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Predicting New Hampshire Indoor Radon Concentrations from Geologic Information and Other Covariates

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ABSTRACT

Generalized geologic province information and data on house construction were used to predict indoor radon concentrations in New Hampshire (NH). A mixed-effects regression model was used to predict the geometric mean (GM) short-term radon concentrations in 259 NH towns. Bayesian methods were used to avoid over-fitting and to minimize the effects of small sample variation within towns. Data from a random survey of short-term radon measurements, individual residence building characteristics, along with geologic unit information, and average surface radium concentration by town, were variables used in the model. Predicted town GM short-term indoor radon concentrations for detached houses with usable basements range from 34 Bq/m^3 (1 pCi/l) to 558 Bq/m^3 (15 pCi/l), with uncertainties of about 30%. A geologic province consisting of glacial deposits and marine sediments, was associated with significantly elevated radon levels, after adjustment for radium concentration, and building type. Validation and interpretation of results are discussed.

Introduction

Radon, a radioactive gas known to cause lung cancer when present in high concentrations, is a product of radium decay in rocks and soil. Indoor radon concentrations are necessarily strongly affected by soil radium content, soil permeability, and other geologic parameters, as well as by other parameters that control the ease with which soil gas can enter the building, the ventilation rate, and other building characteristics. However, direct quantitative measurements of important parameters, including geologic parameters, are not generally available, and a physical model to predict indoor concentrations based on such measurements is unavailable in any case. Past efforts to use geologic information to predict radon levels or to estimate "potential" radon concentrations have usually relied on correlation of geologic features (typically generalized geologic provinces) with measured indoor radon concentrations. Although some of these efforts have produced useful results, they have suffered from shortcomings such as the use of ad hoc scoring methods, failure to make testable quantitative predictions, inadequate or inconsistent handling of small sample variation in observed radon concentrations, and lack of measures of model fit and validation.

We have used a statistical technique known as Bayesian mixed effects regression to investigate the predictive power of geologic information in combination with other data related to indoor radon measurements. Bayesian methods have previously been applied to prediction of indoor radon concentrations based on data collected in relatively sparse radon screening measurement surveys (Price and others 1995, Price and Nero 1995, and Revzan and others 1996). These techniques can be used to estimate parameters, such as the geometric mean (GM), that describe radon concentration distributions in selected areas. Predictions based on statistical models of this type have the potential to provide guidance as to which geographic areas require the most urgent attention for such measures as intensive radon monitoring or mitigation.

The analysis and results presented in this paper use Bayesian mixed-effects regression analyses to predict the geometric mean indoor radon concentration for each of the 259 "towns" in the state of New Hampshire (NH). Note that in New Hampshire a "town" is a political unit similar to what is known as a "township" in some other states; some "towns" contain more than one village.

Data Used for Predictive Radon Modeling in New Hampshire

Data used in the analysis include: 1) radon "screening" measurements in 1814 dwellings selected from a stratified random sample of the state's housing stock, 2) physical characteristics of each building collected via questionnaire during the survey (Pirie and Hanington 1989; Pirie and Hanington 1990), 3) radium content of the surface soil (Duval and others 1989), and 4) the underlying geologic characteristics of the ground upon which each New Hampshire town lies, as identified by the U.S Geological Survey (USGS) in a modified radon geology map for the state (Gundersen and Schumann 1993).

The New Hampshire Radon Survey

Due to concern about the public health risks associated with the exposure to radon in residences, the state of New Hampshire, Division of Public Health Services conducted a stratified random survey of short-term indoor radon concentrations, or "screening measurements," during the winter months of 1988–1990 (Pirie and Hanington 1989; Pirie and Hanington 1990). Screening measurements are intended to quickly and inexpensively determine indoor radon concentrations. They are typically conducted in the basements of houses during winter when the house is relatively well sealed, so they tend to overestimate annual-average living-area radon concentrations by a factor of 1.5 to 3. We use the screening data because they are available and because they are expected to show approximately the same spatial patterns that would be present in long-term living-area measurements. Adjustment to calibrate screening data to predict annual-average living-area concentrations is possible in principle (Price and Nero 1996), but has not been attempted for these data.

The NH survey sample was stratified by 1) town or city population and 2) the predominant bedrock uranium content. Participating households were selected at random from within each stratum, based on telephone directory lists. Households which agreed to participate were mailed a radon screening measurement kit containing a charcoal absorption detector, a set of instructions, and a survey questionnaire. The participants were asked to expose the detector for three days on the "lowest livable level" of their home, which was usually a basement. The questionnaire obtained information about the home's construction, heating sources and usage, water supply, etc. Exposed charcoal detectors and completed questionnaires were returned by mail to the analytical laboratory and survey office, respectively. Further details of the survey design, implementation and results can be found in the survey report cited above.

Over the three winters during which the survey was conducted, 1814 dwellings in 232 of New Hampshire's 259 towns were monitored. Overall, 27 towns were unsampled, five towns had only one measurement, and the median number of measurements per town is 6. Only 10 towns had 30 or more measurements. For both the state as a whole and for individual towns, measurements appear to be approximately lognormally distributed (i.e. the logarithms of the measurements are normally distributed). The number of observations and the observed town geometric means (GM) and geometric standard deviations (GSD) for a randomly selected set of towns are presented in Table 1.

Fig. 1 depicts the distribution of the 1814 screening measurements. The calculated GM and GSD of these measurements are 81 Bq/m^3 (2.2 pCi/L) and 3.01, respectively. About 31% of the homes had measurements above 150 Bq/m^3 (4 pCi/L), the Environmental Protection Agency's (EPA) recommended "action level" for remediation. (Recall, however, that screening measurements substantially overstate long-term living-area concentrations.) About 2.5% had measurements above 740 Bq/m^3 (20 pCi/L). The distribution of screening measurements is approximately lognormal. Superimposed on Fig. 1 is a lognormal curve of GM=81 Bq/m^3 and GSD=3.01.

Table 2 summarizes some characteristics of homes included in the survey. Water source is a potentially important variable, since water drawn from a drilled artesian well can contain high concentrations of dissolved radon, which can escape into the house whenever water is used (Nazaroff and others 1987). Radon concentrations of 4×10^6 Bq/m^3 or more have been observed in some NH artesian water. All of the regression analyses presented in this work have been conducted on data from the subset of 1775 homes for which water supply information was reported.

Surface Radium Content and Geological Attributes of New Hampshire

The town-average surface radium content of the soil for each town was derived from digital maps from data from the National Uranium Resource Evaluation (NURE), which were processed (Duval and others 1989) to correct for various problems with the raw data. NURE measurements have been found to be correlated with mean indoor radon concentrations (Price 1996; Price and others 1995). Fig. 2 depicts the relationship between the natural logarithm of NH town radium levels (equivalent U, ppm) from NURE and the natural logarithm of town GM screening measurements (Bq/m^3). The vertical error bars indicate $\pm$ 1 classical standard error in the log of the town's GM. The line through the plotted points was calculated using an ordinary least-squares regression (R^2=0.09). The poor correlation, evident in the plot and the low value of R^2, is at least partly attributable to the small sample sizes within towns, since the observed town GMs vary substantially about the true town GMs. Thus even if NURE predicted the *true* town GMs perfectly (i.e., the GM that would be found if every home in every town were to be monitored), a low correlation with *observed* GMs is expected. One of the goals of the present analysis is to determine the extent to which various explanatory variables, including NURE, can be used to predict the *true* town GMs.

A digitized map of generalized geologic provinces in New Hampshire was obtained from the US Geological Survey (USGS). The provinces are based on both bedrock geology and soil characteristics, and were developed by Linda Gundersen and Randall Schumann of the USGS from their radon geology map for the state (Gundersen and Schumann, 1993). Table 3 lists the 9 geologic provinces which were used in our work, and Fig. 3 shows them on a map of New Hampshire.

The geologic data were processed through a Geographic Information System (GIS) and superimposed over the town boundaries, and the prevalence of each geologic unit within each town was calculated. Many towns were found to lie on two or more geologic units. For example, the prevalence of geologic units 541, 544, and 561 in Alexandria, NH is 48%, 13%, and 39%, respectively.

Statistical Modeling Techniques

Observed distributions of indoor radon concentrations often appear to be approximately lognormal, irrespective of scale (Nero and others 1986, Nero and others 1990; Price 1996; Price and others 1995).

This is probably because the indoor radon concentration in a home can be written as a product of factors that are not perfectly correlated (soil radium content, times emanation fraction, times soil permeability, etc.), so that the logarithm of the indoor radon concentration can be written as a sum of logarithms of separately variable factors. The central limit theorem then implies that, if the individual factors are sufficiently variable across the population of homes, the resulting distribution of indoor concentrations will tend towards a normal distribution (in log-space). Fig. 4 shows quartile plots of the logarithm of the concentration measurements in the six NH towns with more than 25 measurements. Vertical scales for the different towns have been shifted to avoid overlap. On this type of plot, measurements that are normally distributed fall on a straight line, with the slope of the line determined by the standard deviation. It can be seen from Fig. 5 that the log radon measurements in these towns are approximately normally distributed, with mildly variable standard deviations; in untransformed space, these towns have approximately lognormal distributions of measurements, with similar GSDs.

Fig. 5 presents all of the observed town GSDs as a function of the number of observations made in each town. The weighted mean within-town GSD for all of the measured towns in the state is 2.6. If every town had the same true GSD of 2.6, then 95% of the observed GSDs would fall between the two lines on the figure. The figure suggests that most of the observed variation in GSDs is due to small sample sizes within towns. In other words, it looks as though given enough measurements, the GSD of the measured town screening measurements within most towns would be close to 2.6 for all towns in the state.

Based on previous experience (Price and others 1996, for example), the included explanatory variables are expected to have multiplicative rather than additive effects on indoor radon concentrations, so it is computationally convenient to work with the logarithms of the radon measurements rather than with the measurements themselves. We would like to write the log of the radon measurement in each home as a sum of terms associated with various explanatory variables (NURE, presence of a basement, geologic type, what town the home is in, etc.) plus residual variation. A slight complication is that the geologic type associated with each home is unknown, since the only location information available is the town that the home is in, and many towns contain more than one geologic type. In the absence of specific geologic knowledge associated with each home, we fit a model in which each home's prediction has a contribution from each of the geologic types present in the town, weighted by prevalence, as explained below.

The full statistical model used to predict individual short-term radon measurement (SRM) radon concentrations follows: we write Y_{ij}, the natural logarithm of the SRM in home i, which is in town j, as

$$Y_{ij} = X_{ij} \cdot \beta + \sum_{k} f_{jk} \alpha_k + \theta_j + \varepsilon_{ij} \tag{1}$$

where X_{ij} is a vector of explanatory variables for home i in town j, and is known. The explanatory variables in X are: the logarithm of the town-average NURE measurements, the building type, above/below-grade measurement location, presence of below-grade living space, presence of a forced-air heating system, the domestic water supply, and a constant term. The vector β is composed of coefficients associated with the explanatory variables in X; these coefficients are to be estimated from the data. The parameters $\{\alpha_k\}$ are the "geologic unit effects" (discussed below), and allow some geologic units to have generally higher or lower radon levels than others do, by amounts to be estimated from the data; f_{jk} is the fraction of town j that is composed of geologic type k, and is determined from digitized town and geology maps. The parameters $\{\theta_j\}$ are called the "town effects" (discussed below), and represent the amount by which the logarithms of the true town GMs differ from the predicted values based on the other explanatory variables in the model. Finally, the logarithm of the radon measurement generally differs from its predicted value by a residual, ε_{ij}.

The building type, forced-air heating, measurement location, and water supply variables were entered into the model as indicator ("dummy") variables. For example, there is a variable that takes the value of one for homes that are apartments, and zero for homes that are not apartments. A second variable takes the value of one for mobile homes, and zero for others. The coefficients associated with these variables indicate the extent to which measurements in apartments and mobile homes, respectively, are elevated or depressed relative to detached homes. Thus, a negative coefficient for "apartment" implies that indoor radon measurements in apartments are lower than single family detached homes, after controlling for all of the other variables in the model.

We assume the effects associated with the various geologic units are drawn from a normal distribution (in log space) with mean 0 and unknown variance τ^2 to be estimated from the data:

$$\alpha_k \sim N\left(0, \tau^2\right) \tag{2}$$

The assumption that the geologic unit effects distribution has mean zero does not reduce the generality of the model, since any overall shift in the distribution of geologic unit effects is absorbed by the constant term in the model (see Equation 1). This is also true for the town effects, discussed below. The normality assumption is chosen partly for computational convenience, but also for substantive reasons: we expect a few geologic units to be associated with highly elevated or depressed indoor radon concentrations, while most of the rest are more "typical", there is no *a priori* reason to expect the distribution of geologic effects to be bimodal, for example. Model appropriateness and validation will be discussed in a later section.

We assume the town effects are drawn from a normal distribution with mean 0 and unknown variance δ^2 to be estimated from the data:

$$\theta_j \sim N\left(0, \delta^2\right) \tag{3}$$

The role of the town effects $\{\theta\}$ is as follows. If the regression coefficients $\{\beta\}$ and all of the geologic unit effects $\{\alpha\}$ were known, then the town effect would be the residual between the town's true SRM ln(GM) and the prediction based on the other explanatory variables. If the town effects were all close to zero, this would indicate that the predictions based on the other explanatory variables were very accurate; if, on the other hand, town effects tended to be large, this would indicate that predictions based on the other explanatory variables are subject to large errors. In the present analysis, the town effect estimate fulfills two roles: first, it allows the final prediction of the true SRM values to be directly influenced by the observed town GM — if the prediction from the explanatory variables alone differs from the observation, the final prediction for the town (including the town effect) is a weighted average of the regression-predicted value and the observed value. Secondly, the typical size of the town effects (as described by the variance δ^2, or related measures) provides an estimate of the variation of the true SRM GMs about their regression-predicted values.

We assume that after the best coefficients for the explanatory variables β, α, and θ are obtained, residual error ε_{ij} for a home is drawn from the distribution:

$$\varepsilon_{ij} \sim N\left(0, \sigma^2\right) \tag{4}$$

For any particular model, the variance of the residuals, σ^2, provides an indication of model fit at the individual house level, with smaller values of σ^2 indicating better predictions of individual house log radon measurements.

The town effects variance δ^2 and the residual variance σ^2 address different aspects of model fit. The town effects variance addresses the issue of how well individual town means are predicted by the explanatory variables, but ignores within-town variance. The residual variance addresses the issue of how well the measurements in individual homes are predicted; since town effects are included in all of our models, this parameter summarizes the variation between homes within towns, and ignores variation between towns.

In practice, of course, we do not know the values of the regression coefficients $\{\beta\}$, the geologic unit effects $\{\alpha\}$, nor the town effects $\{\theta\}$. We can, however, obtain estimates of all of these parameters through use of a statistical technique known as a mixed-effects regression. A Bayesian mixed effects regression is analogous to an ordinary multivariate regression, except that some parameters — town effects $\{\theta\}$ and geologic unit effects $\{\alpha\}$, in this case — are not allowed to vary independently, but are assumed drawn from common distributions. Such parameters are known as "random effects", as opposed to the "fixed effects" $\{\beta\}$ which are treated independently of one another as in a conventional regression; the combination of both types of parameters leads to the terminology "mixed effects regression." The term "random effects" does not imply that the coefficients are actually assigned at random, merely that *except* for the data included in the model we have no knowledge that tells us which effects should be high and which should be low. Knowledge of the distribution from which parameters are drawn can help substantially in minimizing the effects of statistical noise due to small sample sizes. For example, if we know the range in which most town GMs fall, we have some knowledge about the range of likely values even for a town with no observations at all. Detailed discussion of Bayesian mixed effects regression is outside the scope of the present paper. For a discussion of the topic as a whole, including computational and validation issues, see

Gelman and others (1995); for an application to predicting county radon concentrations in Minnesota, see Price and others (1995).

The assumptions that town effects and geologic unit effects are drawn from common distributions for which the variance is estimated from the data reduce the danger of substantially "overfitting" the model. In a conventional regression, if we include a regression coefficient for every town the coefficients will be estimated so that the model fits the observed town GMs exactly. Such a model would fit the data perfectly but would be highly unsatisfactory on substantive grounds. For example, consider Langdon, NH. This town had only one observation, which was 174 Bq/m^3 (4.7 pCi/L). Choosing a town effect estimate so that the GM of this county is predicted to be 174 Bq/m^3 would fit the data perfectly. However, such a prediction would not be appropriate since it does not take into account the possibility that the sampled home had a particularly high, or low, measurement compared to true town GM. In fact, most towns in the same geologic unit have observed GMs much lower than 174 Bq/m^3, so it seems likely that the single measured home in Langdon, NH, had a higher radon concentration than is typical in the town, at least for the period over which it was monitored. Bayes's theorem, upon which the approach of random (and mixed) effects regression is based, puts this idea on a firm statistical footing. Generally, the posterior estimate for a random effect is a compromise between the best-fit value and the expected value based on the distributional information for the parameter.

As it happens, under our model if the values of the variance parameters (τ^2, σ^2, and δ^2) were known exactly, the posterior estimates and uncertainties in the geologic effects, town effects, and other coefficients could be determined analytically from the data. But of course, the variance values are themselves uncertain. In order to correctly incorporate this uncertainty, we perform many different regressions, each with a different set of variances. We use a Monte Carlo method to sample from the distribution of likely values of the variance components, and then determine the regression coefficients (and individual town and geologic effects estimates) given that set of variance values (see Gelman and others 1995). Performing this procedure many times (several hundred) allows us to take account of the uncertainties in the distributional parameters and the regression coefficients.

The outcome of the series of regressions is a set of estimates, one for each simulation, for every geologic province coefficient, every town coefficient, and every regression coefficient. Through coefficients of large magnitude, more influence is given to very reliable indicators of the true measurements, and through coefficients of small magnitude less influence is given to less reliable indicators. The method empirically determines the coefficients that best fit the data given the constraints of the model, and repetition of the regression with different variance estimates allows us to incorporate our uncertainty in each of the parameters. The variation in the estimates of a particular parameter indicates the uncertainty of that parameter's value. A set of posterior predictions, having a distribution of several hundred estimates for each observation, can be created by multiplying each vector of coefficients by the matrix of predictive variables used in the model.

Parameter Estimates from the Model

Table 4 presents coefficient estimates resulting from a number of Bayesian mixed-effects model fits incorporating different classes of variables. The regressions do not generate a point estimate for each coefficient, but rather a distribution of possible parameter values. The table shows the mean of this distribution, for each coefficient, and the last row in Table 4 consists of the standard errors of the estimates for the full model described above, denoted in the table as Model 10. The uncertainties for the other models are similar to those of Model 10.

Rather than showing all 232 town coefficients and 9 geologic unit coefficients, we show only the estimated variance of their distributions, in Table 4. By definition (equations 2 and 3), the mean of these distributions is zero.

Although we are most interested in the results of the full model presented above, we have performed a variety of fits using various combinations of explanatory variables, to explore the extent to which the individual classes of variables add predictive power. The simplest model summarized in Table 4 is that of a mixed effects regression of the observed log SRM against a constant term, with town effects. This model uses the radon measurements alone to estimate the between-town variation in log radon measurements, without controlling for any explanatory variables. The expression for this model is:

$$Y_{ij} = \beta_0 + \theta_j + \varepsilon_{ij} \tag{5}$$

As described in equation 3, values of θ are assumed to have been drawn from a normal distribution with a mean 0 and a variance of δ^2 which is unknown. The estimate for β_0 was 4.43±0.05 (in units of ln(Bq/m^3)), and that for δ^2 is 0.28±0.05. In untransformed space, this model assumes that the town GM radon concentrations are themselves drawn from a lognormal distribution. The parameter estimates suggest that the distribution of town GMs has a GM of exp(4.43) = 84 Bq/m^3 (2.27 pCi/L) and a GSD of exp(δ) =1.7. The town effects coefficients produced by the procedure can be used to predict the individual town radon GM and GSD, though with considerable less precision than those from the models that include additional explanatory variables.

Measures of Model Fit

As discussed above, the variance of the residuals, σ^2, can be used to assess the predictive error of the models. (Note that within the context of Eq. 5, which contains no individual-house variables, σ^2 can be thought of as the "within-town variance.") This quantity can be used to make a comparative measure of model fit for predictions at the individual house level similar to the "R^2" used in conventional regressions (Bryk and Raudenbush 1992; Price and others 1995). This "effective" R^2 can be expressed as:

$$R^2_{indiv} \cong 1 - \frac{\sigma^2_m}{\sigma^2_{SRM}} \tag{6}$$

where

σ^2_m = unexplained variance of true ln(SRM) in model m, and

σ^2_{SRM} = total variance of true ln(SRM).

Although the actual unexplained variance and the total variances are unknown, we can estimate them from the data. σ^2_m can be estimated by the value σ^2 derived from the random-effects regression fits of a model. The total variance, σ^2_{SRM}, can be estimated as the calculated variance of the actual ln(Rn) values for the

NH survey The total variance of the measured ln(Rn) values in the 1774 homes in the analysis is 1 2 R^2_{indiv} for Model 1, as shown in Table 4, is 0 18 This indicates that about 18% of the variation in the logarithms of the individual radon measurements is attributable to variation between towns, with the remainder being due to variation within towns

The variance in town effects (δ^2) gives an indication of the extent to which the difference between towns is explained by the variation in the explanatory variables This metric can be thought of as the "unexplained between-town variance " Model 2, which includes NURE, is defined by

$$Y_{ij} = \beta_0 + \beta_1 \cdot \log(NURE_j) + \theta_j + \varepsilon_{ij} \tag{7}$$

The estimated town effects variance is δ^2 of 0 22±0 04 (corresponding to a GSD of town effects of exp(0 22)=1 6), slightly lower than the value of 0 28±0 04 for Model 1 Recall that the town effects (θ) proxy for sources of town-to-town variation that aren't included in the model As such, we wouldn't expect town-level explanatory variables such as NURE to explain within-town variance Addition of a variable could enormously improve the model fit at the town level (*i e* could improve prediction of the town GMs) without changing R^2_{indiv}

The change in between-town variance can be utilized in the metric R^2_{town} to compare how well different models predict the town mean log radon measurements This metric, also presented in Table 4, similar to R^2_{indiv}, is simply

$$R^2_{town} \equiv 1 - \frac{\delta^2_m}{\delta^2_{town-only}} \tag{8}$$

where

δ^2_m = unexplained between-town variance in model *m*, and

$\delta^2_{town-only}$ = total variance of town-effects, *i e*, variance in Model 1 which has only town-effects and a constant Model 2 (Eq 7) has an R^2_{town} of 0 19 This value can be compared to the simple R^2 of 0 09 from the regression of ln(NURE) against observed ln(GM SRM). The increase from 0.09 to 0.19 can be attributed to a reduction of the sample-noise present in the simple regression.

Model 9 shows a slight improvement in the prediction of indoor radon levels through the addition of the water-source variables. Model 10, the full model as described by Eq. 1, includes all of the fixed-effect variables, the town-effects and geologic unit effects. Adding the geologic unit effects to Model 9 does not change the R^2_{indiv}, which represents residual variation after controlling for all of the explanatory variables. This is expected, since within any town all of the homes are assigned the same combination of geologic contributions, in the absence of geologic information at the individual house level. However, Model 10 does show substantially reduced town effects compared with Model 9, indicating that the geologic variables do help explain the variation of radon measurements between towns.

The estimated town effects variance is considerably lower for models that include the geologic units than for other models, although the unexplained variation is still substantial. For example, consider a town from which we have no monitoring data (and thus no information on its town effect), but for which we do have explanatory variables, including geologic information, for homes in the town. We expect our prediction of the town's GM to be off by about a factor of $\exp(\delta)=1.46$ (see Model 10 in Table 4), a considerable improvement over the factor of about 1.6 or 1.7 that would obtain without the geologic information, but still a large expected error.

The estimated town coefficients, which represent the amount by which mean town log radon measurements differ from the value expected from the explanatory variables, are not presented fully in this paper. In Model 10, they range from -0.56±0.23 for Belmont, NH, to 0.86±0.36 for Clarksville, NH. In each case the listed standard error is a posterior interval rather than a classical confidence interval, an important distinction that is discussed below in the context of the geologic effects. A full listing of these coefficients is available on the internet web-site of the HIGH-RADON PROJECT at "http://eetd.lbl.gov/IEP/high-radon/hr.html."

Posterior Predictions of Town Geometric Means

The last column in Table 1 presents predicted town GM SRM values calculated from the posterior distribution of Model 10 for a random selection of NH towns. Fig. 6 is a map of NH towns with shading used to depict the posterior predictions of GM SRM values for all 249 towns in New Hampshire, for detached single-family dwellings with occupied basements (*i.e.*, MH = AP = BGL = MAG = 0, see Table 4 for identification of these abbreviations), no forced-air furnace (i.e., FAF = 0), and a municipal water source (*i.e.*, DW = SW = 0).

We present the results in this way since for most towns we have little data on the fraction of homes in each town that are not single-family detached homes, that have forced-air heating, etc. All we know are these fractions for the homes in our data set, and the small number of homes in most towns makes the true fractions very uncertain. Predicted GMs for other types of homes can be calculated by multiplying by the appropriate factor. For example, from Table 1 the posterior prediction of GM SRM for single-family detached dwellings with municipal water and no forced air furnace in Madison, NH is 214 ×/÷1.39 Bq/m^3 (note multiplicative error). To calculate the predicted GM for, say, mobile homes that use municipal water and have no forced-air heating—and which are typically measured above grade, with no below ground living space—we must adjust for several factors: mobile homes have different (lower) measurements than do single-family homes, homes without below-ground living space have somewhat different (lower) measurements than homes with below-ground living space, and measurements above grade are substantially lower than those made below grade. No adjustment is necessary for heating type or water supply, since these match the "standard" home for which the predictions were generated. The required adjustment factor, then, (from Table 4) is:

$$\exp(-0.73_{MH} - 0.52_{MAG} - 0.10_{BGL}) = e^{-1.35} = 0.26. \qquad (9)$$

Thus the predicted GM for mobile homes in Madison is 0.25 x 214 Bq/m^3 = 55 Bq/m^3. Note the subscripts in Eq. 9 refer to the variable for which the coefficients apply.

The lowest and highest predicted NH town GMs are for Antrim (34 Bq/m^3 or 1 pCi/l, GSD = 1.3) and Harts Location (558 Bq/m^3 or 15 pCi/l, GSD = 1.5), respectively. Posterior predictions for all towns in NH using Model 10 are listed in Appendices I and II.

Model Validation

In this section, two methods are used to assess how well the assumptions of the model agree with the data. The first method, known as "posterior predictive checks," simulates data from the full-Bayes model and compares them to the observed data. The second method, "cross-validation", is to run a full-Bayes model-fit on a subset of the data and use the results to predict the radon concentrations in the homes which were excluded from the dataset. A comparison of the predicted radon levels in these houses to the observed levels provides a measure of how well the model is working.

Posterior Predictive Checks.

Bayesian posterior predictive checks have been suggested as a means to assess the ability of a model to produce realistic simulations. Such checks can point out possible model violations, as described fully by Gelman and others (Gelman and others 1995). Posterior predictive checks are performed as follows: we fit our statistical model to the data, thus generating predictions and uncertainties for each of the coefficients (including geologic and town effects) as well as for the variance components. Using these predictions, we then generate predictions for each home's SRM as well as the uncertainty in this quantity. Sampling from this distribution for each home, we construct an "imputed" or "simulated" data set. This data set can be thought of as another "possible" data set that could have occurred, if another New Hampshire survey were conducted with another set of homes with the same explanatory variables (but not necessarily the same radon levels) as the homes in the actual survey. A significant discrepancy between the simulated data set and the actual data may indicate a model violation.

This procedure may seem circular, after all, our predictions and variance estimates are generated from the data, so aren't our imputed data guaranteed to agree with the actual data? The answer is no, because the predictions and variance estimates do *not* just depend on the data, but also on the assumptions built into the model. If these assumptions are seriously in error—for instance, if the normality assumptions or the assumption of uniform variance in the residuals are very wrong—the imputed data can differ greatly from the actual data. Some test statistics are more sensitive than others to such model choices, with effects of non-normality or heteroskedasticity (non-uniform variance) showing up most strongly in the tails of the distribution of measurements.

Fig. 7 presents a set of histograms of the distributions of imputed 10th, 20th, 50th, 80th, 90th, and 99th percentile SRM values from model 10 (using 1200 draws from the posterior distribution). These histograms depict the variation in predicted values at their percentile. For example the upper-left histogram in Fig. 7 shows that under the model, the 10th percentile measured value is expected to be within about 1 or 2 Bq/m^3 of 21 Bq/m^3. Each histogram in the Figure has a solid vertical line indicating the appropriate percentile of the observed SRM distribution. The 10th percentile SRM in the actual data is just over 22 Bq/m^3, which is towards the high side of the expected distribution from the model: As the "Bayes p-value" of 0.13 shown on the figure indicates, in only 13% of the simulation draws was the imputed 10th percentile SRM as high or higher than in the actual data. So if the model is "correct" then a 10th percentile measurement as high as 22 Bq/m^3 is somewhat unusual. Extreme Bayes p-values — very close to 0 or 1 — indicate that the observed data are very unexpected under the model, suggesting the model assumptions may be substantially incorrect. This may or may not represent a serious problem, depending on the sensitivity of the parameters of interest to the problems with the model.

In the present case, the imputation does not indicate any serious problems for the bulk of the data: although the Bayes p-values for the 10th, 50th, and 80th percentile are all further from 0.5 than we would like, the discrepancies between imputation and actual data are very small in absolute terms, of the order of a few Bq/m^3. For example, in the case of the 50th percentile, the Bayesian p-value of 0.94 indicates a fairly poor fit, but on substantive grounds this means little since the 5 Bq/m^3 (0.1 pCi/L) difference between the observed value (approx. 82 Bq/m^3) and the median of the posterior distribution (approx. 87 Bq/m^3) is of no consequence.

However, the posterior predictions from Model 10 clearly underestimate the observed 99th percentile of radon concentrations. In fact, in only about 0.2% of the 1200 simulations was the 99th percentile SRM as high as the observed value of 1550 Bq/m^3! On substantive grounds too, the model also appears to fail to capture these tail concentrations: the difference between the median prediction and the true value is several hundred Bq/m^3. The model assumptions that lead to these discrepancies are probably either the assumption of homoskedasticity (equal variance) within towns and geologic provinces, or the assumption of normality of the residuals in log space. This problem only occurs for the highest few percent of measurements, and this failure of the model to predict individual high homes does not have a significant influence on the estimates of the individual town and geologic unit effects because these estimates are dominated by the bulk of the data. Modifying the model to better fit the high tail of measurements would lead to increased computational complexity and would make the results harder to interpret, without substantially affecting the estimates of the geologic effects and town effects. For this reason, we retain our simple model and merely note that it would not be a good idea to use the model to predict, say, the number of homes in the state in which a screening measurement would exceed 750 Bq/m^3 (20 pCi/L).

Model Validation Using a Restricted Data Set.

In order to search for other violations of the model, a validation data set was created by randomly removing 80% of the data from the 27 towns in the NH survey with 15 or more observations. Most of these towns are from the highly populous southeastern portion of the state. Model 10 was re-run with the reduced dataset (N=1320). Observed data from the remaining 455 homes from the previously well-sampled towns were then available for comparison with the model predictions for those homes. Fig. 8 shows the relationship between observed and predicted town mean ln(radon) values for the validation dataset. The diagonal line in this figure represents the theoretical perfect fit (slope =1, intercept =0) for this comparison. The observed mean for each town was calculated directly from the full survey dataset. The predicted mean for each town was calculated from the individual town means from the 1200 sets of posterior predictions. The error bars for each point (each town mean) in the figure indicate ± 1 standard error, calculated from the distribution of posterior predictions.

Note that only 3 of the 27 distributions of predicted mean town ln(radon) levels are more than 1 standard error away from the observed value, in contrast to the 9 or so towns that are expected to differ by that amount. That is, the model appears to predict better than it is expected to (the stated uncertainties are too large). Investigation of the model predictions reveals the explanation: most of the well-sampled towns are from the southeastern part of the state, and one of the two major geologic types there—G438, Granitic plutons, metasedimentary rocks—shows much less relative variation between towns than do other geologic types. The model assumes that all geologics are equally variable (in log space), and thus expects more unpredicted variation between towns than is present in this geology. The model could be modified (with difficulty) to estimate separately the town effects variances for the different geologies, but we have not performed this task. Only two geologic provinces show evidence of substantially atypical variability: G438 (mentioned above) which is much less variable than most geologies, and G546, Jurassic-Cretaceous rocks of the White Mountain and New England-Quebec igneous succession, which is much more variable than most for reasons discussed below. Fitting a more complicated heteroskedastic (multi-variance) model would provide somewhat better predictions for towns in these provinces, but the main effect would to change the error estimates: posterior intervals for towns in G438 would be narrowed, and those for G546 would be widened, compared to the predictions generated from Model 10.

Interpretation of Geologic Unit Effects

The posterior estimates and uncertainties of the geologic coefficients $\{\alpha\}$ from Model 10 are included in Table 3. Recall that NURE is included in this model, so these geologic coefficients represent the effects of the various geologic provinces after accounting for variation in surficial radium concentration. Many of the coefficient estimates are consistent with zero, in the sense that the standard errors overlap zero. However, there is an important distinction between the posterior intervals given in the table and classical confidence intervals. The classical estimates would be chosen to best fit the data, and overlap of the confidence intervals with zero might then lead one to conclude that the estimates are not "significant" and that all of the true coefficients are likely to be closer to zero, and might all be zero. The posterior estimates shown in the table, though, have *already* been "pulled" towards zero through Bayes' theorem (recall that the posterior estimate is a compromise between the best-fit estimate and the mean of the distribution of effects, which is zero in this case). If the statistical model we have fit is appropriate then each coefficient's true value is equally likely to be higher or lower than the posterior estimate. Thus the fact that many of the standard errors overlap zero should *not* be taken to indicate that the geologic effects overall are small.

At least two of the nine provinces are associated with large effects on radon measurements. The "glacial deposits and marine sediments" geology (ID number G411) is present in the Northeastern part of the state and has a high estimated coefficient (1.09 ± 0.52), indicating a substantial association with elevated indoor radon levels even after controlling for other risk factors such as NURE, building type, water source, town effects, etc. Town GM radon levels for this province can be expected to be a factor of about $\exp(1.1) = 3$ times higher than the average for other towns with similar house construction and surface uranium concentration. Silurian-Devonian intrusive rocks (ID numbers G541) are associated with substantially depressed levels, about a factor of 0.4 times as high as the median geology. Two additional provinces (ID numbers G546 and G561) are also associated with substantial estimated effects. Several of the other geologies might also have large effects, but the available data are not sufficient to determine which (if any). Fig. 3 maps the geographic distribution of geologic units with an index signifying the strength of the coefficients.

The variance of the distribution from which the geologic province effects were drawn, τ^2, is quite uncertain, with values as low as 0.2 and as high as 1.4 being marginally consistent with the data. However, the exact value of this variance is not particularly informative in the present case, since only a small number (9) of different geologies are present—we are interested in the effects associated with these particular geologies, not the distribution of "possible" geologic effects from which these values were drawn. The assumed "hyperdistribution" from which the geologic effects were drawn is a mathematical fiction that helps us quantify our uncertainties in the individual geologic effects.

The fact that some of the geologic provinces are associated with substantially elevated (or depressed) radon concentrations does not necessarily mean that this association is causal. With a few exceptions the geologic provinces tend to be fairly compact and localized, so that any strong spatial correlation could manifest itself in large geologic province effects. In fact, a fit that uses the 10 counties in New Hampshire instead of the geologies — primarily a spatial assignment, although the county boundaries do follow some geographic features — performs as well statistically as the fit based on the 9 geologic provinces. The county boundaries allow for one important feature that the geologic provinces do not: in the central latitudes of the state, towns east of roughly the North-South mid-line have much higher radon measurements than do towns west of that line, even after controlling for NURE and house construction variables. The division between high and low town GMs corresponds approximately with the border between Carroll and Grafton counties, so the county variables can capture this difference in radon levels. In contrast, geologic province G544 contains both low-radon towns to the west and high-radon towns to the east. This is the reason for the large town effects variance within this geologic province, which was noted above: the high and low towns cannot be predicted from geologic province, NURE, and the other explanatory variables, so the only remaining source of variability is the town effects. On the other hand, elsewhere in the state the geologic province boundaries perform better than do the county boundaries, particularly in the southeastern and northern portions of the state. In short, the reason for the demonstrated relationship between the geologic provinces and the indoor radon measurements is unclear.

Also unclear is the extent to which the abnormally low variance between towns in geology G438 is causal, as opposed to being the result of, say, more uniform housing stock in this fairly urbanized area.

Discussion and Conclusions

A Bayesian mixed-effects linear regression model has been used to predict short-term indoor radon concentrations in residences in New Hampshire with more precision than would be possible using available monitoring data only. Validation checks indicate minor model violations that do not strongly influence estimates of the main parameters of interest, which are town GMs and the coefficients associated with geologic provinces and other explanatory variables. Geologic indicator variables were included in some models, as were predictive variables that are believed to be directly related to indoor radon concentrations: town-average NURE data are a measure of radium in the surface soil. Structure-specific variables including building type, below-grade living space, forced air furnace heating system, and water supply source are all variables likely to affect the equilibrium indoor radon concentration were included. Since radon generally enters through the lowest floor of the home measurements are expected to be higher if made below grade.

One assumption upon which the model relies is that the distributions of indoor radon concentrations within towns are lognormally distributed. This assumption appears to be approximately correct, based on the observed distributions of several well-sampled NH towns. The posterior predictions of town GMs from the full model range from 34 Bq/m^3 (1 pCi/l) with a multiplicative uncertainty of 1.3, to 571 Bq/m^3 (15 pCi/l) with a multiplicative uncertainty of 1.5.

Model 2, in which only town effects and NURE are used to predict short-term radon concentrations, suggests that NURE alone does considerably better than suggested by the value of R^2 in the conventional regression, with $R^2_{town} = 0.18$ compared to the conventional regression estimate of $R^2 = 0.09$: the observed GMs vary about their true values due to small sample variation, depressing the conventional R^2 estimate.

Addition of geologic province information, as in Models 3,4, and 10, substantially improves prediction of the town GMs compared to models that exclude this information, with R^2_{town} of the order of 0.5 rather than between 0.15 to 0.20. However, as discussed previously the reasons for this improvement are not clear—it may be that the geologic provinces merely capture some spatial variation that is due to other causes not included in the model.

There is considerable variation in town GMs across the state—indeed, as can be seen from the estimate of R^2_{indiv} in the first line in Table 4, about 18% of the variance in individual house log radon measurements is attributable to the fact that some towns have elevated concentrations compared to others. Improvements in the fit are also seen when characteristics of the individual homes are included, leading to an increase of R^2_{indiv} from 0.18 to 0.26. The coefficient estimates are reasonable in both magnitude and direction: measurements made above grade are considerably lower than those made below grade, and measurements in mobile homes are much lower than those in single family homes, by a factor of exp(0.73) = 2, compared to single-family homes monitored above grade. The coefficient of 0.20±0.06 for drilled wells indicates that residences with this water source have 20 percent higher radon values than those with a municipal water source.

Forced air furnaces are associated with about a 15% reduction in measured concentrations, but this finding must be interpreted with some care. For instance, it may be that forced-air furnaces cause increased mixing between the basement and upper floors, and are thus associated with decreased radon concentrations in the basement (where most measurements were made) while increasing concentrations upstairs. The present data, which do not contain measurements for multiple floors in the same homes, are not sufficient for us to investigate this possibility.

The approach used in this paper has provided a means to use radon survey screening data and other explanatory variables to more precisely predict short-term indoor radon concentrations and town GMs. Bayesian modeling helps reduce the effects of sampling variation allowing more precision than possible in analyses based on available monitoring alone. The approach appears to work well in predicting short-term indoor radon distributions at a scale as small as individual towns. The inclusion of geologic unit information has been useful in identifying high-radon areas and serves to increase the predictive power of the model, but results are ambiguous with respect to the causal relationship between the included geologic province information and the radon measurements. More information on the development and results of these methods is available through e-mail to "high-radon@lbl.gov," or via the world-wide web as discussed above.

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Tables

Table 1. Summary of screening data, average surficial uranium concentrations (NURE) and posterior predictions from Model 10 for 25 of the 259 New Hampshire towns.

Town Name	Number of Homes	Observed GM (Bq/m^3)	Observed GSD	NURE (ppm U)	Predicted GM for Houses with Basement (Bq/m^3)
WINDSOR	1	15		2.02	40 ×/÷1.44
WEBSTER	7	31	2.04	1.58	38 ×/÷1.31
CENTER HARBOR	2	32	1.38	1.77	45 ×/÷1.39
WARNER	6	36	3.98	1.99	53 ×/÷1.32
MONROE	4	43	1.78	1.49	51 ×/÷1.35
SUPEE	5	43	2.22	3.01	73 ×/÷1.32
MEREDITH	12	46	2.38	2.17	77 ×/÷1.28
HINSDALE	11	51	2.44	1.88	82 ×/÷1.26
ALEXANDRIA	4	59	1.25	2.84	74 ×/÷1.33
NEWINGTON	5	59	2.95	2.50	104 ×/÷1.33
ANDOVER	7	65	4.93	2.11	78 ×/÷1.29
EASTON	4	76	2.02	1.46	62 ×/÷1.37
BENTON	5	77	2.56	1.74	82 ×/÷1.32
MONT VERNON	6	78	3.00	3.33	79 ×/÷1.31
GOFFSTOWN	14	82	2.49	3.15	96 ×/÷1.24
PEMBROKE	9	88	3.26	1.76	87 ×/÷1.28
WINCHESTER	5	92	8.10	1.93	80 ×/÷1.34
MIDDLETON	4	127	2.12	2.87	123 ×/÷1.36
COLEBROOK	8	127	2.89	1.40	90 ×/÷1.31
CANDIA	13	141	2.25	3.28	129 ×/÷1.26
BRADFORD	2	159	5.42	1.66	59 ×/÷1.41
STARK	4	270	6.06	2.07	168 ×/÷1.36
GORHAM	16	323	3.45	2.56	281 ×/÷1.24
WOODSTOCK	4	427	2.23	2.43	129 ×/÷1.39
MADISON	4	589	2.04	3.59	209 ×/÷1.39

Table 2. Summary of home types in the New Hampshire radon survey.

	number of homes	percent of homes
Total number of homes surveyed	1814	100
Number of homes in the present analysis	1775	98
single family detached	1661	91
multi family	85	5
mobile home	68	4
basement is used as living space	565	31
no basement or basement not lived in	1249	69
monitored in basement (if any)	1485	82
no basement or not monitored in basement	329	18
forced-air heating system	584	32
other heating system	1230	68
town water supply	639	35
shallow well	305	18
drilled well	831	46
unknown	39	2
home built before 1900	266	15
1900–1950	267	15
1950–1974	439	24
1974–1990	842	46

Table 3. Groupings of New Hampshire geologic types used in the radon predictive model and geologic effects coefficients derived from Model 10.

USGS Geologic Type ID Number	USGS Geologic Type Description	Geologic effects Model 10 (mean ± std err.)
G411	Glacial deposits and marine sediments	1.10 ± 0.52
G438	Granitic plutons, metasedimentary rocks (phyllites, carboceous slates and schists).	-0.26 ± 0.39
G541	Silurian-Devonian intrusive rocks ranging from gabbro to granite.	-0.88 ± 0.37
G542	Cambrian-Early Ordovician metamorphic rocks	0.02 ± 0.99
G544	Devonian-Carboniferous two-mica granite.	0.22 ± 0.42
G546	Jurassic-Cretaceous rocks of the White Mountain and New England-Quebec igneous succession.	0.47 ± 0.42
G561	Late Cambrian-Early Devonian metamorphic rocks	-0.47 ± 0.36
G591	Middle to Late Ordovician intrusive rocks ranging from gabbro to syenite in composition	0.05 ± 0.40
G640	Early Ordovician intrusive rocks ranging from gabbro to granodiorite.	-0.08 ± 0.36

Table 4. Coefficient estimates from Bayesian mixed-effects models for various combinations of explanatory variables.

		Coefficients of fixed-effects explanatory variables[a]								Variances					Measures of fit	
Model Number	Const. ln(Bq/m^3)	Log of NURE	Mobile Home [MH]	Apart-ment [AP]	Meas. Above Grade [MAG]	Below Ground Living Space [BGL]	Forced Air Furnace [FAF]	Drilled Well [DW]	Shallow Well/ Spring [SW]	Town Effects δ^2	Town Effects $\exp(\delta)$	Geologic Effects τ^2	Geologic Effects $\exp(\tau)$	Residuals σ^2	Effective[b] R^2 resid. R^2_{resid}	Effective[c] R^2 town R^2_{town}
1	4.43									0.28	1.69			0.99	0.18	
2	3.70	0.87								0.22	1.61			0.99	0.18	0.19
3	4.82									0.14	1.45	1.37	3.23	0.99	0.17	0.51
4	4.22	0.67								0.11	1.40	0.93	2.62	1.00	0.17	0.60
5	3.79	0.93	-0.79	-0.09	-0.53					0.23	1.62			0.90	0.25	0.16
6	3.80	0.94	-0.80	-0.09	-0.55	-0.07				0.23	1.62			0.90	0.25	0.16
7	3.83	0.92	-0.73	-0.08	-0.53		-0.12			0.23	1.62			0.90	0.25	0.17
8	3.86	0.93	-0.73	-0.09	-0.55	-0.08	-0.13			0.24	1.62			0.90	0.25	0.15
9	3.80	0.91	-0.75	-0.09	-0.54	-0.09	-0.13	0.16	-0.06	0.25	1.65			0.89	0.26	0.10
10	4.23	0.74	-0.73	-0.12	-0.52	-0.10	-0.12	0.20	0.01	0.14	1.45	0.67	2.67	0.90	0.26	0.50
SD[f] Model 10	0.39	0.18	0.14	0.11	0.07	0.05	0.05	0.06	0.08	0.03		1.18		0.03		

[a]Coefficients are actually the mean of 1200 coefficients estimates made using Full Bayes regressions. Bracketed codes are abbreviations for the variables which are used in the text.

[b]Effective R^2 resid.(R^2_{resid}) is calculated as 1-[(var(residuals)/var(ln(observed radon screening measurements))].

[c]Effective R^2 town (R^2_{town}) is calculated as 1-[(δ^2 from Model n)/(δ^2 from Model 1)].

[d]Standard deviation of 1200 coefficient estimates made using Full Bayes regression Model 10.

Figure Captions

Fig. 1. The distribution of short-term radon screening measurements collected in the New Hampshire Radon Survey.

Fig. 2. The natural logarithm of observed geometric mean short-term radon screening measurements for 232 towns in New Hampshire plotted against town-average surficial uranium concentrations (NURE).

Fig. 3. Geologic map of New Hampshire with distinct geologic units indexed by random-effects geological unit coefficients from Model 10.

Fig. 4. The distributions of natural logarithm of measured indoor radon screening measurements for six New Hampshire towns. Note that the values of these observations have been shifted by constant amounts in order to superimpose them on the same figure.

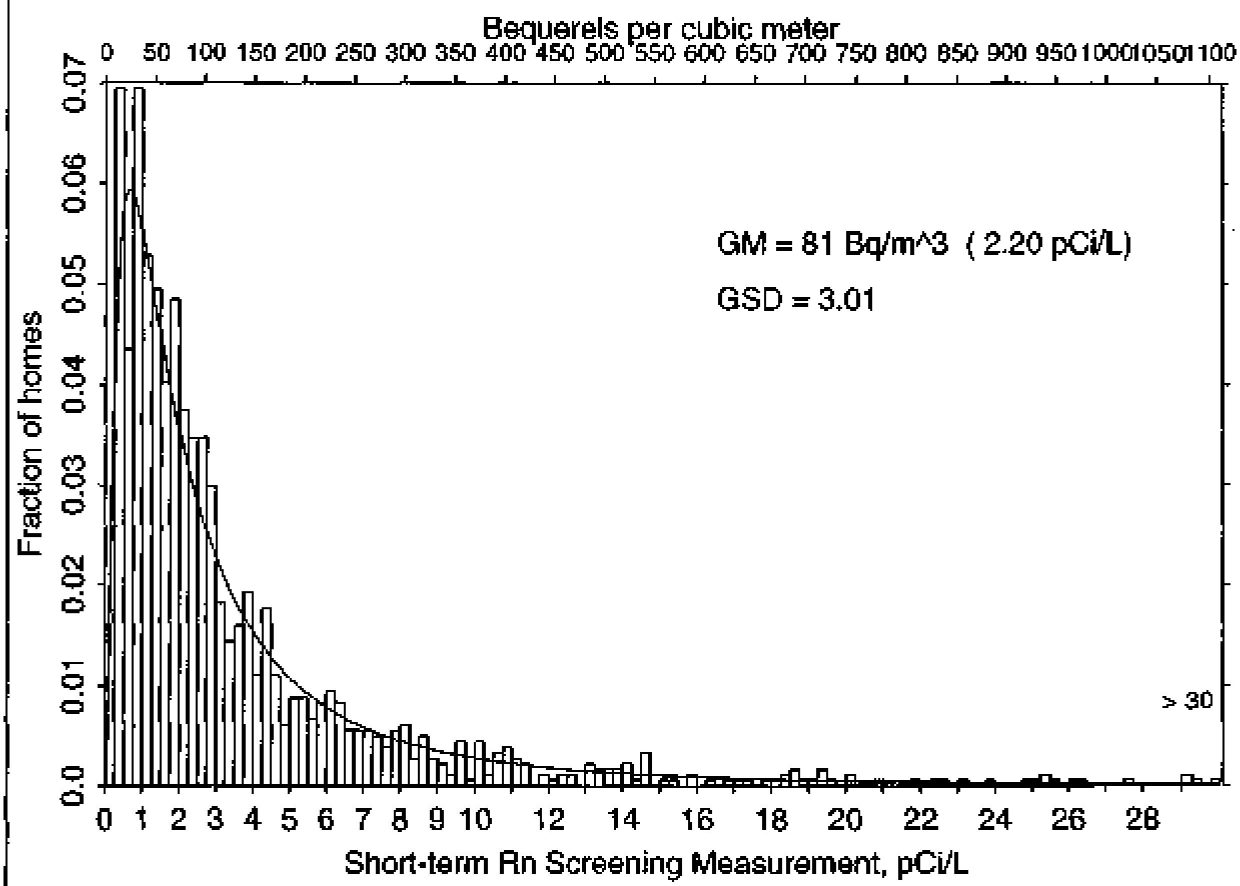
Fig. 5. Observed town geometric standard deviations of short-term radon screening measurements plotted as a function of the number of observations in each town. The superimposed curves indicate the 95% confidence interval for the hypothesis that the true GSD for New Hampshire town radon screening measurements is 2.6.

Fig. 6. Posterior predictions of town GM indoor radon concentrations for homes with basements. The GM values presented here are not the "true" values but only one of many possible sets of predictions drawn at random from distributional data.

Fig. 7. Bayesian posterior predictive checks of 1200 simulations of Model 10. The predictive checks have been conducted at the 10th, 20th, 50th, 80th, 90th, and 99th percentiles. The superimposed line on each histogram of posterior distributions is the measurement at the percentile in question, from the New Hampshire Radon Survey. The *p-value* presented is a Bayesian *p-value*, as discussed in the text. For example, the first plot shows that the 10th-percentile measurement was about 22 Bq/m^3 (vertical line), and that simulated data from the posterior distribution had 10th-percentile values between about 19 and 22.5 Bq/m^3 most of the time (histogram), with the simulated 10th-percentile value exceeding the actual measurement about 11% of the time (p=0.11).

Fig. 8. A *validation set* was created by removing 80% of the data from the 27 best-sampled towns in the New Hampshire Radon Survey. This figure shows predicted town ln(geometric mean) radon concentrations (using Model 10) for these towns, plotted as a function of the towns' observed radon concentrations as indicated by the validation set. The diagonal line represents a "perfect fit". The error bars indicate one standard error of the distributions of predicted town concentrations.

Figures



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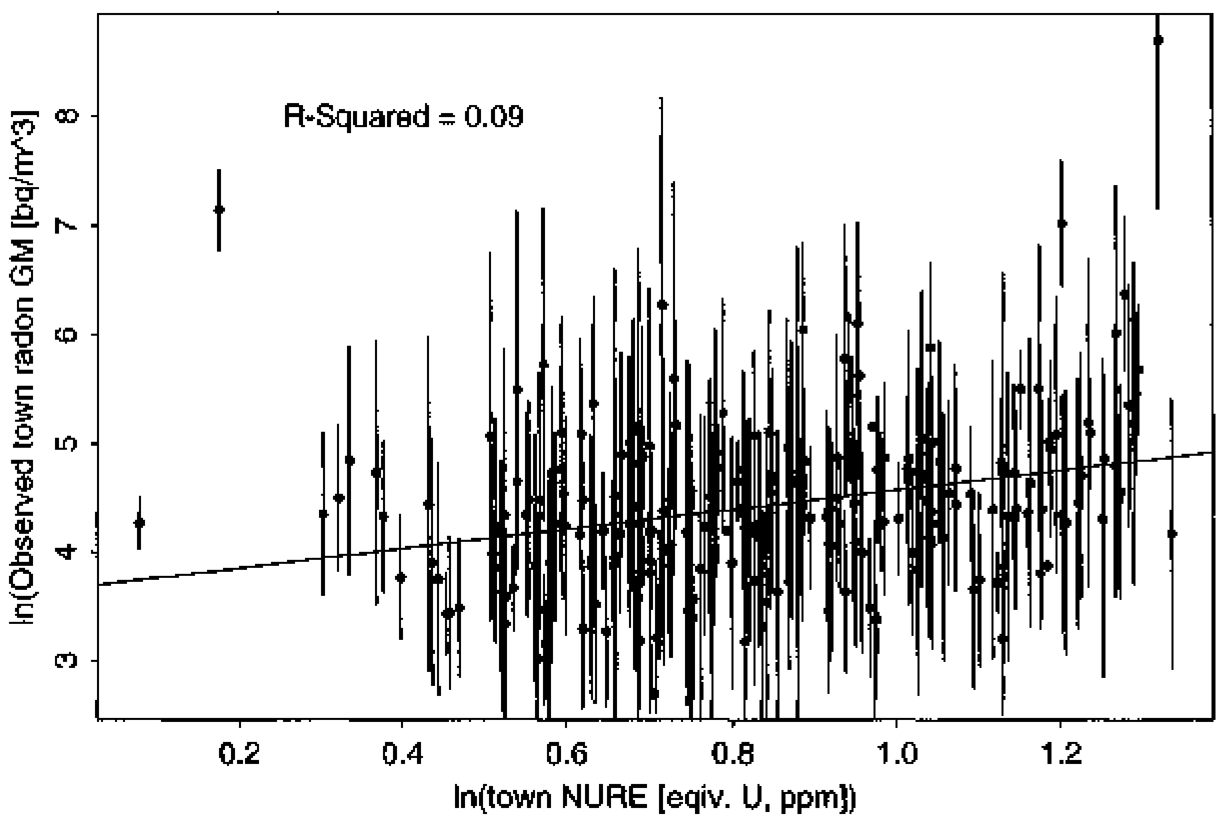

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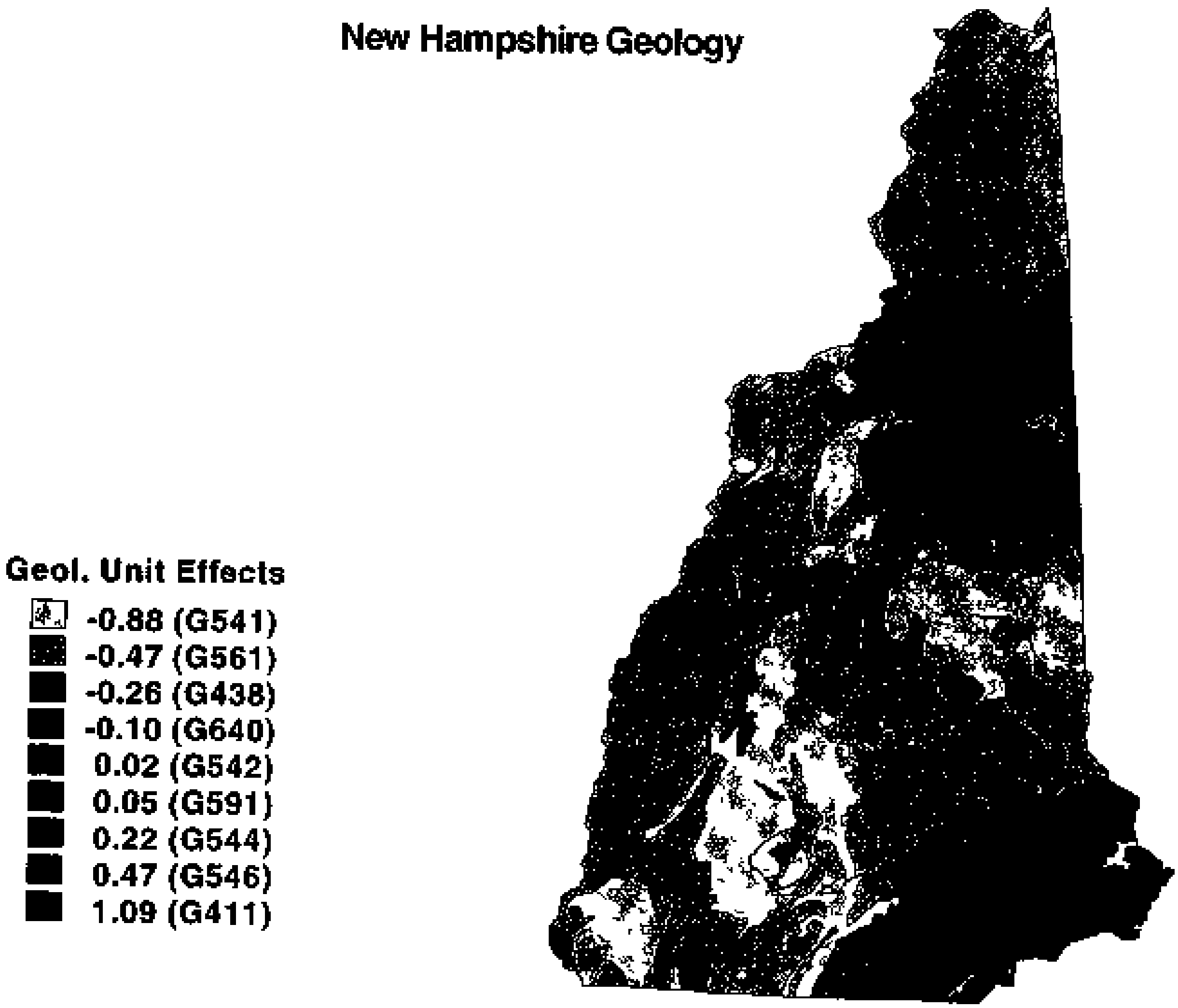
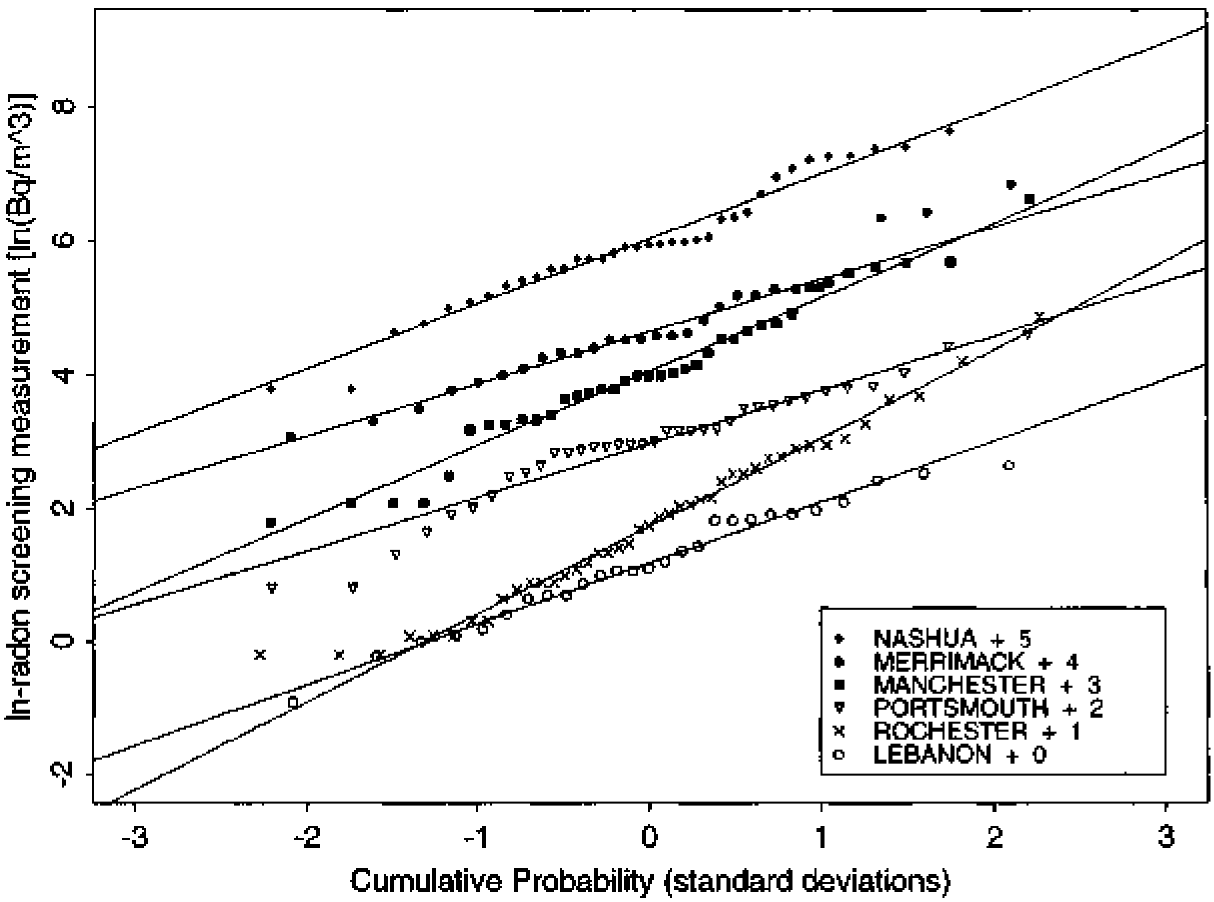


Fig. 3. Geologic map of New Hampshire with distinct geologic units indexed by random-effects geological unit coefficients from Model 10



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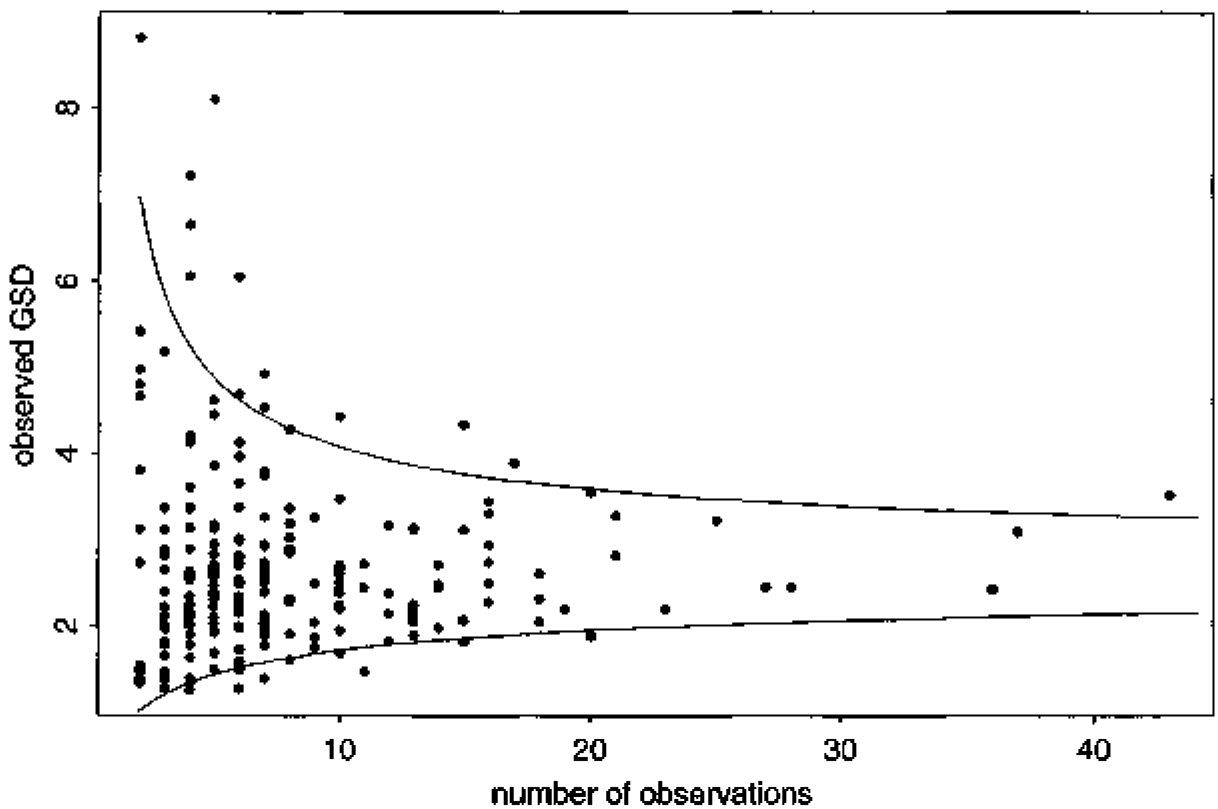

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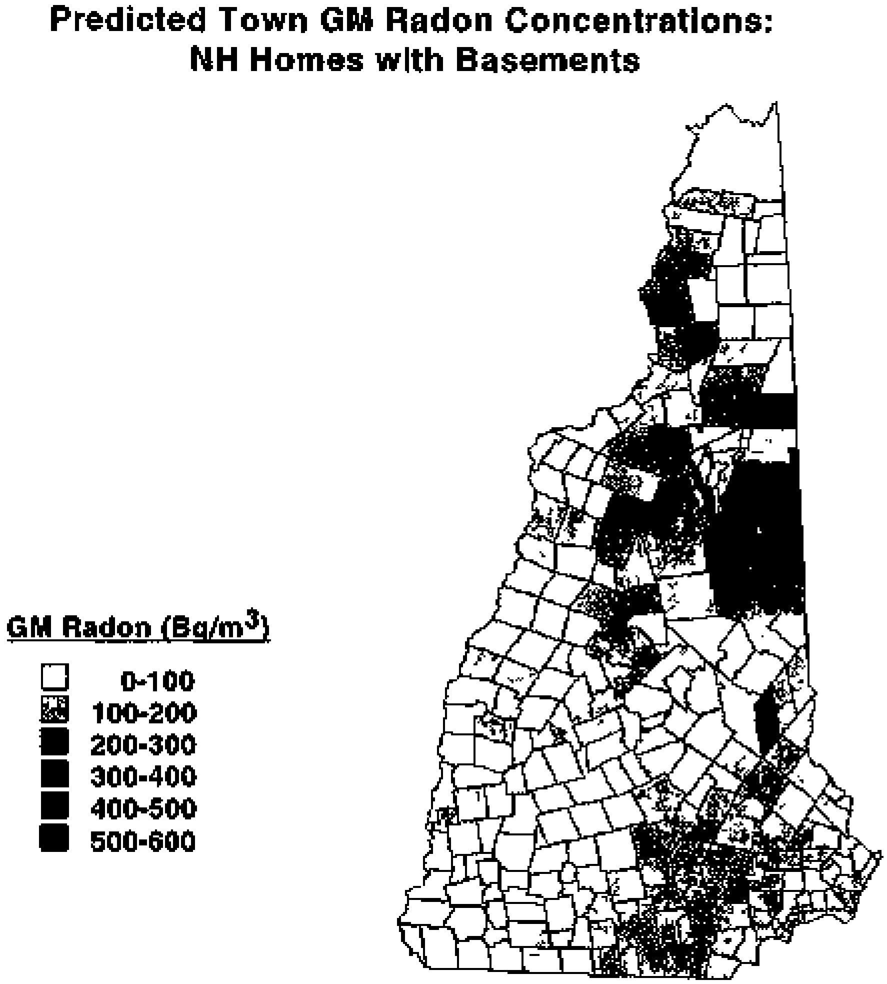

Fig. 6. Posterior predictions of town GM indoor radon concentrations for homes with basements The GM values presented here are not the "true" values but only one of many possible sets of predictions drawn at random from distributional data

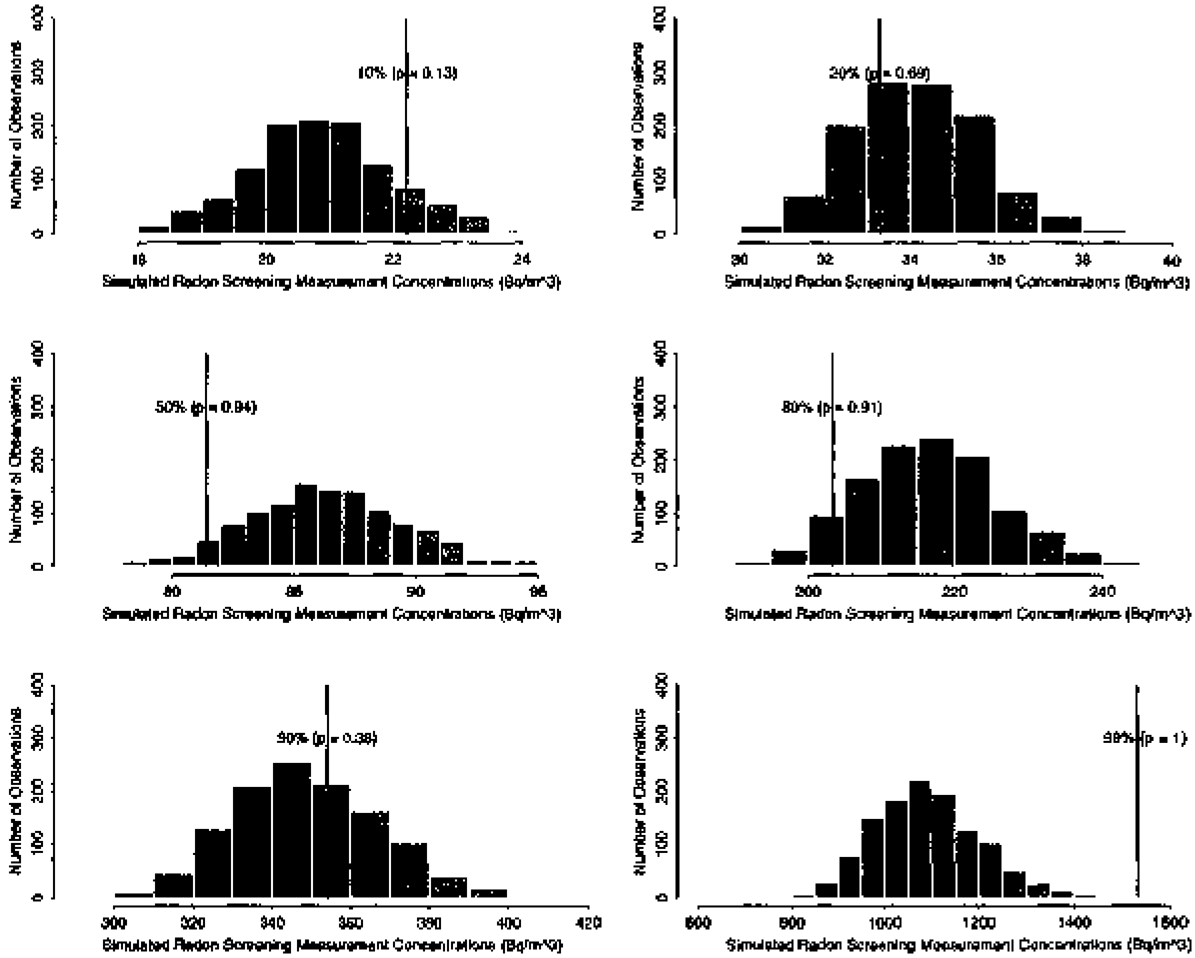

Fig. 7. Bayesian posterior predictive checks of 1200 simulations of Model 10. The predictive checks have been conducted at the 10th, 20th, 50th, 80th, 90th, and 99th percentiles. The superimposed line on each histogram of posterior distributions is the measurement at the percentile in question, from the New Hampshire Radon Survey. The *p-value* presented is a Bayesian *p-value*, as discussed in the text. For example, the first plot shows that the 10th-percentile measurement was about 22 Bq/m^3 (vertical line), and that simulated data from the posterior distribution had 10th-percentile values between about 19 and 22.5 Bq/m^3 most of the time (histogram), with the simulated 10th-percentile value exceeding the actual measurement about 11% of the time (p=0.11).

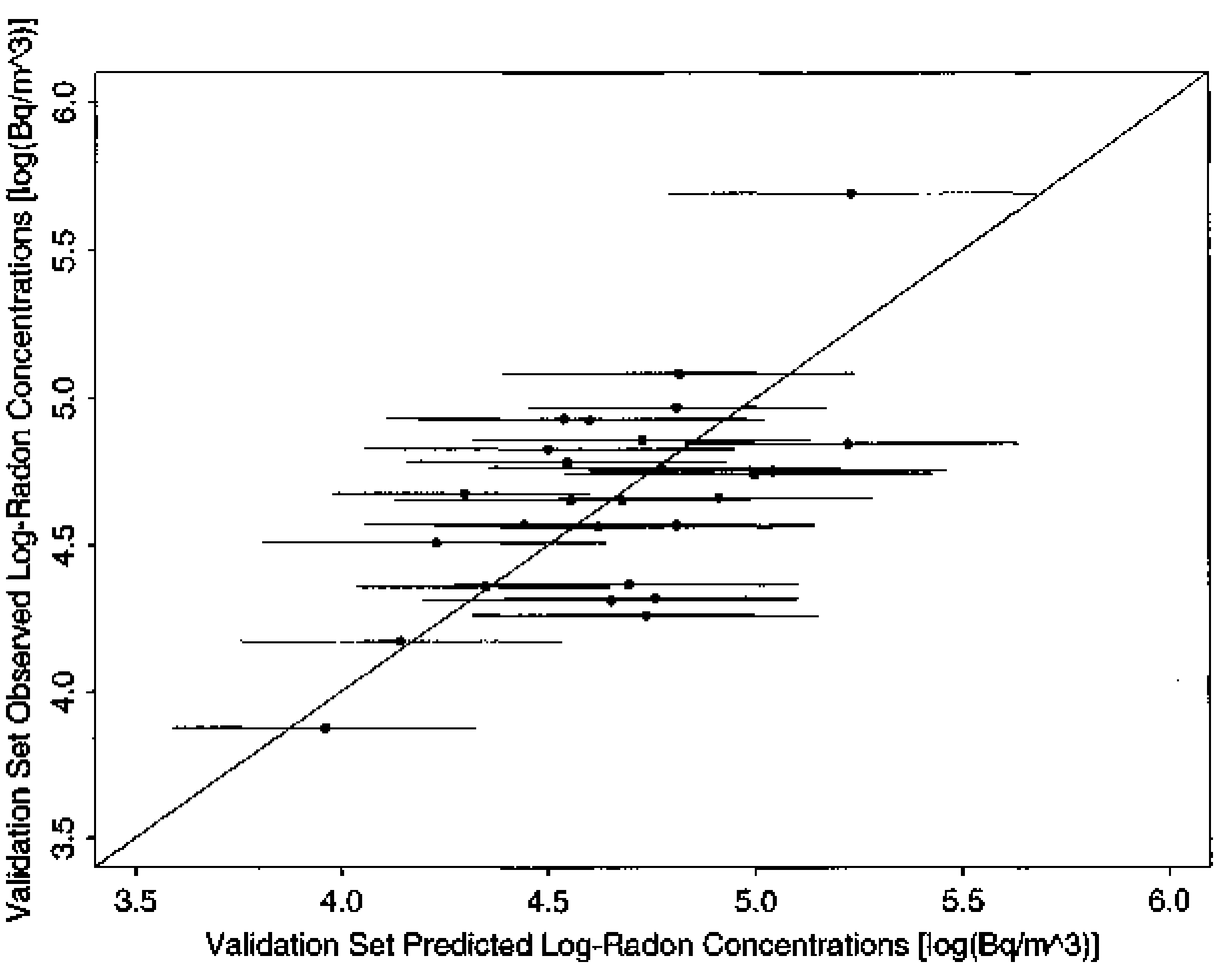

Fig. 8. A *validation set* was created by removing 80% of the data from the 27 best-sampled towns in the New Hampshire Radon Survey. This figure shows predicted town ln(geometric mean) radon concentrations (using Model 10) for these towns, plotted as a function of the towns' observed radon concentrations as indicated by the validation set. The diagonal line represents a "perfect fit". The error bars indicate one standard error of the distributions of predicted town concentrations.

Appendix I. Posterior predictions from Model 10 for the 232 towns included in the NH State Radon Survey. These predictions are for the Town GMs of detached single-family dwellings with occupied basements, no FAF, and a municipal water source.

Town	Predicted GM for Houses with Basement (Bq/m^3) (x/+ std err)	Town	Predicted GM for Houses with Basement (Bq/m^3) (x/+ std err)
ACWORTH	80 (1.31)	CONCORD	99 (1.22)
ALBANY	289 (1.42)	CONWAY	269 (1.31)
ALEXANDRIA	74 (1.33)	CORNISH	63 (1.32)
ALLENSTOWN	168 (1.31)	CROYDON	112 (1.33)
ALSTEAD	74 (1.34)	DALTON	87 (1.33)
ALTON	73 (1.33)	DANBURY	71 (1.39)
AMHERST	113 (1.23)	DANVILLE	185 (1.34)
ANDOVER	78 (1.29)	DEERFIELD	122 (1.23)
ANTRIM	34 (1.32)	DEERING	43 (1.31)
ASHLAND	118 (1.32)	DERRY	112 (1.21)
ATKINSON	118 (1.22)	DIXVILLE	60 (1.39)
AUBURN	111 (1.25)	DORCHESTER	69 (1.35)
BARNSTEAD	115 (1.37)	DOVER	96 (1.27)
BARRINGTON	111 (1.28)	DUBLIN	60 (1.33)
BARTLETT	273 (1.41)	DUMMER	80 (1.37)
BATH	72 (1.30)	DUNBARTON	127 (1.30)
BEDFORD	137 (1.22)	DURHAM	101 (1.27)
BELMONT	55 (1.27)	EAST KINGSTON	140 (1.31)
BENNINGTON	38 (1.31)	EASTON	62 (1.37)
BENTON	82 (1.32)	EATON	238 (1.36)
BERLIN	123 (1.19)	EFFINGHAM	90 (1.36)
BETHLEHEM	141 (1.33)	ELLSWORTH	70 (1.43)
BOSCAWEN	50 (1.39)	ENFIELD	93 (1.34)
BOW	97 (1.23)	EPPING	123 (1.30)
BRADFORD	59 (1.41)	EPSOM	95 (1.35)
BRENTWOOD	119 (1.31)	ERROL	78 (1.39)
BRIDGEWATER	97 (1.35)	EXETER	79 (1.28)
BRISTOL	94 (1.31)	FARMINGTON	146 (1.32)
BROOKFIELD	105 (1.36)	FITZWILLIAM	70 (1.34)
BROOKLINE	96 (1.33)	FRANCESTOWN	48 (1.37)
CAAN	140 (1.33)	FRANCONIA	96 (1.37)
CAMPTON	74 (1.30)	FRANKLIN	78 (1.24)
CANDIA	129 (1.26)	FREEDOM	193 (1.41)
CANTERBURY	77 (1.33)	FREMONT	113 (1.35)
CARROLL	169 (1.34)	GILFORD	90 (1.28)
CENTER HARBOR	45 (1.39)	GILMANTON	72 (1.31)
CHARLESTOWN	77 (1.31)	GILSUM	85 (1.32)
CHATHAM	289 (1.45)	GOFFSTOWN	96 (1.24)
CHESTER	148 (1.28)	GORHAM	281 (1.24)
CHESTERFIELD	68 (1.31)	GOSHEN	67 (1.31)
CHICHESTER	104 (1.40)	GRANTHAM	80 (1.34)
CLAREMONT	71 (1.30)	GREENFIELD	65 (1.32)
CLARKSVILLE	112 (1.44)	GREENLAND	95 (1.30)
COLEBROOK	90 (1.31)	GREENVILLE	108 (1.33)
COLUMBIA	160 (1.39)	GROTON	71 (1.36)

Appendix II. Posterior predictions from Model 10 for the 27 towns not included in the NH State Radon Survey. These predictions are for the Town GMs of detached single-family dwellings with occupied basements, no FAF, and a municipal water source.

Town	Predicted GM for Houses with Basement (Bq/m^3) (×/÷ std err)
ATKINSON & GILMANTON	38 (2.40)
BEANS GRANT	68 (1.76)
BEANS PURCHASE	71 (1.83)
CAMBRIDGE	35 (2.18)
CHANDLERS PURCHASE	54 (1.75)
CRAWFORDS PURCHASE	72 (2.18)
CUTTS GRANT	58 (1.82)
DIXS GRANT	36 (2.37)
ERVINGS LOCATION	33 (1.72)
GRAFTON	52 (1.89)
GREENS GRANT	66 (1.87)
HADLEYS PURCHASE	87 (2.07)
HALES LOCATION	100 (2.39)
KILKENNY	76 (2.30)
LOW&BURBANKS	52 (1.67)
MARTINS LOCATION	62 (1.68)
MILLSFIELD	44 (1.76)
NEWFIELDS	74 (2.40)
ODELL	47 (1.62)
PINKHAM'S GRANT	60 (1.88)
SARGENTS PURCHASE	66 (1.78)
SECOND COLLEGE	38 (2.38)
SUCCESS	34 (1.62)
THOMPSON&MESERVE	65 (1.77)
UNORGANIZED TERRITORY	91 (2.11)
WATERVILLE VALLEY	81 (2.02)
WENTWORTHS LOCATION	36 (2.10)