

# Package ‘betareg’

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**Title** Beta Regression

## Description

Beta regression for modeling beta-distributed dependent variables on the open unit interval (0, 1), e.g., rates and proportions, see Cribari-Neto and Zeileis (2010) <[doi:10.18637/jss.v034.i02](https://doi.org/10.18637/jss.v034.i02)>.

Moreover, extended-support beta regression models can accommodate dependent variables with boundary observations at 0 and/or 1. For the classical beta regression model, alternative specifications are

provided: Bias-corrected and bias-

reduced estimation, finite mixture models, and recursive partitioning for

beta regression, see Grün, Kosmidis, and Zeileis (2012) <[doi:10.18637/jss.v048.i11](https://doi.org/10.18637/jss.v048.i11)>.

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**Author** Achim Zeileis [aut, cre] (<<https://orcid.org/0000-0003-0918-3766>>),  
Francisco Cribari-Neto [aut] (<<https://orcid.org/0000-0002-5909-6698>>),  
Bettina Grün [aut] (<<https://orcid.org/0000-0001-7265-4773>>),  
Ioannis Kosmidis [aut] (<<https://orcid.org/0000-0003-1556-0302>>),  
Alexandre B. Simas [ctb] (earlier version by),  
Andrea V. Rocha [ctb] (earlier version by)

**Maintainer** Achim Zeileis <[Achim.Zeileis@R-project.org](mailto:Achim.Zeileis@R-project.org)>

**Repository** CRAN

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 Beta01

*Create a Zero- and/or One-Inflated Beta Distribution*


---

### Description

Class and methods for zero- and/or one-inflated beta distributions in regression specification using the workflow from the **distributions3** package.

### Usage

Beta01(mu, phi, p0 = 0, p1 = 0)

**Arguments**

mu	numeric. The mean of the beta distribution (on the open unit interval).
phi	numeric. The precision parameter of the beta distribution.
p0	numeric. The probability for an observation of zero (often referred to as zero inflation).
p1	numeric. The probability for an observation of one (often referred to as one inflation).

**Details**

The zero- and/or one-inflated beta distribution is obtained by adding point masses at zero and/or one to a standard beta distribution.

Note that the support of the standard beta distribution is the open unit interval where values of exactly zero or one cannot occur. Thus, the inflation jargon is rather misleading as there is no probability that could be inflated. It is rather a hurdle or two-part (or three-part) model.

**Value**

A Beta01 distribution object.

**See Also**

[dbeta01](#), [BetaR](#)

**Examples**

```
## package and random seed
library("distributions3")
set.seed(6020)

## three beta distributions
X <- Beta01(
  mu = c(0.25, 0.50, 0.75),
  phi = c(1, 1, 2),
  p0 = c(0.1, 0, 0),
  p1 = c(0, 0, 0.3)
)

X

## compute moments of the distribution
mean(X)
variance(X)

## support interval (minimum and maximum)
support(X)

## simulate random variables
random(X, 5)
```

```

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ])
hist(x[2, ])
hist(x[3, ])

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0.25, 0.5, 0.75)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## point mass probabilities (if any) on boundary
cdf(X, 0, lower.tail = TRUE)
cdf(X, 1, lower.tail = FALSE)

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

```

**Description**

Density, distribution function, quantile function, and random generation for the zero- and/or one-inflated beta distribution in regression parameterization.

**Usage**

```

dbeta01(x, mu, phi, p0 = 0, p1 = 0, log = FALSE)

pbeta01(q, mu, phi, p0 = 0, p1 = 0, lower.tail = TRUE, log.p = FALSE)

qbeta01(p, mu, phi, p0 = 0, p1 = 0, lower.tail = TRUE, log.p = FALSE)

rbeta01(n, mu, phi, p0 = 0, p1 = 0)

```

**Arguments**

x, q	numeric. Vector of quantiles.
p	numeric. Vector of probabilities.
n	numeric. Number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
mu	numeric. The mean of the beta distribution (on the open unit interval).
phi	numeric. The precision parameter of the beta distribution.
p0	numeric. The probability for an observation of zero (often referred to as zero inflation).
p1	numeric. The probability for an observation of one (often referred to as one inflation).
log, log.p	logical. If TRUE, probabilities p are given as $\log(p)$ .
lower.tail	logical. If TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$ .

**Details**

The zero- and/or one-inflated beta distribution is obtained by adding point masses at zero and/or one to a standard beta distribution.

Note that the support of the standard beta distribution is the open unit interval where values of exactly zero or one cannot occur. Thus, the inflation jargon is rather misleading as there is no probability that could be inflated. It is rather a hurdle or two-part (or three-part) model.

**Value**

`dbeta01` gives the density, `pbeta01` gives the distribution function, `qbeta01` gives the quantile function, and `rbeta01` generates random deviates.

**See Also**

[dbetar](#), [Beta01](#)

**Description**

Class and methods for 4-parameter beta distributions in regression specification using the workflow from the **distributions3** package.

**Usage**

```
Beta4(mu, phi, theta1 = 0, theta2 = 1 - theta1)
```

**Arguments**

mu	numeric. The mean of the beta distribution that is extended to support [theta1, theta2].
phi	numeric. The precision parameter of the beta distribution that is extended to support [theta1, theta2].
theta1, theta2	numeric. The minimum and maximum, respectively, of the 4-parameter beta distribution. By default a symmetric support is chosen by theta2 = 1 - theta1 which reduces to the classic beta distribution because of the default theta1 = 0.

**Details**

The distribution is obtained by a linear transformation of a beta-distributed random variable with intercept theta1 and slope theta2 - theta1.

**Value**

A Beta4 distribution object.

**See Also**

[dbeta4](#), [BetaR](#)

**Examples**

```
## package and random seed
library("distributions3")
set.seed(6020)

## three beta distributions
X <- Beta4(
  mu = c(0.25, 0.50, 0.75),
  phi = c(1, 1, 2),
  theta1 = c(0, -0.1, -0.1),
  theta2 = c(1, 1.1, 1.5)
)
```

```
X

## compute moments of the distribution
mean(X)
variance(X)

## support interval (minimum and maximum)
support(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ])
hist(x[2, ])
hist(x[3, ])

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0.25, 0.5, 0.75)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)
```

**Description**

Density, distribution function, quantile function, and random generation for the 4-parameter beta distribution in regression parameterization.

**Usage**

```
dbeta4(x, mu, phi, theta1 = 0, theta2 = 1 - theta1, log = FALSE)
```

```
pbeta4(q, mu, phi, theta1 = 0, theta2 = 1 - theta1, lower.tail = TRUE, log.p = FALSE)
```

```
qbeta4(p, mu, phi, theta1 = 0, theta2 = 1 - theta1, lower.tail = TRUE, log.p = FALSE)
```

```
rbeta4(n, mu, phi, theta1 = 0, theta2 = 1 - theta1)
```

**Arguments**

x, q	numeric. Vector of quantiles.
p	numeric. Vector of probabilities.
n	numeric. Number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
mu	numeric. The mean of the beta distribution that is extended to support <code>[theta1, theta2]</code> .
phi	numeric. The precision parameter of the beta distribution that is extended to support <code>[theta1, theta2]</code> .
theta1, theta2	numeric. The minimum and maximum, respectively, of the 4-parameter beta distribution. By default a symmetric support is chosen by <code>theta2 = 1 - theta1</code> which reduces to the classic beta distribution because of the default <code>theta1 = 0</code> .
log, log.p	logical. If TRUE, probabilities p are given as <code>log(p)</code> .
lower.tail	logical. If TRUE (default), probabilities are <code>P[X &lt;= x]</code> otherwise, <code>P[X &gt; x]</code> .

**Details**

The distribution is obtained by a linear transformation of a beta-distributed random variable with intercept `theta1` and slope `theta2 - theta1`.

**Value**

`dbeta4` gives the density, `pbeta4` gives the distribution function, `qbeta4` gives the quantile function, and `rbeta4` generates random deviates.

**See Also**

[dbetar](#), [Beta4](#)



betamix

*Finite Mixtures of Beta Regression for Rates and Proportions***Description**

Fit finite mixtures of beta regression models for rates and proportions via maximum likelihood with the EM algorithm using a parametrization with mean (depending through a link function on the covariates) and precision parameter (called phi).

**Usage**

```
betamix(formula, data, k, subset, na.action, weights, offset,
        link = c("logit", "probit", "cloglog", "cauchit", "log",
                "loglog"), link.phi = "log",
        control = betareg.control(...), cluster = NULL,
        FLXconcomitant = NULL, FLXcontrol = list(), verbose = FALSE,
        nstart = if (is.null(cluster)) 3 else 1, which = "BIC",
        ID, fixed, extra_components, ...)

extraComponent(type = c("uniform", "betareg"), coef, delta,
               link = "logit", link.phi = "log")
```

**Arguments**

formula	symbolic description of the model (of type $y \sim x$ or $y \sim x   z$ ; for details see <a href="#">betareg</a> ).
data, subset, na.action	arguments controlling formula processing via <a href="#">model.frame</a> .
weights	optional numeric vector of integer case weights.
offset	optional numeric vector with an a priori known component to be included in the linear predictor for the mean.
k	a vector of integers indicating the number of components of the finite mixture; passed in turn to the k argument of <a href="#">stepFlexmix</a> .
link	character specification of the link function in the mean model (mu). Currently, "logit", "probit", "cloglog", "cauchit", "log", "loglog" are supported. Alternatively, an object of class "link-glm" can be supplied.
link.phi	character specification of the link function in the precision model (phi). Currently, "identity", "log", "sqrt" are supported. The default is "log" unless formula is of type $y \sim x$ where the default is "identity" (for backward compatibility). Alternatively, an object of class "link-glm" can be supplied.
control	a list of control arguments specified via <a href="#">betareg.control</a> .
cluster	Either a matrix with k columns of initial cluster membership probabilities for each observation; or a factor or integer vector with the initial cluster assignments of observations at the start of the EM algorithm. Default is random assignment into k clusters.

FLXconcomitant	concomitant variable model; object of class FLXP. Default is the object returned by calling <code>FLXPconstant</code> . The argument <code>FLXconcomitant</code> can be omitted if formula is a three-part formula of type $y \sim x \mid z \mid w$ , where $w$ specifies the concomitant variables.
FLXcontrol	object of class "FLXcontrol" or a named list; controls the EM algorithm and passed in turn to the <code>control</code> argument of <code>flexmix</code> .
verbose	a logical; if TRUE progress information is shown for different starts of the EM algorithm.
nstart	for each value of $k$ run <code>stepFlexmix</code> <code>nstart</code> times and keep only the solution with maximum likelihood.
which	number of model to get if $k$ is a vector of integers longer than one. If character, interpreted as number of components or name of an information criterion.
ID	grouping variable indicating if observations are from the same individual, i.e. the component membership is restricted to be the same for these observations.
fixed	symbolic description of the model for the parameters fixed over components (of type $\sim x \mid z$ ).
extra_components	a list containing objects returned by <code>extraComponent()</code> .
...	arguments passed to <code>betareg.control</code> .
type	specifies if the component follows a uniform distribution or a beta regression model.
coef	a vector with the coefficients to determine the midpoint of the uniform distribution or names list with the coefficients for the mean and precision of the beta regression model.
delta	numeric; half-length of the interval of the uniform distribution.

### Details

The arguments and the model specification are similar to `betareg`. Internally `stepFlexmix` is called with suitable arguments to fit the finite mixture model with the EM algorithm. See Grün et al. (2012) for more details.

`extra_components` is a list where each element corresponds to a component where the parameters are fixed a-priori.

### Value

An object of class "flexmix" containing the best model with respect to the log likelihood or the one selected according to which if  $k$  is a vector of integers longer than 1.

### Author(s)

Bettina Grün and Achim Zeileis

## References

- Cribari-Neto F, Zeileis A (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. doi:10.18637/jss.v034.i02
- Grün B, Kosmidis I, Zeileis A (2012). Extended Beta Regression in R: Shaken, Stirred, Mixed, and Partitioned. *Journal of Statistical Software*, **48**(11), 1–25. doi:10.18637/jss.v048.i11
- Grün B, Leisch F (2008). FlexMix Version 2: Finite Mixtures with Concomitant Variables and Varying and Constant Parameters. *Journal of Statistical Software*, **28**(4), 1–35. doi:10.18637/jss.v028.i04
- Leisch F (2004). FlexMix: A General Framework for Finite Mixture Models and Latent Class Regression in R. *Journal of Statistical Software*, **11**(8), 1–18. doi:10.18637/jss.v011.i08

## See Also

[betareg](#), [flexmix](#), [stepFlexmix](#)

## Examples

```
options(digits = 4)

## data with two groups of dyslexic and non-dyslexic children
data("ReadingSkills", package = "betareg")

suppressWarnings(RNGversion("3.5.0"))
set.seed(4040)
## try to capture accuracy ~ iq relationship (without using dyslexia
## information) using two beta regression components and one additional
## extra component for a perfect reading score
rs_mix <- betamix(accuracy ~ iq, data = ReadingSkills, k = 3,
  nstart = 10, extra_components = extraComponent(type = "uniform",
  coef = 0.99, delta = 0.01))

## visualize result
## intensities based on posterior probabilities
prob <- 2 * (posterior(rs_mix)[cbind(1:nrow(ReadingSkills),
  clusters(rs_mix))] - 0.5)
## associated HCL colors
col0 <- hcl(c(260, 0, 130), 65, 45, fixup = FALSE)
col1 <- col0[clusters(rs_mix)]
col2 <- hcl(c(260, 0, 130)[clusters(rs_mix)], 65 * abs(prob)^1.5,
  95 - 50 * abs(prob)^1.5, fixup = FALSE)
## scatter plot
plot(accuracy ~ iq, data = ReadingSkills, col = col2, pch = 19,
  cex = 1.5, xlim = c(-2, 2))
points(accuracy ~ iq, data = ReadingSkills, cex = 1.5, pch = 1,
  col = col1)
## fitted lines
iq <- -30:30/10
cf <- rbind(coef(rs_mix, model = "mean", component = 1:2),
  c(qlogis(0.99), 0))
for(i in 1:3)
```

```

lines(iq, plogis(cf[i, 1] + cf[i, 2] * iq), lwd = 2,
      col = col0[i])

## refit the model including a concomitant variable model using the
## dyslexia information with some noise to avoid complete separation
## between concomitant variable and component memberships
set.seed(4040)
w <- rnorm(nrow(ReadingSkills),
          c(-1, 1)[as.integer(ReadingSkills$dyslexia)])

## The argument FLXconcomitant can be omitted when specifying
## the model via a three part formula given by
## accuracy ~ iq | 1 | w
## The posteriors from the previously fitted model are used
## for initialization.
library("flexmix")
rs_mix2 <- betamix(accuracy ~ iq, data = ReadingSkills,
  extra_components = extraComponent(type = "uniform",
  coef = 0.99, delta = 0.01), cluster = posterior(rs_mix),
  FLXconcomitant = FLXPmultinom(~w))
coef(rs_mix2, which = "concomitant")
summary(rs_mix2, which = "concomitant")

```

---

BetaR

*Create a Beta Regression Distribution*


---

## Description

Class and methods for beta distributions in regression specification using the workflow from the **distributions3** package.

## Usage

```
BetaR(mu, phi)
```

## Arguments

mu	numeric. The mean of the beta distribution.
phi	numeric. The precision parameter of the beta distribution.

## Details

Alternative parameterization of the classic beta distribution in terms of its mean  $\mu$  and precision parameter  $\phi$ . Thus, the distribution provided by BetaR is equivalent to the [Beta](#) distribution with parameters  $\alpha = \mu * \phi$  and  $\beta = (1 - \mu) * \phi$ .

## Value

A BetaR distribution object.

**See Also**

[dbetar](#), [Beta](#)

**Examples**

```
## package and random seed
library("distributions3")
set.seed(6020)

## three beta distributions
X <- BetaR(
  mu = c(0.25, 0.50, 0.75),
  phi = c(1, 1, 2)
)

X

## compute moments of the distribution
mean(X)
variance(X)
skewness(X)
kurtosis(X)

## support interval (minimum and maximum)
support(X)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ])
hist(x[2, ])
hist(x[3, ])

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0.25, 0.5, 0.75)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses (except at censoring points)
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
```

```

## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

```

---

betar

*The Beta Distribution in Regression Parameterization*


---

## Description

Density, distribution function, quantile function, and random generation for the beta distribution in regression parameterization.

## Usage

```

dbetar(x, mu, phi, log = FALSE)

pbetar(q, mu, phi, lower.tail = TRUE, log.p = FALSE)

qbetar(p, mu, phi, lower.tail = TRUE, log.p = FALSE)

rbetar(n, mu, phi)

```

## Arguments

x, q	numeric. Vector of quantiles.
p	numeric. Vector of probabilities.
n	numeric. Number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.
mu	numeric. The mean of the beta distribution.
phi	numeric. The precision parameter of the beta distribution.
log, log.p	logical. If TRUE, probabilities p are given as $\log(p)$ .
lower.tail	logical. If TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$ .

## Details

This is the reparameterization of the beta distribution with mean  $\mu$  and precision  $\phi$ , as employed in beta regression. The classic parameterization of the beta distribution is obtained by setting  $\text{shape1} = \mu * \phi$  and  $\text{shape2} = (1 - \mu) * \phi$ , respectively.

**Value**

dbetar gives the density, pbetar gives the distribution function, qbetar gives the quantile function, and rbetar generates random deviates.

**See Also**

[dbeta](#), [BetaR](#)

---

 betareg

*Beta Regression for Rates and Proportions*


---

**Description**

Fit beta regression models for rates and proportions via maximum likelihood using a parametrization with mean (depending through a link function on the covariates) and precision parameter (called phi).

**Usage**

```
betareg(formula, data, subset, na.action, weights, offset,
        link = c("logit", "probit", "cloglog", "cauchit", "log", "loglog"),
        link.phi = NULL, type = c("ML", "BC", "BR"), dist = NULL, nu = NULL,
        control = betareg.control(...), model = TRUE,
        y = TRUE, x = FALSE, ...)
```

```
betareg.fit(x, y, z = NULL, weights = NULL, offset = NULL,
           link = "logit", link.phi = "log", type = "ML", control = betareg.control(),
           dist = NULL, nu = NULL)
```

**Arguments**

formula	symbolic description of the model, either of type $y \sim x$ (mean submodel, constant precision) or $y \sim x \mid z$ (submodels for both mean and precision); for details see below.
data, subset, na.action	arguments controlling formula processing via <a href="#">model.frame</a> .
weights	optional numeric vector of case weights.
offset	optional numeric vector with an a priori known component to be included in the linear predictor for the mean. In <code>betareg.fit</code> , <code>offset</code> may also be a list of two offsets for the mean and precision equation, respectively.
link	character specification of the link function in the mean model ( $\mu$ ). Currently, "logit", "probit", "cloglog", "cauchit", "log", "loglog" are supported. Alternatively, an object of class "link-glm" can be supplied.
link.phi	character specification of the link function in the precision model ( $\phi$ ). Currently, "identity", "log", "sqrt" are supported. The default is "log" unless formula is of type $y \sim x$ where the default is "identity" (for backward compatibility). Alternatively, an object of class "link-glm" can be supplied.

type	character specification of the type of estimator. Currently, maximum likelihood ("ML"), ML with bias correction ("BC"), and ML with bias reduction ("BR") are supported.
dist	character specification of the response distribution. Usually, this does not have to be set by the user because by default the classical "beta" distribution is used when all observations for the dependent variable are in (0, 1). In the presence of boundary observations (0 or 1, which cannot be accommodated by "beta") the extended-support beta mixture distribution ("xbetax") is used. Additionally, <code>dist = "xbeta"</code> can be used with fixed exceedence parameter <code>nu</code> , mostly for testing and debugging purposes.
nu	numeric. The fixed value of the expected exceedence parameter <code>nu</code> in case the extended-support beta mixture distribution is used. By default, <code>nu</code> does not need to be specified and is estimated if needed. So setting <code>nu</code> is mostly for profiling and debugging.
control	a list of control arguments specified via <code>betareg.control</code> .
model, y, x	logicals. If TRUE the corresponding components of the fit (model frame, response, model matrix) are returned. For <code>betareg.fit</code> , <code>x</code> should be a numeric regressor matrix and <code>y</code> should be the numeric response vector (with values in (0,1)).
z	numeric matrix. Regressor matrix for the precision model, defaulting to an intercept only.
...	arguments passed to <code>betareg.control</code> .

## Details

Beta regression as suggested by Ferrari and Cribari-Neto (2004) and extended by Simas, Barreto-Souza, and Rocha (2010) is implemented in `betareg`. It is useful in situations where the dependent variable is continuous and restricted to the unit interval (0, 1), e.g., resulting from rates or proportions. It is modeled to be beta-distributed with parametrization using mean and precision parameter (called  $\mu$  and  $\phi$ , respectively). The mean  $\mu$  is linked, as in generalized linear models (GLMs), to the explanatory variables through a link function and a linear predictor. Additionally, the precision parameter  $\phi$  can be linked to another (potentially overlapping) set of regressors through a second link function, resulting in a model with variable dispersion (see Cribari-Neto and Zeileis 2010). Estimation is performed by default using maximum likelihood (ML) via `optim` with analytical gradients and starting values from an auxiliary linear regression of the transformed response. Subsequently, the `optim` result may be enhanced by an additional Fisher scoring iteration using analytical gradients and expected information. Alternative estimation methods are bias-corrected (BC) or bias-reduced (BR) maximum likelihood (see Grün, Kosmidis, and Zeileis 2012). For ML and BC the Fisher scoring is just a refinement to move the gradients even closer to zero and can be disabled by setting `fsmaxit = 0` in the control arguments. For BR the Fisher scoring is needed to solve the bias-adjusted estimating equations.

In the beta regression as introduced by Ferrari and Cribari-Neto (2004), the mean of the response is linked to a linear predictor described by  $y \sim x_1 + x_2$  using a link function while the precision parameter  $\phi$  is assumed to be constant. Simas et al. (2009) suggest to extend this model by linking  $\phi$  to an additional set of regressors ( $z_1 + z_2$ , say): In `betareg` this can be specified in a formula of type  $y \sim x_1 + x_2 \mid z_1 + z_2$  where the regressors in the two parts can be overlapping. In the precision model (for  $\phi$ ), the link function `link.phi` is used. The default is a "log" link unless no precision



model is specified. In the latter case (i.e., when the formula is of type  $y \sim x_1 + x_2$ ), the "identity" link is used by default for backward compatibility.

Kosmidis and Zeileis (2024) introduce a generalization of the classic beta regression model with extended support  $[0, 1]$ . Specifically, the extended-support beta distribution ("xbeta") leverages an underlying symmetric four-parameter beta distribution with exceedence parameter  $\nu$  to obtain support  $[-\nu, 1 + \nu]$  that is subsequently censored to  $[0, 1]$  in order to obtain point masses at the boundary values 0 and 1. The extended-support beta mixture distribution ("xbetax") is a continuous mixture of extended-support beta distributions where the exceedence parameter follows an exponential distribution with mean  $\nu$  (rather than a fixed value of  $\nu$ ). The latter "xbetax" specification is used by default in case of boundary observations at 0 and/or 1. The "xbeta" specification with fixed  $\nu$  is mostly for testing and debugging purposes.

A set of standard extractor functions for fitted model objects is available for objects of class "betareg", including methods to the generic functions `print`, `summary`, `plot`, `coef`, `vcov`, `logLik`, `residuals`, `predict`, `terms`, `model.frame`, `model.matrix`, `cooks.distance` and `hatvalues` (see `influence.measures`), `gleverage` (new generic), `estfun` and `bread` (from the **sandwich** package), and `coeftest` (from the **lmtest** package).

See `predict.betareg`, `residuals.betareg`, `plot.betareg`, and `summary.betareg` for more details on all methods.

The main parameters of interest are the coefficients in the linear predictor of the mean model. The additional parameters in the precision model ( $\phi$ ) can either be treated as full model parameters (default) or as nuisance parameters. In the latter case the estimation does not change, only the reported information in output from `print`, `summary`, or `coef` (among others) will be different. See also `betareg.control`.

The implemented algorithms for bias correction/reduction follow Kosmidis and Firth (2010). Technical note: In case, either bias correction or reduction is requested, the second derivative of the inverse link function is required for `link` and `link.phi`. If the two links are specified by their names (as done by default in `betareg`), then the "link-glm" objects are enhanced automatically by the required additional `d2mu.deta` function. However, if a "link-glm" object is supplied directly by the user, it needs to have the `d2mu.deta` function or, for backward compatibility, `dmu.deta`.

The original version of the package was written by Alexandre B. Simas and Andrea V. Rocha (up to version 1.2). Starting from version 2.0-0 the code was rewritten by Achim Zeileis.

## Value

`betareg` returns an object of class "betareg", i.e., a list with components as follows. For classic beta regressions (`dist = "beta"`) several elements are lists with the names "mean" and "precision" for the information from the respective submodels. For extended-support beta regressions (`dist = "xbetax" or "xbeta"`), the corresponding names are "mu" and "phi" because they are not exactly the mean and precision due to the censoring in the response variable.

`betareg.fit` returns an unclassed list with components up to converged.

<code>coefficients</code>	a list with elements "mean" (or "mu") and "precision" (or "phi") containing the coefficients from the respective submodels and for extended-support beta regressions an additional element "nu",
<code>residuals</code>	a vector of raw residuals (observed - fitted),
<code>fitted.values</code>	a vector of fitted means,

<code>optim</code>	output from the <code>optim</code> call for maximizing the log-likelihood(s),
<code>method</code>	the method argument passed to the <code>optim</code> call,
<code>control</code>	the control arguments passed to the <code>optim</code> call,
<code>start</code>	the starting values for the parameters passed to the <code>optim</code> call,
<code>weights</code>	the weights used (if any),
<code>offset</code>	a list of offset vectors used (if any),
<code>n</code>	number of observations,
<code>nobs</code>	number of observations with non-zero weights,
<code>df.null</code>	residual degrees of freedom in the null model (constant mean and dispersion), i.e., $n - 2$ ,
<code>df.residual</code>	residual degrees of freedom in the fitted model,
<code>phi</code>	logical indicating whether the precision ( <code>phi</code> ) coefficients will be treated as full model parameters or nuisance parameters in subsequent calls to <code>print</code> , <code>summary</code> , <code>coef</code> etc.,
<code>loglik</code>	log-likelihood of the fitted model,
<code>vcov</code>	covariance matrix of all parameters in the model,
<code>pseudo.r.squared</code>	pseudo R-squared value (squared correlation of linear predictor and link-transformed response),
<code>link</code>	a list with elements "mean" (or "mu") and "precision" (or "phi") containing the link objects for the respective submodels,
<code>converged</code>	logical indicating successful convergence of <code>optim</code> ,
<code>call</code>	the original function call,
<code>formula</code>	the original formula,
<code>terms</code>	a list with elements "mean" (or "mu"), "precision" (or "phi") and "full" containing the terms objects for the respective models,
<code>levels</code>	a list with elements "mean" (or "mu"), "precision" (or "phi") and "full" containing the levels of the categorical regressors,
<code>contrasts</code>	a list with elements "mean" (or "mu") and "precision" (or "phi") containing the contrasts corresponding to levels from the respective models,
<code>model</code>	the full model frame (if <code>model = TRUE</code> ),
<code>y</code>	the response proportion vector (if <code>y = TRUE</code> ),
<code>x</code>	a list with elements "mean" (or "mu") and "precision" (or "phi") containing the model matrices from the respective models (if <code>x = TRUE</code> ).

## References

- Cribari-Neto F, Zeileis A (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. doi:10.18637/jss.v034.i02
- Ferrari SLP, Cribari-Neto F (2004). Beta Regression for Modeling Rates and Proportions. *Journal of Applied Statistics*, **31**(7), 799–815.

Grün B, Kosmidis I, Zeileis A (2012). Extended Beta Regression in R: Shaken, Stirred, Mixed, and Partitioned. *Journal of Statistical Software*, **48**(11), 1–25. doi:10.18637/jss.v048.i11

Kosmidis I, Firth D (2010). A Generic Algorithm for Reducing Bias in Parametric Estimation. *Electronic Journal of Statistics*, **4**, 1097–1112.

Kosmidis I, Zeileis A (2024). Extended-Support Beta Regression for [0, 1] Responses. 2409.07233, *arXiv.org E-Print Archive*. doi:10.48550/arXiv.2409.07233

Simas AB, Barreto-Souza W, Rocha AV (2010). Improved Estimators for a General Class of Beta Regression Models. *Computational Statistics & Data Analysis*, **54**(2), 348–366.

### See Also

[summary.betareg](#), [predict.betareg](#), [residuals.betareg](#), [Formula](#)

### Examples

```
options(digits = 4)

## Section 4 from Ferrari and Cribari-Neto (2004)
data("GasolineYield", package = "betareg")
data("FoodExpenditure", package = "betareg")

## Table 1
gy <- betareg(yield ~ batch + temp, data = GasolineYield)
summary(gy)

## Table 2
fe_lin <- lm(I(food/income) ~ income + persons, data = FoodExpenditure)
library("lmtest")
bptest(fe_lin)
fe_beta <- betareg(I(food/income) ~ income + persons, data = FoodExpenditure)
summary(fe_beta)

## nested model comparisons via Wald and LR tests
fe_beta2 <- betareg(I(food/income) ~ income, data = FoodExpenditure)
lrtest(fe_beta, fe_beta2)
waldtest(fe_beta, fe_beta2)

## Section 3 from online supplements to Simas et al. (2010)
## mean model as in gy above
## precision model with regressor temp
gy2 <- betareg(yield ~ batch + temp | temp, data = GasolineYield)

## MLE column in Table 19
summary(gy2)

## LRT row in Table 18
lrtest(gy, gy2)
```

---

betareg.control      *Control Parameters for Beta Regression*

---

### Description

Various parameters that control fitting of beta regression models using [betareg](#).

### Usage

```
betareg.control(phi = TRUE, method = "BFGS", maxit = 5000,
  gradient = NULL, hessian = FALSE, trace = FALSE, start = NULL,
  fsmaxit = 200, fstol = 1e-8, quad = 20, ...)
```

### Arguments

phi	logical indicating whether the precision parameter phi should be treated as a full model parameter (TRUE, default) or as a nuisance parameter.
method	characters string specifying the method argument passed to <a href="#">optim</a> . Additionally, method = "nllminb" can be used to employ <a href="#">nllminb</a> , instead.
maxit	integer specifying the maxit argument (maximal number of iterations) passed to <a href="#">optim</a> .
trace	logical or integer controlling whether tracing information on the progress of the optimization should be produced (passed to <a href="#">optim</a> ).
gradient	logical. Should the analytical gradient be used for optimizing the log-likelihood? If set to FALSE a finite-difference approximation is used instead. The default of NULL signals that analytical gradients are only used for the classical "beta" distribution but not for "xbetax" or "xbeta".
hessian	logical. Should the numerical Hessian matrix from the <a href="#">optim</a> output be used for estimation of the covariance matrix? By default the analytical solution is employed. For details see below.
start	an optional vector with starting values for all parameters (including phi).
fsmaxit	integer specifying maximal number of additional (quasi) Fisher scoring iterations. For details see below.
fstol	numeric tolerance for convergence in (quasi) Fisher scoring. For details see below.
quad	numeric. The number of quadrature points for numeric integration in case of dist = "xbetax" is used in the beta regression.
...	arguments passed to <a href="#">optim</a> .

## Details

All parameters in `betareg` are estimated by maximum likelihood using `optim` with control options set in `betareg.control`. Most arguments are passed on directly to `optim`, and `start` controls how `optim` is called.

After the `optim` maximization, an additional (quasi) Fisher scoring can be performed to further enhance the result or to perform additional bias reduction. If `fsmxit` is greater than zero, this additional optimization is performed and it converges if the threshold `fstol` is attained for the cross-product of the step size.

Starting values can be supplied via `start` or estimated by `lm.wfit`, using the link-transformed response. Covariances are in general derived analytically. Only if `type = "ML"` and `hessian = TRUE`, they are determined numerically using the Hessian matrix returned by `optim`. In the latter case no Fisher scoring iterations are performed.

The main parameters of interest are the coefficients in the linear predictor of the model and the additional precision parameter `phi` which can either be treated as a full model parameter (default) or as a nuisance parameter. In the latter case the estimation does not change, only the reported information in output from `print`, `summary`, or `coef` (among others) will be different. See also examples.

## Value

A list with the arguments specified.

## See Also

[betareg](#)

## Examples

```
options(digits = 4)

data("GasolineYield", package = "betareg")

## regression with phi as full model parameter
gy1 <- betareg(yield ~ batch + temp, data = GasolineYield)
gy1

## regression with phi as nuisance parameter
gy2 <- betareg(yield ~ batch + temp, data = GasolineYield, phi = FALSE)
gy2

## compare reported output
coef(gy1)
coef(gy2)
summary(gy1)
summary(gy2)
```

---

betatree	<i>Beta Regression Trees</i>
----------	------------------------------

---

**Description**

Fit beta regression trees via model-based recursive partitioning.

**Usage**

```
betatree(formula, partition,
         data, subset = NULL, na.action = na.omit, weights, offset, cluster,
         link = "logit", link.phi = "log", control = betareg.control(),
         ...)
```

**Arguments**

formula	symbolic description of the model of type $y \sim x$ or $y \sim x \mid z$ , specifying the variables influencing mean and precision of $y$ , respectively. For details see <a href="#">betareg</a> .
partition	symbolic description of the partitioning variables, e.g., $\sim p1 + p2$ . The argument partition can be omitted if formula is a three-part formula of type $y \sim x \mid z \mid p1 + p2$ .
data, subset, na.action, weights, offset, cluster	arguments controlling data/model processing passed to <a href="#">mob</a> .
link	character specification of the link function in the mean model ( $\mu$ ). Currently, "logit", "probit", "cloglog", "cauchit", "log", "loglog" are supported. Alternatively, an object of class "link-glm" can be supplied.
link.phi	character specification of the link function in the precision model ( $\phi$ ). Currently, "identity", "log", "sqrt" are supported. Alternatively, an object of class "link-glm" can be supplied.
control	a list of control arguments for the beta regression specified via <a href="#">betareg.control</a> .
...	further control arguments for the recursive partitioning passed to <a href="#">mob_control</a> .

**Details**

Beta regression trees are an application of model-based recursive partitioning (implemented in [mob](#), see Zeileis et al. 2008) to beta regression (implemented in [betareg](#), see Cribari-Neto and Zeileis 2010). See also Grün et al. (2012) for more details.

Various methods are provided for "betatree" objects, most of them inherit their behavior from "mob" objects (e.g., print, summary, coef, etc.). The plot method employs the [node\\_bivplot](#) panel-generating function.

**Value**

betatree() returns an object of S3 class "betatree" which inherits from "modelparty".

## References

- Cribari-Neto F, Zeileis A (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. doi:10.18637/jss.v034.i02
- Grün B, Kosmidis I, Zeileis A (2012). Extended Beta Regression in R: Shaken, Stirred, Mixed, and Partitioned. *Journal of Statistical Software*, **48**(11), 1–25. doi:10.18637/jss.v048.i11
- Zeileis A, Hothorn T, Hornik K (2008). Model-Based Recursive Partitioning. *Journal of Computational and Graphical Statistics*, **17**(2), 492–514.

## See Also

[betareg](#), [betareg.fit](#), [mob](#)

## Examples

```
options(digits = 4)
suppressWarnings(RNGversion("3.5.0"))

## data with two groups of dyslexic and non-dyslexic children
data("ReadingSkills", package = "betareg")
## additional random noise (not associated with reading scores)
set.seed(1071)
ReadingSkills$x1 <- rnorm(nrow(ReadingSkills))
ReadingSkills$x2 <- runif(nrow(ReadingSkills))
ReadingSkills$x3 <- factor(rnorm(nrow(ReadingSkills)) > 0)

## fit beta regression tree: in each node
## - accurcay's mean and precision depends on iq
## - partitioning is done by dyslexia and the noise variables x1, x2, x3
## only dyslexia is correctly selected for splitting
bt <- betatree(accuracy ~ iq | iq, ~ dyslexia + x1 + x2 + x3,
  data = ReadingSkills, minsize = 10)
plot(bt)

## inspect result
coef(bt)
if(require("strucchange")) sctest(bt)
## IGNORE_RDIFF_BEGIN
summary(bt, node = 2)
summary(bt, node = 3)
## IGNORE_RDIFF_END

## add a numerical variable with relevant information for splitting
ReadingSkills$x4 <- rnorm(nrow(ReadingSkills), c(-1.5, 1.5)[ReadingSkills$dyslexia])

bt2 <- betatree(accuracy ~ iq | iq, ~ x1 + x2 + x3 + x4,
  data = ReadingSkills, minsize = 10)
plot(bt2)

## inspect result
coef(bt2)
if(require("strucchange")) sctest(bt2)
```

```
## IGNORE_RDIFF_BEGIN
summary(bt2, node = 2)
summary(bt2, node = 3)
## IGNORE_RDIFF_END
```

---

CarTask

*Partition-Primed Probability Judgement Task for Car Dealership*

---

### Description

In this study participants were asked to judge how likely it is that a customer trades in a coupe or that a customer buys a car from a specific salesperson out of four possible salespersons.

### Usage

```
data("CarTask", package = "betareg")
```

### Format

A data frame with 155 observations on the following 3 variables.

`task` a factor with levels Car and Salesperson indicating the condition.

`probability` a numeric vector of the estimated probability.

`NFCCscale` a numeric vector of the NFCC scale.

### Details

All participants in the study were undergraduate students at The Australian National University, some of whom obtained course credit in first-year Psychology for their participation in the study.

The NFCC scale is a combined scale of the Need for Closure and Need for Certainty scales which are strongly correlated.

For task the questions were:

**Car** What is the probability that a customer trades in a coupe?

**Salesperson** What is the probability that a customer buys a car from Carlos?

### Source

Taken from Smithson et al. (2011) supplements.

### References

Smithson, M., Merkle, E.C., and Verkuilen, J. (2011). Beta Regression Finite Mixture Models of Polarization and Priming. *Journal of Educational and Behavioral Statistics*, **36**(6), 804–831. doi:10.3102/1076998610396893

Smithson, M., and Segale, C. (2009). Partition Priming in Judgments of Imprecise Probabilities. *Journal of Statistical Theory and Practice*, **3**(1), 169–181.



## Examples

```
data("CarTask", package = "betareg")
library("flexmix")
car_betamix <- betamix(probability ~ 1, data = CarTask, k = 3,
  extra_components = list(extraComponent(type = "uniform", coef = 1/2,
    delta = 0.01), extraComponent(type = "uniform", coef = 1/4, delta = 0.01)),
  FLXconcomitant = FLXPmultinom(~ task))
```

---

FoodExpenditure	<i>Proportion of Household Income Spent on Food</i>
-----------------	---

---

## Description

Data on proportion of income spent on food for a random sample of 38 households in a large US city.

## Usage

```
data("FoodExpenditure", package = "betareg")
```

## Format

A data frame containing 38 observations on 3 variables.

**food** household expenditures for food.

**income** household income.

**persons** number of persons living in household.

## Source

Taken from Griffiths et al. (1993, Table 15.4).

## References

Cribari-Neto, F., and Zeileis, A. (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. doi:10.18637/jss.v034.i02

Ferrari, S.L.P., and Cribari-Neto, F. (2004). Beta Regression for Modeling Rates and Proportions. *Journal of Applied Statistics*, **31**(7), 799–815.

Griffiths, W.E., Hill, R.C., and Judge, G.G. (1993). *Learning and Practicing Econometrics* New York: John Wiley and Sons.

## See Also

[betareg](#)

**Examples**

```

data("FoodExpenditure", package = "betareg")

## Ferrari and Cribari-Neto (2004)
## Section 4
fe_lin <- lm(I(food/income) ~ income + persons, data = FoodExpenditure)
library("lmtest")
bptest(fe_lin)

## Table 2
fe_beta <- betareg(I(food/income) ~ income + persons, data = FoodExpenditure)
summary(fe_beta)

```

---

GasolineYield

*Estimation of Gasoline Yields from Crude Oil*


---

**Description**

Operational data of the proportion of crude oil converted to gasoline after distillation and fractionation.

**Usage**

```
data("GasolineYield", package = "betareg")
```

**Format**

A data frame containing 32 observations on 6 variables.

**yield** proportion of crude oil converted to gasoline after distillation and fractionation.

**gravity** crude oil gravity (degrees API).

**pressure** vapor pressure of crude oil (lbf/in2).

**temp10** temperature (degrees F) at which 10 percent of crude oil has vaporized.

**temp** temperature (degrees F) at which all gasoline has vaporized.

**batch** factor indicating unique batch of conditions gravity, pressure, and temp10.

**Details**

This dataset was collected by Prater (1956), its dependent variable is the proportion of crude oil after distillation and fractionation. This dataset was analyzed by Atkinson (1985), who used the linear regression model and noted that there is “indication that the error distribution is not quite symmetrical, giving rise to some unduly large and small residuals” (p. 60).

The dataset contains 32 observations on the response and on the independent variables. It has been noted (Daniel and Wood, 1971, Chapter 8) that there are only ten sets of values of the first three explanatory variables which correspond to ten different crudes and were subjected to experimentally controlled distillation conditions. These conditions are captured in variable batch and the data were ordered according to the ascending order of temp10.

## Source

Taken from Prater (1956).

## References

Atkinson, A.C. (1985). *Plots, Transformations and Regression: An Introduction to Graphical Methods of Diagnostic Regression Analysis*. New York: Oxford University Press.

Cribari-Neto, F., and Zeileis, A. (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. doi:10.18637/jss.v034.i02

Daniel, C., and Wood, F.S. (1971). *Fitting Equations to Data*. New York: John Wiley and Sons.

Ferrari, S.L.P., and Cribari-Neto, F. (2004). Beta Regression for Modeling Rates and Proportions. *Journal of Applied Statistics*, **31**(7), 799–815.

Prater, N.H. (1956). Estimate Gasoline Yields from Crudes. *Petroleum Refiner*, **35**(5), 236–238.

## See Also

[betareg](#)

## Examples

```
## IGNORE_RDIFF_BEGIN
data("GasolineYield", package = "betareg")

gy1 <- betareg(yield ~ gravity + pressure + temp10 + temp, data = GasolineYield)
summary(gy1)

## Ferrari and Cribari-Neto (2004)
gy2 <- betareg(yield ~ batch + temp, data = GasolineYield)
## Table 1
summary(gy2)
## Figure 2
par(mfrow = c(3, 2))
plot(gy2, which = 1, type = "pearson", sub.caption = "")
plot(gy2, which = 1, type = "deviance", sub.caption = "")
plot(gy2, which = 5, type = "deviance", sub.caption = "")
plot(gy2, which = 4, type = "pearson", sub.caption = "")
plot(gy2, which = 2:3)
par(mfrow = c(1, 1))

## exclude 4th observation
gy2a <- update(gy2, subset = -4)
gy2a
summary(gy2a)
## IGNORE_RDIFF_END
```

---

`gleverage`*Generalized Leverage Values*

---

**Description**

Compute the generalized leverages values for fitted models.

**Usage**

```
gleverage(model, ...)
```

**Arguments**

<code>model</code>	a model object.
<code>...</code>	further arguments passed to methods.

**Value**

`gleverage` is a new generic for computing generalized leverage values as suggested by Wei, Hu, and Fung (1998). Currently, there is only a method for `betareg` models, implementing the formulas from Rocha and Simas (2011) which are consistent with the formulas from Ferrari and Cribari-Neto (2004) for the fixed dispersion case.

Currently, the vector of generalized leverages requires computations and storage of order  $n \times n$ .

**References**

- Ferrari SLP, Cribari-Neto F (2004). Beta Regression for Modeling Rates and Proportions. *Journal of Applied Statistics*, **31**(7), 799–815.
- Rocha AV, Simas AB (2011). Influence Diagnostics in a General Class of Beta Regression Models. *Test*, **20**(1), 95–119. doi:10.1007/s117490100189z
- Wei BC, Hu, YQ, Fung WK (1998). Generalized Leverage and Its Applications. *Scandinavian Journal of Statistics*, **25**, 25–37.

**See Also**

[betareg](#)

**Examples**

```
options(digits = 4)
data("GasolineYield", package = "betareg")
gy <- betareg(yield ~ batch + temp, data = GasolineYield)
gleverage(gy)
```

---

ImpreciseTask

*Imprecise Probabilities for Sunday Weather and Boeing Stock Task*

---

## Description

In this study participants were asked to estimate upper and lower probabilities for event to occur and not to occur.

## Usage

```
data("ImpreciseTask", package = "betareg")
```

## Format

A data frame with 242 observations on the following 3 variables.

`task` a factor with levels Boeing stock and Sunday weather.

`location` a numeric vector of the average of the lower estimate for the event not to occur and the upper estimate for the event to occur.

`difference` a numeric vector of the differences of the lower and upper estimate for the event to occur.

## Details

All participants in the study were either first- or second-year undergraduate students in psychology, none of whom had a strong background in probability or were familiar with imprecise probability theories.

For the Sunday weather task see [WeatherTask](#). For the Boeing stock task participants were asked to estimate the probability that Boeing's stock would rise more than those in a list of 30 companies.

For each task participants were asked to provide lower and upper estimates for the event to occur and not to occur.

## Source

Taken from Smithson et al. (2011) supplements.

## References

Smithson M, Merkle EC, Verkuilen J (2011). Beta Regression Finite Mixture Models of Polarization and Priming. *Journal of Educational and Behavioral Statistics*, **36**(6), 804–831. doi:10.3102/1076998610396893

Smithson M, Segale C (2009). Partition Priming in Judgments of Imprecise Probabilities. *Journal of Statistical Theory and Practice*, **3**(1), 169–181.

## Examples

```
data("ImpreciseTask", package = "betareg")
library("flexmix")
wt_betamix <- betamix(location ~ difference * task, data = ImpreciseTask, k = 2,
  extra_components = extraComponent(type = "betareg", coef =
    list(mean = 0, precision = 8)),
  FLXconcomitant = FLXPmultinom(~ task))
```

---

LossAversion

*(No) Myopic Loss Aversion in Adolescents*

---

## Description

Data from a behavioral economics experiment assessing the extent of myopic loss aversion among adolescents (mostly aged 11 to 19).

## Usage

```
data("LossAversion", package = "betareg")
```

## Format

A data frame containing 570 observations on 7 variables.

**invest** numeric. Average proportion of tokens invested across all 9 rounds.

**gender** factor. Gender of the player (or team of players).

**male** factor. Was (at least one of) the player(s) male (in the team)?

**age** numeric. Age in years (averaged for teams).

**treatment** factor. Type of treatment: long vs. short.

**grade** factor. School grades: 6-8 (11-14 years) vs. 10-12 (15-18 years).

**arrangement** factor. Is the player a single player or team of two?

## Details

Myopic loss aversion is a phenomenon in behavioral economics, where individuals do not behave economically rationally when making short-term decisions under uncertainty. Example: In lotteries with positive expected payouts investments are lower than the maximum possible (loss aversion). This effect is enhanced for short-term investments (myopia or short-sightedness).

The data in LossAversion were collected by Matthias Sutter and Daniela Glätzle-Rützler (Universität Innsbruck) in an experiment with high-school students in Tyrol, Austria (Schwaz and Innsbruck). The students could invest  $X$  tokens (0-100) in each of 9 rounds in a lottery. The payouts were  $100 + 2.5 * X$  tokens with probability  $1/3$  and  $100 - X$  tokens with probability  $2/3$ . Thus, the expected payouts were  $100 + 1/6 * X$  tokens. Depending on the treatment in the experiment, the investments could either be modified in each round (treatment: "short") or only in round 1, 4, 7 (treatment "long"). Decisions were either made alone or in teams of two. The tokens were converted

to monetary payouts using a conversion of EUR 0.5 per 100 tokens for lower grades (Unterstufe, 6-8) or EUR 1.0 per 100 tokens for upper grades (Oberstufe, 10-12).

From the myopic loss aversion literature (on adults) one would expect that the investments of the players (either single players or teams of two) would depend on all factors: Investments should be

- lower in the short treatment (which would indicate myopia),
- higher for teams (indicating a reduction in loss aversion),
- higher for (teams with) male players,
- increase with age/grade.

See Glätzle-Rützler et al. (2015) for more details and references to the literature. In their original analysis, the investments are analyzed using a panel structure (i.e., 9 separate investments for each team). Here, the data are averaged across rounds for each player, leading to qualitatively similar results. The full data along with replication materials are available in the Harvard Dataverse.

Kosmidis and Zeileis (2024) revisit the data using extended-support beta mixture (XBX) regression, which can simultaneously capture both the probability of rational behavior and the mean amount of loss aversion.

### Source

Glätzle-Rützler D, Sutter M, Zeileis A (2020). Replication Data for: No Myopic Loss Aversion in Adolescents? - An Experimental Note. *Harvard Dataverse*, UNF:6:6hVtbHavJAFYfL7dDI7jqA==. [doi:10.7910/DVN/IHFZAK](https://doi.org/10.7910/DVN/IHFZAK)

### References

Glätzle-Rützler D, Sutter M, Zeileis A (2015). No Myopic Loss Aversion in Adolescents? – An Experimental Note. *Journal of Economic Behavior & Organization*, **111**, 169–176. [doi:10.1016/j.jebo.2014.12.021](https://doi.org/10.1016/j.jebo.2014.12.021)

Kosmidis I, Zeileis A (2024). Extended-Support Beta Regression for [0, 1] Responses. 2409.07233, *arXiv.org E-Print Archive*. [doi:10.48550/arXiv.2409.07233](https://doi.org/10.48550/arXiv.2409.07233)

### See Also

[betareg](#)

### Examples

```
options(digits = 4)

## data and add ad-hoc scaling (a la Smithson & Verkuilen)
data("LossAversion", package = "betareg")
LossAversion <- transform(LossAversion,
  invests = (invest * (nrow(LossAversion) - 1) + 0.5)/nrow(LossAversion))

## models: normal (with constant variance), beta, extended-support beta mixture
la_n <- lm(invest ~ grade * (arrangement + age) + male, data = LossAversion)
summary(la_n)
```

```

la_b <- betareg(invests ~ grade * (arrangement + age) + male | arrangement + male + grade,
  data = LossAversion)
summary(la_b)

la_xbx <- betareg(invest ~ grade * (arrangement + age) + male | arrangement + male + grade,
  data = LossAversion)
summary(la_xbx)

## coefficients in XBX are typically somewhat shrunken compared to beta
cbind(XBX = coef(la_xbx), Beta = c(coef(la_b), NA))

## predictions on subset: (at least one) male players, higher grades, around age 16
la <- subset(LossAversion, male == "yes" & grade == "10-12" & age >= 15 & age <= 17)
la_nd <- data.frame(arrangement = c("single", "team"), male = "yes", age = 16, grade = "10-12")

## empirical vs fitted E(Y)
la_nd$mean_emp <- aggregate(invest ~ arrangement, data = la, FUN = mean)$invest
la_nd$mean_n <- predict(la_n, la_nd)
la_nd$mean_b <- predict(la_b, la_nd)
la_nd$mean_xbx <- predict(la_xbx, la_nd)
la_nd

## visualization: all means rather similar
la_mod <- c("Emp", "N", "B", "XBX")
la_col <- unname(palette.colors())[c(1, 2, 4, 4)]
la_lty <- c(1, 5, 5, 1)
matplot(la_nd[, paste0("mean_", tolower(la_mod))], type = "l",
  col = la_col, lty = la_lty, lwd = 2, ylab = "E(Y)", main = "E(Y)", xaxt = "n")
axis(1, at = 1:2, labels = la_nd$arrangement)
legend("topleft", la_mod, col = la_col, lty = la_lty, lwd = 2, bty = "n")

## empirical vs. fitted P(Y > 0.95)
la_nd$prob_emp <- aggregate(invest >= 0.95 ~ arrangement, data = la, FUN = mean)$invest
la_nd$prob_n <- pnorm(0.95, mean = la_nd$mean_n, sd = summary(la_n)$sigma, lower.tail = FALSE)
la_nd$prob_b <- 1 - predict(la_b, la_nd, type = "probability", at = 0.95)
la_nd$prob_xbx <- 1 - predict(la_xbx, la_nd, type = "probability", at = 0.95)
la_nd[, -(5:8)]

## visualization: only XBX works well
matplot(la_nd[, paste0("prob_", tolower(la_mod))], type = "l",
  col = la_col, lty = la_lty, lwd = 2, ylab = "P(Y > 0.95)", main = "P(Y > 0.95)", xaxt = "n")
axis(1, at = 1:2, labels = la_nd$arrangement)
legend("topleft", la_mod, col = la_col, lty = la_lty, lwd = 2, bty = "n")

```



**Description**

Data with responses of naive mock jurors to the conventional conventional two-option verdict (guilt vs. acquittal) versus a three-option verdict setup (the third option was the Scottish 'not proven' alternative), in the presence/absence of conflicting testimonial evidence.

**Usage**

```
data("MockJurors", package = "betareg")
```

**Format**

A data frame containing 104 observations on 3 variables.

**verdict** factor indicating whether a two-option or three-option verdict is requested. (A sum contrast rather than treatment contrast is employed.)

**conflict** factor. Is there conflicting testimonial evidence? (A sum contrast rather than treatment contrast is employed.)

**confidence** jurors degree of confidence in his/her verdict, scaled to the open unit interval (see below).

**Details**

The data were collected by Daily (2004) among first-year psychology students at Australian National University. Smithson and Verkuilen (2006) employed the data scaling the original confidence (on a scale 0–100) to the open unit interval:  $((\text{original\_confidence}/100) * 103 - 0.5) / 104$ .

The original coding of conflict in the data provided from Smithson's homepage is -1/1 which Smithson and Verkuilen (2006) describe to mean no/yes. However, all their results (sample statistics, histograms, etc.) suggest that it actually means yes/no which was employed in MockJurors.

**Source**

Example 1 from Smithson and Verkuilen (2006) supplements.

**References**

Deady S (2004). The Psychological Third Verdict: 'Not Proven' or 'Not Willing to Make a Decision'? *Unpublished honors thesis*, The Australian National University, Canberra.

Smithson M, Verkuilen J (2006). A Better Lemon Squeezer? Maximum-Likelihood Regression with Beta-Distributed Dependent Variables. *Psychological Methods*, **11**(7), 54–71.

**See Also**

[betareg](#), [ReadingSkills](#), [StressAnxiety](#)

## Examples

```

data("MockJurors", package = "betareg")
library("lmtest")

## Smithson & Verkuilen (2006, Table 1)
## variable dispersion model
## (NOTE: numerical rather than analytical Hessian is used for replication,
## Smithson & Verkuilen erroneously compute one-sided p-values)
mj_vd <- betareg(confidence ~ verdict * conflict | verdict * conflict,
  data = MockJurors, hessian = TRUE)
summary(mj_vd)

## model selection for beta regression: null model, fixed dispersion model (p. 61)
mj_null <- betareg(confidence ~ 1 | 1, data = MockJurors)
mj_fd <- betareg(confidence ~ verdict * conflict | 1, data = MockJurors)
lrtest(mj_null, mj_fd)
lrtest(mj_null, mj_vd)
## McFadden's pseudo-R-squared
1 - as.vector(logLik(mj_null)/logLik(mj_vd))

## visualization
if(require("lattice")) {
  histogram(~ confidence | conflict + verdict, data = MockJurors,
    col = "lightgray", breaks = 0:10/10, type = "density")
}

## see demo("SmithsonVerkuilen2006", package = "betareg") for more details

```

---

plot.betareg

*Diagnostic Plots for betareg Objects*

---

## Description

Various types of standard diagnostic plots can be produced, involving various types of residuals, influence measures etc.

## Usage

```

## S3 method for class 'betareg'
plot(x, which = 1:4,
  caption = c("Residuals vs indices of obs.", "Cook's distance plot",
    "Generalized leverage vs predicted values", "Residuals vs linear predictor",
    "Half-normal plot of residuals", "Predicted vs observed values"),
  sub.caption = paste(deparse(x$call), collapse = "\n"), main = "",
  ask = prod(par("mfcol")) < length(which) && dev.interactive(),
  ..., type = "quantile", nsim = 100, level = 0.9)

```

**Arguments**

x	fitted model object of class "betareg".
which	numeric. If a subset of the plots is required, specify a subset of the numbers 1:6.
caption	character. Captions to appear above the plots.
sub.caption	character. Common title-above figures if there are multiple.
main	character. Title to each plot in addition to the above caption.
ask	logical. If TRUE, the user is asked before each plot.
...	other parameters to be passed through to plotting functions.
type	character indicating type of residual to be used, see <a href="#">residuals.betareg</a> .
nsim	numeric. Number of simulations in half-normal plots.
level	numeric. Confidence level in half-normal plots.

**Details**

The plot method for [betareg](#) objects produces various types of diagnostic plots. Most of these are standard for regression models and involve various types of residuals, influence measures etc. See Ferrari and Cribari-Neto (2004) for a discussion of some of these displays.

The which argument can be used to select a subset of currently six supported types of displays. The corresponding element of caption contains a brief description. In some more detail, the displays are: Residuals (as selected by type) vs indices of observations (which = 1). Cook's distances vs indices of observations (which = 2). Generalized leverage vs predicted values (which = 3). Residuals vs linear predictor (which = 4). Half-normal plot of residuals (which = 5), which is obtained using a simulation approach. Predicted vs observed values (which = 6).

**References**

Cribari-Neto F, Zeileis A (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. [doi:10.18637/jss.v034.i02](https://doi.org/10.18637/jss.v034.i02)

Ferrari SLP, Cribari-Neto F (2004). Beta Regression for Modeling Rates and Proportions. *Journal of Applied Statistics*, **31**(7), 799–815.

**See Also**

[betareg](#)

**Examples**

```
data("GasolineYield", package = "betareg")

gy <- betareg(yield ~ gravity + pressure + temp10 + temp, data = GasolineYield)

par(mfrow = c(3, 2))
plot(gy, which = 1:6)
par(mfrow = c(1, 1))
```

---

predict.betareg      *Prediction Method for betareg Objects*

---

## Description

Extract various types of predictions from beta regression models: First, GLM-style predictions on the scale of responses in (0, 1) or the scale of the linear predictor are provided. Second, various quantities based on the predicted beta distributions are available, e.g., moments, quantiles, probabilities, densities, etc.

## Usage

```
## S3 method for class 'betareg'
predict(object, newdata = NULL,
        type = c("response", "link", "precision", "variance", "parameters",
                "distribution", "density", "probability", "quantile"),
        na.action = na.pass, at = 0.5, elementwise = NULL, ...)
```

## Arguments

object	fitted model object of class "betareg".
newdata	optionally, a data frame in which to look for variables with which to predict. If omitted, the original observations are used.
type	character indicating type of predictions: fitted means of the response (default, "response" or equivalently "mean"), corresponding linear predictor ("link"), fitted precision parameter phi ("precision"), fitted variances of the response ("variance"), all "parameters" of the response distribution, or the corresponding "distribution" object (using the infrastructure from <b>distributions3</b> ). Finally, standard functions for the distribution can be evaluated (at argument at, see below), namely the "density" (or equivalently "pdf"), the "quantile" function, or the cumulative "probability" (or equivalently "cdf").
na.action	function determining what should be done with missing values in newdata. The default is to predict NA.
at	numeric vector at which the predictions should be evaluated if type specifies a function that takes an additional argument.
elementwise	logical. Should each element of the distribution only be evaluated at the corresponding element of at (elementwise = TRUE) or at all elements in at (elementwise = FALSE). Elementwise evaluation is only possible if the number of observations is the same as the length of at and in that case a vector of the same length is returned. Otherwise a matrix is returned. The default is to use elementwise = TRUE if possible, and otherwise elementwise = FALSE.
...	further arguments when type specifies a function, e.g., type = "density" with log = TRUE computes log-densities (or log-likelihoods).

## Details

Each prediction for a `betareg` model internally first computes the parameters for all observations in `newdata` (or the original data if `newdata` is missing). These parameters correspond to a predicted beta distribution (or extended-support beta distribution) for each observation. Then the actual predictions can also be moments of the distributions or standard quantities such as densities, cumulative probabilities, or quantiles. The latter are computed with the d/p/q functions such as `dbetar` (or `dxbetax` or `dxbeta`).

## Value

Either a vector or matrix of predictions with the same number of observations as rows in `newdata`.

## Examples

```
options(digits = 4)

data("GasolineYield", package = "betareg")

gy2 <- betareg(yield ~ batch + temp | temp, data = GasolineYield)

cbind(
  predict(gy2, type = "response"),
  predict(gy2, type = "link"),
  predict(gy2, type = "precision"),
  predict(gy2, type = "variance"),
  predict(gy2, type = "quantile", at = c(0.25, 0.5, 0.75))
)

## evaluate cumulative probabilities for (small) new data set
gyd <- GasolineYield[c(1, 5, 10), ]

## CDF at 0.1 for each observation
predict(gy2, newdata = gyd, type = "probability", at = 0.1)

## CDF at each combination of 0.1/0.2 and observations
predict(gy2, newdata = gyd, type = "probability", at = c(0.1, 0.2))

## CDF at elementwise combinations of 0.1/0.2/0.3 and observations
predict(gy2, newdata = gyd, type = "probability", at = c(0.1, 0.2, 0.3))
predict(gy2, newdata = gyd, type = "probability", at = c(0.1, 0.2, 0.3), elementwise = TRUE)

## CDF at all combinations of 0.1/0.2/0.3 and observations
predict(gy2, newdata = gyd, type = "probability", at = c(0.1, 0.2, 0.3), elementwise = FALSE)
```

**Description**

Data for assessing the contribution of non-verbal IQ to children's reading skills in dyslexic and non-dyslexic children.

**Usage**

```
data("ReadingSkills", package = "betareg")
```

**Format**

A data frame containing 44 observations on 3 variables.

**accuracy** numeric. Reading score with maximum restricted to be 0.99 rather than 1 (see below).

**dyslexia** factor. Is the child dyslexic? (A sum contrast rather than treatment contrast is employed.)

**iq** numeric. Non-verbal intelligence quotient transformed to z-scores.

**accuracy1** numeric. Unrestricted reading score with a maximum of 1 (see below).

**Details**

The data were collected by Pammer and Kevan (2004) and employed by Smithson and Verkuilen (2006). The original reading accuracy score was transformed by Smithson and Verkuilen (2006) so that accuracy is in the open unit interval (0, 1) and beta regression can be employed. First, the original accuracy was scaled using the minimal and maximal score (a and b, respectively) that can be obtained in the test:  $accuracy1 = (original\_accuracy - a) / (b - a)$  (a and b are not provided). Subsequently, accuracy was obtained from accuracy1 by replacing all observations with a value of 1 with 0.99.

Kosmidis and Zeileis (2024) propose to investigate the original unrestricted accuracy1 variable using their extended-support beta mixture regression.

**Source**

Example 3 from Smithson and Verkuilen (2006) supplements.

**References**

- Cribari-Neto F, Zeileis A (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. [doi:10.18637/jss.v034.i02](https://doi.org/10.18637/jss.v034.i02)
- Grün B, Kosmidis I, Zeileis A (2012). Extended Beta Regression in R: Shaken, Stirred, Mixed, and Partitioned. *Journal of Statistical Software*, **48**(11), 1–25. [doi:10.18637/jss.v048.i11](https://doi.org/10.18637/jss.v048.i11)
- Kosmidis I, Zeileis A (2024). Extended-Support Beta Regression for [0, 1] Responses. 2409.07233, *arXiv.org E-Print Archive*. [doi:10.48550/arXiv.2409.07233](https://doi.org/10.48550/arXiv.2409.07233)
- Pammer K, Kevan A (2004). The Contribution of Visual Sensitivity, Phonological Processing and Non-Verbal IQ to Children's Reading. *Unpublished manuscript*, The Australian National University, Canberra.
- Smithson M, Verkuilen J (2006). A Better Lemon Squeezer? Maximum-Likelihood Regression with Beta-Distributed Dependent Variables. *Psychological Methods*, **11**(7), 54–71.

**See Also**

[betareg](#), [MockJurors](#), [StressAnxiety](#)

**Examples**

```
options(digits = 4)
data("ReadingSkills", package = "betareg")

## Smithson & Verkuilen (2006, Table 5)
## OLS regression
## (Note: typo in iq coefficient: 0.3954 instead of 0.3594)
rs_ols <- lm(qlogis(accuracy) ~ dyslexia * iq, data = ReadingSkills)
summary(rs_ols)
## Beta regression (with numerical rather than analytic standard errors)
## (Note: Smithson & Verkuilen erroneously compute one-sided p-values)
rs_beta <- betareg(accuracy ~ dyslexia * iq | dyslexia + iq,
  data = ReadingSkills, hessian = TRUE)
summary(rs_beta)

## Extended-support beta mixture regression (Kosmidis & Zeileis 2024)
rs_xbx <- betareg(accuracy1 ~ dyslexia * iq | dyslexia + iq, data = ReadingSkills)
summary(rs_xbx)

## Coefficients in XBX are typically somewhat shrunken compared to beta
cbind(XBX = coef(rs_xbx), Beta = c(coef(rs_beta), NA))

## Visualization
plot(accuracy1 ~ iq, data = ReadingSkills, col = c(4, 2)[dyslexia], pch = 19)
nd <- data.frame(dyslexia = "no", iq = -30:30/10)
lines(nd$i, predict(rs_xbx, nd), col = 4)
lines(nd$i, predict(rs_beta, nd), col = 4, lty = 5)
lines(nd$i, plogis(predict(rs_ols, nd)), col = 4, lty = 3)
nd <- data.frame(dyslexia = "yes", iq = -30:30/10)
lines(nd$i, predict(rs_xbx, nd), col = 2)
lines(nd$i, predict(rs_beta, nd), col = 2, lty = 5)
lines(nd$i, plogis(predict(rs_ols, nd)), col = 2, lty = 3)
legend("topleft", c("Dyslexia: no", "Dyslexia: yes", "OLS", "XBX", "Beta"),
  lty = c(0, 0, 3, 1, 5), pch = c(19, 19, NA, NA, NA), col = c(4, 2, 1, 1, 1), bty = "n")

## see demo("SmithsonVerkuilen2006", package = "betareg") for further details
```

---

residuals.betareg

*Residuals Method for betareg Objects*


---

**Description**

Extract various types of residuals from beta regression models: raw response residuals (observed - fitted), Pearson residuals (raw residuals scaled by square root of variance function), deviance residuals (scaled log-likelihood contributions), and different kinds of weighted residuals suggested by Espinheira et al. (2008).

**Usage**

```
## S3 method for class 'betareg'
residuals(object, type = c("quantile",
  "deviance", "pearson", "response", "weighted", "sweighted", "sweighted2"),
  ...)
```

**Arguments**

object	fitted model object of class "betareg".
type	character indicating type of residuals.
...	currently not used.

**Details**

The default residuals (starting from version 3.2-0) are quantile residuals as proposed by Dunn and Smyth (1996) and explored in the context of beta regression by Pereira (2017). In case of extended-support beta regression with boundary observations at 0 and/or 1, the quantile residuals for the boundary observations are randomized.

The definitions of all other residuals are provided in Espinheira et al. (2008): Equation 2 for "pearson", last equation on page 409 for "deviance", Equation 6 for "weighted", Equation 7 for "sweighted", and Equation 8 for "sweighted2".

Espinheira et al. (2008) recommend to use "sweighted2", hence this was the default prior to version 3.2-0. However, these are rather burdensome to compute because they require operations of  $O(n^2)$  and hence are typically prohibitively costly in large sample. Also they are not available for extended-support beta regression. Finally, Pereira (2017) found quantile residuals to have better distributional properties.

**References**

- Cribari-Neto F, Zeileis A (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. doi:10.18637/jss.v034.i02
- Dunn PK, Smyth GK (1996). Randomized Quantile Residuals. *Journal of Computational and Graphical Statistics*, **5**(3), 236–244. doi:10.2307/1390802
- Espinheira PL, Ferrari SLP, Cribari-Neto F (2008). On Beta Regression Residuals. *Journal of Applied Statistics*, **35**(4), 407–419. doi:10.1080/02664760701834931
- Ferrari SLP, Cribari-Neto F (2004). Beta Regression for Modeling Rates and Proportions. *Journal of Applied Statistics*, **31**(7), 799–815. doi:10.1080/0266476042000214501
- Pereira GHA (2017). On Quantile Residuals in Beta Regression. *Communications in Statistics – Simulation and Computation*, **48**(1), 302–316. doi:10.1080/03610918.2017.1381740
- Kosmidis I, Zeileis A (2024). Extended-Support Beta Regression for [0, 1] Responses. 2409.07233, *arXiv.org E-Print Archive*. doi:10.48550/arXiv.2409.07233

**See Also**

[betareg](#)



**Examples**

```
options(digits = 4)

data("GasolineYield", package = "betareg")

gy <- betareg(yield ~ gravity + pressure + temp10 + temp, data = GasolineYield)

gy_res <- cbind(
  "quantile" = residuals(gy, type = "quantile"),
  "pearson" = residuals(gy, type = "pearson"),
  "deviance" = residuals(gy, type = "deviance"),
  "response" = residuals(gy, type = "response"),
  "weighted" = residuals(gy, type = "weighted"),
  "sweighted" = residuals(gy, type = "sweighted"),
  "sweighted2" = residuals(gy, type = "sweighted2")
)
pairs(gy_res)

cor(gy_res)
```

---

StressAnxiety

*Dependency of Anxiety on Stress*

---

**Description**

Stress and anxiety among nonclinical women in Townsville, Queensland, Australia.

**Usage**

```
data("StressAnxiety", package = "betareg")
```

**Format**

A data frame containing 166 observations on 2 variables.

**stress** score, linearly transformed to the open unit interval (see below).

**anxiety** score, linearly transformed to the open unit interval (see below).

**Details**

Both variables were assessed on the Depression Anxiety Stress Scales, ranging from 0 to 42. Smithson and Verkuilen (2006) transformed these to the open unit interval (without providing details about this transformation).

**Source**

Example 2 from Smithson and Verkuilen (2006) supplements.

**References**

Smithson M, Verkuilen J (2006). A Better Lemon Squeezer? Maximum-Likelihood Regression with Beta-Distributed Dependent Variables. *Psychological Methods*, **11**(7), 54–71.

**See Also**

[betareg](#), [MockJurors](#), [ReadingSkills](#)

**Examples**

```
data("StressAnxiety", package = "betareg")
StressAnxiety <- StressAnxiety[order(StressAnxiety$stress),]

## Smithson & Verkuilen (2006, Table 4)
sa_null <- betareg(anxiety ~ 1 | 1,
  data = StressAnxiety, hessian = TRUE)
sa_stress <- betareg(anxiety ~ stress | stress,
  data = StressAnxiety, hessian = TRUE)
summary(sa_null)
summary(sa_stress)
AIC(sa_null, sa_stress)
1 - as.vector(logLik(sa_null)/logLik(sa_stress))

## visualization
attach(StressAnxiety)
plot(jitter(anxiety) ~ jitter(stress),
  xlab = "Stress", ylab = "Anxiety",
  xlim = c(0, 1), ylim = c(0, 1))
lines(lowess(anxiety ~ stress))
lines(fitted(sa_stress) ~ stress, lty = 2)
lines(fitted(lm(anxiety ~ stress)) ~ stress, lty = 3)
legend("topleft", c("lowess", "betareg", "lm"), lty = 1:3, bty = "n")
detach(StressAnxiety)

## see demo("SmithsonVerkuilen2006", package = "betareg") for more details
```

---

summary.betareg

*Methods for betareg Objects*

---

**Description**

Methods for extracting information from fitted beta regression model objects of class "betareg".

**Usage**

```
## S3 method for class 'betareg'
summary(object, phi = NULL, type = "quantile", ...)

## S3 method for class 'betareg'
```

```

coef(object, model = c("full", "mean", "precision"), phi = NULL, ...)
## S3 method for class 'betareg'
vcov(object, model = c("full", "mean", "precision"), phi = NULL, ...)
## S3 method for class 'betareg'
bread(x, phi = NULL, ...)
## S3 method for class 'betareg'
estfun(x, phi = NULL, ...)

```

### Arguments

object, x	fitted model object of class "betareg".
phi	logical indicating whether the parameters in the precision model (for phi) should be reported as full model parameters (TRUE) or nuisance parameters (FALSE). The default is taken from object\$phi.
type	character specifying type of residuals to be included in the summary output, see <a href="#">residuals.betareg</a> .
model	character specifying for which component of the model coefficients/covariance should be extracted. (Only used if phi is NULL.)
...	currently not used.

### Details

A set of standard extractor functions for fitted model objects is available for objects of class "betareg", including methods to the generic functions [print](#) and [summary](#) which print the estimated coefficients along with some further information. The [summary](#) in particular supplies partial Wald tests based on the coefficients and the covariance matrix. As usual, the [summary](#) method returns an object of class "summary.betareg" containing the relevant summary statistics which can subsequently be printed using the associated [print](#) method.

A [logLik](#) method is provided, hence [AIC](#) can be called to compute information criteria.

### References

- Cribari-Neto F, Zeileis A (2010). Beta Regression in R. *Journal of Statistical Software*, **34**(2), 1–24. [doi:10.18637/jss.v034.i02](https://doi.org/10.18637/jss.v034.i02)
- Ferrari SLP, Cribari-Neto F (2004). Beta Regression for Modeling Rates and Proportions. *Journal of Applied Statistics*, **31**(7), 799–815.
- Simas AB, Barreto-Souza W, Rocha AV (2010). Improved Estimators for a General Class of Beta Regression Models. *Computational Statistics & Data Analysis*, **54**(2), 348–366.

### See Also

[betareg](#)

### Examples

```

options(digits = 4)

data("GasolineYield", package = "betareg")

```

```

gy2 <- betareg(yield ~ batch + temp | temp, data = GasolineYield)

summary(gy2)
coef(gy2)
vcov(gy2)
logLik(gy2)
AIC(gy2)

coef(gy2, model = "mean")
coef(gy2, model = "precision")
summary(gy2, phi = FALSE)

```

---

WeatherTask

*Weather Task with Priming and Precise and Imprecise Probabilities*


---

### Description

In this study participants were asked to judge how likely Sunday is to be the hottest day of the week.

### Usage

```
data("WeatherTask", package = "betareg")
```

### Format

A data frame with 345 observations on the following 3 variables.

**priming** a factor with levels two-fold (case prime) and seven-fold (class prime).

**eliciting** a factor with levels precise and imprecise (lower and upper limit).

**agreement** a numeric vector, probability indicated by participants or the average between minimum and maximum probability indicated.

### Details

All participants in the study were either first- or second-year undergraduate students in psychology, none of whom had a strong background in probability or were familiar with imprecise probability theories.

For priming the questions were:

**two-fold** [What is the probability that] the temperature at Canberra airport on Sunday will be higher than every other day next week?

**seven-fold** [What is the probability that] the highest temperature of the week at Canberra airport will occur on Sunday?

For eliciting the instructions were if

**precise** to assign a probability estimate,

**imprecise** to assign a lower and upper probability estimate.

**Source**

Taken from Smithson et al. (2011) supplements.

**References**

Smithson M, Merkle EC, Verkuilen J (2011). Beta Regression Finite Mixture Models of Polarization and Priming. *Journal of Educational and Behavioral Statistics*, **36**(6), 804–831. doi:[10.3102/1076998610396893](https://doi.org/10.3102/1076998610396893)

Smithson M, Segale C (2009). Partition Priming in Judgments of Imprecise Probabilities. *Journal of Statistical Theory and Practice*, **3**(1), 169–181.

**Examples**

```
data("WeatherTask", package = "betareg")
library("flexmix")
wt_betamix <- betamix(agreement ~ 1, data = WeatherTask, k = 2,
  extra_components = extraComponent(type = "betareg", coef =
    list(mean = 0, precision = 2)),
  FLXconcomitant = FLXPmultinom(~ priming + eliciting))
```

---

 XBeta

---

*Create an Extended-Support Beta Distribution*


---

**Description**

Class and methods for extended-support beta distributions using the workflow from the **distributions3** package.

**Usage**

```
XBeta(mu, phi, nu = 0)
```

**Arguments**

mu	numeric. The mean of the underlying beta distribution on $[-nu, 1 + nu]$ .
phi	numeric. The precision parameter of the underlying beta distribution on $[-nu, 1 + nu]$ .
nu	numeric. Exceedence parameter for the support of the underlying beta distribution on $[-nu, 1 + nu]$ that is censored to $[0, 1]$ .

**Details**

In order to obtain an extended-support beta distribution on  $[0, 1]$  an additional exceedence parameter  $nu$  is introduced. If  $nu > 0$ , this scales the underlying beta distribution to the interval  $[-nu, 1 + nu]$  where the tails are subsequently censored to the unit interval  $[0, 1]$  with point masses on the boundaries 0 and 1. Thus,  $nu$  controls how likely boundary observations are and for  $nu = 0$  (the default), the distribution reduces to the classic beta distribution (in regression parameterization) without boundary observations.

**Value**

A XBeta distribution object.

**See Also**

[dxbeta](#), [BetaR](#)

**Examples**

```
## package and random seed
library("distributions3")
set.seed(6020)

## three beta distributions
X <- XBeta(
  mu = c(0.25, 0.50, 0.75),
  phi = c(1, 1, 2),
  nu = c(0, 0.1, 0.2)
)

X

## compute moments of the distribution
mean(X)
variance(X)

## support interval (minimum and maximum)
support(X)

## it is only continuous when there are no point masses on the boundary
is_continuous(X)
cdf(X, 0)
cdf(X, 1, lower.tail = FALSE)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ])
hist(x[2, ])
hist(x[3, ])

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0.25, 0.5, 0.75)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)
```

```

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses (except at censoring points)
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

```

---

 xbeta

*The Extended-Support Beta Distribution*


---

## Description

Density, distribution function, quantile function, and random generation for the extended-support beta distribution (in regression parameterization) on  $[0, 1]$ .

## Usage

```

dxbeta(x, mu, phi, nu = 0, log = FALSE)

pxbeta(q, mu, phi, nu = 0, lower.tail = TRUE, log.p = FALSE)

qxbeta(p, mu, phi, nu = 0, lower.tail = TRUE, log.p = FALSE)

rxbeta(n, mu, phi, nu = 0)

```

## Arguments

x, q	numeric. Vector of quantiles.
p	numeric. Vector of probabilities.
n	numeric. Number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.
mu	numeric. The mean of the underlying beta distribution on $[-nu, 1 + nu]$ .

<code>phi</code>	numeric. The precision parameter of the underlying beta distribution on $[-nu, 1 + nu]$ .
<code>nu</code>	numeric. Exceedence parameter for the support of the underlying beta distribution on $[-nu, 1 + nu]$ that is censored to $[0, 1]$ .
<code>log, log.p</code>	logical. If TRUE, probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	logical. If TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$ .

### Details

In order to obtain an extended-support beta distribution on  $[0, 1]$  an additional exceedence parameter  $nu$  is introduced. If  $nu > 0$ , this scales the underlying beta distribution to the interval  $[-nu, 1 + nu]$  where the tails are subsequently censored to the unit interval  $[0, 1]$  with point masses on the boundaries 0 and 1. Thus,  $nu$  controls how likely boundary observations are and for  $nu = 0$  (the default), the distribution reduces to the classic beta distribution (in regression parameterization) without boundary observations.

### Value

`dxbeta` gives the density, `pxbeta` gives the distribution function, `qxbeta` gives the quantile function, and `rxbeta` generates random deviates.

### See Also

[dbetar](#), [XBeta](#)

---

XBetaX

*Create an Extended-Support Beta Mixture Distribution*

---

### Description

Class and methods for extended-support beta distributions using the workflow from the **distributions3** package.

### Usage

```
XBetaX(mu, phi, nu = 0)
```

### Arguments

<code>mu</code>	numeric. The mean of the underlying beta distribution on $[-nu, 1 + nu]$ .
<code>phi</code>	numeric. The precision parameter of the underlying beta distribution on $[-nu, 1 + nu]$ .
<code>nu</code>	numeric. Mean of the exponentially-distributed exceedence parameter for the underlying beta distribution on $[-nu, 1 + nu]$ that is censored to $[0, 1]$ .



## Details

The extended-support beta mixture distribution is a continuous mixture of extended-support beta distributions on  $[0, 1]$  where the underlying exceedence parameter is exponentially distributed with mean  $\nu$ . Thus, if  $\nu > 0$ , the resulting distribution has point masses on the boundaries 0 and 1 with larger values of  $\nu$  leading to higher boundary probabilities. For  $\nu = 0$  (the default), the distribution reduces to the classic beta distribution (in regression parameterization) without boundary observations.

## Value

A `XBetaX` distribution object.

## See Also

[dxbetax](#), [XBeta](#)

## Examples

```
## package and random seed
library("distributions3")
set.seed(6020)

## three beta distributions
X <- XBetaX(
  mu = c(0.25, 0.50, 0.75),
  phi = c(1, 1, 2),
  nu = c(0, 0.1, 0.2)
)

X

## compute moments of the distribution
mean(X)
variance(X)

## support interval (minimum and maximum)
support(X)

## it is only continuous when there are no point masses on the boundary
is_continuous(X)
cdf(X, 0)
cdf(X, 1, lower.tail = FALSE)

## simulate random variables
random(X, 5)

## histograms of 1,000 simulated observations
x <- random(X, 1000)
hist(x[1, ])
hist(x[2, ])
hist(x[3, ])
```

```

## probability density function (PDF) and log-density (or log-likelihood)
x <- c(0.25, 0.5, 0.75)
pdf(X, x)
pdf(X, x, log = TRUE)
log_pdf(X, x)

## cumulative distribution function (CDF)
cdf(X, x)

## quantiles
quantile(X, 0.5)

## cdf() and quantile() are inverses (except at censoring points)
cdf(X, quantile(X, 0.5))
quantile(X, cdf(X, 1))

## all methods above can either be applied elementwise or for
## all combinations of X and x, if length(X) = length(x),
## also the result can be assured to be a matrix via drop = FALSE
p <- c(0.05, 0.5, 0.95)
quantile(X, p, elementwise = FALSE)
quantile(X, p, elementwise = TRUE)
quantile(X, p, elementwise = TRUE, drop = FALSE)

## compare theoretical and empirical mean from 1,000 simulated observations
cbind(
  "theoretical" = mean(X),
  "empirical" = rowMeans(random(X, 1000))
)

```

---

 xbetax

*The Extended-Support Beta Mixture Distribution*


---

## Description

Density, distribution function, quantile function, and random generation for the extended-support beta mixture distribution (in regression parameterization) on  $[0, 1]$ .

## Usage

```

dxbetax(x, mu, phi, nu = 0, log = FALSE, quad = 20)

pxbetax(q, mu, phi, nu = 0, lower.tail = TRUE, log.p = FALSE, quad = 20)

qxbetax(p, mu, phi, nu = 0, lower.tail = TRUE, log.p = FALSE, quad = 20,
  tol = .Machine$double.eps^0.7)

rxbetax(n, mu, phi, nu = 0)

```

**Arguments**

<code>x, q</code>	numeric. Vector of quantiles.
<code>p</code>	numeric. Vector of probabilities.
<code>n</code>	numeric. Number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<code>mu</code>	numeric. The mean of the underlying beta distribution on $[-nu, 1 + nu]$ .
<code>phi</code>	numeric. The precision parameter of the underlying beta distribution on $[-nu, 1 + nu]$ .
<code>nu</code>	numeric. Mean of the exponentially-distributed exceedence parameter for the underlying beta distribution on $[-nu, 1 + nu]$ that is censored to $[0, 1]$ .
<code>log, log.p</code>	logical. If TRUE, probabilities <code>p</code> are given as $\log(p)$ .
<code>lower.tail</code>	logical. If TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$ .
<code>quad</code>	numeric. The number of quadrature points for numeric integration of the continuous mixture. Alternatively, a matrix with nodes and weights for the quadrature points can be specified.
<code>tol</code>	numeric. Accuracy (convergence tolerance) for numerically determining quantiles based on <a href="#">uniroot</a> and <code>pxbetax</code> .

**Details**

The extended-support beta mixture distribution is a continuous mixture of extended-support beta distributions on  $[0, 1]$  where the underlying exceedence parameter is exponentially distributed with mean  $nu$ . Thus, if  $nu > 0$ , the resulting distribution has point masses on the boundaries 0 and 1 with larger values of  $nu$  leading to higher boundary probabilities. For  $nu = 0$  (the default), the distribution reduces to the classic beta distribution (in regression parameterization) without boundary observations.

**Value**

`dxbetax` gives the density, `pxbetax` gives the distribution function, `qxbetax` gives the quantile function, and `rxbetax` generates random deviates.

**See Also**

[dxbeta](#), [XBetaX](#)

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