bandwidth and technology level.

Each SPi will announce a price p_i for its service and seek to maximize its revenue. Each customer is identified as x, $x \in [0, 1]$, with a reservation price for the service denoted by an inverse demand function $P(x)$. We assume that $P(x)$ is linear and given by

$$P(x) = 1 - x. \tag{1}$$

A customer x will accept service if and only if the delivered price is no greater than its reservation price $P(x)$ and seek to obtain service from the SP with the lowest delivered price. SP1 and SP2 will choose their service prices, i.e., p_1 and p_2, to compete for the customers if SP1 has not made a commitment not to compete. Let x_1 and x_2 denote the mass of customers that choose the service of SP1 and SP2, respectively. Given the SPs' resource sharing policy and prices, the customers' choices will reach a Wardrop Equilibrium (WE) [24], [25]. Specifically, if both SPs serve the market, their delivered prices should be the same. If any SP has a higher delivered price, it serves no customers.

To capture the WE in different resource sharing policies, we define a variable α, where $\alpha = 1$ indicates that SP1 offers primary access and $\alpha = 0$ indicates that SP1 offers secondary access. We characterize the Wardrop Equilibrium conditions as follows:

$$\begin{cases} \frac{x_1+\alpha x_2}{B} + p_1 \geq 1 - x_1 - x_2, \\ \frac{x_1+x_2}{B} + p_2 \geq 1 - x_1 - x_2, \\ x_1(1 - x_1 - x_2 - \frac{x_1+\alpha x_2}{B} - p_1) = 0, \\ x_2(1 - x_1 - x_2 - \frac{x_1+x_2}{B} - p_2) = 0, \\ x_1, x_2 \geq 0, \;\; x_1 + x_2 \leq 1. \end{cases} \tag{2}$$

The equilibria will fall into one of three possible cases: (i) the *competition case*, where both SP1 and SP2 serve customers, (ii) the *agent case*, where only SP2 serves customers, and (iii) the *monopoly case*, where only SP1 serves customers. It can be verified that there always exists a unique Wardrop Equilibrium given service prices p_1, p_2, and resource sharing policy α. We denote the number of customers of each SP that satisfies the Wardrop Equilibrium condition as $x_1^{\mathrm{WE}}(\alpha, p_1, p_2)$ and $x_2^{\mathrm{WE}}(\alpha, p_1, p_2)$.

If SP1 commits not to enter after SP2 enters, SP2 will serve as a new monopolist in stage III if it signs the contract. In this case, $x_1 = 0$ and x_2 needs to satisfy $\frac{x_2}{B} + p_2 = 1 - x_2$ at equilibrium. We use another variable C to indicate SP1's commitment state. If $C = 1$, SP1 commits not to compete with SP2. If $C = 0$, SP1 makes no such commitment.

If SP2 does not accept the contract that SP1 offers, then $x_2 = 0$ and SP1 remains a monopolist. In this case, x_1 needs to satisfy $\frac{x_1}{B} + p_1 = 1 - x_1$. We use a variable $S = 1$ to denote that SP2 signs the contract, and $S = 0$ to indicate that SP2 does not sign the contract.

Combining these cases, we derive SP1's and SP2's market shares in stage III given the decisions in the former stages and service prices. We denote them by the following vector:

$$\begin{aligned} &\mathbf{x}(\alpha, C, S, p_1, p_2) = \\ &[S \cdot (1 - C) \cdot x_1^{WE}(\alpha, p_1, p_2) + (1 - S)\frac{(1 - p_1)B}{B + 1}, \\ &S \cdot C\frac{(1 - p_2)B}{B + 1} + S \cdot (1 - C)x_2^{WE}(\alpha, p_1, p_2)]. \end{aligned}$$

B. SPs' Profits and Revenue Sharing

In addition to the prices the SPs charge for service, the profit of each SP will depend on the payment scheme used by SP1 to collect revenue from SP2 and also on the added value obtained by SP2 for both its own and SP1's customers. We consider two payment schemes between SP1 and SP2: the flat-rate payment scheme and the usage-based payment scheme. In the flat-rate payment scheme, the MVNO pays a fixed fee, denoted as A, to the MNO, and the MNO has the option of making a commitment about whether it would enter the market later. In the usage-based payment scheme, SP2 pays SP1 according to SP2's total traffic. This is also called a wholesale price scheme, which is commonly used in the current MVNO market [7]. We use p_d to denote the wholesale price that SP1 charges SP2 for each customer served by SP2.

In terms of the added value gained by the MVNO, we consider two components: a unit added value of k_2 that SP2 obtains from each customer that uses its service and a unit added value of k_1 that it obtains from each user of SP1's service. We assume that $k_2 \geq k_1$, as the MVNO can always get equal or more added value from its own customers. We can model the different types of MVNOs discussed in Sect. I by considering different values of k_1 and k_2.

Thus, the SPs' profits are as follows:

$$\Pi_1(x_1, x_2, p_1, A, p_d) = p_1 x_1 + A + p_d x_2, \tag{3}$$

$$\Pi_2(x_1, x_2, p_2, A, p_d) = p_2 x_2 + k_1 x_1 + k_2 x_2 - A - p_d x_2. \tag{4}$$

Under the flat-rate payment scheme, we have $p_d = 0$, and under the usage-based payment scheme, we have $A = 0$.[3]

C. Problem Formulation

In this subsection, we formulate the analysis of the three-stage game using backward induction. In stage III, given SP2's signing decision (i.e., S) in stage II and the contract variables (i.e., p_d, A, α, and C), SP1 solves the following problem to maximize its profit:

$$\begin{aligned} \max_{p_1 \in \mathbf{R}} \;\; & x_1 p_1 + S(A + p_d x_2) \\ \text{s.t. } & [x_1, x_2] = \mathbf{x}(\alpha, C, S, p_1, p_2). \end{aligned} \tag{5}$$

At the same time, SP2 solves

$$\begin{aligned} \max_{p_2 \in \mathbf{R}} \;\; & S(p_2 x_2 + k_2 x_2 - (A + p_d x_2)) + k_1 x_1 \\ \text{s.t. } & [x_1, x_2] = \mathbf{x}(\alpha, C, S, p_1, p_2). \end{aligned} \tag{6}$$

They will achieve a price equilibrium where neither of them can unilaterally change its service price to increase its profit. Note that there could be multiple equilibria in this stage. Since SP1 owns the spectrum and infrastructure, it has more market power. We assume that it can select the price equilibrium that satisfies the equilibrium condition when there are multiple equilibria. Thus, SP1 will select the price equilibrium that maximizes its profit in this stage and we denote this price equilibrium by a vector $\mathbf{p}^{\mathrm{PE}}(p_d, C, \alpha, S)$.[4]

[3]Our work can be easily extended to the two-part tariff payment scheme by considering non-zero A and p_d. However, the flat-rate payment scheme and the usage-based payment scheme are two most commonly used payment schemes in the current MVNO market, according to [7].

[4]A will not affect the optimization problems (5) and (6) in stage III.

In stage II, given the contract, SP2 solves the following problem:

$$\max_{S\in\{0,1\}} \; S(p_2x_2 + k_2x_2 - (A + p_dx_2)) + k_1x_1 \qquad (7)$$
$$\text{s.t. } [x_1, x_2] = \mathbf{x}(\alpha, C, S, p_1, p_2),$$
$$[p_1, p_2] = \mathbf{p}^{\mathrm{PE}}(p_d, C, \alpha, S).$$

We denote the optimal signing decision as $S^*(p_d, A, C, \alpha)$.

In stage I, SP1 will choose the contract that maximizes its profit, considering SP2's signing and pricing decision in stages II and III. It solves the following problem:

$$\max \; x_1p_1 + S(A + p_dx_2) \qquad (8)$$
$$\text{s.t. } p_d \cdot A = 0,$$
$$S = S^*(p_d, A, C, \alpha),$$
$$[p_1, p_2] = \mathbf{p}^{\mathrm{PE}}(p_d, C, \alpha, S),$$
$$[x_1, x_2] = \mathbf{x}(\alpha, C, S, p_1, p_2),$$
$$\text{var. } p_d \in \mathbf{R}, A \in \mathbf{R}, C \in \{0, 1\}, \alpha \in \{0, 1\}. \qquad (9)$$

Note that C and α are binary decision variables and either one of p_d and A needs to be 0 at the equilibrium. Thus, we will separately discuss these cases and compare SP1's profit in each case. We will first focus on the flat-rate case and compare the two resource sharing policies in Sect. III and Sect. IV. Then, we focus on the usage-based case and compare the two resource sharing policies in Sect. V and Sect. VI. Before getting into these, we first give the analysis of the benchmark case, where SP1 and SP2 do not cooperate, as this analysis of the benchmark case is the same for all cooperation modes. If SP1 or SP2 cannot profit more than the benchmark case, the cooperation contract will not be offered or signed in any cooperation mode.

D. Benchmark Case

If SP1, the MNO, cannot profit more by cooperating with SP2, it should not cooperate. This can be realized by setting A or p_d high so that SP2 will refuse to cooperate, i.e., $S = 0$. We denote this non-cooperation case as the benchmark case.[5] In this case, SP1 is the monopolist and solves the following problem:

$$\max_{p_1} \Pi_1 = p_1x_1$$
$$\text{s.t. } \frac{x_1}{B} + p_1 = 1 - x_1.$$

We will have the following results at the equilibrium:[6]

$$p_1^b = \frac{1}{2}, \qquad x_1^b = \frac{B}{2(1+B)}, \qquad (10)$$
$$\Pi_1^b = \frac{B}{4(1+B)}, \qquad \Pi_2^b = \frac{k_1B}{2(1+B)}. \qquad (11)$$

[5] This is different from the monopoly case introduced before. In this benchmark case, SP1 does not cooperate with SP2 at all, so SP2 cannot provide wireless service. In the monopoly case, only SP1 serving the market is a result of the price competition between SP1 and SP2. Hence, SP1's prices in the benchmark case and monopoly case will be different.

[6] Here, the superscript "b" denotes the price equilibrium in the benchmark case, where SP1 refuses any mode of cooperation with SP2.

III. PRIMARY ACCESS WITH FLAT-RATE PAYMENT

In this section, we consider the scenario with primary access and a flat rate payment, i.e., $p_d = 0$ and $\alpha = 1$. We analyze the cases where $C = 1$ and $C = 0$ separately and get the following theorem.

Theorem 1. *If SP1 offers primary access with a flat-rate payment, there exists a win-win cooperation and SP1 should commit not to enter in stage III. SP1 will get a profit of* $\frac{((1+k_2)^2-2k_1)B}{4(1+B)}$ *by setting the flat fee to this amount.*

We first analyze the case where SP1 commits not to enter in stage III and SP2 accepts the contract. By committing not to enter, SP1 can charge a larger flat fee from SP2 as they avoid a price war, and SP2 can simply maximize its profit Π_2 as a monopolist if it accepts the contract. In this case, $S = 1$ and SP2's problem in Stage III is as follows:

$$\max_{p_2} \; p_2x_2 + k_2x_2 - A$$
$$\text{s.t. } \frac{x_2}{B} + p_2 = 1 - x_2.$$

This yields $p_2^{\mathrm{PE}}(0, 1, 1, 1) = \frac{1-k_2}{2}$ and $\Pi_2 = \frac{(1+k_2)^2B}{4(1+B)} - A$. Comparing this to Π_2^b, the maximum flat fee SP1 can charge to ensure SP2's signing is

$$\frac{((1 + k_2)^2 - 2k_1)B}{4(1 + B)}. \qquad (12)$$

It can be easily verified that this flat fee is greater than Π_1^b. Thus, SP1 should cooperate with SP2.

We next analyze the case where SP1 does not commit. In this case, SP1 and SP2 will compete for customers in stage III, and the Wardrop Equilibrium conditions in (2) lead to

$$x_1^{WE}(1, p_1, p_2) = \begin{cases} \frac{B(1-p_1)}{1+B}, \text{ if } p_1 < p_2, \\ \frac{B(1-p_2)}{2(1+B)}, \text{ if } p_1 = p_2, \\ 0, \text{ if } p_1 > p_2, \end{cases}$$

and

$$x_2^{WE}(1, p_1, p_2) = \begin{cases} \frac{B(1-p_2)}{1+B}, \text{ if } p_1 > p_2, \\ \frac{B(1-p_1)}{2(1+B)}, \text{ if } p_1 = p_2, \\ 0, \text{ if } p_1 < p_2. \end{cases}$$

Here, we assume that SP1 and SP2 split the market evenly when they offer the same service and price.[7] Each SP's profit as a function of p_1 and p_2 is then given as follows:

$$\Pi_1(p_1, p_2) = \begin{cases} p_1\frac{B(1-p_1)}{1+B} + A, \text{ if } p_1 < p_2, \\ p_1\frac{B(1-p_2)}{2(1+B)} + A, \text{ if } p_1 = p_2, \\ A, \text{ if } p_1 > p_2. \end{cases}$$

$$\Pi_2(p_1, p_2) = \begin{cases} (p_2 + k_2)\frac{B(1-p_2)}{1+B} - A, \text{ if } p_2 < p_1, \\ (p_2 + k_2 + k_1)\frac{B(1-p_1)}{2(1+B)} - A, \text{ if } p_2 = p_1, \\ -A, \text{ if } p_2 > p_1. \end{cases}$$

[7] In this case, the Wardrop equilibrium is not unique.

From these, it can be seen that there can never be a price equilibrium where both SPs charge positive prices as they would both have an incentive to lower their prices to increase their profit. The only possible price equilibrium is that SP2 serves the whole market with a price slightly lower than 0, which leads to a profit $\frac{k_2 B}{B+1} - A$. Thus, SP1's profit without commitment is $\frac{(2k_2-k_1)B}{2(B+1)}$, which is lower than what SP1 can get when SP1 makes a commitment not to enter (as shown in (12)). Hence, SP1 should commit not to enter in the contract when offering primary access of its band resource to SP2 with a flat rate (where the fee should be set as in (12)).

IV. Secondary Access with Flat-rate Payment

In this section, we discuss the case where SP1 offers SP2 secondary access to its network with a flat-rate payment. Note that, if SP1 commits not to enter the market in stage III, SP2 will actually get primary access, which is the same as the scenario discussed in Sect. III.[8] Hence, under a flat-rate payment, offering secondary access should be no worse than primary access for SP1. More interestingly, we will show that in some cases, allowing secondary access may bring more profit for SP1 if both SP1 and SP2 serve the market in stage III. We first derive SP1's and SP2's market shares given their prices by setting $\alpha = 0$ in (2) as follows:

$$x_1^{WE}(0,p_1,p_2)=\begin{cases}\frac{B}{1+B}(1-p_1), & \text{if } p_1 < p_2,\\ \frac{B^2}{1+B}p_2 - Bp_1 + \frac{B}{1+B}, & \text{if } p_2 \le p_1 \le \frac{1+Bp_2}{1+B},\\ 0, & \text{if } p_1 > \frac{1+Bp_2}{1+B},\end{cases} \tag{13}$$

$$x_2^{WE}(0,p_1,p_2)=\begin{cases}0, & \text{if } p_2 > p_1\\ B(p_1-p_2), & \text{if } \frac{1+B}{B}p_1 - \frac{1}{B} \le p_2 \le p_1,\\ \frac{B}{1+B}(1-p_2), & \text{if } p_2 < \frac{1+B}{B}p_1 - \frac{1}{B}.\end{cases} \tag{14}$$

The above three cases correspond to the three competition cases introduced in Sect. II-A: monopoly case, competition case, and agent case. The price equilibrium in stage III cannot be in the monopoly case, as SP2 can always make its price a bit lower than SP1 to increase its profit. In the agent case, SP1 ends up serving no customers. Thus, all its profit comes from SP2's flat-rate payment. This payment should not be greater than that generated by making a commitment, where SP2 optimizes its revenue without competition. Hence, SP1 may get a higher profit by competing with SP2 only in the competition case. We introduce the following theorem.

Theorem 2. *If SP1 offers SP2 secondary access with a flat-rate payment and competes with SP2 in stage III, SP1 and SP2 will set prices*

$$p_1^{\mathrm{PE}}(0,0,0,1) = \frac{B^2 k_1}{(1+B)(4+3B)} + \frac{2-Bk_2}{4+3B},$$
$$p_2^{\mathrm{PE}}(0,0,0,1) = \frac{1+2Bk_1-2k_2-2Bk_2}{4+3B},$$

and both will serve customers if and only if

$$k_2 < \frac{2}{B} + \frac{Bk_1}{1+B}. \tag{15}$$

If condition (15) does not hold, the price equilibrium is not in the competition case, and SP1 should commit not to compete. Similar to Sect. III, SP1 will set the flat fee so that SP2 can get a slightly higher profit than that in the benchmark case. The resulting flat fee is

$$A = x_2(p_2+k_2) + k_1 x_1 - \Pi_2^b,$$

and the total profit of SP1 is then $\Pi_1 = x_1 p_1 + A$, where $[p_1, p_2] = \mathbf{p}^{\mathrm{PE}}(0,0,0,1)$ and $[x_1, x_2] = \mathbf{x}^{WE}(0, p_1^{\mathrm{PE}}(0,0,0,1), p_2^{\mathrm{PE}}(0,0,0,1))$. Comparing Π_1 with that in the commitment case (i.e., (12)), we have the following lemma.

Lemma 1. *If SP1 offers SP2 secondary access with a flat-rate payment, SP1 should serve customers in stage III if the band resource B is small and $k_2 < 2k_1 + \frac{1}{4}$.*

Due to SP2's added value, SP1 may get a larger profit by charging SP2 via the flat fee when staying out of the market in stage III. The reason is that the SPs avoid a price war and this increases SP2's profit. However, there are also benefits for SP1 to compete because of the use of secondary access. In our model, the second-tier customers will not degrade the quality of service of the first-tier customers. Thus, adding SP2 improves the utilization of the band and attracts more customers. The intuition of Lemma 1 is that when the band resource is limited, the secondary access helps improve the utilization of the band. The condition $k_2 < 2k_1 + \frac{1}{4}$ indicates that when SP2's unit added values from SP1's and SP2's customers are close, SP1 can benefit more from the tiered usage of the resources than from making a commitment. However, when the band resource is abundant, the competition between SP1 and SP2 will lead to overuse of the spectrum and a large decrease of SP1's gain from the wireless service. In this case, SP1 will prefer not to compete with SP2 and make a commitment not to enter the market in stage I.

We next consider two special cases. The first case is $k_1 = 0$, where SP2 only has added value from its own customers (which applies to the brand-oriented MVNOs introduced in Sect. I). The second case is $k_1 = k_2$, where SP2's added value only depends on the size of the overall wireless market (which applies to the access MVNOs introduced in Sect. I).

Corollary 1. *If SP1 offers SP2 secondary access with a flat-rate payment and the band resource is limited, SP1 should choose to serve customers in stage III when (i) $k_1 = 0$, $k_2 < \frac{1}{4}$; or (ii) $k_1 = k_2$.*

This is a corollary of Lemma 1, implying that SP1 should keep serving customers if (i) SP2 is a brand-oriented MVNO with relative small added value, or (ii) SP2 is an access MVNO.

Lemma 2. *Suppose that $k_1 = 0$, if SP1 offers SP2 secondary access with a flat-rate payment, SP1 should commit not to enter in stage III when $k_2 > \frac{1}{4}$ or $B > \frac{2\sqrt{10}-2}{9}$.*

[8] To keep consistency, we still allow commitment in the analysis. However, readers should note that SP1's requirement for keeping its own traffic's priority makes no difference, as it will not enter the market in stage III.

This lemma gives two conditions under either of which SP1 should commit not to enter the market for a brand-oriented MVNO. The first condition is that SP2's unit added value from its own customers is greater than $\frac{1}{4}$; the second condition is that the MNO's band resource is greater than $\frac{1}{4}$. The intuition is that there are two factors that incentivize SP1 to commit. The first factor is a large k_2, which brings a large profit to SP1 via the flat fee A. The second factor is a large B, which makes competition more harmful to SP1's profit.

V. Primary Access with Usage-based Payment

In Sect. III and IV, we considered the flat-rate payment scheme under two different resource sharing policies. In this section and Sect. VI, we consider the usage-based payment scheme where SP2 pays SP1 according to the amount of customers it serves. In this section, we again consider primary access. We set $\alpha = 1$ and $A = 0$, and consider $C = 1$ and $C = 0$ separately to solve Problem (5)-(8). We compare SP1's optimal profit in the cases when $C = 1$ and $C = 0$. We introduce the following theorem.

Theorem 3. *If SP1 offers primary access with usage-based payment scheme, there exists a unique market equilibrium.*
If $k_1 < \frac{1}{2}$ or $k_2 \geq k_1 + \sqrt{2k_1} - 1 + \sqrt{2k_1(\sqrt{2k_1} - 1)}$, SP1 should compete in stage III. At the equilibrium,

$$p_d^* = \frac{1 - k_1 + k_2}{2}, \quad p_1^* = \overline{\frac{1 + k_1 - k_2}{2}}, \quad p_2^* = \frac{1 + k_1 - k_2}{2}.$$

This leads to $\Pi_1 = \frac{B(1-k_1+k_2)^2}{4(1+B)}$.[9]
If $3k_1 - 1 \leq k_2 < k_1 + \sqrt{2k_1} - 1 + \sqrt{2k_1(\sqrt{2k_1} - 1)}$, SP1 will commit not to enter. If $k_2 \leq 3k_1 - 1$, there is no difference between making a commitment or not for SP1. The equilibrium of these two cases will be:

$$p_d^* = 1 + k_2 - \sqrt{2k_1}, \qquad p_2^* = 1 - \frac{\sqrt{2k_1}}{2},$$

and p_1^* needs to be greater than $\frac{1}{2}$ if no commitment is made. This leads to $\Pi_1 = \frac{B\sqrt{k_1}(1+k_2-\sqrt{2k_1})}{\sqrt{2}(1+B)}$.

We include the detailed proof in our online technical report [26]. The insights here are when the unit added values from customers of both SPs are small, there exists no win-win cooperation under primary access with usage-based payment. When the unit added value from the customers of SP1 is small, but the unit added value from the customers of SP2 is large, SP1 should remain free to compete with SP2 in stage III. Actually, in this case, although SP1 announces a service price, it serves no customers in stage III. SP1 announces this price as a threat of entry, and makes SP2 lower its price to serve more customers. As SP2's payment to SP1 is based on the number of customers SP2 serves, SP1 gets more profit by remaining free to compete. However, when k_1 and k_2 are close, SP1's price threat is less powerful, because SP2 will have a large profit by just having added value from SP1's customers. In this case, SP1's price threat no longer matters when SP2 optimizes its profit in stage III. Thus, a commitment should do no worse than competing for SP1 when the unit added values from customers of both SPs are large and close to each other.

VI. Secondary Access with Usage Based Payment

In this section, we consider the case when SP1 offers secondary access with a usage-based payment scheme. If SP1 commits, it will be the same as the primary access case studied in Sect. V-A. If SP1 does not commit, SP1 may still end up serving no customers, but its price will be a "threat of entry" for SP2. We are only interested in the equilibrium in the competition and agent cases, since the equilibrium in the monopoly case cannot lead to a profit larger than that in the benchmark case for SP1. Applying $\alpha = 0$, $C = 0$ and $A = 0$ to Problem (5)-(8), we can use a similar method to solve the equilibrium using backward induction.

Due to space limit, we focus on an interesting special case of k_1, k_2, and B, and include more details in our technical report [26]. In Sect. VII, we will provide more numerical analysis about SP1's optimal profit.

Lemma 3. *If $k_1 = 0$ and $k_2 < \frac{3}{3B+2}$, the equilibrium will be the competition type. The optimal p_d that maximizes SP1's profit is:*

$$p_d^* = \frac{4 + 9B}{2(8 + 9B)} + \frac{4 + 5B}{8 + 9B} k_2. \tag{16}$$

As for the service prices at equilibrium, $p_1^{\mathrm{PE}}(p_d^*, 0, 0, 1)$ can be derived by substituting (16) in

$$\frac{(k_1 - k_2)B^2 + B(2 - k_2) + 2}{(1 + B)(4 + 3B)} + \frac{(3B + 2)p_d^*}{4 + 3B},$$

and $p_2^{\mathrm{PE}}(p_d^*, 0, 0, 1)$ can be derived by substituting (16) in

$$\frac{1 + 2B(k_1 - k_2) - 2k_2}{4 + 3B} + \frac{3(1 + B)p_d^*}{4 + 3B}.$$

The intuition is that when k_2 and the band resource are relatively small, the advantage of secondary access is noteworthy. This is because: (i) when k_2 is small, the loss of added value is insignificant; (ii) when the band resource is limited, increasing the band resource efficiency is important.

VII. Comparisons

By comparing SP1's profit calculated in Sect. III-Sect. V, we are able to get SP1's optimal resource sharing policy and payment scheme. In this section, we introduce both analytical results and numerical examples to show the comparison. We first introduce the following theorem.

Theorem 4. *Let $\Pi_1^{Pri-Flat}$, $\Pi_1^{Pri-Usage}$, and $\Pi_1^{Sec-Flat}$ be SP1's profit when it chooses to offer primary access with a flat-rate payment, primary access with a usage-based payment, and secondary access with a flat-rate payment with the appropriate option of commitment in each case. We have $\Pi_1^{Sec-Flat} \geq \Pi_1^{Pri-Flat} \geq \Pi_1^{Pri-Usage}$.*

[9] We use an upper bar on a variable to denote a value that is greater than (and arbitrarily close to) this variable.

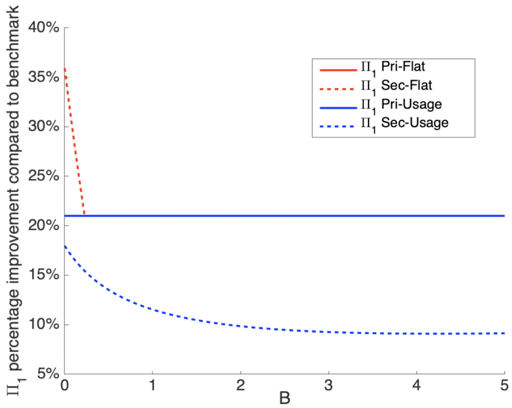

Fig. 1: SP1's profit improvement when facing the entry of a brand-oriented MVNO with $k_1 = 0$ and $k_2 = 0.1$.

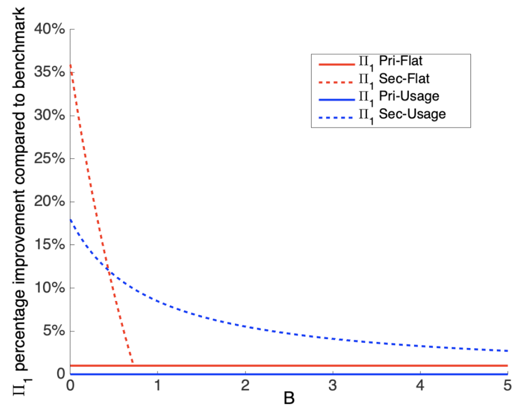

Fig. 2: SP1's profit improvement when facing the entry of an access MVNO with $k_1 = 0.1$ and $k_2 = 0.1$.

This theorem indicates that it is sufficient for SP1 to consider only two of the four possible combinations: (i) offering secondary access with a flat-rate payment or (ii) offering secondary access with a usage-based payment. First, under the flat-rate payment scheme, offering secondary access always leads to equal or more profit to SP1 than offering primary access. Under the flat-rate payment scheme, applying secondary access may increase SP1's profit when (i) the band resource is limited and (ii) SP2's unit added value from its own customers is not much larger than the unit added value from SP1's customers. Moreover, under primary access with a flat-rate payment, committing not to enter always brings more profit to SP1. As we also allow SP1 to offer secondary access with commitment, offering secondary access should be no worse than primary access under the flat-rate payment scheme. Second, if SP1 chooses to provide primary access to SP2, the flat-rate payment is always better than the usage-based payment for SP1. This can be shown by comparing SP1's profit analyzed in Theorem 1 with SP1's profit shown in (9).

Theorem 5. *In the optimal cooperation mode, the SPs' profits, customer surplus and social welfare do not decrease after the entry of SP2.*

This is intuitive as if SP1 and SP2 cooperate, they must both benefit from it. Customer surplus may increase as SP2 might have a lower service price. Thus, social welfare as the sum of SPs' profits and customer surplus should not decrease.

Next, we analyze SP1's choice of different cooperation modes when facing the different types of MVNOs introduced in Sect. I.

A. Brand-oriented MVNOs

If SP2 is a brand-oriented MVNO, it only profits from its own customers. In Fig. 1, we assume that $k_2 = 0.1$ and plot the improvement in SP1's profit over the benchmark case against B. Observe that secondary access with a flat-rate payment gives the most profit for all B. When the band resource is limited, SP1 should compete with SP2 in stage III to take advantage of the secondary resource sharing. When the band resource is not scarce, SP1 should commit not to enter. In this case, providing primary access with either revenue sharing scheme generates the same profit as secondary access with a flat-rate payment for SP1 because of the commitment. This is consistent with Lemma 1 and Corollary 1. Also, note that offering primary access with a usage-based payment gives the same result when the band resource is not scarce. For all the cooperation contracts SP1 might offer, SP1 extracts nearly all of SP2's profits.

B. Access MVNOs

An access MVNO's profit is only affected by the number of wireless users, i.e., $k_1 = k_2$. For this case, in Fig. 2, we plot SP1's improvement in profit over the benchmark versus B when $k_1 = k_2 = 0.1$. From the figure, we observe that SP1's profit is always maximized under secondary access without commitment. When the band resource is limited, SP1 should charge a flat fee to extract SP2's added value as much as possible. However, when the band resource is not scarce, SP1 should choose a usage-based payment scheme. The intuition is that when the band resource is abundant, competition could cause significant revenue loss. A usage-based payment takes advantage of revenue sharing and reduces the loss caused by competition, because SP1 also profits from SP2's customers. In this case, SP2 has an increased profit compared to the benchmark case. Note that SP1 will choose a usage-based payment to ease the competition only if k_1 and k_2 are relatively small. Since SP2 has an added value per customer, letting SP2 enter the market causes a drop in the service price, which leads to more customers being served. If k_1 and k_2 are large, the gain from these new customers is large. In this case, SP1 will prefer a flat-rate payment, since this enables it to extract almost all of SP2's added value. To avoid the revenue loss caused by competition, SP1 would rather commit not to enter in stage III. Using a usage-based payment can also reduce competition, however, this shares some of the revenue from added value to SP2, which reduces SP1's profit. An example with $k_1 = k_2 = 0.4$ is shown in Fig. 3.

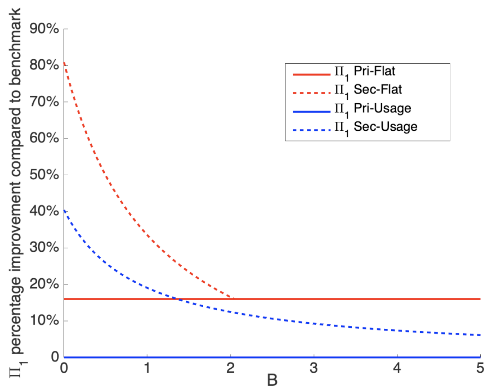

Fig. 3: SP1's profit improvement when facing the entry of an access MVNO with $k_1 = 0.4$ and $k_2 = 0.4$.

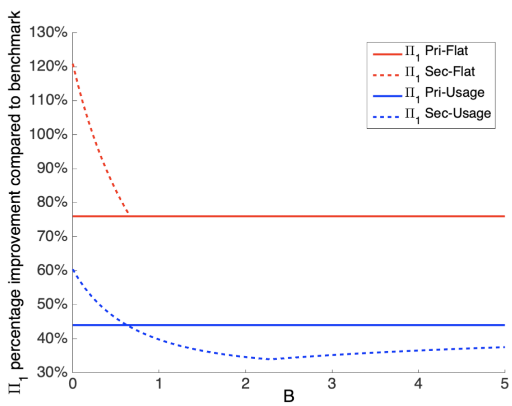

Fig. 4: SP1's profit improvement when facing the entry of a hybrid MVNO with $k_1 = 0.4$ and $k_2 = 0.6$.

C. Hybrid MVNOs

If SP2 is a hybrid MVNO, its original revenue depends on the wireless access. Becoming an MVNO will bring it even more profit per customer it serves. For example, some big E-commerce companies or companies selling mobile devices may profit from better bundling and advertising when serving as MVNOs. The values of k_1 and k_2 depend on how much these companies rely on wireless services and how much more they can profit per customer as MVNOs. Here, we give an example with $k_1 = 0.4$ and $k_2 = 0.6$ in Fig. 4. As shown in Fig. 4, SP1 should again choose to offer secondary access with a flat-rate payment. Note that this is the same conclusion as in the scenario in Fig. 1.

VIII. Conclusions

We considered a model for sharing spectrum and infrastructure between an MNO and an MVNO where the MVNO has additional added value from wireless customers. Two different approaches for sharing resources and two different pricing approaches were considered. We found that if the MNO offers primary access, it should commit not to compete with the MVNO when they cooperate. With an option of commitment, the secondary sharing scheme will always be preferred by an MNO. The pricing scheme that SP1 should choose depends on the available band resource and the MVNO's added value.

There are several ways this work could be extended, including considering multiple MVNOs and MNOs, studying models where the MVNO's added value is private information and considering different pools of customers for different SPs.

References

[1] "Size of the global MVNO market from 2012 to 2022," https://www.statista.com/statistics/671623/global-mvno-market-size/, February 2017.

[2] "Mobile virtual network operator (MVNO) market (2018-2023)," *Mordor Intelligence*, Feb, 2018.

[3] Engo, K., E. Befring, and K. Tilley, "An entrant strategy for an MVNO based on communities," *Telektronikk*, 2001, 39-45.

[4] J. Lehikoinen, P. Pont, Y. Sent, "Virtually mobile: What drives MVNO success," *Telecom, Media & High Tech Extranet*

[5] Do Van Thanh, "The mobile Virtual Network Operator concept: Truth and Myths," *Telektronikk*, 2001, 97(4): 3-6.

[6] Mobile Virtual Network Operator (MVNO) Market Report, http://www.grandviewresearch.com/industry-analysis/mobile-virtual-network-operator-mvno-market.

[7] C. Tinine, " MVNOs 2.0: New pricing models in a data-driven environment," *Analysis Mason*, 2015

[8] T. Nguyen, H. Zhou, R. Berry, M. Honig, and R. Vohra, "The cost of free spectrum," *Operations Research*, vol. 64, no. 6, pp. 1217-1229, 2017.

[9] T. Nguyen, et al., "The impact of additional unlicensed spectrum on wireless services competition," *IEEE DySPAN*, 2011.

[10] C. Liu and R. Berry, "Competition with shared spectrum," *IEEE DySPAN*, 2014.

[11] Y. Zhu, R. A. Berry, "Contract as entry barriers in unlicensed spectrum," *IEEE INFOCOM SDP*, May. 2017.

[12] M. Lotfi, and S. Sarkar, "The economics of competition and cooperation between MNOs and MVNOs," *Information Sciences and Systems (CISS)*, 2017.

[13] L. Zheng, et al., "An economic analysis of wireless network infrastructure sharing," *WiOpt*, 2017.

[14] L. Zheng, et al., "Economic viability of a virtual ISP," *INFOCOM*, 2017.

[15] Y. Zhu, H. Yu, R. A. Berry, C.Liu, "Cross-Network prioritized sharing: an added value MVNO's perspective," *IEEE INFOCOM*, 2019.

[16] J. Elias, F. Martignon, L. Chen, E. Altman, "Joint operator pricing and network selection game in cognitive radio networks: equilibrium, system dynamics and price of anarchy," *IEEE Transactions on Vehicular Technology*, vol. 62, issue 9, pages 1-14, 2013.

[17] R. Saruthirathanaworakun and J. Peha, "Dynamic primary-secondary spectrum sharing with cellular systems," *CROWNCOM*, 2010.

[18] F. Zhang, and W. Zhang, "Competition between wireless service providers: Pricing, equilibrium and efficiency," *WiOpt*, 2013.

[19] P. Maill, B. Tuffin, and J. Vigne, "Competition between wireless service providers sharing a radio resource," *International Conference on Research in Networking*, 2012.

[20] L. Cadre, Hlne, M. Bouhtou, and B. Tuffin, "A pricing model for a mobile network operator sharing limited resource with a mobile virtual network operator," *International Workshop on Internet Charging and QoS Technologies*, 2009.

[21] L. Duan, L. Gao, and J. Huang, "Contract-based cooperative spectrum sharing," *IEEE DySPAN*, 2011.

[22] R. Berry, M. Honig, V. Subramanian, T. Nguyen, and R. Vohra, "Market structures for wireless services with shared spectrum," *Allerton Conference*, 2013.

[23] L. Gao, X. Wang, Y. Xu and Q. Zhang, "Spectrum trading in cognitive radio networks: A contract-theoretic modeling approach," in *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 843-855, April 2011.

[24] J. G. Wardrop, "Some theoretical aspects of road traffic research," *ICE Proceedings: Engineering Divisions*, vol. 1, no. 4, 1952.

[25] Mt J Smith, "The existence, uniqueness and stability of traffic equilibria," *Transportation Research Part B: Methodological* (1979): 295-304.

[26] https://www.dropbox.com/s/i39bc4mvgzemv41/appendix.pdf?dl=0