

Stability properties of delay tolerant networks with buffered relay nodes

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Abstract—We study stability properties of single-source single-destination delay tolerant networks with random packet arrivals and buffered relay nodes, using source spray and wait routing. We derive the stability threshold, the supremum of arrival rates for which the source queue is stable, as a function of the buffer space at the relays. In particular, we show that the stability threshold only doubles as the relays’ buffer size increases from one to infinity for a network without packet delivery feedback. For the system without packet delivery feedback, we propose lower bounds for the average queueing delay and average delivery delay for packets and compare with simulations. We also obtain the stability threshold numerically for a network with instantaneous packet delivery feedback.

I. INTRODUCTION

Delay tolerant networks (DTNs) are comprised of nodes connected by links which have intermittent connectivity. DTN routing and scheduling protocols such as Epidemic [1], Spray and Wait [2], Spray and Focus [3], Maxprop [4], Rapid [5], and Prophet [6] use the store and copy paradigm for reliable packet delivery. Such DTN routing protocols copy packets from the source to multiple *relay nodes* so that there is diversity in the packets’ path to the destination. This leads to an improved probability of packet delivery within a target time and average delivery delay. However, copying of packets from a source to a relay node requires the relay to have enough storage or buffer space to store the packet copies until it can be delivered or copied again. An important engineering problem is provisioning the storage or buffer space at the nodes, which can be used to tradeoff various quality of service metrics of the network.

In this paper, we consider the effect of storage or buffer size of the relays on performance metrics such as queue stability region, average queueing delay, and average delivery delay for a DTN with a single source and destination of packets. The DTN uses the source Spray and Wait (SW) routing protocol [2] to copy packets to mobile relay nodes which have finite buffer capacity to hold the packets. We obtain conditions for stability of the source queue, an analytical lower bound for the average queueing delay experienced by packets at the source queue, and a lower bound for the average delivery delay of packets.

We assume that the source generates packets at the epochs of a Poisson point process. In contrast to prior work on DTNs which have assumed *lone packet* models (only a single packet exists within the network at a time), we consider queueing

stability and queueing delay for DTNs with random arrivals as a function of the buffer size at relay nodes. We consider Source Spray and Wait (SW) protocol [1] which copies a single packet K times to relays. The SW protocol consists of a spray phase and a wait phase. In the spray phase the source copies the packet to the first K relays that it meets. But a relay that is carrying the packet cannot copy it to another relay. After the source copies the packet to K relays, the protocol enters the wait phase which continues until the packet reaches the destination. In our prior work [7] we had studied stability and delay performance for a similar DTN model but with extremely resource limited relay nodes which have a unit buffer capacity. In this paper, we consider stability and delay performance when all relay nodes have a buffer size of B . We obtain that the stability region of the DTN at most doubles when the buffer size B increases to infinity from one.

Lee et al. [8] study the asymptotic scaling of stability region and delay in a DTN as the number of nodes N increases. We note that this paper provides non-asymptotic results compared to the scaling laws which were obtained in [8]. Also, [8] assumes that all relays share a common buffer, while we assume per-relay buffers which are more realistic. Herdtner et al. [9] consider the effect of finite buffers on the sum throughput of a mobile adhoc network and shows that the sum throughput degrades substantially compared to systems with infinite buffers. Mahendran et al. [10] propose an asymptotic large-deviations based scheme to provision relay buffer sizes in a DTN so that performance is close to a DTN with infinite relay buffer size. Using our stability results we also show that a buffer size of 19 is enough to achieve 95% of the maximum achievable throughput.

We note that prior work on DTNs with packet arrivals (e.g., see [2], [4], [5], [6]) have used simulation tools (such as ONE [11]) to understand the effect of protocol parameters on quality of service for packets. Ramaiyan et al. [12] considered a vehicular DTN with $K = 1$, in which packet need not be copied to the first relay. They study the problem of discovering the optimal relay on the go. Groenevelt et al. [13] modeled epidemic relaying and two-hop relaying using Markov chains. They derived the average delay and the number of copies generated until the time of delivery. Zhang et al. [14] developed a unified framework based on ordinary differential equations (ODEs) to study epidemic routing and its variants. Altman et al. [15] addressed the optimal relaying problem for a class of *monotone relay strategies* which includes epidemic relaying and two-hop relaying. Singh et al. [16] considered

the tradeoff between delivery delay and number of copies in a DTN where a single packet has to be transmitted to multiple destinations. However, in this paper we analyze the case where multiple packets need to be transmitted to a single destination.

Outline and contributions: We present the DTN model in Section II. We also state the definitions of the stability region and the delay metrics in this section. We obtain the stability region (threshold in the case of single source-destination) for the DTN without packet delivery feedback in Section III. The characterization of the stability threshold as a function of DTN parameters and especially the buffer size is our primary contribution in this paper. We also discuss the similarities and differences between our results and [8] in this section. We also obtain the stability region for a DTN with unit buffer size but with feedback of packet delivery information in Section IV; this is a secondary contribution. We then consider the delay performance of the DTN without packet delivery feedback in Section V. We obtain an analytical lower bound on the average queueing delay of packets as well as a lower bound on the average delivery delay which form secondary contributions. Simulations of queueing delay and delivery delay in the DTN and their comparisons with our obtained bounds are also presented in Section V.

II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider a continuous time mobile ad-hoc network model similar to that in [16] and [7]. Our presentation of the model is similar to that in our prior work [7]. The time index is denoted as t . We assume that the network consists of $N+2$ mobile nodes, comprising of a source node, a destination node, and N relay nodes. The source is assumed to have fixed length packets arriving to it at the random points of a Poisson point process of rate λ . The packets have to be transferred to the destination and are assumed to be queued in an infinite length buffer at the source. The source transfers the data packets to the destination through the mobile relay nodes using two-hop relaying. We assume that the source does not meet the destination directly*. We say that a relay has met the source/destination when the relay node comes within the communication range of the source/destination.

For improving the delivery delay, we assume that the source copies each packet to at most K separate relays. We assume that a relay has a finite buffer which can hold at most $B \in \mathbb{Z}_+$ packets. When a source meets a relay, the source finds out whether the relay has a free buffer space or not. If the relay has a free buffer space, then the source finds out the list of packet copies that the relay is not currently carrying but which are there in the source queue and copies one copy of such a packet to the relay. A relay with a free buffer space is said to be susceptible, while a relay which has a packet copy is said to be infected by that packet.

A packet is delivered when any of relays infected with that packet first meets the destination and copies the packet to

*This assumption is valid if we have large number of relay nodes, since the ratio of number of source-relay meetings to the number of source-destination meetings is $O(\frac{1}{N})$ which is very small.

the destination. We consider two cases: (a) *no-feedback* - we assume that there is no feedback mechanism employed by the destination by which the source and the infected relays would get information about packet delivery, and (b) *instant-feedback* - we assume that there is instantaneous feedback about packet delivery from the destination to the source and relay. In the no-feedback case, we assume that a packet is removed from the source buffer just after K copies have been made and the relay buffer space is freed after the relay copies that packet's copy to the destination. In the instant-feedback case, we assume that a packet is removed from the source and relay buffers when it is first delivered to the destination.

We assume that the intermeeting time processes of relay nodes with the source and destination nodes are independent, with each intermeeting time distributed as an exponential random variable with rate β (an assumption motivated by [13]). For the r^{th} relay the buffer size at time t is denoted as $B^r(t)$.

We note that a state space description of the evolution of the DTN system is complex; the state description contains the identities of packets in the source queue as well as the number of packet copies that are yet to be copied to the relays in addition to the identities of the packets that are carried by all the relays. This state evolves according to the spray and wait protocol described above, with the state changing at points of packet arrivals, source-relay meetings, and relay-destination meetings.

Our objective is to obtain analytical insights into the stability of the source queue as well as the different types of delays experienced by the packets arriving to the source. We denote the number of packets queued up at the source buffer at time t as $Q(t)$. The average queue length is then defined as

$$\bar{q}_q = \limsup_{\tau \rightarrow \infty} \frac{1}{\tau} \int_{t=0}^{\tau} \mathbb{E}Q(t) dt,$$

where the expectation is with respect to the randomness of arrivals, intermeeting times, and choice of packets when copying. We say that the source queue is stable if $\bar{q}_q < \infty$. We define the stability threshold λ^* as $\lambda^* = \sup \{ \lambda : \bar{q}_q < \infty \}$. The stability region is the set of arrival rates $[0, \lambda^*)$.

We note that the i^{th} packet experiences a waiting delay from its time of arrival till the first copy is made to a susceptible relay. In the no-feedback case, we note that the i^{th} packet stays in the source queue until K copies have been made. We define the queueing delay $D_{q,i}$ experienced by the i^{th} packet as the total waiting time at the source node until the packet has been copied to K susceptible relays. In the instant-feedback case, the queueing delay is the minimum of the time from packet arrival to either the packet delivery or the time till K copies have been made. We define the delivery delay $D_{d,i}$ as the time from when the first copy is made to the earliest time when any relay infected with a copy of this packet meets the destination. The performance metrics that we are interested in are: (a) the average queueing delay \bar{d}_q , and (b) the average delivery delay \bar{d}_d which are defined as the packet averages of $D_{q,i}$ and $D_{d,i}$ respectively over all packets.

In the following sections, we obtain the stability threshold λ^* for the no-feedback and instant-feedback cases. Then, for the no-feedback case, we obtain an analytical lower bound for \bar{d}_q and a lower bound for \bar{d}_d .

III. STABILITY THRESHOLD FOR NO-FEEDBACK

In this section we characterize λ^* for the no-feedback system. We first obtain an upper bound on λ^* by considering a *saturated queue* system, i.e., a system with a source queue which always has packets. We note that the average number of packet copies made from the source queue or average number of packet copies delivered to the destination under the saturated queue assumption is an upper bound to $K\lambda^*$, since this is the maximum rate of service or maximum throughput of packet copies. We note that under the saturated queue assumption each relay's buffer length process $B^r(t)$ evolves independently according to a CTMC with transition rates as in Figure 1. The embedded Markov chain (EMC) corresponding to the above CTMC has transition probabilities which are also shown in Figure 1. We note that the EMC is periodic. Suppose

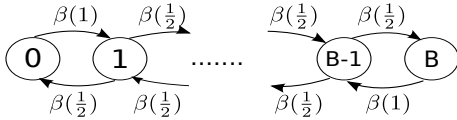


Fig. 1: The transition diagram for the CTMC and its EMC for the $B^r(t)$ process under saturated queue assumption for no-feedback. The transition rates for the CTMC are shown along with the transition probabilities (in parenthesis) for the EMC.

$\hat{\pi}(0), \hat{\pi}(1), \dots, \hat{\pi}(B)$ denote the stationary distribution of the EMC. Then it can be shown that

$$\hat{\pi}(0) = \hat{\pi}(B) = \frac{1}{2B}, \text{ and } \hat{\pi}(b) = \frac{1}{B}, \forall b \in \{1, \dots, B-1\}.$$

For the EMC (or a $B^r(t)$), we note that whenever a down-transition occurs there is a packet copy delivery from relay r to the destination (or whenever an up-transition occurs there is a packet copy from the source to relay r). Assuming that each down-transition leads to a unit reward, the average number of down-transitions per time or the average number of packet copies per time is given by the time average reward associated with the EMC. The time average reward associated with the EMC can be obtained using Markov renewal reward theorem [17, Section D.3.3] as

$$\frac{\hat{\pi}(B) + \sum_{b=1}^{B-1} \frac{\hat{\pi}(b)}{2}}{\hat{\pi}(0) + \frac{\hat{\pi}(B)}{\beta} + \sum_{b=1}^{B-1} \frac{\hat{\pi}(b)}{2\beta}} = \frac{B}{B+1}\beta.$$

We note that the factor $\frac{1}{2}$ multiplies $\hat{\pi}(b)$ in the numerator since only one out of the two transitions contributes to the reward. We note that since there are N relays whose $B^r(t)$ processes evolve independently of each other, an upper bound on the packet copy throughput is given by $\frac{B}{B+1}\beta N$. Since this is an upper bound on the saturation throughput for packet copies, we have that $\lambda^* \leq \frac{B}{B+1} \frac{\beta N}{K}$. We now show that this upper bound is achievable arbitrarily closely by showing that

the average queue length $\bar{q}_q < \infty$ for $\lambda < \frac{B}{B+1} \frac{\beta N}{K}$. We have the following result

Theorem III.1. *The stability region for the no-feedback system is $\left\{ \lambda : \lambda < \lambda^* = \frac{B}{B+1} \frac{\beta N}{K} \right\}$, where λ^* is the stability threshold.*

The proof is presented in the extended version of this paper [18]. We note that for $B = 1$ we recover the stability region result obtained in our previous work [7]. For $B = 1$ we have that the stability threshold is $\frac{\beta N}{2K}$. As $B \rightarrow \infty$ we have that the stability threshold is $\frac{\beta N}{K}$ which is twice what we have for $B = 1$; thus the stability threshold only doubles when the buffer size increases from one to infinity. In fact, in order to achieve 95% of the maximum value of stability threshold, a buffer size of only 19 is required. We note that Lee et al. [8] showed that the stability threshold is $\Theta(N\beta)$ for $B = \infty$ while we obtain that it is exactly $N\beta$ for $K = 1$. Using a pooled-buffer assumption [8] also showed that the stability threshold is $\Theta\left(\frac{B}{B+\rho} N\beta\rho\right)$ (for a constant load factor ρ) while we show that it is exactly $\frac{B}{B+1} N\beta$ for $K = 1$.

IV. STABILITY PROPERTIES FOR INSTANT-FEEDBACK

In this section we discuss the stability properties of the instant-feedback DTN. In this case we have results only for the case where the relays have $B = 1$.

We first obtain an upper bound on the stability threshold by using a saturated queue assumption as in the case of the system without feedback. Under the saturated queue assumption, we have that the average rate of packet copies or the saturation throughput of packet copies can be obtained from a CTMC $N_r(t)$, where $N_r(t)$ is the number of relay nodes which are carrying a packet at time t . The transition diagram of $N_r(t)$ is shown in Figure 2. We note that the left to right transitions at

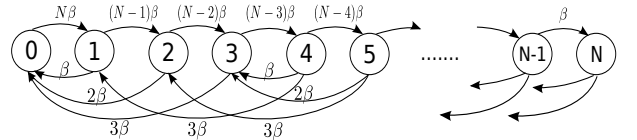


Fig. 2: The transition diagram for the $N_r(t)$ CTMC for $K = 3$ under saturated queue assumption for instant-feedback.

rate $n\beta$ in state n are due to any $N-n$ relays without a packet copy (that is a free buffer space since $B = 1$) meeting the source and obtaining a packet copy (there is always a packet to be copied since the queue is assumed to be saturated). An important feature of the $N_r(t)$ process is that $N_r(t)$ is also the number of packet copies which are carried by the relays at a time t (since $B = 1$). Suppose $N_r(t) = n$ and $n = lK + m$, then we can infer that there are m copies of a packet and K copies of l different packets in transit. Now since there is instantaneous feedback, when a relay meets the destination, the $N_r(t)$ process has a right to left transition of two types. Again with $N_r(t) = n$ we observe that with rate $m\beta$, any relay which is carrying one of the m copies of a packet will meet the destination and then there is a right to left transition of m

steps. Also with rate $lK\beta$ any of the relays that are carrying one of the K copies of one of the l packets will meet the destination and then there is a right to left transition of K steps. This is illustrated in Figure 2 for $K = 3$.

Suppose we associate a reward of 1 unit with every down-transition in the $N_r(t)$ process. Then the time average reward is average saturation throughput of packets out of the saturated source queue and is our upper bound on the stability threshold. We obtain this time average saturation throughput numerically by solving for the stationary distribution of the EMC associated with $N_r(t)$, associating a reward with downtransitions, and using Markov renewal reward theorem as in the no-feedback case. However, a closed form expression is yet to be obtained for the upper bound in this case. We compare the throughput obtained for instant-feedback systems to no-feedback for the same N , β , and K values in Figure 3. Naturally the saturation throughput for systems with instant-feedback is more than for no-feedback. Although the plots suggest that the saturation throughput with instant-feedback is a constant multiple of that with no-feedback, it is not so; there is still a residual variation with K . We note that even

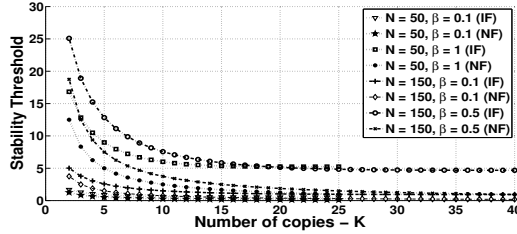


Fig. 3: Comparison of the saturation throughput (stability threshold) for instant-feedback systems (IF) and no-feedback (NF) systems

though the upper bound is not obtained analytically, it can be shown that the upper bound is achievable arbitrarily closely using a multislot Lyapunov drift proof as in the no-feedback case. Intuitively, this is because once the source queue length is large, then the average service rate out of the source is described by $N^r(t)$ as in the no-feedback case. Because of space constraints, here we state without proof that any packet arrival rate which is strictly less than the above saturation throughput can be stabilized.

V. DELAY ANALYSIS FOR NO-FEEDBACK

A. Queueing delay

For systems with no-feedback we obtain a lower bound on the queueing delay of packets at the source queue by considering another system with the same arrival rate λ but with infinite relay buffer size.

We consider the arrival process of packet copies rather than packets to the source queue. From the system model discussed in Section II we have that the source queue has a batch Poisson arrival process of packet copies with rate λ and batch size of K . The packet copy arrival rate is λK . From Theorem III.1 we have that $\lambda^* = \frac{N\beta}{K}$ when $B \rightarrow \infty$. We note that when the buffer size is infinite and the source queue is saturated,

each time a relay meets a source, there is a packet copy to the relay (since a saturated source queue implies that there is a packet copy which is not being currently carried by the relay). Suppose we consider a scenario where the arrival rate $\lambda \approx \frac{N\beta}{K}$. Then we can assume that almost always there is a packet copy whenever a relay meets a source. Then the service time seen by a packet copy is exponentially distributed with rate $N\beta$, and each packet copy's service time is independent of another. Thus the source queueing process can be approximated for λ close to $\frac{N\beta}{K}$ by a $M^{[K]}/M/1$ system; i.e., a $M/M/1$ queue with batch arrivals of size K and exponential service times of rate $N\beta$. The average queue length of packet copies is then [19, Chapter 2, Section 2.10.1]

$$\bar{q}_q = \frac{\rho(K+1)}{2(1-\rho)}, \rho = \frac{\lambda K}{N\beta}.$$

An approximation to the average queue length of packets is then \bar{q}_q/K . The average queueing delay \bar{d}_q of packets is then $\bar{q}_q/K\lambda$ using Little's law. We note that the above approximation for the queueing delay could be applied to study the queueing delay performance of buffered systems where B is large. For extremely constrained relays (i.e. $B = 1$) we have presented delay analysis in [7].

In Figures 4 and 5 we compare the average queueing delay, obtained using simulations, for systems with $B = 1, 5, 10, 100$, and $B = 500$ with the approximation that we have obtained above. We observe that the approximate average queueing delay for $B = \infty$ is a lower bound to the average queue lengths for finite B for the same λ . We also compare

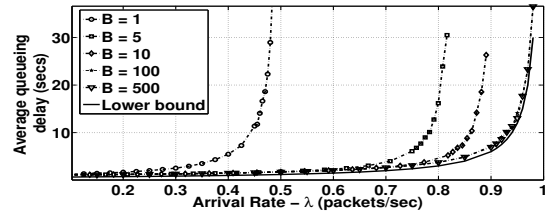


Fig. 4: The average queueing delay experienced by the packets at the source for a DTN with $N = 50$, $K = 5$, and $\beta = 0.1$. The average delay for maximum buffer sizes $B = 1, 5, 10, 100$, and 500 are compared with the analytical average queue length (lower bound) obtained for infinite buffer sizes. The stability threshold is 1 for the infinite buffer case.

the average queueing delay, obtained using simulations, for systems with $B = 100$, $K = 5$, $\beta = 0.1$ and arrival rates of 0.8 and 0.85 with the approximation as a function of the number of relay nodes N in Figure 6. We observe that the analytical lower bound is close to the simulated values. We compare the average queueing delay, obtained using simulations, for systems with $B = 100$, $N = 50$ and 100 , $\beta = 0.1$ and arrival rate of 0.7 with the analytical lower bound as a function of the number of copies K in Figure 7. The deviation of the average queueing delay from the lower bound for larger K can be explained as follows. For larger K for a fixed N , in the actual system the nodes would have to wait to meet a relay

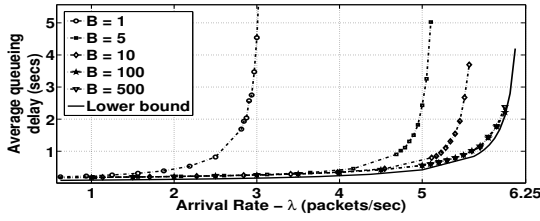


Fig. 5: The average queuing delay experienced by the packets at the source for a DTN with $N = 250$, $K = 20$, and $\beta = 0.5$. The average delay for maximum buffer sizes of $B = 1, 5, 10, 100$, and 500 are compared with the analytical average queue length (lower bound) obtained for infinite buffer sizes. The stability threshold is 6.25 for the infinite buffer case.

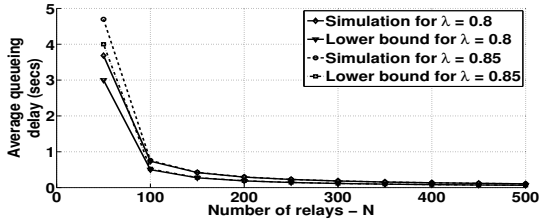


Fig. 6: The average queuing delay experienced by the packets at the source as a function of the number of relay nodes N , for a DTN with $B = 100$, $K = 5$, $\beta = 0.1$, and arrival rates of 0.8 and 0.85 .

which is not carrying a duplicate copy of a packet. However, the lower bound does not take care of this particular feature of the actual system.

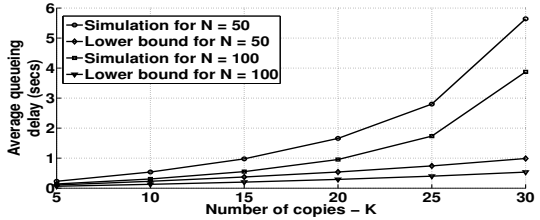


Fig. 7: The average queuing delay experienced by the packets at the source as a function of the number of copies K , for a DTN with $B = 100$, $\lambda = 0.7$, $\beta = 0.5$, and number of nodes N being 50 and 100 .

B. Delivery delay

Another component of the delay is the average delivery delay, which is the expected time taken from the first source to relay packet copy till the first relay to destination packet delivery. A lower bound on the average delivery delay can be obtained using the CTMC shown in Figure 8. The CTMC's state is the number of unique relays to which copies of a single packet has been made. The CTMC models the evolution of this number in a network with infinite buffers and under the assumption that there is only one packet in the network at a time (lone packet assumption). The average delivery delay for that lone packet is the average time taken for the CTMC starting in state 1 to hit the special terminating state ϕ . When

the number of relays is j , the maximum rate at which the packet will be delivered will be $j\beta$ since there are j relays which can meet the destination to deliver a packet copy. The use of this maximum rate is one of the reasons why we obtain a lower bound on the delivery delay in our original system. The actual delivery would happen only if at a meeting this particular packet is chosen amongst all the packets to be copied to the destination; under the lone packet assumption this is true. We also note that when the number of relays is j there are $(N - j)$ other relays which the source can meet to make a new copy of the packet (since all relays are assumed to have infinite buffer space, such a copy can happen). Suppose $t(j)$

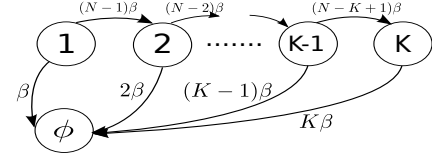


Fig. 8: The transition diagram for the CTMC modelling the number of unique relays to which a packet has been copied. The expected time to hit state ϕ starting from state 1 is the expected delivery delay of the packet under the lone packet assumption.

represents the expected time to hit ϕ starting from j . We have that $t(1)$ is the expected delivery delay. The following set of equations can be obtained by considering the transitions of the CTMC in Figure 8.

$$t(j) = \frac{1}{N\beta} + \frac{N-j}{N}t(j+1),$$

for all $j \in \{1, \dots, K-1\}$. We also note that $t(K) = \frac{1}{K\beta}$. We note that we spend on average $\frac{1}{N\beta}$ time in every state $j \in \{1, \dots, K-1\}$. Then, we directly hit ϕ with probability j/N from j , or with probability $(N-j)/N$ we go to $j+1$ from which the expected time to hit ϕ is $t(j+1)$.

For the system under consideration, the average delivery delay would differ from the lower bound due to two reasons: (a) the rate at which more copies of the packet is made, once the first copy has been made would be smaller than assumed since the relays have finite buffers and would be carrying other packet copies, and (b) the rate at which a copy would be delivered to the destination when a relay meets the destination would be less since a particular packet is only one amongst the packets that are currently carried by the relay, so the delivery of a particular packet happens with probability less than one at a relay-destination meeting. We compare the simulated delivery delay with the lower bound as a function of λ in Figure 9. We observe that as stated before, the delivery delay for the system is close to the lower bound at small arrival rates (since the bound assumes a lone packet). We note that the average delivery delay does not grow unbounded as a function of λ (it is always bounded above by $\frac{1}{\beta}$ since this is the average delay with just one relay). As λ increases (consider $B = 1$) we observe that the delivery delay approaches its maximum value. We compare the simulated delivery delay with the lower bound as a function of N in Figure 10. We

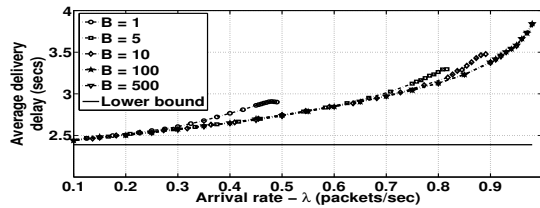


Fig. 9: The average delivery delay experienced by the packets at the source for a DTN with $N = 50$, $K = 5$, and $\beta = 0.1$. The average delay for maximum buffer sizes $B = 1, 5, 10, 100$, and 500 are compared with the lower bound under the lone packet assumption.

find that the simulated values and the lower bound coincide. For large N and $B = 100$ we expect that the assumptions made in our derivation of the bound hold. In Figure 11 we

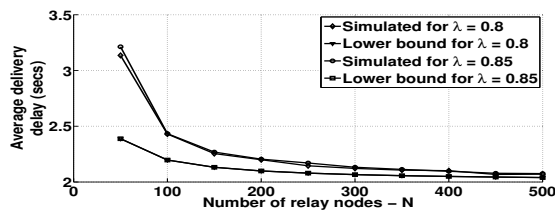


Fig. 10: The average delivery delay experienced by the packets at the source as a function of the number of relay nodes N . We consider a DTN with $B = 100$, $K = 5$, $\beta = 0.1$, and arrival rates of 0.8 and 0.85 .

compare the simulated delivery delay with the lower bound as a function of K for a fixed N . For a fixed N as K increases, the source might not meet relays which are not carrying a duplicate copy of the packet at the transitions rates assumed in the CTMC shown in Figure 8. We observe that as K increases the simulated delivery delay deviates from the lower bound.

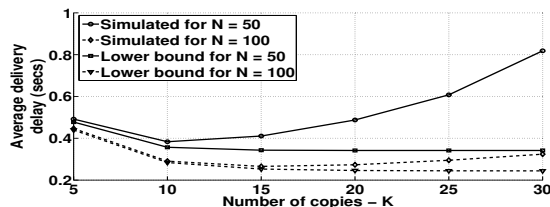


Fig. 11: The average delivery delay experienced by the packets at the source as a function of the number of copies K . We consider a DTN with $B = 100$, $\lambda = 0.7$, $\beta = 0.5$, and number of nodes N being 50 and 100 .

VI. CONCLUSIONS

In this paper, we considered the stability properties of a single source single destination DTN with multiple relay nodes employing two hop relaying. We considered the effect of the buffer size employed at the relay on the stability threshold and obtained that the stability threshold is at most doubled for a DTN with infinite buffer relays as compared to a DTN with unit buffer relays. We employed a multi-slot Lyapunov drift criterion for obtaining the stability threshold. We also studied

the average queueing delay and average delivery delay in systems with buffered relays and found that simple analytical approximations for the average queueing delay can be obtained for the case where the buffer size is large (compared to our previous work [7] where we considered unit buffer size). We also obtained a lower bound on the average delivery delay, which is observed to be close to the actual when the number of nodes N is large.

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