Queuing Behavior of Queue-Backlog-Based Random Access Scheduling in the Many-Channel Regime

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***Abstract*—Efficient and distributed scheduling algorithms are essential to garner the full potential of wireless systems with multiple channels, e.g., OFDM systems. Recently, a group of random access scheduling algorithms have been proposed to achieve throughput optimality with a single non-fading channel. While these distributed algorithms can readily be extended to a multi-channel setting, the analysis of their queuing behavior and delay performance is non-trivial in the many-channel regime, since the state space of schedules grows exponentially with the number of channels. In this paper, we first generalize these distributed random access algorithms from a single-channel setting to a multi-channel setting for fully connected networks. In an attempt to characterize the delay performance of this new algorithm, we introduce a novel *equivalent deterministic single-queue system* and show that the queuing behavior of individual communication links under our algorithm converges to the equivalent queue system as the number of channels grows. By studying this equivalent queue system, we derive a closed-form approximation for the data queue backlogs/delays at the steady state in the many-channel regime.**

I. Introduction

Wireless systems with multiple channels, e.g., OFDM systems, have important applications in next-generation networks such as WiMAX, 4G cellular networks, cognitive radio networks, etc. Efficient and distributed algorithms are essential to garner the full potential of these systems. A number of centralized algorithms [2]-[5] have been developed for multi-channel systems to achieve throughput optimality. However, these algorithms need centralized computation with high time complexity, and thus are not suitable for practical distributed implementations. Several suboptimal low-complexity distributed algorithms have been proposed, e.g., in [6][7], which only support a fraction of the capacity region in general. Recently, a class of queue-backlog-based random access scheduling algorithms [8]-[12] have been proposed for single channel settings. These random access algorithms yield simple distributed implementations [13] and have been proved to achieve throughput optimality with non-fading channels.

Throughput and delay are two important metrics to analyze the quality of service (QoS) in wireless networks. Throughput optimality of the aforementioned distributed algorithms has been proved in [8]-[12], and their average queue backlog/delay performance has been studied in [12][14][19]. Specifically, through a mixing-time analysis, it is shown in [14] that an upper-bound for the time-averaged data queue backlog is of exponential order in the number of communication links in the network, and an upper-bound of polynomial order in the number of links is provided in [12] for a fraction of the capacity region. A lower-bound of expected data queue backlogs has been derived in [19]. However, these (average) delay bounds are not tight in general. In fact, no closed-form analysis/estimation of the queue backlog/delay performance of these random access algorithms is available in the literature, since interactions between links lead to complicated queuing behaviors and make such characterization intractable. Not surprisingly, queuing behavior and delay analysis of multi-channel extensions of these distributed algorithms is non-trivial and remains an open research problem, since the state space of schedules grows exponentially with the number of channels.

With the above motivation, we study the *transient queuing behavior* of multi-channel random access algorithms for a fully connected wireless network (e.g., WLAN and cellular networks). To this end, we develop a novel equivalent queue system to model and show that the queuing behavior of individual links converges to the equivalent queue system as the number of channels grows. Note that transient queuing behavior (including transient evolution of service rates) implies the ability to analyze the transient delay performance in addition to the steady-state study of the queuing behavior and the expected delay performance of the wireless system.

In this paper, we are especially interested in the behavior of a system with a fixed number of links where the number of channels scales over a constant capacity bandwidth. This approach is motivated and supported by the properties and trends of recently deployed wireless systems such as WiMAX. The physical service area in which high data rates are provided to links is generally limited, which means the number of links is roughly upper bounded (e.g., in WiMAX pico and femto cells [22]), as well. On the other hand, the same systems operate over hundreds of orthogonal channels. So, the focus of our investigation is the behavior of a system with a large number of channels over a given finite frequency band, serving a given constant population size. Note that the growing number of channels represents diminishing bandwidth per channel, where the sum of all bands is constant. This scenario can be justified by the fact that inter-symbol interference (ISI) can be significantly reduced in an OFDM system by transmitting data in parallel over a large number of low-rate (sub)channels [23].

In this work, we first generalize the random access algorithms from a single-channel setting [8]-[12] to a multi-channel one, denoted as *multi-channel random access algorithm*. The multi-channel random access algorithm is shown to be throughput-optimal for *any finite number of non-fading channels* under the time-scale separation assumption.[1] With the introduction of the novel *equivalent deterministic single-queue system*, the salient features of our work can be listed as follows:

(1) Assuming (asymptotically) uniform arrival rates, we show that the queuing behavior of the network asymptotically (with respect to the number of channels) approaches that of the equivalent deterministic single-queue system. Law-of-large-numbers (LLNs) results have been established in the many-channel regime for the queue backlog and the scheduled service rate, which are governed by simple deterministic dynamics. These results are the first of their kind for random access scheduling, which yield a simplified and scalable characterization of the individual queuing behavior in the many-channel regime.

(2) By studying the equivalent deterministic single-queue system, we find a *closed-form* approximation for the queue backlog in its steady state in the many-channel regime. While the delay bounds derived in [12][14][19] for a single-channel setting are generally not tight, we show that the closed-form steady state queue backlog/delay approximation becomes accurate as the number of channel increases. Furthermore, in resonance with the findings in [18], [19], [20], we show for a fully connected network that the more aggressive the weight function[2] is, the smaller the asymptotic queue length becomes in the many-channel regime.[3]

(3) Based on a steady state study, the multi-channel random access algorithm is proved to be asymptotically (with respect to the number of channels) throughput-optimal *without the time-scale separation assumption*.

Via numerical evaluations, we validate the accuracy of the equivalent queue system representation and show that the queue backlogs/delays indeed converge to the derived closed-form steady state results in the many-channel regime.

[1]The time-scale separation assumption (i.e., the underlying Markov chain of the scheduling algorithm converges instantaneously to its steady state) has been employed in [8][9] and justified in [15][16].

[2]Weight functions are employed to determine the transmission probability in the random access algorithms.

[3]Results have been provided in [18], [19], and [20], respectively, on the optimality, a lower-bound on queue backlogs, and the stability for random access algorithms in a single-channel setting. In comparison, we provide a *closed-form* queue backlog approximation for random access algorithms in the many-channel regime.

The rest of the paper is organized as follows: We introduce the multi-channel random access algorithm in Section II and introduce the equivalent queue system that asymptotically represents the queuing behavior of the network under the random access algorithm in Section III. A steady state study is carried out in Section IV that introduces a closed-form approximation for the queue backlogs and service rates under the random access algorithm. We present the numerical results in Section V and conclude our work in Section VI.

II. Network Model and the Multi-Channel Random Access Algorithm

For simplicity, we introduce the notations $\xrightarrow{P}_N$ and $\xrightarrow{L}_N$ to denote convergence in probability and convergence in law (or convergence in distribution) [24], respectively, as $N \to \infty$.

A. Network Elements

We consider a time-slotted fully connected single-hop wireless network where M communication links contend for N orthogonal channels. Each link i maintains a data queue $q_i(t)$ updated at the beginning of a time slot $t = 0, 1, 2, ...,$ with $i = 1, 2, ..., M$. Let $A_i(t)$ be the amount of data (in unit of bits) arriving at queue i at the beginning of a time slot t. We assume that the number of links does not scale with the number of channels, which typically applies to WiMAX pico-cell, and femto-cell scenarios [22].

We assume that each channel is non-fading and always available with a capacity (i.e., maximum data rate per time slot) $\frac{C}{N}$, where C denotes the total capacity of the considered wireless system. We denote by $\mu_{ij}(t) \in \{0, 1\}$ the schedule of link i, $i = 1, 2, ..., M$, over channel j, $j = 1, 2, ..., N$, at time slot t. Specifically, $\mu_{ij}(t) = 1$ if link i is scheduled over channel j; $\mu_{ij}(t) = 0$, otherwise. We consider an OFDM mechanism, i.e., one link can transmit over multiple channels in a time slot. Since we have assumed a fully connected network, simultaneous transmissions over the same channel will cause interference at all nodes and, hence, each channel can only be allocated to one link, i.e., $\sum_i \mu_{ij}(t) \leq 1$, $\forall t$. Thus, the queue of each link evolves as, $\forall i$,

$$q_i(t) = [q_i(t-1)+A_i(t-1)-\frac{C}{N}\sum_{j=1}^{N}\mu_{ij}(t-1)]^+, \forall t \geq 0, \quad (1)$$

where $[\cdot]^+ \triangleq \max\{\cdot, 0\}$.

B. Multi-Channel Random Access Algorithm

In this section, we introduce a distributed throughput-optimal multi-channel random access algorithm, which is a generalization of the single-channel random access algorithms [9]-[13]. For *each* time slot t, the algorithm is composed of two parts: *Exchange Phase* and *Scheduling Phase*.

The exchange phase is scheduled at the beginning of time slot t and is composed of M mini-slots reserved for (control) message exchange. Each link is assigned a dedicated mini-slot out of the M mini-slots. Transmissions during mini-slots use the entire spectrum C (i.e., channels $\{1, ..., N\}$) to minimize the length of mini-slots, which we assume is negligible compared to that of a unit time slot. Specifically, the transmitter of each link i broadcasts the following three binary vectors to all other nodes (i.e., the receiver of link i, and the transmitters and the receivers of all the other links) during its dedicated mini-slot: $\left((\mu_{ij}(t-1))_{j=1}^N, (a_{ij}(t))_{j=1}^N, (p_{ij}(t))_{j=1}^N\right)$, where $(\mu_{ij}(t-1))_{j=1}^N$ denotes the schedules of link i at the previous time slot $t-1$. The *contention variables* $a_{ij}(t)$ are independent over link i and channel j with

$$a_{ij}(t) = \begin{cases} 1, \text{ w.p. } \beta, \\ 0, \text{ w.p. } 1-\beta. \end{cases}$$

The contention probability $0 < \beta < 1$ is typically chosen as $\frac{1}{M}$ [17]. The *transmission variables* $p_{ij}(t)$ are independent over links i and i.i.d. over channels j with $p_{ij}(t) = \begin{cases} 1, \text{ w.p. } \dfrac{h(q_i(t-1))}{1+h(q_i(t-1))} \\ 0, \text{ w.p. } \dfrac{1}{1+h(q_i(t-1))} \end{cases}$, where we denote the transmission weight function by $h: [0,\infty) \to [0,\infty)$. These binary vectors $\left((\mu_{ij}(t-1))_{j=1}^N, (a_{ij}(t))_{j=1}^N, (p_{ij}(t))_{j=1}^N\right)$ are received by all links in the single-hop network and used to determine the transmission schedules for individual links. Note that the vector of contention variables $(p_{ij}(t))_{j=1}^N$ will be used only by the transmitter i and its intended receiver in the scheduling phase.

After the exchange phase, the schedule $\mu_{ij}(t)$, for any given link i and channel j, is determined in the scheduling phase based on the Glauber dynamics in a single channel setting [9]-[13]. Specifically, the schedule $\mu_{ij}(t)$ for any given link i and channel j at time slot t depends on the following three conditions:

Condition (i): The "contention" of link i for channel j is successful, i.e., $a_{ij}(t)\prod_{k\neq i}(1-a_{kj}(t)) = 1$.

Condition (ii): $\sum_{k\neq i}\mu_{kj}(t-1) = 0$, i.e., no other links were allocated channel j in the previous time slot.

Condition (iii): The transmission variable $p_{ij}(t) = 1$.

The scheduling phase is performed locally at each node (i.e., the transmitter and the receiver of each link i) as follows:

Scheduling Phase

The transmitter and the receiver of each link i determine $\mu_{ij}(t)$, $j = 1, ..., N$, according to the following:

If Conditions (i), (ii), and (iii) hold,

then $\mu_{ij}(t) = 1$ (i.e., channel j is allocated to link i for time slot t);

Else if Condition (i) does not hold,

then $\mu_{ij}(t) = \mu_{ij}(t-1)$;

Otherwise, $\mu_{ij}(t) = 0$.

From the multi-channel random access algorithm introduced above, we conclude, for any given link i and any given channel j,

$$\begin{aligned} &\mu_{ij}(t) \\ =&a_{ij}(t)\prod_{k\neq i}(1-a_{kj}(t))\left(1-\sum_{k\neq i}\mu_{kj}(t-1)\right)p_{ij}(t) \\ &+\left(1-a_{ij}(t)\prod_{k\neq i}(1-a_{kj}(t))\right)\mu_{ij}(t-1). \end{aligned} \tag{2}$$

The first term on the RHS of (2) corresponds to the case when Conditions (i)(ii)(iii) hold, and the second term on the RHS of (2) corresponds to the case when Condition (i) does not hold.

Since both the transmitter and the receiver of link i have a copy of the schedule vector $(\mu_{ij}(t))_{j=1}^N$ when the scheduling phase ends, they will tune to the set of channels $\{j : \mu_{ij}(t) = 1\}$ for data transmission in the remaining time slot.

In the following theorem, we state the throughput optimality of the above random access algorithm.

Theorem 1: Assume that the transmission weight function takes the form of $h(x) = e^{f(x)}$ with $f(x)$ satisfying the following two properties:

(A1) $f : [0,\infty) \to [0,\infty)$ is nondecreasing and continuous with $\lim_{x\to\infty} f(x) = \infty$.

(A2) Given any $M_1, M_2 > 0$ and any $\epsilon' > 0$ arbitrarily small, there exists $M_3 > 0$ such that for all $x > M_3$: $f(x)(1-\epsilon') \leq f(x-M_1) \leq f(x+M_2) \leq f(x)(1+\epsilon')$.

If the arrival process is stationary and bounded above (i.e., $A_i(t) \leq A_{max}$, $\forall t, \forall i$, for some sufficiently large $A_{max} > 0$), under the time-scale separation assumption [8] (which is employed in [8][9] and justified in [15][16]), the algorithm is throughput-optimal[4] for any given $N \geq 1$.

Proof: Theorem 1 is proved in [26]. ■

Random access algorithms proposed in [8]-[11] have been proved to be throughput-optimal in a single channel setting. However, there are few works analyzing the queue backlog/delay performance for these throughput-optimal algorithms other than the order results [12][14] on the upper-bounds and [19] on the lower-bounds of the expected queue backlogs, which are not tight in general. In Section III, we show that the queue backlogs $(q_i(t))$ and service rates asymptotically converge to an equivalent deterministic single-queue system. In Section IV, by studying the steady state of the equivalent queue, we find a *closed-form* approximation for the steady state queue backlogs/delays in the many-channel regime.

III. Asymptotic Queuing Behavior under the Many-Channel Regime

Before we present the asymptotic queuing behavior under the multi-channel random access algorithm in Theorem 2, we introduce the following limit law assumption on arrival processes:

$$A_i(t) \xrightarrow{P}_N \alpha, \forall i, \forall t, \tag{3}$$

[4]We say that an algorithm is *throughput-optimal* if it can stabilize any arrival rate vector within the capacity region (which is the closure of all arrival rate vectors that can be stably supported by the network) [1].

where $\alpha > 0$ can be interpreted as the arrival rate normalized by the number of channels. The arrival process defined above is general: It can be non-stochastic, dependent over links or even bursty (with the constraint (3) satisfied). The arrival process does not necessarily depend on the number of channels, e.g., $A_i(t) = \alpha$, $\forall i$, $\forall t$ (i.e., constant homogenous arrivals). Some examples of arrival processes are given with numerical results in [26]. Note that the analysis can be readily extended to the model with assumption (3) relaxed as $A_i(t) \xrightarrow{P}_N \alpha(t), \forall i, \forall t$, where $\alpha(t)$ is deterministic for each time slot t.

For analytical simplicity, we initialize the system at $t = -1$ as follows:

$$q_i(-1) = 0,\ \mu_{ij}(-1) = 0,\ \text{and}\ A_i(-1) \xrightarrow{P}_N \alpha, \forall i, j. \tag{4}$$

Then, we show in the following theorem that the queuing behavior of individual links under the multi-channel random access algorithm converges to an equivalent *deterministic single-queue* system as the number of channels grows.

Theorem 2: Assume that the transmission weight function h is invertible and assumption (3) holds. There exist an *equivalent deterministic queue* $q(t)$ and an *equivalent schedule variable* $v(t)$ for each time slot t, such that the following four arguments (denoted by I(t), II(t), III(t), and IV(t)) hold $\forall t$ under the multi-channel random access algorithm:
The data queue backlogs converge in probability to the equivalent deterministic queue $q(t)$ for each link:

$$\text{I}(t)\text{: } q_i(t) \xrightarrow{P}_N q(t), \forall i. \tag{5}$$

The schedules of individual links converge in law to $v(t)$:

$$\text{II}(t)\text{: } \mu_{ij}(t) \xrightarrow{L}_N v(t), \forall i, j. \tag{6}$$

The following law-of-large-numbers result holds for each link:

$$\text{III}(t)\text{: } \frac{1}{N}\sum_{j=1}^{N} \mu_{ij}(t) \xrightarrow{P}_N \mathbb{E}\{v(t)\}, \forall i. \tag{7}$$

Schedules of different channels become asymptotically mutually independent. Specifically, for any given two links $i_1, i_2 \in \mathcal{L}$, and any two distinct channels $j_1 \neq j_2$, the scheduling decisions are independent, i.e., $\forall k_1, k_2 \in \{0, 1\}$,

$$\begin{aligned}\text{IV}(t)\text{: } &\lim_{N\to\infty} P\{\mu_{i_1 j_1}(t) = k_1, \mu_{i_2 j_2}(t) = k_2\} \\ &= P\{v(t) = k_1\}P\{v(t) = k_2\}.\end{aligned} \tag{8}$$

The dynamics of the equivalent queue and the equivalent schedule variable are defined as

$$q(t) = [q(t-1) + \alpha - C\mathbb{E}\{v(t-1)\}]^+, t \geq 0, \tag{9}$$

$$v(t) = V_1(t)v(t-1) + V_2(t)(1 - v(t-1)), t \geq 0, \tag{10}$$

with the initial states

$$q(-1) = 0 \text{ and } v(-1) = 0. \tag{11}$$

$V_1(t)$ and $V_2(t)$ are mutually independent random variables, independent over time slot t, defined in the following:

$$V_1(t) = \begin{cases} 1,\ \text{w.p. } F_1 + (2 - M)F_0(q(t-1)), \\ 0, \text{otherwise}, \end{cases}$$

$$V_2(t) = \begin{cases} 1,\ \text{w.p. } F_0(q(t-1)), \\ 0,\ \text{w.p. } 1 - F_0(q(t-1)), \end{cases}$$

where for notational simplicity we define

$$F_1 \triangleq 1 - \beta(1-\beta)^{M-1}$$

$$F_0(x) \triangleq \beta(1-\beta)^{M-1}\frac{h(x)}{1+h(x)}.$$

Proof: The proof of Theorem 2 is provided in Appendix A. ■

In [26], we also verify that $V_1(t)$ is a valid random variable, i.e., $0 < F_1 + (2-M)F_0(q(t-1)) < 1$. Note that different from Theorem 1, the assumptions of stationary arrival processes and time-scale separation are *not required* in Theorem 2.

Remark 1: Theorem 2 states that the queuing behavior of each *individual* link converges to an equivalent *deterministic single-queue* system as the number of channels goes to infinity. Specifically, according to (5) and (7), the queuing behavior of each link i $\left(q_i(t), \frac{C}{N}\sum_{j=1}^{N}\mu_{ij}(t)\right)$ (i.e., queue backlog and scheduled service rate) converges asymptotically in the number of channels to the equivalent queue system $(q(t), C\mathbb{E}\{v(t)\})$. By taking expectation on both sides of (10), we know that the equivalent queue system $(q(t), C\mathbb{E}\{v(t)\})$ can be updated *deterministically* and *independent of individual links* with (9) and

$$\begin{aligned}\mathbb{E}\{v(t)\} = [&F_1 + (1-M)F_0(q(t-1))]\mathbb{E}\{v(t-1)\} \\ &+ F_0(q(t-1)).\end{aligned} \tag{12}$$

Thus, the easy updates of $(q(t), C\mathbb{E}\{v(t)\})$ yield a simplified and scalable characterization of the queuing behavior for individual links under the multi-channel random access algorithm.

In Section IV, we will study the steady state of the equivalent queue system $(q(t), C\mathbb{E}\{v(t)\})$, which becomes the (asymptotic) steady state of each link under the multi-channel random access algorithm according to Theorem 2. In the following corollary, we present the LLN results for the aggregated queue backlog and aggregated service rate under the multi-channel random access algorithm.

Corollary 1: Given $q(t)$ and $v(t)$ updated as in Theorem 2, the following LLN results hold for any time slot t:

$$\frac{1}{M}\sum_{i=1}^{M} q_i(t) \xrightarrow{P}_N q(t),$$

$$\frac{C}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N} \mu_{ij}(t) \xrightarrow{P}_N C\mathbb{E}\{v(t)\}.$$

Corollary 1 directly follows (5) and (7). Note that $\frac{1}{M}\sum_i q_i(t)$ and $\frac{C}{MN}\sum_i\sum_j \mu_{ij}(t)$ can be considered as the average queue occupancy (which represents the congestion level/delay performance) and average service rate under the multi-channel random access algorithm, respectively.

IV. A Steady State Study

We have shown that the equivalent queue system $(q(t), C\mathbb{E}\{v(t)\})$ represents the queuing behavior of individual links in the many-channel regime according to Theorem 2 and Corollary 1. We now study the steady state result for $(q(t), C\mathbb{E}\{v(t)\})$, i.e., the case when $t \to \infty$, in an attempt to approximate the steady state of the queue backlogs under the multi-channel random access scheduling algorithm. By analyzing the steady state of $q(t)$, we show that the multi-channel random access algorithm is asymptotically (with respect to N) throughput-optimal *without the time-scale separation assumption*, which is required for Theorem 1 to hold.

Since the maximal stabilizable normalized arrival rate cannot exceed $\frac{C}{M}$ as $N \to \infty$ [5], we only analyze the case for $\alpha < \frac{C}{M}$. We assume that the following limit exists: [6]

$$(q, v) \triangleq \lim_{t\to\infty} (q(t), C\mathbb{E}\{v(t)\}), \text{ with } 0 < q \leq \infty. \quad (13)$$

Note that $q = \infty$ stands for the case when the normalized arrival rate α cannot be stabilized by the algorithm. We say that the multi-channel random access algorithm is *asymptotically throughput-optimal* if the steady state q of the equivalent queue $q(t)$ corresponding to the algorithm is finite (i.e., $q < \infty$) for all $0 < \alpha < \frac{C}{M}$.

By taking the limit (over time) of both sides of (9) and (12), we obtain the steady state in its closed form

$$(q, v) = \left(h^{-1}\left(\frac{\alpha}{C - \alpha M}\right), \alpha\right).$$

Since $\alpha < \frac{C}{M}$, we can find $\epsilon \triangleq C - \alpha M > 0$ such that

$$0 < q = h^{-1}\left(\frac{\alpha}{\epsilon}\right) < \infty. \quad (14)$$

The steady-state equivalent queue backlogs that correspond to different weight functions are listed in Table I.

Remark 2: ϵ can be considered as the closeness of the traffic load to the optimality. We know from (5) that the data queue backlogs converge in probability to $q(t)$. Therefore, when the number of channels N becomes large, the individual queue backlogs in the steady state converge to the steady state of the equivalent queue q. We can see from Table I that more aggressive weight functions lead to smaller delays. For example, $h(x) = e^x - 1$ results in a better delay performance than x and $\log(x+1)$. This is in accordance with the findings in a single channel setting [19], where a lower-bound on the expected queue backlog is derived with respect to weight functions. We note that while a more aggressive weight function reduces delays, it can potentially aggravate the *temporal starvation* [21] when there is *a limited number of channels*, i.e., links can undergo prolonged periods of inactivity followed by a prolonged period of activity, leading to bursty service and undesirable jitter performance. The study of this tradeoff between delay and temporal starvation will be one of our future works.

[5]Recall assumption (3) and that the total channel capacity is C. We must have $\alpha \leq \frac{C}{M}$ as $N \to \infty$.

[6]We have shown in [26] that (q, v) is indeed a stable equilibrium of the system $(q(t), C\mathbb{E}\{v(t)\})$. We also show through extensive numerical results in Section V and [26] that the queuing behavior of individual links $\left(q_i(t), \frac{C}{N}\sum_{j=1}^{N}\mu_{ij}(t)\right)$ converges to (q, v) in time, for sufficiently large N.

Since channel resources are shared by M links, the interval of $0 < \alpha < \frac{C}{M}$ denotes the stabilizable range of (normalized) arrival rates. From (14), the steady state value q is finite for any $\alpha < \frac{C}{M}$, and hence, the random access algorithm is asymptotically (with respect to N) throughput-optimal under the assumption that $\lim_{t\to\infty}(q(t), C\mathbb{E}\{v(t)\})$ exists. This asymptotic result is summarized in the following proposition:

Proposition 1: The multi-channel random access algorithm is asymptotically throughput-optimal with respect to N under assumption (13).

We note that Proposition 1 is a steady state outcome of the convergence results I(t)-IV(t) in Theorem 2, which *hold for each time slot* t. Thus, like Theorem 2, Proposition 1 does not require the assumptions of stationary arrival processes and the time-scale separation.

In [26], we further show that $\left(h^{-1}\left(\frac{\alpha}{\epsilon}\right), \alpha\right)$ is indeed a stable equilibrium of the system $(q(t), C\mathbb{E}\{v(t)\})$.

TABLE I
STEADY STATE EQUIVALENT QUEUE BACKLOG q WITH DIFFERENT WEIGHT FUNCTIONS h

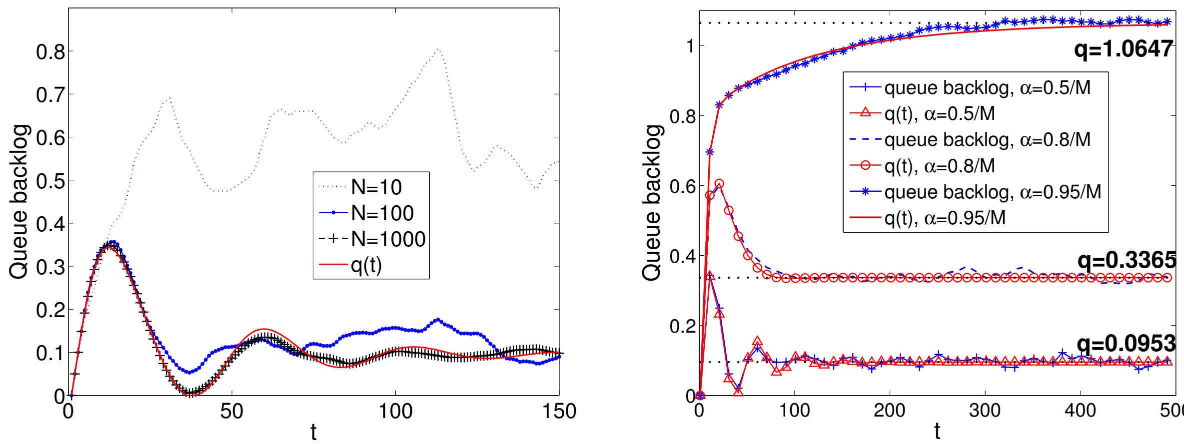
$h(x)$	$e^x - 1$	x	$\log(x+1)$
q	$\log(\frac{\alpha}{\epsilon}+1)$	$\frac{\alpha}{\epsilon}$	$e^{\frac{\alpha}{\epsilon}} - 1$

V. Numerical Results

In this section, we provide extensive numerical results for the multi-channel random access algorithm. For analytical simplicity, we assume a unit capacity system, i.e., $C = 1$. Following the steady-state analysis in Section IV, we only consider the case $\alpha < \frac{1}{M}$ since the maximal stabilizable normalized arrival rate cannot exceed $\frac{1}{M}$ as $N \to \infty$. The contention probability is fixed as $\beta = \frac{1}{M}$ [17]. The equivalent queue system $(q(t), \mathbb{E}\{v(t)\})$ is updated deterministically according to dynamics (9)(12). In the numerical evaluations, we focus on the following metric which represents the *aggregated* algorithm performance on delay: The average queue backlog $\left(\frac{1}{M}\sum_{i=1}^{M} q_i(t)\right)$. Simulation results on another aggregated metric, average service rate $\left(\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\mu_{ij}(t)\right)$, have been presented in [26], as well.

A. Convergence of Queue Backlogs and Scheduled Service Rates with Increasing N

In Figure 1(a), we illustrate the simulation results of the evolution of actual queue backlogs with different number of channels N, compared to the equivalent queue $q(t)$. Figure 1(a) shows that the queue dynamics of the aggregated links approaches the equivalent queue $q(t)$ as the number of channels increases, where we note that $C = 1$ in the simulations. In the numerical evaluations, we consider $M = 10$ links (corresponding to a typical WiMAX femto cell scenario [22]).



(a) Different N, $\alpha = 0.5/M$

(b) Different α, $N = 1000$

Fig. 1. Queue dynamics under the multi-channel random access algorithm

Specifically, we consider a scenario with normalized arrival rate $\alpha = 0.5/M$, the weight function $h(x) = e^x - 1$, and identical arrival processes $A_i(t) = \alpha$, $\forall i$ $\forall t$. It is observed in Figure 1(a) that the accuracy of the approximation of the average queue backlog by $q(t)$ increases as N increases, which coincides with the convergence results in Corollary 1. Since the dynamics of $q(t)$ is deterministic and independent of the queuing behavior of individual links, the equivalent queue system provides a simplified but accurate estimation for the actual queuing behavior under the multi-channel random access algorithm in the many-channel regime.

We note that $N = 1000$ is a practical choice for OFDM-based wireless systems. For example, the number of sub-carriers can be up to 2048 in WiMAX [25]. Thus, in the following numerical evaluations, we consider the case of $N = 1000$.

B. Effect of Different Normalized Arrival Rates α

We compare the queue dynamics under different normalized arrival rates α in Figure 1(b). Specifically, we consider a scenario with $M = 10$ links, $N = 1000$ channels, weight function $h(x) = e^x - 1$, and arrival processes $A_i(t) = \alpha$, $\forall i$ $\forall t$. Note that $\alpha = \frac{0.95}{M}$ corresponds to a normalized arrival rate achieving 95% of the maximum stabilizable capacity. In conformance with Corollary 1, the equivalent queue $q(t)$ accurately tracks the (average) queue backlogs of the multi-channel random access algorithm. As expected, the queue backlogs increase with a growing normalized arrival rate α. In addition, while tracking the queuing behavior of the multi-channel random access algorithm, $q(t)$ indeed converges to the steady state q over time, as illustrated in Figure 1(b). We also observe in Figure 1(b) that the convergence of $q(t)$ to steady state is slower with a larger traffic load (i.e., a larger α).

C. Effect of Different Weight Functions

In Figure 2(a), we compare the multi-channel random access algorithm with different weight functions h: $h(x) = e^x - 1$, $h(x) = x$, and $h(x) = \log(x+1)$. Specifically, we consider a scenario with $M = 10$ links, $N = 1000$ channels, $A_i(t) = \alpha$, $\forall i$ $\forall t$, and a heavy load $\alpha = 0.8/M$. The results again show that, while tracking the average queue backlog, $q(t)$ converges to and attains its limit q as theoretically calculated in Table I. In conformance with Table I, the more aggressive the weight function h is, the higher congestion level (illustrated as the average queue backlog in Figure 2(a)) the system experiences.

D. Queuing Behavior with Different Number of Links

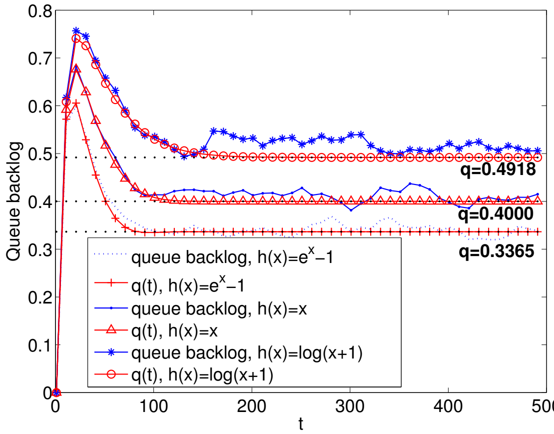
In this section, we illustrate the queuing behavior for different number of links in Figure 2(b), where we consider $N = 1000$ channels, weight function $h(x) = e^x - 1$, normalized arrival rate $\alpha = \frac{0.75}{M}$, and i.i.d. arrivals with

$$A_i(t) = \alpha - rand(1) \times \frac{0.2\alpha}{\sqrt{N}}, \forall i, \forall t,$$

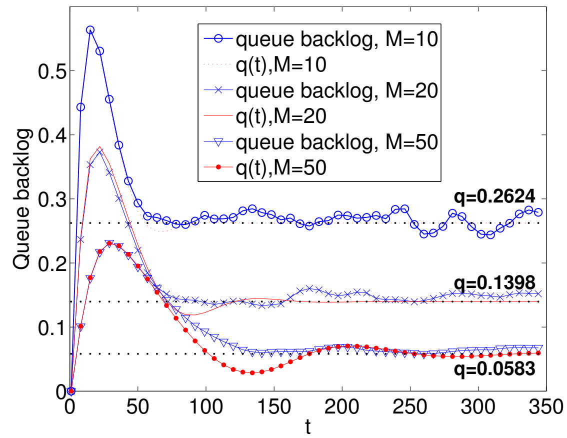
where $rand(1)$ outputs a random value uniformly distributed over the interval $(0, 1)$ independently across time slots and links. For $M = 10, 20, 50$, which are typical values for pico and femto cell scenarios [22], we observe that the data queue backlogs are correctly tracked by $q(t)$ and fluctuates around the steady state q as $t \to \infty$. Thus, we conclude that, for all considered M values, the evolution of the original system is well presented by the equivalent single-queue system. Note that under this simulation setting, the steady state $q = \log(\frac{3}{M} + 1)$, according to (14). Hence, a larger number of links leads to smaller delays (represented by queue lengths in Figure 2(b)) when the traffic load is fixed (i.e., $\alpha M = 0.75$).

VI. Conclusions and Future Works

Our work aims to better understand the delay performance and the fundamental properties of queuing dynamics for random access algorithms in a many-channel regime. Specifically, in this paper, we generalize a class of throughput-optimal random access algorithms to a multi-channel setting. We show that the individual queuing behavior under the multi-channel



(a) Different weight function, $\alpha = 0.8/M$, $M = 10$



(b) Different number of links M, $h(x) = e^x - 1$, $\alpha = 0.75/M$

Fig. 2. Queue dynamics under the multi-channel random access algorithm

random access algorithm converges to an equivalent deterministic single-queue system. By analyzing this equivalent queue system, we find a closed-form approximation for the steady state queue backlogs in the many-channel regime.

To show the efficacy of our model, we will perform testbed implementations of the multi-channel random access algorithm in our future work. The LLN results presented in Theorem 2 are based on assumption (3) of asymptotically uniform (normalized) arrival rates and the fully connected network topology (where the interference set is uniform for all links). In our future work, we will also study the queuing behavior for its convergence to an equivalent M-queue system, in a multi-hop topology where each of the M links has a unique interference set and an individual arrival rate. By analyzing the equivalent M-queue system, we will study the throughput and delay performance of the random access algorithm in a more general setting.

REFERENCES

[1] L. Georgiadis, M. Neely and L.Tassiulas, "Resource allocation and cross-Layer control in wireless networks", in *Foundations and Trends in Networking*, pp. 1-149, 2006.

[2] M. Hajiaghayi, M. Dong, and B. Liang, "Optimal channel assignment and power allocation for dual-hop multi-channel multi-user relaying", in *Proc. INFOCOM 2011 Mini-Conference*, pp. 76-80, April 2011.

[3] S. Bodas, S. Shakkotai, L. Ying, and R. Srikant, "Low-complexity scheduling algorithms for multi-channel downlink wireless networks", in *Proc. IEEE INFOCOM 2010*, pp. 1-9, March 2010.

[4] S. Bodas, S. Shakkotta, L. Ying, and R. Srikant, "Scheduling for small delay in multi-rate multi-channel wireless networks", in *Proc. INFOCOM 2011*, pp. 1251-1259, April 2011.

[5] M. Sharma and X. Lin, "OFDM downlink scheduling for delay-optimality: many-channel many-source asymptotics with general arrival processes", in *2011 Information Theory and Applications Workshop*, pp. 1-10, February 2011.

[6] X. Lin and S. Rasool, "A distributed joint channel-assignment, scheduling and routing algorithm for multi-channel Ad Hoc wireless networks", in *Proc. INFOCOM 2007*, pp. 1118-1126, May 2007.

[7] B. Han, V. Kumar, Madhav. Marathe, S. Parthasarathy, and A. Srinivasan, "Distributed strategies for channel allocation and scheduling in software-defined radio networks", *Proc. INFOCOM 2009*, 1521-1529, April 2009.

[8] L. Jiang and J. Walrand,"a distributed algorithm for throughput and utility maximization in wireless networks", in *IEEE/ACM Transactions on Networking*, vol. 18, no.3, pp. 960-972, June 2010.

[9] J. Ni, B. Tan, and R. Srikant, "Q-CSMA: queue-length based CSMA/CA algorithms for achieving maximum throughput and low delay in wireless networks", in *Proc. IEEE INFOCOM 2010 Mini-Conference*, pp. 1-5, April 2010.

[10] S. Rajagopalan, D. Shah, and J. Shin, "Network adiabatic theorem: an efficient randomized protocol for contention resolution", in *SIGMETRICS'09: Proceeding of the eleventh international joint conference on measurement and modeling of computer systems*, pp. 133-144, 2009.

[11] J. Ghaderi and R. Srikant, "On the design of efficient CSMA algorithms for wireless networks", in *Proc. IEEE conference on Decision and Control (CDC 2010)*, pp. 954-959, December 2010.

[12] L. Jiang, M. Leconte, J. Ni, R. Srikant, and J. Walrand, "Fast mixing of parallel Glauber dynamics and low-delay CSMA scheduling", in *IEEE INFOCOM 2011 Mini-Conference*, pp. 371-375, April 2011.

[13] D. Xue and E. Ekici, "Distributed utility-optimal scheduling with finite buffers", in *Proc. WiOpt 2012*, pp. 278-285, May 2012.

[14] M. Lotfinezhad and P. Marbach, "Throughput-optimal random access with order-optimal delay", in *Proc. IEEE INFOCOM 2011*, pp. 2867-2875, April 2011.

[15] A. Proutiere, Y. Yi, T. Lan, and M. Chiang, "Resource allocation over network dynamics without timescale separation", in *Proc. IEEE INFOCOM 2010 Mini-Conference*, pp. 1-5, March 2010.

[16] L. Jiang and J. Walrand, "Convergence and stability of a distributed CSMA algorithm for maximal network throughput", in *IEEE CDC'09*, pp. 4840-4845, 2009.

[17] J. Ghaderi and R. Srikant, "Effect of access probabilities on the delay performance of Q-CSMA algorithms", in *IEEE INFOCOM 2012*, pp. 2068-2076, March 2012.

[18] M. Feuillet, A. Proutire, and P. Robert, "Random capture algorithms fluid limits and stability", in *Information Theory and Applications Workshop (ITA) 2010*, pp. 1-4, Jan. 2010.

[19] N. Bouman, S. Borst, J. Leeuwaarden, "Achievable delay performance in CSMA networks", in *Forty-Ninth Annual Allerton Conference*, pp. 384-391, September 2011.

[20] J. Ghaderi, S. Borst, P. Whiting, "Backlog-based random access in wireless networks: fluid limits and instability issues", in *WiOpt 2012*, pp. 15-22, May 2012.

[21] K. Lam, C. Chau, M. Chen, and S. Liew, "Mixing time and temporal starvation of general CSMA networks with multiple frequency agility", in *Proc. of 2012 IEEE International Symposium on Information Theory (ISIT 2012)*, July, 2012.

[22] Fujitsu Microelectronics America, inc., "Achieving cost-effective broadband coverage with WiMAX micro, pico and femto base stations", White Paper, Oct. 2008.

[23] S. Ye, R. Blum, and L. Cimini, "Adaptive OFDM systems with imperfect channel state information", in *IEEE Transactions on Wireless Communications*, vol. 5, no. 11, pp. 3255-3265, Nov. 2006.

[24] A. Karr, *Probability*, Springer-Verlag, New York, 1993.

[25] IEEE 802.16e Task Group (Mobile WirelessMAN): http://www.ieee802.org/16/tge

[26] D. Xue and E. Ekici, "Queuing behavior of queue-backlog-based random access scheduling in the Many-Channel Regime", Technical Report, Ohio State University, available: http://www.ece.osu.edu/∼xued/multichannel.pdf

APPENDIX A
PROOF OF THEOREM 2

Theorem 2 can be proved by mathematical induction over time slot t. With the initialization (4)(11), I(-1), II(-1), III(-1), and, IV(-1) trivially hold as the base case. Suppose I$(t-1)$, II$(t-1)$, III$(t-1)$, and, IV$(t-1)$ hold, which we call as induction hypothesis. We prove that the induction step holds. Specifically, we prove I(t), II(t), and III(t) hold in the following subsections, and prove IV(t) holds in [26], which completes the proof for Theorem 2.

A. Proof of I(t)

From the induction hypothesis I($t-1$) and III($t-1$), we have

$$\begin{aligned}&q_i(t-1)+A_i(t-1)-\frac{C}{N}\sum_{j=1}^{N}\mu_{ij}(t-1)\\ &\xrightarrow{P}_N q(t-1)+\alpha-C\mathbb{E}\{v(t-1)\}.\end{aligned}$$

Thus, we conclude that I(t) holds, i.e., $q_i(t)\xrightarrow{P}_N q(t),\forall i$, by queue dynamics (1)(9) and the continuity of the function $[\cdot]^+$.

B. Proof of II(t)

Taking the conditional expectation on both sides of (2) leads to

$$\begin{aligned}&\mathbb{E}\{\mu_{ij}(t)|(\mu_{kj}(t-1))_{k=1}^{M},q_i(t-1)\}\\ =&F_0(q_i(t-1))-\sum_{k\neq i}F_0(q_i(t-1))\mu_{kj}(t-1)\\ &+F_1\mu_{ij}(t-1).\end{aligned} \tag{15}$$

Further taking the expectation of both sides of (15), we get

$$\begin{aligned}&\mathbb{E}\{\mu_{ij}(t)\}=\mathbb{E}\{F_0(q_i(t-1))\}\\ &-\sum_{k\neq i}\mathbb{E}\{F_0(q_i(t-1))\mu_{kj}(t-1)\}+F_1\mathbb{E}\{\mu_{ij}(t-1)\}.\end{aligned} \tag{16}$$

From the induction hypothesis I($t-1$) and II($t-1$), we have

$$(q_i(t-1),\mu_{kj}(t-1))\xrightarrow{L}_N (q(t-1),v(t-1)).$$

By applying the continuous mapping theorem [24], we find that

$$F_0(q_i(t-1))\mu_{kj}(t-1)\xrightarrow{L}_N F_0(q(t-1))v(t-1).$$

By the bounded convergence theorem [24], we further obtain

$$\lim_{N\to\infty}\mathbb{E}\{F_0(q_i(t-1))\mu_{kj}(t-1)\}=F_0(q(t-1))\mathbb{E}\{v(t-1)\}.$$

Similarly, we can obtain

$$\begin{aligned}&\lim_{N\to\infty}\mathbb{E}\{F_0(q_i(t-1))\}=F_0(q(t-1)),\\ &\lim_{N\to\infty}F_1\mathbb{E}\{\mu_{ij}(t-1)\}=F_1\mathbb{E}\{v(t-1)\}.\end{aligned}$$

Applying the above results to (16), we conclude

$$\begin{aligned}&\lim_{N\to\infty}P\{\mu_{ij}(t)=1\}=\lim_{N\to\infty}\mathbb{E}\{\mu_{ij}(t)\}\\ =&F_0(q(t-1))-(M-1)F_0(q(t-1))\mathbb{E}\{v(t-1)\}\\ &+F_1\mathbb{E}\{v(t-1)\}\\ =&F_0(q(t-1))+\left[(1-M)F_0(q(t-1))+F_1\right]\mathbb{E}\{v(t-1)\}\\ =&\mathbb{E}\{v(t)\}=P\{v(t)=1\},\end{aligned}$$

where the second to last equality follows from (12). Hence, II(t) holds, i.e., $\mu_{ij}(t)\xrightarrow{L}_N v(t),\forall i,j$, by definition of convergence in law.

C. Proof of III(t)

We first take the variance of $\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)$, for any given link i:

$$\begin{aligned}&Var\left\{\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)\right\}\\ &=N^{-2}\sum_{j=1}^{N}Var\{\mu_{ij}(t)\}\\ &\quad+N^{-2}\sum_{j,k=1,\dots,N,j\neq k}Cov\{\mu_{ij}(t),\mu_{ik}(t)\}\\ &=N^{-1}Var\{\mu_{i1}(t)\}+\frac{N-1}{N}Cov\{\mu_{i1}(t),\mu_{i2}(t)\} \qquad (17)\\ &\xrightarrow{N\to\infty}0, \qquad (18)\end{aligned}$$

where (17) follows the exchangeability of $\mu_{i1}(t),...,\mu_{iN}(t)$, and (18) is implied from

$$\lim_{N\to\infty}Cov(\mu_{i1}(t),\mu_{i2}(t))=0$$

since we have proved IV(t) in [26], i.e., $\mu_{i1}(t)$ and $\mu_{i2}(t)$ are asymptotically mutually independent.

Employing Chebyshev's inequality to $Var\{\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)\}$, we have $\forall\epsilon>0$:

$$P\left\{\left|\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)-\mathbb{E}\{\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)\}\right|\geq\epsilon\right\}\xrightarrow{N\to\infty}0.$$

Thus, by the exchangeability and the definition of convergence in probability, we conclude

$$\begin{aligned}&\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)-\mathbb{E}\{\mu_{i1}(t)\}\\ =&\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)-\mathbb{E}\{\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)\}\xrightarrow{P}_N 0.\end{aligned}$$

Employing II(t) (proved in Appendix A-B) to the above convergence result yields

$$\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)\xrightarrow{P}_N \mathbb{E}\{v(t)\},$$

which completes the proof of III(t).