# Optimal distributed scheduling for single-hop wireless networks

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***Abstract*—We consider the problem of optimal distributed scheduling for delay minimization in single-hop wireless networks. We focus on static scheduling policies, where the CSMA channel access rates are determined by the long-run traffic statistics, but not the instantaneous queue states. Such static scheduling is preferable over dynamic scheduling policies like max-weight when the traffic flows are heterogeneous. In this paper, we formulate the problem of optimizing the channel access rates of different links subject to an upper bound on the access rate of each link. This is a hard non-convex optimization. We propose an approximate solution that is asymptotically optimal in the limit as the maximum permissible channel access rate grows to infinity. We also study the role of the intra-queue scheduling policy. Specifically, we consider two policies: first come first served (FCFS) and pre-emptive last come first served (PLCFS). Analogous to the case of an M/G/1 queue, we show that PLCFS is preferable to FCFS for highly variable flows.**

## I. INTRODUCTION

The design of link scheduling policies for wireless networks has received widespread attention in the research community over the past decades. Scheduling in wireless networks is complicated by interference considerations, which allow only a subset of the links to be active at any time. The challenge then is to design distributed medium access protocols that provide high throughput as well as low delay.

An important design goal for wireless scheduling policies is *throughput optimality*. A policy is said to be throughput optimal if the set of arrival rates it can stably support is maximal. Broadly, throughput optimal scheduling policies for wireless networks fall into two distinct categories.

**Dynamic policies:** In this class of policies, the link scheduling is governed by the queue lengths in the network. The starting point for the literature on dynamic throughput optimal scheduling is the classical max-weight policy proposed by Tassiulas and Ephremides [1], [2]. There is now a substantial body of literature on evaluating and generalizing the max-weight policy in a variety of settings; see, for example, [3]–[9]. In the context of this paper, two aspects of the max-weight class of policies are worth noting. Firstly, these policies do not require learning/knowledge of arrival statistics. Secondly, this class of policies exhibit fairness issues in asymmetric traffic settings. Specifically, when the network sees a mix of heavy-tailed (highly bursty) and light-tailed (less bursty) traffic, it can be shown that the max-weight policy can induce heavy-tailed delays on the light-tailed flows [10]–[12]. This is because (relatively frequent) bursts into the heavy-tailed queues can cause contending light-tailed queues to be starved of service for extended periods of time.

**Static policies:** In this class of policies, each flow is associated with a desired service rate, and the scheduling parameters are set such that these service rates are achieved whenever feasible; see, for example, [13]–[16]. These policies are static in the sense that the scheduling does not depend on the state of the queues in the network. Note that static policies do require learning/knowledge of traffic parameters, in order to ensure that all queues are stable. However, it is important to note that static policies address the above mentioned fairness issue in asymmetric traffic settings, since the service process of each queue is insensitive to the instantaneous queue occupancies.

One issue which, to the best of our knowledge, has remained unaddressed in the study of static scheduling policies is *how to set the scheduling parameters in order to minimize delay.* The present paper seeks to address this question. Indeed, most of the prior work on static throughput optimal scheduling assumes an exogenously defined service rate for each flow. However, delay minimization entails *setting* the service rate of each flow, subject to both queue stability constraints as well as network capacity constraints.

Specifically, we consider a single-hop wireless network, where only one link can be active at any time. Links contend for service according to a distributed, asynchronous, CSMA protocol. Our model allows for asymmetry in the arrival process of each link, in terms of the file/job arrival rate, as well as the file/job size distribution. In this setting, we formulate the problem of optimizing the CSMA parameters, namely, the channel access rates, so as to minimize the average response time. Since it is impractical for nodes to implement arbitrarily high access rates, we impose the constraint that the channel access rate of each transmitter is bounded from above. It turns out that the above optimization is non-convex, and therefore hard to solve exactly. Accordingly, we propose an approximate solution that is easy to compute, amenable to distributed evaluation, and asymptotically optimal as the maximum channel access rate grows to infinity. Numerical results show that the proposed solution is near-optimal even for moderate bounds on the channel access rate.

Additionally, we consider the effect of the intra-queue scheduling policy, which determines which job from the queue is served when the corresponding link is active. We consider two intra-queue scheduling policies: first come first served (FCFS) and pre-emptive last come first served (PLCFS).

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Analogous to the case of an M/G/1 queue, we show that PLCFS is preferable to FCFS for highly variable flows. Finally, we consider the optimization of the channel access rates when a subset of links employs FCFS intra-queue scheduling, while the remaining links employ PLCFS scheduling. We provide an easily computable approximate solution for this non-convex optimization that is asymptotically optimal as the maximum channel access rate grows to infinity.

This paper is organised as follows. In Section II, we describe our model and state some preliminary results. In Section III, we consider the problem of optimizing channel access rates. Next, in Section IV, we study the effect of the intra-queue scheduling policy. We present numerical simulations in Section V, and conclude in Section VI.

## II. Model and Preliminaries

In this section, we describe our system model and derive some preliminary results.

**Medium access model:** We consider a wireless network composed of $l$ links, labeled $1, 2, \cdots, l$. We assume that the links interfere with one another, so that only one link may be active at any time. The links get served as per the following asynchronous CSMA protocol, which was first proposed in [13]. The transmitter of Link $i$ maintains an independent exponential clock of rate $R_i$. In other words, the clock-tick instants at the transmitter of Link $i$ form a Poisson process of rate $R_i$. When a clock tick occurs at the transmitter of Link $i$, it commences transmission if the channel is sensed to be idle. Once transmission commences on any link, it continues for a period of time that is (independently) exponentially distributed with rate $\mu$.

Note that under the above model, collisions do not occur (with probability one).[1] Moreover, it is easy to see that the link activations follow a continuous time Markov chain, as depicted in Figure 1.

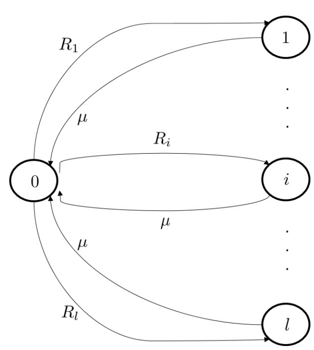



Fig. 1: Markov chain describing link activations

Here, State $i$ corresponds to Link $i$ being active, and State 0 corresponds to a the channel being idle. Define $Z := \sum_{i=1}^{l} R_i + \mu$. Clearly, the stationary distribution of this Markov chain is given by

$$\pi_i = \frac{R_i}{Z} \quad (1 \leq i \leq l),$$
$$\pi_0 = \frac{\mu}{Z}.$$

This implies of course that the service rate of Link $i$ equals $\frac{R_i}{Z}$.

In this paper, we treat the vector of channel access rates $R = (R_i, 1 \leq i \leq l)$ as a control parameter, to be optimized so as to minimize the average delay in the network. Specifically, we optimize $R$ under the constraint $R_i \leq r \; \forall \; i$. Here, $r$ is an upper bound on the channel probe rate of each link, and may be interpreted as a physical constraint of the wireless transmitters.[2]

**Traffic model:** We assume that Link $i$ is equipped with an infinite buffer. Jobs/files arrive for transmission at Link $i$ according to a Poisson process of rate $\lambda_i$. The service times of jobs/files on Link $i$ are i.i.d., with $S_i$ denoting a generic service time requirement on Link $i$. Note that the service time of a job/file is simply the time required to transmit it over the channel. Thus, $\rho_i := \lambda_i \mathbb{E}[S_i]$ is the traffic intensity at Link $i$. Let $\rho = (\rho_i, \; 1 \leq i \leq l)$. Since Link $i$ is active intermittently, we assume that jobs/files may be pre-empted and served over multiple activity periods as needed. Moreover, a single activity period may support multiple jobs. It is then easy to see that the queue of Link $i$ is stable if and only if $\rho_i < \frac{R_i}{Z}$.

We make the following remarks about the model.

1) Our model allows for a different service time distribution for each link. This allows us to capture heterogeneous traffic flows. For example, a subset of links may have bursty traffic flows (characterized by highly variable service time distributions).
2) The service process of each queue is independent of its instantaneous state. In other words, we consider *static* scheduling. In contrast, scheduling policies like *max-weight* [1] and its variants are *dynamic*, in that scheduling decisions are dependent on the states of the different queues in the network.

**Rate region:** We now characterize the rate region, which is the set of traffic rate vectors that can be stably supported by the network. Define $\mathcal{R} = \{R \in \mathbb{R}^l : \; R_i \in [0, r] \; \forall \; i\}$. The rate region for our system is given by

$$\Theta_r = \{\tilde{\rho} \in \mathbb{R}^l_+ : \; \exists \; R \in \mathcal{R} \text{ such that } \tilde{\rho}_i < \frac{R_i}{Z} \; \forall \; i\}.$$

Throughout, we assume that $\rho \in \Theta_r$. Clearly, for $\Theta_r \subseteq \Theta_{r'}$ for $r < r'$. Moreover, $\Theta_r \subset \Theta$, where

$$\Theta = \{\tilde{\rho} \in \mathbb{R}^l_+ : \; \sum_{i=1}^{l} \tilde{\rho}_i < 1\}$$

is the rate region when the channel access rates are unconstrained.

[1] Such *idealized* CSMA modeling is standard in the literature; see, for example, [8], [13], [16].

[2] Such an upper bound is also considered in [13]. Also, it turns out that the optimization of the channel access rates to minimize delay is ill-posed without an upper bound on the access rates.

**Delay characterization:** We conclude this section with a characterization of the average (stationary) response time for jobs on each link under our model. The response time of a job is the interval between its arrival and its departure. Let $T_i^{\text{FCFS}}$ denote the stationary response time corresponding to Link $i$, assuming first-come-first-served (FCFS) scheduling of jobs.[3]

**Lemma 1.** *If* $\rho_i < \frac{R_i}{Z}$,

$$\mathbb{E}[T_i^{FCFS}] = \frac{\frac{1}{\mu}\left(1 - \frac{(Z+\mu)}{Z^2}R_i\right)}{\frac{R_i}{Z} - \rho_i} + \frac{\lambda_i \mathbb{E}[S_i^2]}{\frac{2R_i}{Z}\left(\frac{R_i}{Z} - \rho_i\right)} + \frac{\mathbb{E}[S_i]}{\frac{R_i}{Z}}.$$

The proof of Lemma 1 is given in Appendix A.

## III. Optimizing channel access rates for minimum delay

In this section, we consider the problem of optimizing the channel access rates in order to minimize the average (stationary) job response time. In this, we impose the natural constraint that all channel access rates are bounded from above. However, it turns out that the above optimization problem is non-convex, and therefore hard to solve. We propose an approximate solution, which is proved to be asymptotically optimal as the upper bound $r$ on the channel access rates grows to infinity (see Theorem 1). Moreover, the proposed approximation is easy to compute. Specifically, it is the solution of a convex network utility maximization problem [17]; efficient algorithms (including distributed implementations) are available for this class of optimization problems [17], [18].

We assume throughout this section that all links employ FCFS scheduling.[4] If all links are stable, then the average (stationary) job response time is given by

$$\frac{1}{\sum_{i=1}^{l} \lambda_i} \sum_{i=1}^{l} \lambda_i \mathbb{E}[T_i^{\text{FCFS}}].$$

Note that by Little's law, minimizing the above is equivalent to minimizing the long-run average number of jobs in the system. Formally, the optimization problem we consider is the following.[5]

$$\begin{aligned} \text{min.} \quad & \sum_{i=1}^{l} \frac{\frac{\lambda_i}{\mu}(1 - \frac{(Z+\mu)R_i}{Z^2})}{\frac{R_i}{Z} - \rho_i} + \frac{\lambda_i^2 \mathbb{E}[S_i^2]}{\frac{2R_i}{Z}(\frac{R_i}{Z} - \rho_i)} + \frac{Z\rho_i}{R_i} \\ \text{s.t.} \quad & R_i \leq r \ \forall i \qquad\qquad (F) \\ & R_i \geq 0 \ \forall i \end{aligned}$$

Recall that $Z = \sum_{i=1}^{l} R_i + \mu$. Also, note that the optimization $(F)$ has the implicit constraint that $\rho_i < \frac{R_i}{Z}$ for all $i$, so that all queues are stable.[6] It is important to note that performing the optimization $(F)$ requires that we know/learn the following traffic statistics for each link $i$: The arrival rate $\lambda_i$, as well as the first and second moments of the job/file size $S_i$. Clearly, each link can learn these quantities from observed arrival stream.

It is easy to show that $(F)$ is a non-convex optimization, making an optimal solution computationally intractable. In the remainder of this section, we develop an approximate solution of $(F)$, which is both easily computable, and also asymptotically optimal (as $r \to \infty$).

Let $f(\cdot)$ denote the objective function of $(F)$, and let $f_r^*$ denote the optimal value of $(F)$. The following observation will be useful.

**Lemma 2.** *If* $f(R) < \infty$, *then* $f(\beta R) < f(R)$ *for any* $\beta > 1$.

The proof of this lemma is elementary and is omitted. Now, consider the related optimization

$$\begin{aligned} \text{min.} \quad & \sum_{i=1}^{l} \frac{\frac{\lambda_i}{\mu}(1 - \frac{(Z+\mu)R_i}{Z^2})}{\frac{R_i}{Z} - \rho_i} + \frac{\lambda_i^2 \mathbb{E}[S_i^2]}{\frac{2R_i}{Z}(\frac{R_i}{Z} - \rho_i)} + \frac{Z\rho_i}{R_i} \\ \text{s.t.} \quad & \sum_{i=1}^{l} R_i \leq r \qquad\qquad (F') \\ & R_i \geq 0 \ \forall i \end{aligned}$$

Note that we have replaced the constraint $\|R\|_\infty \leq r$ in $(F)$ by the constraint $\|R\|_1 \leq r$. It turns out that $(F')$ is also non-convex, but can be convexified using the following observation.

**Lemma 3.** *Any solution* $R^*$ *of* $(F')$ *satisfies*

$$\sum_{i=1}^{l} R_i^* = r.$$

*Proof.* The statement is a trivial consequence of Lemma 2. □

Lemma 3 allows us to transform the inequality constraint $\|R\|_1 \leq r$ in $(F')$ to the equality constraint $\|R\|_1 = r$. Now, making the the substitution $R_i = r\alpha_i$, the problem reduces to the following.

$$\begin{aligned} \text{min.} \quad & \sum_{i=1}^{l} \frac{\frac{\lambda_i}{\mu}(1 - \frac{\alpha_i r(r+2\mu)}{(r+\mu)^2})}{\frac{\alpha_i r}{r+\mu} - \rho_i} + \frac{\lambda_i^2 \mathbb{E}[S_i^2]}{2\frac{\alpha_i r}{r+\mu}(\frac{\alpha_i r}{r+\mu} - \rho_i)} + \frac{\rho_i}{\frac{\alpha_i r}{r+\mu}} \\ \text{s.t.} \quad & \sum_{i=1}^{l} \alpha_i = 1 \qquad\qquad (F'') \\ & \alpha_i \geq 0 \ \forall i \end{aligned}$$

It is easy to show that $(F'')$ is a convex optimization problem. Moreover, since the objective function is separable with respect to the components of $\alpha = (\alpha_i, \ 1 \leq i \leq l)$, we note that $(F'')$ may be interpreted as a network utility maximization problem [17], with the $i$th term in the objective denoting the 'cost function' of Link $i$, and the constraint $\sum_{i=1}^{l} \alpha_i = 1$ representing a capacity constraint. As a result, $(F'')$ can be

[3] While the channel access rates define the *inter-queue* scheduling policy, FCFS is the *intra-queue* scheduling policy. Note that the response time distribution depends on both.

[4] We consider the effect of the intra-queue scheduling policies in Section IV.

[5] WLOG, we omit the constant factor of $\frac{1}{\sum_{i=1}^{l} \lambda_i}$ from the objective function.

[6] We interpret the objective function value to be $\infty$ if the queue stability conditions are not satisfied.

solved efficiently, in a distributed manner, using standard techniques [17], [18].

We now develop an approximate solution of $(F)$ in terms of the solution of $(F'')$. Let $\alpha^*(r)$ denote the optimum of $(F'')$. The proposed approximation $\hat{R} = (\hat{R}_i,\ 1 \leq i \leq l)$ is defined as follows.

$$\hat{R}_i := \frac{\alpha_i^*(r)}{\|\alpha^*(r)\|_\infty} r \qquad (1 \leq i \leq l)$$

Note that $\hat{R}$ depends on $r$, although we do not make this dependence explicit. We first show that $\hat{R}$ is a feasible solution of $(F)$.

**Lemma 4.** *$\hat{R}$ is a feasible solution of $(F)$. In particular, the allocation $\hat{R}$ keeps all queues stable.*

*Proof.* Since $0 \leq \frac{\alpha_i^*(r)}{\|\alpha^*\|_\infty} \leq 1$, we have $0 \leq \hat{R}_i \leq r$ for all $i$.

For stability, we need the following inequality to hold:

$$\frac{\hat{R}_i}{\sum_{i=0}^{l} \hat{R}_i + \mu} = \frac{\frac{\alpha_i^*(r)}{\|\alpha^*\|_\infty} r}{\frac{r}{\|\alpha^*\|_\infty} + \mu} > \rho_i$$

We have

$$\frac{\frac{\alpha_i^*(r)}{\|\alpha^*\|_\infty} r}{\frac{r}{\|\alpha^*\|_\infty} + \mu} = \frac{\alpha_i^*(r) r}{r + \|\alpha^*\|_\infty \mu} \geq \frac{\alpha_i^*(r) r}{r + \mu}, \tag{1}$$

where the last inequality holds because $\|\alpha^*\|_\infty \leq 1$. Moreover, as $\alpha^*(r)$ is the solution to the problem $F''$, we have

$$\frac{\alpha_i^*(r) r}{r + \mu} > \rho_i,$$

which proves (1). $\square$

We are now ready to state the main result of this section, which asserts the asymptotic optimality of $\hat{R}$.

**Theorem 1.** *$\hat{R}$ is asymptotically optimal as $r \to \infty$, i.e.,*

$$\lim_{r \to \infty} f(\hat{R}) = \lim_{r \to \infty} f_r^*.$$

*Proof.* Let $f_r'^*$ denote the optimal value of $(F')$. It is easy to see that $f_r'^*$ is strictly decreasing in $r$, and is bounded from below. Let $f'^* := \lim_{r \to \infty} f_r'^*$.

Now, since

$$\|R\|_1 \leq r \Rightarrow \|R\|_\infty \leq r \Rightarrow \|R\|_1 \leq lr,$$

it follows that

$$f_{lr}'^* \leq f_r^* \leq f_r'^*.$$

This implies

$$\lim_{r \to \infty} f_r^* = f'^*. \tag{2}$$

Next, note that from Lemma 3, $r\alpha^*(r)$ is the optimal solution of $(F')$. Moreover, $\hat{R} = \frac{r}{\|\alpha^*(r)\|_\infty} \alpha^*(r)$. It follows from Lemma 2 that

$$f(\hat{R}) \leq f(r\alpha^*(r)) = f_r'^*.$$

Moreover, since $\|\hat{R}\|_1 = \frac{r}{\|\alpha^*(r)\|_\infty}$, we have

$$f(\hat{R}) \geq f_{r/\|\alpha^*(r)\|_\infty}'^*.$$

Combining the above inequalities, we have

$$f_{r/\|\alpha^*(r)\|_\infty}'^* \leq f(\hat{R}) \leq f_r'^*.$$

Taking limits as $r \to \infty$, and noting that $\lim_{r \to \infty} r/\|\alpha^*(r)\|_\infty = \infty$, we conclude that

$$\lim_{r \to \infty} f(\hat{R}) = f'^*. \tag{3}$$

The statement of the theorem follows from (2) and (3). $\square$

To conclude, in this section, we consider the problem of optimizing the channel access rates to minimize the average job/file response time. For this non-convex optimization, we propose an approximate solution in terms of the solution of a convex network utility maximization problem, which can be solved efficiently in a distributed fashion. Also, we show that the proposed allocation of channel access rates is asymptoticaly optimal, in the limit as the maximum channel access rate $r$ grows to infinity. Even through our analytical guarantee for the proposed solution only holds in the limit as $r \uparrow \infty$, our numerical experiments in Section V demonstrate that the proposed solution is nearly optimal even for moderate values of $r$.

## IV. The Effect of the Intra-Queue Scheduling Policy

In the previous section, we considered the optimization of the *inter-queue* scheduling, determined by the channel access rates. In this, we assumed FCFS scheduling within each queue. In the present section, we focus on the role of the *intra-queue* scheduling policy. Specifically, we analyse performance under an alternate intra-queue scheduling policy, namely PLCFS. The motivation for considering PLCFS is the following well known result for the M/G/1 queue: FCFS results in a lower mean response time when the job size distribution exhibits low variability, while PLCFS results in a lower mean response time when the job size distribution is highly variable [19].

We begin by characterizing the mean response time for a link under PLCFS (Lemma 5). Interestingly, the condition for PLCFS to produce a lower mean response time than FCFS in our system is identical to the condition for the M/G/1 queue (Lemma 6). We then consider the problem of optimizing the channel access rates assuming PLCFS scheduling within each queue. As before, this is a non-convex optimization, and we obtain an asymptotically optimal (as $r \to \infty$) approximation for this problem (Theorem 2). Remarkably, this approximation has a closed form. Next, we consider the general setting wherein a subset of queues employ PLCFS scheduling, while the remaining employ FCFS. As in Section III, we provide an asymptotically optimal (as $r \to \infty$) approximation for this non-convex optimization, in terms of the solution of a convex network utility maximization problem (Theorem 3).

Let $T_i^{\text{PLCFS}}$ denote the stationary response time on Link $i$ under PLCFS intra-queue scheduling.

**Lemma 5.** *If* $\rho_i < \frac{R_i}{Z}$,

$$\mathbb{E}\left[T_i^{PLCFS}\right] = \frac{\frac{1}{\mu}\left(1 - \frac{(Z+\mu)}{Z^2}R_i\right) + \mathbb{E}[S_i]}{\frac{R_i}{Z} - \rho_i}.$$

Lemma 5 can be proved using the same line of arguments as Lemma 1; we omit the proof due to space constraints. A key point to note is that unlike in the case of PLCFS, the mean response time under PLCFS does not depend on the second moment of the job size distribution. Next, we compare the mean response time under FCFS and PLCFS. Define, for a non-negative random variable $X$, the squared coefficient of variation (SCV) $C_X^2 := \frac{Var(X)}{\mathbb{E}[X]^2}$. Note that the SCV is a normalized metric of the variability of $X$ [19].

**Lemma 6.** *Assuming* $\rho_i < \frac{R_i}{Z}$,

$$\mathbb{E}\left[T_i^{PLCFS}\right] < \mathbb{E}\left[T_i^{FCFS}\right] \iff C_{S_i}^2 > 1.$$

The proof follows easily from the delay characterizations in Lemmas 1 and 5; we omit the details. Note that the above comparison of the mean response time under PLCFS and FCFS in our system is identical to that in an M/G/1 queue [19]. It follows from Lemma 6 that to minimize the mean response time for the system, one should employ FCFS on links with lightly variable traffic (file size SCV $\leq 1$), and PLCFS on links with more variable traffic (file size SCV $> 1$).

### A. All links use PLCFS

We now consider the problem of optimizing the channel access rates with PLCFS scheduling within each queue. This case is appealing for the following reasons. Firstly, to optimize the link access rates with PLCFS intra-queue scheduling, we need to only learn the mean job size (recall that for FCFS, we need to also learn the second moment). Consequently, the resulting solution is also insensitive to the variability of the different job size distributions. Secondly, our approximation in this case has a closed form.

Formally, the optimization problem under consideration is:

$$\begin{aligned} \text{min.} \quad & \sum_{i=1}^{l} \frac{\frac{\lambda_i}{\mu}(1 - \frac{(Z+\mu)}{Z^2}R_i) + \rho_i}{\frac{R_i}{Z} - \rho_i} \\ \text{s.t.} \quad & R_i \leq r \ \forall i \qquad\qquad (P) \\ & R_i \geq 0 \ \forall i \end{aligned}$$

As before, this can be shown to be a non-convex optimization. Let $p(\cdot)$ denote the objective function of $(P)$, and let $p_r^*$ the optimal value.

We now state the proposed approximation. Define the vector $\alpha^* \in \mathbb{R}_+^l$ as

$$\alpha_i^*(r) := \left(\frac{r+\mu}{r}\right)\rho_i + \left(\frac{r+\mu}{r}\right)\frac{\left((\frac{r}{r+\mu}) - \rho_t\right)}{c(r)}\sqrt{c_i(r)},$$

where

$$\rho_t = \sum_{i=1}^{l} \rho_i,$$
$$c_i(r) = \frac{r}{\lambda_t(r+\mu)}\left(\frac{\lambda_i}{\mu}\left(1 - \frac{(r+2\mu)}{(r+\mu)}\rho_i\right) + \rho_i\right),$$
$$c(r) = \sum_{i=1}^{l} \sqrt{c_i(r)}.$$

The proposed approximation $\hat{R}$ is defined in terms of the vector $\alpha^*$ as follows.

$$\hat{R}_i = \frac{\alpha_i^*(r)}{\|\alpha^*\|_\infty} r \qquad (1 \leq i \leq l).$$

The following theorem asserts the asymptotic optimality of $\hat{R}$.[7]

**Theorem 2.** $\hat{R}$ *is a feasible point of* $(P)$. *Moreover,* $\hat{R}$ *is asymptotically optimal as* $r \to \infty$, *i.e.,*

$$\lim_{r\to\infty} p(\hat{R}) = \lim_{r\to\infty} p_r^*.$$

We omit the proof due to space constraints.

### B. A subset of links use PLCFS

In this section, we consider the problem of optimizing the channel access rates when a subset $\mathcal{F}$ of links employs FCFS scheduling, while the remaining links employ PLCFS. As before, this problem is non-convex, and we provide an approximate solution that is asymptotically optimal (as $r \to \infty$) in terms of the solution of a convex network utility maximization problem. While we allow $\mathcal{F}$ to be an arbitrary subset of $\mathcal{L}$, note that from Lemma 6, it follows that the mean response time for the system is minimized by taking $\mathcal{F}$ to be the set of links with job size distributions having SCV $\leq 1$.

The problem of optimizing the mean response time is formulated as follows.

$$\begin{aligned} \text{min.} \quad & \sum_{i\in\mathcal{F}} \frac{\frac{\lambda_i}{\mu}(1 - \frac{(Z+\mu)R_i}{Z^2})}{\frac{R_i}{Z} - \rho_i} + \frac{\lambda_i^2\mathbb{E}[S_i^2]}{\frac{2R_i}{Z}(\frac{R_i}{Z} - \rho_i)} + \frac{Z\rho_i}{R_i} \\ & + \sum_{i\in\mathcal{F}\setminus\mathcal{L}} \frac{\frac{\lambda_i}{\mu}(1 - \frac{(Z+\mu)}{Z^2}R_i) + \rho_i}{\frac{R_i}{Z} - \rho_i} \\ \text{s.t.} \quad & R_i \leq r \ \forall i \qquad\qquad (G) \\ & R_i \geq 0 \ \forall i \end{aligned}$$

This optimization is easily seen to be non-convex. Let $g(\cdot)$ denote the objective function of $(G)$, and let $g_r^*$ denote its optimal value.

[7] Note the abuse of notation in our reuse of the symbols $\alpha^*$ and $\hat{R}$.

Following the line of argument in Section III, we propose an approximate solution to $(G)$ in terms of the solution of the following optimization.

$$\begin{aligned}
\min.\ & \sum_{i\in\mathcal{F}} \frac{\frac{\lambda_i}{\mu}\big(1-\frac{\alpha_i r(r+2\mu)}{(r+\mu)^2}\big)}{\frac{\alpha_i r}{r+\mu}-\rho_i} + \frac{\lambda_i^2\mathbb{E}[S_i^2]}{2\frac{\alpha_i r}{r+\mu}\big(\frac{\alpha_i r}{r+\mu}-\rho_i\big)} + \frac{\rho_i}{\frac{\alpha_i r}{r+\mu}} \\
& + \sum_{i\in\mathcal{F}\setminus\mathcal{L}} \frac{\frac{\lambda_i}{\mu}\big(1-\frac{(r+2\mu)r}{(r+\mu)^2}\alpha_i\big)+\rho_i}{\frac{\alpha_i r}{r+\mu}-\rho_i} \\
\text{s.t.}\ & \sum_{i=1}^{l}\alpha_i = 1\ \forall i \qquad\qquad (G'') \\
& \alpha_i \geq 0\ \forall i
\end{aligned}$$

Let $\alpha^*$ denote the optimal solution of $(G'')$. As before, our approximation $\hat{R}$ is defined in terms of $\alpha^*$ as follows.[7]

$$\hat{R}_i = \frac{\alpha_i^*(r)}{\|\alpha^*\|_\infty} r \qquad (1 \leq i \leq l).$$

The following result establishes the asymptotic optimality of $\hat{R}$.

**Theorem 3.** *$\hat{R}$ is a feasible point of $(G)$. Moreover, $\hat{R}$ is asymptotically optimal as $r\to\infty$, i.e.,*

$$\lim_{r\to\infty} g(\hat{R}) = \lim_{r\to\infty} g_r^*.$$

The proof of Theorem 3 uses the same line of argument as the proof of Theorem 1. We omit the details due to space constraints.

## V. Numerical experiments

In this section, we evaluate the scheduling policies proposed in Sections III and IV via simulations. For our experiments, we set $l=3$, $\mu=1$, and $\lambda_i=0.1$ for all $i$.

### A. Quality of Approximation

We first quantify the suboptimality of the approximation proposed in Section III for the optimization $(F)$. We take the job size distribution on each link to be exponential with mean 2, and assume FCFS scheduling on each link. Figure (2) shows the average response time under the proposed approximation, as well as a lower bound on average response time under the exact solution, as a function of the the upper bound $r$ on the channel access rates. The lower bound is computed as the optimal value of the optimization $(F'')$ with the constraint $\sum_{i=1}^{3} R_i = 3r$; note that this problem has a feasible region which is a superset of that corresponding to $(F)$. Note that the suboptimality of the proposed approximation shrinks as $r$ increases (as suggested by Theorem 1). Moreover, the suboptimality is negligible even for moderate values of $r$.

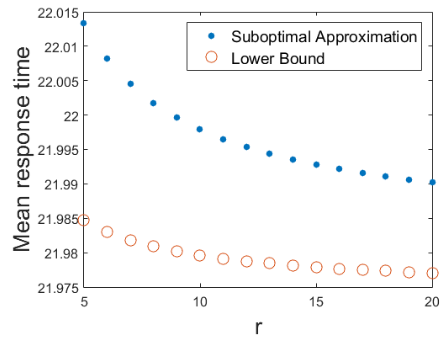



Fig. 2: Suboptimality of Approximation

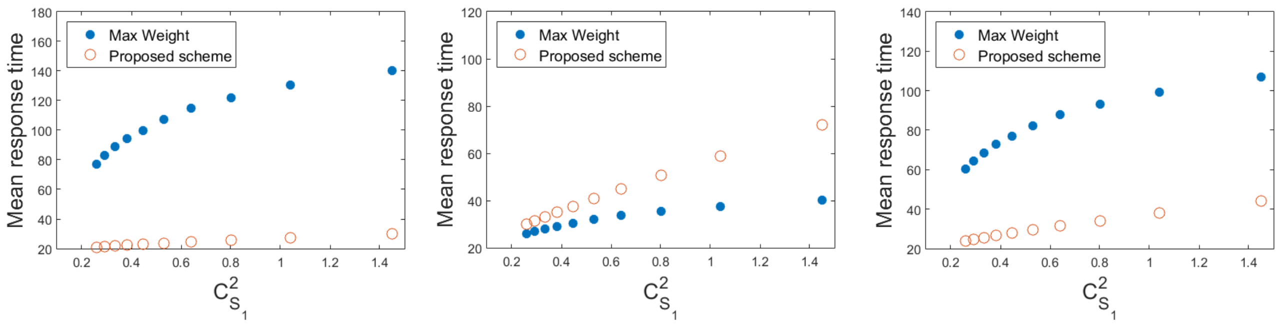
### B. Static vs Dynamic Scheduling

In this section, we compare the delay characteristics of the proposed static scheduling strategy with the (dynamic) max-weight scheduling policy [1]. We assume FCFS intra-queue scheduling, and set $r=10$. For this comparison, we introduce heterogeneity in the arrival processes. Specifically, we assume a Pareto job size distribution on Link 1. The Pareto distribution is heavy-tailed, and is commonly used to model highly variable phenomenon. A Pareto random variable $X$ is characterized by the following tail distribution function, for $b,\alpha>0$.

$$P(X>x) = \begin{cases} \left(\frac{b}{x}\right)^{\alpha} & \text{if } x\geq b \\ 1 & \text{otherwise} \end{cases}$$

We hold the mean of the Pareto job size distribution equal to 8, and study the effect of increasing SCV (i.e., increasing burstiness). The job size distributions corresponding to the remaining links are taken to be exponential with mean 0.2. The mean response time the overall system, for the heavy-tailed queue alone, and for the light-tailed queues alone, are shown in Figure 3. We make the following observations.

1) The mean response time is increasing in the SCV $C_{S_1}^2$ of the (heavy-tailed) job size distribution corresponding to Link 1. This is to be expected, since the mean response time corresponding to Link 1 is an increasing function of $C_{S_1}^2$. Moreover, as $C_{S_1}^2$ increases, the delay of the light-tailed links grows as well, since the static policy increases the service rate of Link 1 in order to minimize the overall mean response time.
2) As expected, the proposed scheme outperforms the max-weight policy on overall mean response time. Moreover, while the heavy-tailed queue sees a lower mean response time under max-weight, the proposed static policy provides a far lower mean response time to the light-tailed traffic. This is because the max-weight policy tends to throttle the light-tailed queues whenever there is a large arrival into the heavy-tailed queue [10]–[12]. On the other hand, the proposed static policy isolates light-tailed traffic from the burstiness of the heavy-tailed flows, since the service process of each queue is independent of the queue occupancies.




(a) Mean response time for the links with light-tailed arrivals

(b) Mean response time for the link with heavy-tailed arrivals

(c) Overall mean response time

Fig. 3: Dynamic Vs Static Scheduling Policy

### C. Intra-queue Scheduling

In this section we demonstrate that it is optimal to choose each intra-queue scheduling policy according to the SCV of the corresponding job size distribution. Using the same setup as in the previous section, we vary the SCV of $S_1$ and compare the mean response time under two schemes: i) A scheme that employs FCFS on all links, ii) A *switching* scheme that employs FCFS on a link if the SCV of the corresponding job size distribution is $\leq 1$, and PLCFS otherwise. The results are shown in Figure 4. As expected, the switched scheme performs better, and moreover leads to mean response time that is insensitive to $C^2_{S_1}$ when $C^2_{S_1} > 1$.

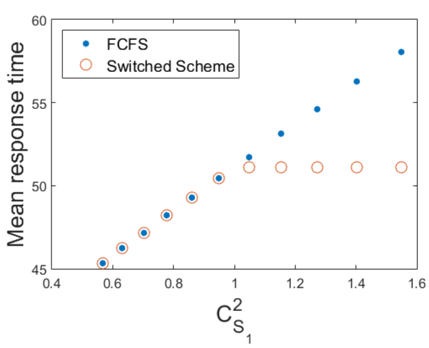



Fig. 4: Delay

## VI. Concluding Remarks

In this paper, we consider the problem of optimizing the CSMA channel access rates in a single-hop wireless network in order to minimize the mean response time (a.k.a. file transmission time). While this problem is itself non-convex, we provide an approximate solution that is asymptotically optimal as the maximum channel access rate grows to infinity. This work motivates generalizations along the following dimensions. Firstly, it would be interesting to consider multi-hop wireless networks with general interference constraints. In this setting, one would want to develop distributed mechanisms for adapting the CSMA access rates to minimize delay. Secondly, note that the present paper assumes an *idealised CSMA* system with no collisions. It would be interesting to generalize this work to a more realistic model that captures collisions (for example, along the lines of [20]).

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## Appendix A
## Proof of Lemma 1

This section is devoted to the proof of Lemma 1.

Under our model, the service process of Link $i$ may be interpreted as that of an M/G/1 queue with an intermittently available server. Specifically, the server availability follows a semi-Markov process, availability periods being exponentially distributed with rate $\mu$, and unavailability periods distributed as the interval between a departure from State $i$ and the next arrival into State $i$ for the Markov chain shown in Figure 1. Let a generic server unavailability period be denoted by $I_i$. The following lemma characterizes the first two moments of the random variable $I_i$.

**Lemma 7.**

$$\mathbb{E}[I_i] = \frac{Z}{\mu R_i} - \mu$$
$$\mathbb{E}[I_i^2] = \frac{2Z}{\mu^2 R_i^2} - \frac{2(Z+\mu)}{\mu^2 R_i}$$

The proof of this lemma is elementary and is omitted.

To obtain the mean (stationary) response time for Link $i$, we note that Link $i$ may be treated as the low priority class in the following (fictitious) two-class priority queueing model analysed in [21]: Jobs of Class $i$ ($i \in \{1,2\}$) arrive according to a Poisson process of rate $\gamma_i$. Each job of Class $i$ has an independent service requirement distributed as $C_i$. Class 1 jobs have pre-emptive priority over Class 2 jobs. It is easy to see that arrivals into Link $i$ in our system can be thought of as the Class 2 arrivals in the above (fictitious) priority queueing model, with $\gamma_2 = \lambda_i$, $C_2 \overset{d}{=} S_i$, and $\gamma_1 = \mu$. Moreover, Class 1 busy periods are distributed as $I_i$. Given this mapping, it is known (see Page 410 in [21]) that

$$\mathbb{E}[T_i^{FCFS}] = \frac{\mathbb{E}[C_2]}{1-\gamma_1\mathbb{E}[C_1]} + \frac{\lambda\mathbb{E}[C^2]}{2(1-\gamma_1\mathbb{E}[C_1])(1-(\gamma_1\mathbb{E}[C_1]+\gamma_2\mathbb{E}[C_2]))}, \tag{4}$$

where $C \overset{d}{=} \frac{\gamma_1}{\gamma_1+\gamma_2}C_1 + \frac{\gamma_1}{\gamma_1+\gamma_2}C_2$.

Thus, to obtain an expression for $\mathbb{E}\left[T_i^{FCFS}\right]$, it remains to compute the first two moments of $C_1$. We do this by equating the first two moments of $I_i$ to the corresponding moments of a Class 1 busy period $B_1$ (which is simply an M/G/1 busy period). Thus,

$$\mathbb{E}[I_i] = \mathbb{E}[B_1] = \frac{\mathbb{E}[C_1]}{1-\gamma_1\mathbb{E}[C_1]},$$

which implies that

$$\mathbb{E}[C_1] = \frac{1}{\mu}\left(1 - \frac{R_i}{Z}\right).$$

Similarly,

$$\mathbb{E}[I_i^2] = \mathbb{E}[B_1^2] = \frac{\mathbb{E}[C_1^2]}{(1-(\gamma_1\mathbb{E}[C_1]+\gamma_2\mathbb{E}[C_2]))^3},$$

which yields

$$\mathbb{E}[C_1^2] = \frac{2}{\mu^2}\left(\frac{R_i}{Z} - \frac{(Z+\mu)R_i^2}{Z^3}\right).$$

Substituting the above expressions for $\mathbb{E}\left[C_1\right]$ and $\mathbb{E}\left[C_1^2\right]$ into (4) gives us the desired result.