The 2017 International Workshop on Resource Allocation, Cooperation and Competition in Wireless Networks (RAWNET)

Making the Case for Dynamic Wireless Infrastructure Sharing: a Techno-Economic Game

Alessandro Lieto*†, Ilaria Malanchini†, Vinay Suryaprakash†, Antonio Capone*
*Dipartimento di Elettronica, Informazione e Bioingegneria Politecnico di Milano, Italy
Email: alessandro.lieto@mail.polimi.it, antonio.capone@polimi.it
†Nokia Bell Labs, Stuttgart, Germany
Email: {ilaria.malanchini, vinay.suryaprakash}@nokia-bell-labs.com

Abstract—**Active sharing of wireless infrastructure can be an effective approach to reduce costs and improve network profitability. However, schemes proposed so far neither guarantee network operators the autonomy to compete and differentiate themselves in various market segments, nor give infrastructure providers the economic resources to keep the network updated in terms of technology and capacity. In this work, we propose a techno-economic model that allows network operators to compete and dynamically select the quality target to deliver to their customers, while simultaneously seeking to maximize their profits. In order to understand the willingness of network operators to participate in such a scenario, we develop a non-cooperative game wherein the Nash Equilibria show the propensity of operators to meet the customers' requirements. This work also points out the importance of retaining independent regulatory bodies, within the new business ecosystem, charged with proposing pricing policies capable of incentivizing investments towards infrastructure upgrades.**

I. INTRODUCTION

The emergence of new technologies and their applications have caused a huge growth in mobile data traffic [1]. Mobile Network Operators (MNOs) are faced with tackling this growth, resulting in an increase in their operating expenditure (OpEx) and capital expenditure (CapEx). Data revenues, however, are not keeping pace with the traffic increase [2] and MNOs, therefore, need to redefine their business modus operandi: inevitably, cost reduction remains the main driver for profitability. *Infrastructure sharing*, according to the Organization for Economic Co-operation and Development (OECD), promises to provide an opportunity to cope with the more stringent requirements of future networks (5G and beyond) while reducing the total expenditure [3]. Following which, many studies analyze the validity and the advantages of several sharing options, focusing either on the economic benefits [4], [5], [6], or on the strategic modeling of infrastructure [7], [8]. One of the main concerns while considering sharing options is coping with the tradeoff between the cost savings and the level of operational autonomy of MNOs [9].

Active sharing, which entails sharing radio access network resources including the spectrum, is the most comprehensive of the sharing methods and allows a significant reduction in costs when compared to passive sharing – as reported by the OECD in [3, Pg. 65]. However, active sharing implies some kind of control on how resources are shared. A popular approach for controlling shared resources is the use of Service Level Agreements (SLAs). Today's SLAs, however, are rather long term and do not help the MNOs cope with short term fluctuations in the traffic their networks carry. Thereby, resulting in scenarios where the MNOs either possess a surfeit of resources or a lack thereof. Therefore, active sharing has to be based on a more dynamic and flexible methodology, wherein MNOs can trade the amount of resources needed to meet their customers' demands in real-time. A first step in this direction was taken in [10], where the authors propose a techno-economic model that allows dynamic (short term) pricing and allocation of network resources. Their model is based on a comprehensive sharing scenario in which multiple Mobile Virtual Network Operators (MVNOs) lease or rent the infrastructure (e.g., wireless spectrum) from an Infrastructure Provider (InP). The dynamism their model permits is due to its ability to allow deviations (from an initial long term SLA) depending on the MVNOs individual budgets as well as their traffic load. However, though [10] incorporates fluctuations in traffic and the MVNOs' budgets, it neither explicitly considers the quality of services provided to the users while modeling fluctuations in traffic, nor analyzes the profitability of the MVNOs. These aspects are precisely what this paper addresses: we provide a method to translate the users' willingness to pay and the quality targets of the MVNOs into time-varying resource requirements, which are in turn used by the InP while allocating network resources to the MVNOs. It is important to note that the use of long term SLAs is no longer necessary in the model we propose. The InP assigns the resources to the MVNOs, based on a proportional fairness criterion for incoming real-time MVNO requests, which in turn depend on their respective profitability evaluation.

In summary, this approach allows the MVNOs to focus on meeting the quality assurances they make to their users rather than on the amount of resources they require. Furthermore, by allowing a differentiation based on the quality of service levels, it enables the MVNOs to target different market segments in terms of user characterization. This in turn results in a diversification of their respective business modus operandi and permits a differentiation of their policies; thereby, addressing one of the main concerns while using active sharing.

This work explores the aforementioned aspects with the use of game theory. Despite many works trying to justify the use of a game theoretic approach for infrastructure sharing,

978-3-901882-90-6/17

most of them focus solely on the benefits of sharing. Authors in [11] and [12] investigate the cost reduction when MVNOs cooperate while taking decisions on network deployment. [13] analyzes the technical advantages in terms of throughput and methods to guarantee fairness. However, unlike this paper, neither of these works highlight the necessity for defining a new business ecosystem, in which MVNOs can retain their autonomy; thereby, addressing MVNOs' desire to differentiate their service policies [14].

The paper is organized as follows: in Section II, we describe the scenario considered and the main features involved. In Section III, we present the approach and the algorithm for determining a solution for the game. Then, in Section IV, the achieved results are shown and finally, our remarks conclude the paper in Section V.

II. SYSTEM MODEL

We consider a multi-agent scenario, where a single infrastructure provider interacts with competitive MVNOs that have to deliver data traffic to end users. We denote the set of the MVNOs by M, and let K be the set of the active users in the network; then, let $K_m \subset K$ be the set of the users of MVNO m. We consider the downlink in a single base station scenario wherein the decisions taken by the base station's scheduler are not directly affected by those in the neighboring base stations, i.e., there is no coordination among base stations.

Let U_{th} be the intended utility (e.g., in terms of spectral efficiency [bps/Hz]) of a generic user for a specific service requested and assume that all users in the area will require the same type of service[1]. Given this assumption, we can estimate the Market Share (MS) of an MVNO as the percentage of users served in the considered area:

$$\mathrm{MS}_m = \frac{|K_m|}{|K|}. \tag{1}$$

We assume that MVNOs set their respective quality targets, $U_{th,m}$, as a product of the intended utility U_{th} and a *quality of service* parameter Q_m, i.e. $U_{th,m} = U_{th} \cdot Q_m$. This quality parameter determines the fraction of utility that the MVNO m decides to provide to its users w.r.t. the one they require. This allows MVNOs to apply quality diversification in the services delivered to their customers. The utility achieved by a user k at time n is defined as a function of the maximum achievable rate, $r_k[n]$, and the number of assigned resources, $x_k[n]$, i.e., $U_k\left(x_k[n], r_k[n]\right)$. Assuming a partitioning of time into elementary slots of duration 1 ms, the scheduler decides to allocate a portion of the total wireless resource, $x_k[n] \geq 0$, at time slot n to the user k. This assignment will depend on the service requested by the user and on the quality target that MVNOs set for the service. The average utility achieved for all the users of a given MVNO m during a time window N is given by

$$U_{ach,m} = \frac{1}{|K_m||N|} \sum_{n \in N} \sum_{k \in K_m} U_k\left(x_k[n], r_k[n]\right). \tag{2}$$

Then, we model the quality experienced by a user by combining the achieved utility[2], U_k, and the tariff imposed by an MVNO. As in [15], we define the *acceptance probability* as the probability that a user k will accept a price p, given the achieved utility, U_k, as:

$$A\left(p, U_k\right) = 1 - \exp(-C p^{-\epsilon} U_k^{\mu}), \tag{3}$$

where C is a normalization factor, and ϵ and μ are microeconomic parameters. With this, the MVNOs can estimate the expected return on investment as well as the profit.

Finally, we assume the existence of a single InP who is not subject to conventional market pressures. This, in turn, allows setting a fixed price per unit of resource by ignoring market driven fluctuations arising from competition between different infrastructure providers. We denote by C_{op}, the operating expenditure (OpEx), and C_{cap}, the capital expenditure (CapEx).

III. FORMULATION AND ANALYSIS

The decisions taken in the ecosystem described in the previous section are twofold. On one hand, the InP has to decide how to allocate the resources among the MVNOs in order to maximize fairness. On the other hand, the MVNOs have to decide how to select the quality parameter Q_m so that their own profit is maximized. While the former is an optimization problem, the latter must account for competition among the MVNOs. Therefore, in this section, we first introduce the Optimal Resource Allocation Model (ORAM) and then the Competitive Quality Selection Game (CQSG).

A. Optimal Resource Allocation Model

Based on the notations defined in Section II, the generic optimization problem at a base station's scheduler can be described by Equations (4a)-(4d):

$$\max \alpha \tag{4a}$$

$$\text{s.t.} \quad \sum_{k \in K} x_k[n] \leq 1, \quad \forall n \in N, \tag{4b}$$

$$U_{ach,m} \leq U_{th,m}, \quad \forall m \in M, \tag{4c}$$

$$\alpha \leq \frac{U_{ach,m}}{U_{th,m}}, \quad \forall m \in M. \tag{4d}$$

The constraint (4b) ensures that the total number of resources assigned is always less than or equal to the total number of resources available in the network. The constraint (4c) guarantees that the InP does not assign more resources to MVNOs than their requested quality targets. Finally, equation (4d) is the proportional fairness condition: each operator gets at least a proportion of the utility target equal to α – independent of the number of users and the quality factor selected. Since the goal of the InP is to maximize the parameter α in (4a), this guarantees fairness among MVNOs. At the same time, the scheduler of a base station is still free to select the best users in

---

[1] Note that extending the model to include diverse traffic types is also possible, but it is beyond the scope of this paper.

[2] Note that the explicit functional dependence described above has been dropped in the interest of notational brevity.

order to efficiently allocate resources. Therefore, the proposed formulation indirectly maximizes the minimum average utility of each MVNO.

Observing equation (4d), we see that MVNOs with higher utility targets and/or larger number of users will need more resources to achieve their targets. Given a shared scenario, this will also affect the quality target of the other MVNOs. Then, MVNOs need to optimize the decisions while trading off the service quality experienced by their users with the total cost of the infrastructure required.

B. Competitive Quality Selection Game

We model competition among MVNOs as a non-cooperative game, where the space of players is defined by M. The strategy set of each player is defined as the set of possible values of the quality parameter $Q_m \in \{0, 0.1, \ldots, 1\}$. We assume that the space of all possible permutations of strategies of the players is the cartesian product of the single strategies of the MVNOs, and denote it by

$$S = \{(q_1, q_2, \ldots, q_n) : q_1, q_2, \ldots, q_n \in Q_m\},$$

where the n-tuple of strategies is defined for $M = \{1, 2, \ldots, n\}$. For each strategy $s \in S$, the infrastructure provider generates a different allocation denoted by $x_{k,s}[n]$, and a given user will get a strategy dependent utility – denoted by $U_{k,s}\left(x_{k,s}[n], r_k[n]\right)$ – depending on the decision taken by the base station scheduler. The objective of the MVNOs is to maximize a profit function, defined as the difference between the payments made by its users, i.e., reward function $R_{m,s}$, and the total expenditure, i.e., $B_{m,s}$, as

$$f_{m,s}(p) = R_{m,s}(p) - B_{m,s}. \tag{5}$$

1) Total expenditure: The InP fixes a price for a unit of resource. Hence, we can define the cost for resource usage as

$$C_{m,s} = \frac{1}{|N|} \cdot (C_{op} + C_{cap}) \sum_{n \in N} \sum_{k \in K_m} x_{k,s}[n], \tag{6}$$

which represents the total amount spent by the m-th MVNO for the resources used in a cell given the strategy s. If the amount of resources available are insufficient to satisfy the utility of each MVNO (i.e., the achieved utility of the MVNOs will be a fraction of the desired threshold, or $\alpha < 1$), the InP can introduce another component of the cost to incentivize the MVNOs to help expand the capacity of the network. We call this *pressure cost* [10], and it corresponds to the total investment necessary for satisfying the targets of all MVNOs. α being a measure of the utility achieved with the available resources, we can assume that the amount of extra resources needed can be evaluated as $\frac{1-\alpha}{\alpha}$. Hence, the pressure cost for a strategy s can be written as

$$C_{\text{pre},s} = \frac{1-\alpha_s}{\alpha_s} \cdot (C_{op} + C_{cap}) \cdot \lambda, \tag{7}$$

where the same price for a unit of resource (in terms of OpEx and CapEx) is assumed, but is scaled by a factor $\lambda > 1$ representing the return on investment for the InP. From the InP point of view, this is the total amount of money the MVNOs have to collect for investing in additional capacity. It is reasonable to assume that this expansion is carried out in the long-run and the individual cost can, therefore, be split according to the market share of the MVNO.

Hence, we can assume that each operator will have an expenditure $B_{m,s}$ equal to:

$$B_{m,s} = C_{m,s} + C_{\text{pre},s} \cdot \text{MS}_m, \qquad \forall m \in M. \tag{8}$$

It is important to recall that α decreases with an increase in the number of users as well as an increase in the selected Q_m value. In the game, we can also figure out if MVNOs – at the equilibrium – are willing to invest in expanding the infrastructure or not.

2) Reward function: In our framework, we model user behavior by evaluating an acceptance probability function that matches the utility received and the price submitted into a satisfaction function for the users. Actually, for introducing an investment incentive for MVNOs, we need to define a more complex reward function, that allows the operators to estimate the advantages or disadvantages of additional investments. For this purpose, the reward function

$$R_{m,s}(p) = \sum_{k \in K_m} p \cdot A\left(p, U_{k,s}\right) + F_{k,s}(p), \tag{9}$$

contains two terms representing two payment criteria:

i) $p \cdot A\left(p, U_{k,s}\right)$ describing the major component of the price paid by the user according to the actual satisfaction (i.e., achieved utility) and weighted by the acceptance probability, where the "optimal" price p is evaluated as the one maximizing the total profit;
ii) $F_{k,s}(p)$ denoting an extra component, that varies according to the quality target promised by the MVNO.

The term in i) is the reference acceptance probability presented in Section II, so we expound the term in ii) given by

$$F_{k,s}(p) = Q_m \cdot \delta \cdot p \cdot A\left(p + Q_m \delta p, U_{th} Q_m\right). \tag{10}$$

By introducing Eq. (10) in the reward function, we assume that MVNOs can charge the users an extra fee that depends on the quality target they promise to deliver to the users. We consider the extra fee to be equal to the product of a percentage $\delta \in [0, 1]$ of the fixed price p and the quality factor Q_m selected. Then, we estimate the willingness of users to pay this amount $(p + Q_m \delta p)$ according to the promised utility $(U_{th} Q_m)$.

Equation (10) plays a slightly different role based on the two cases detailed below:

1) If the resources available in the network are sufficient to achieve the MVNO's threshold ($\alpha = 1$), this term is a way to incentivize MVNOs to push for higher quality for users (short-term investment);
2) If the InP cannot fulfill the requests of the MVNOs (i.e, $\alpha < 1$), the MVNOs need to estimate a potential return on investment for addressing the capacity expansion cost defined earlier (long-term investment).

We characterize the solution of the game by introducing the Nash Equilibrium (NE) in pure strategies [16].

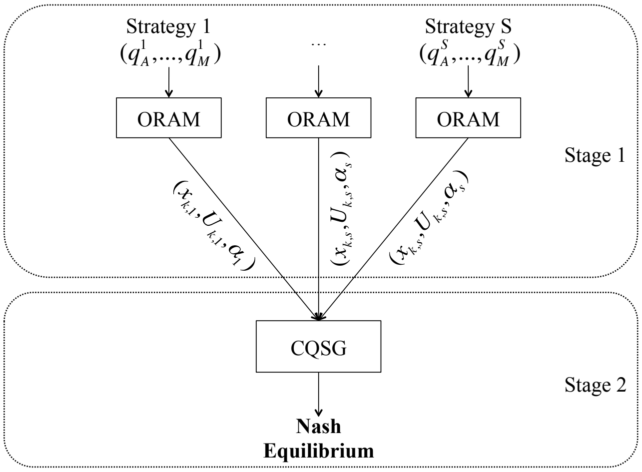


Fig. 1: Two-stage algorithm.

C. Applied algorithm

The formulation described so far points out different goals for the agents involved in this scenario. On one hand, the InP optimizes the resources to be allocated by maximizing the proportional fairness parameter α based on the MVNOs' target utility. On the other hand, the MVNOs try to maximize their profit by analyzing the user satisfaction for any given strategy.

Given this contraposition, we decouple the two objective functions (i.e., those solved by InP and the MVNOs) and solve a two-stage problem as shown in Fig. 1. This procedure allows us to avoid non-linearities in the formulation, i.e., in (4d), and in the evaluation of the *pressure cost*. We solve $|S|$ independent optimization problems (4a)-(4d), where $|S|$ denotes the cardinality of the set of the strategies S. We consider all possible permutations of strategies s as instances of the resource allocation problem and for each of these, we evaluate the cost function described in (8), the reward function in (9), and lastly, the strategy dependent profit function defined in (5). Finally, we derive the NE of the game that, while optimizing the selection of the price p, maximizes the profit for each MVNO m. The existence of an equilibrium has not been proven in this framework; however, the numerical evaluation presented in the next section shows that the algorithm proposed always finds at least one equilibrium point.

IV. SIMULATION RESULTS

In this section, we analyze the MVNOs' strategies at the Nash Equilibrium and compare the results with other (non NE) strategies maximizing the overall profit function.

A. Simulation setup

TABLE I: The applied parameters and their values.

Parameter	Definition	Value
C_{cap}	CapEx Cost	20
C_{op}	OpEx Cost	20
U_{th}	User expected utility	0.5 bps/Hz
δ	Portion of extra price	0.1
λ	InP return on investment	1.5
p	Tariff/month	[0.3, 0.6, …, 30]
$\|M\|$	Number of MVNOs	3
MS_m	Market Share	1/3
N	Duration of simulation	200 ms

We consider the single base station scenario described in Section II and model competition among three MVNOs, i.e., $|M| = 3$. We also assume all the MVNOs have the same number of users and they have the same market share, i.e., $\text{MS}_m = \frac{1}{|M|}$. The users are uniformly distributed throughout the coverage area of the base station and they are simultaneously active in each time slot of the window whose observation of length $N = 200$ time slots, where each time slot is of duration 1 ms. We model the wireless channel by a frequency-flat block fading channel with i.i.d. Rayleigh coefficients – resulting in exponentially distributed random channel gains $|h_k[n]|^2$. Then, we assume that the power received by a generic user is calculated through the Okumura-Hata model, indicating the base station transmitted power (in Watts [W]) by P, and the path-loss exponent by β. Hence, the average Signal-to-Interference-plus-Noise-Ratio (SINR) of user k, SINR_k, can be computed as:

$$\text{SINR}_k = \frac{P \cdot d_k^{-\beta}}{\sigma^2 + I_0}, \tag{11}$$

where d_k is the user's distance from the base station (in meters [m]), σ^2 is the thermal noise, I_0 is the average interference power of the neighboring base stations. Due to the fast fading components, the instantaneous SINR at time slot n is equal to

$$\gamma_k[n] = |h_k[n]|^2 \cdot \text{SINR}_k, \tag{12}$$

which leads to the evaluation of the spectral efficiency of a user k (in bps/Hz) at any time instant n as

$$r_k[n] = \log_2(1 + \gamma_k[n]). \tag{13}$$

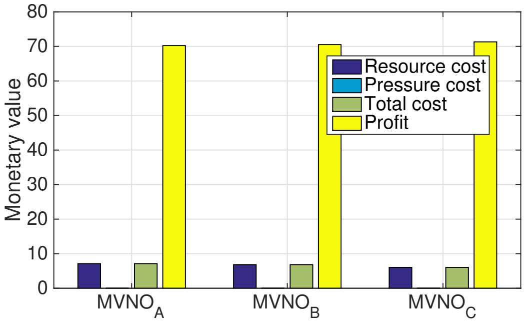
The utility targets are defined in terms of spectral efficiency; hence, we design the utility achieved, U_k, as a linear function of the assigned resources and the maximum achievable rate:

$$U_k\left(x_k[n], r_k[n]\right) = x_k[n] \cdot r_k[n]. \tag{14}$$

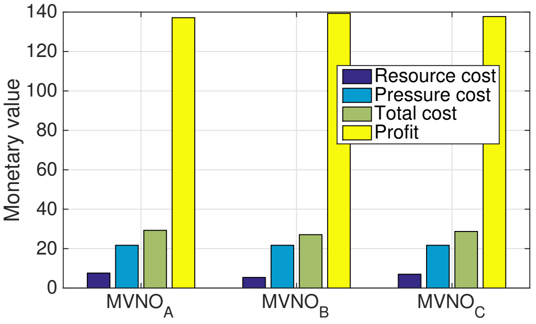
It is important to note that all the costs as well as the pricing tariffs are to be considered purely illustrative (see Table I). They are used for the sole purposes of understanding the key characteristic behavior of the model and not with the intention of trying to estimate realistic profits. Therefore, no monetary units are specified in Table I or in the figures.

B. Results

We consider two different user densities: first, we analyze the strategies played at the equilibrium when the number of users per MVNO is $K_m = 4$, and then, when $K_m = 15$. We observed that in both cases the NE strategy selected by the MVNOs is the one that maximizes the quality of service delivered to their users – denoted by the triple $(1, 1, 1)$. The

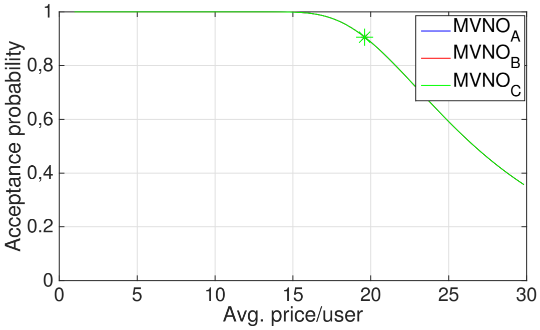


(a) 4 users per MVNO

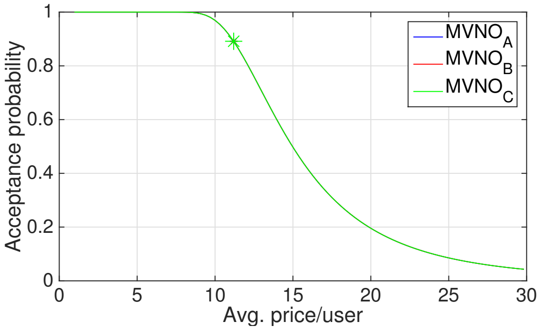


(b) 15 users per MVNO

Fig. 2: Costs and profit of each MVNO at the equilibrium strategy for different user densities.



(a) 4 users per MVNO



(b) 15 users per MVNO

Fig. 3: Acceptance probability of the NE strategy while varying the price. The star indicates the optimal price.

symmetry of the NE strategy is a direct consequence of the symmetry in the MSs.

In Fig. 2, we compare the costs incurred and the profits of individual MVNOs both when the InP can and cannot fulfill the MVNOs' requirements. In Fig. 2(a), the resources available in the network are sufficient to satisfy the target utility of the MVNOs as indicated by the zero-values of the pressure cost. This condition justifies the selection based on maximum quality, i.e., based on users being completely satisfied, and the MVNOs can, therefore, maximize their profit by selecting the optimal price indicated in Fig. 3(a). However, when the total number of users in the cell is increased, the resources are insufficient to meet the target utility. So unlike the previous case, we observe a strictly positive pressure cost (Fig. 2(b)), which is shared equally among the MVNOs owing to the symmetry of the problem. A consequence of resource scarcity is the decrease in the average utility of the users, U_k; this compels MVNOs to adjust the price, as shown in Fig. 3(b), in order to maximize profit[3]. As shown in Fig. 4, the MVNOs could obtain higher profits by selecting a different set of strategies, which do not result in a NE, such as the triple $(0.4, 0.4, 0.4)$. However, since the pressure cost is proportional to the market share and not the chosen quality (cf. (8)), each MVNO tries to take advantage of this scenario by selecting higher quality levels for its users. Hence, as result of this competitive structure, the MVNOs reach an equilibrium point by deciding to jointly invest in network expansion.

Fig. 5 shows how the NE strategy and investments towards the future (i.e., pressure cost) vary while increasing the number of users per MVNO. We observe that there exists a point (at approximately 40 users per MVNO) where the model is unable to sufficiently incentivize additional investments and this leads the MVNOs selecting a NE strategy different from $(1, 1, 1)$. In other words, the resources available in the network are so scarce that the MVNOs deem additional expenditure to be unwarranted. This implies that there exists a minimum amount of resources the InP needs to provide in order to incentivize the MVNOs invest in the infrastructure. Further insights are given by the *Price of Stability* (PoS), defined as the ratio between the profit earned at the NE and the strategy maximizing the individual profit [17]. We observe that the PoS decreases while the number of users increases, which means that the equilibrium strategy deviates from the optimal strategy. Following which, MVNOs decide to move towards a different equilibrium strategy that corresponds to a reduction in their investments; thereby, leading to an increase in the PoS.

V. CONCLUSION

In this paper, we propose an approach to analyze the engagement of multiple MVNOs in a shared infrastructure scenario. This work enables competition among the MVNOs

---

[3] An increase in profit is due to the assumptions made about values of the parameters selected. The absolute values of the profit are not investigated in this work.

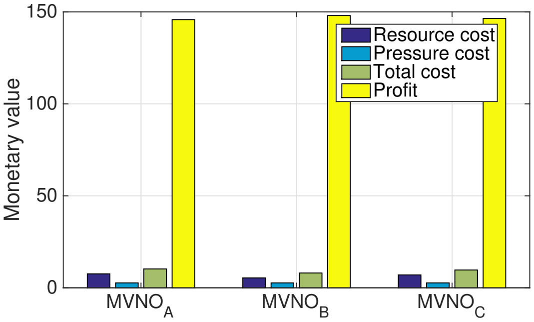


Fig. 4: Costs and profit at non-equilibrium strategy for $K_m = 15$. MVNOs can maximize profit selecting the triple $(0.4, 0.4, 0.4)$.

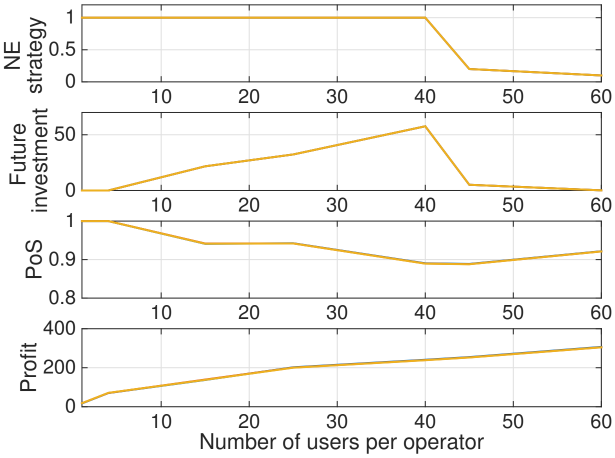


Fig. 5: Nash equilibria with increasing user densities. Note that the curves of the three MVNOs are all superimposed.

based on a differentiation criterion, wherein they are free to select the quality of service levels they want to deliver to their users. MVNOs can estimate the economic advantages of their decision by balancing the costs arising from usage of the infrastructure and the profit earned from their users. This framework also evaluates the willingness of the MVNOs to invest in the shared infrastructure. When faced with a scarcity of resources, the competition based scenario leads the MVNOs to push for additional investments, although it does not represent a strategy that maximizes profit in the short-term. The additional investments are, therefore, directed more towards long-term profitability. This model also finds a limit point at which the amount of extra resources needed is too costly to justify any additional expenditure. This necessitates the evaluation of a more complex business ecosystem wherein the competition among different InPs, maximizing their own profits, could find a means to incentivize additional investments from the InPs themselves and thereby, alleviating some of the burden from the MVNOs.

ACKNOWLEDGEMENT

The work has been supported in part by the Bell Labs Internship program and by the European Union Horizon 2020 Marie Sklodowska-Curie ITN/EID project ACT5G (Grant Agreement No. 643002).

REFERENCES

[1] CISCO, “Cisco Visual Networking Index: Forecast and Methodology, 2015-2020 white paper,” 2015. [Online]. Available: http://www.cisco.com/c/en/us/solutions/collateral/service-provider/visual-networking-index-vni/complete-white-paper-c11-481360.pdf

[2] McKinsey and C. Inc., “Monetizing mobile: making data pay,” 2014. [Online]. Available: http://cs404620.vk.me/u11927290/docs/c94e7ac85fc9/1_McKinsey_Monetizing_mobile_2014-06.pdf

[3] OECD, “Wireless market structures and network sharing,” 2014. [Online]. Available: http://dx.doi.org/10.1787/5jxt46dzl9r2-en

[4] Y.-K. Song, H. Zo, and S. Lee, “Analyzing the economic effect of mobile network sharing in korea,” *ETRI Journal*, vol. 34, no. 3, pp. 308–318, 2012.

[5] F. Berkers, G. Hendrix, I. Chatzicharistou, T. De Haas, and D. Hamera, “To share or not to share?” in *Intelligence in Next Generation Networks (ICIN), 14th International Conference on*, Oct 2010.

[6] D.-E. Meddour, T. Rasheed, and Y. Gourhant, “On the role of infrastructure sharing for mobile network operators in emerging markets,” *Computer Networks*, vol. 55, no. 7, pp. 1576–1591, 2011.

[7] NGMN Alliance, “Suggestions on potential solutions to c-ran,” *White Paper, January*, 2013.

[8] V. Suryaprakash, P. Rost, and G. Fettweis, “Are heterogeneous cloud-based radio access networks cost effective?” *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 10, pp. 2239–2251, Oct 2015.

[9] T. Frisanco, P. Tafertshofer, P. Lurin, and R. Ang, “Infrastructure sharing for mobile network operators; from a deployment and operations view,” in *International Conference on Information Networking*, Jan 2008.

[10] O. U. Akgül, I. Malanchini, V. Suryaprakash, and A. Capone, “Dynamic resource allocation and pricing for shared radio access infrastructure,” in *IEEE International Conference on Communications (ICC)*, May 2017.

[11] A. Antonopoulos, E. Kartsakli, A. Bousia, L. Alonso, and C. Verikoukis, “Energy-efficient infrastructure sharing in multi-operator mobile networks,” *IEEE Communications Magazine*, vol. 53, no. 5, pp. 242–249, May 2015.

[12] L. Cano, A. Capone, G. Carello, M. Cesana, and M. Passacantando, “Cooperative infrastructure and spectrum sharing in heterogeneous mobile networks,” *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 10, pp. 2617–2629, Oct 2016.

[13] I. Malanchini, M. Cesana, and N. Gatti, “Network selection and resource allocation games for wireless access networks,” *IEEE Transactions on Mobile Computing*, vol. 12, no. 12, pp. 2427–2440, Dec 2013.

[14] I. Malanchini and M. Gruber, “How operators can differentiate through policies when sharing small cells,” in *IEEE 81st Vehicular Technology Conference (VTC Spring)*, May 2015.

[15] L. Badia, M. Lindstrom, J. Zander, and M. Zorzi, “Demand and pricing effects on the radio resource allocation of multimedia communication systems,” in *IEEE Global Telecommunications Conference (GLOBECOM)*, vol. 7, Dec 2003, pp. 4116–4121.

[16] J. Nash, “Non-cooperative games,” *Annals of Mathematics*, vol. 54, no. 2, pp. 286–295, 1951.

[17] E. Anshelevich, A. Dasgupta, J. Kleinberg, E. Tardos, T. Wexler, and T. Roughgarden, “The price of stability for network design with fair cost allocation,” in *45th Annual IEEE Symposium on Foundations of Computer Science*, Oct 2004, pp. 295–304.