Multi context logics: a formal framework for structuring knowledge

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Multi context logics (MCL) is a formalism that allow to integrate multiple logical theories (contexts) in a more complex structure called multi context system. In the past 20 years multi context logics have been developed for contexts in propositional logics, first order logics, description logics and temporal logic. The two principles MCL are *locality* and *compatibility*. The principle of locality states that a context axiomatizes in a logical theory a portion of the world, and that every statement entailed by such a theory is intended to hold within such a portion of the world. The principle of compatibility instead states that, since different contexts can describe overlapping portions of world, the theories they contain must be constrained so that they describe compatible situations. Following these two principles, the formal semantics of an MCL is the result of a suitable composition of the semantics associated to each single context. This takes the name of *Local models semantics*. The effects of the two principles above on the inference engines that can be defined on a multi contextual knowledge base are the following: Locality principle implies that inference rules applied to knowledge inside a context (aka, local inference rules) allow to infer local truths; Compatibility principle instead implies that certain facts in a context can be inferred on the base of other facts present on other compatible contexts. This information propagation is formalized via a special type of inter contextual inference rules called *bridge rules*

In general terms, a multi context logic is defined on a family of logical languages $\{L_i\}_{i\in I}$ where each L_i is used to specify what holds in the i-th context. The set I of context indexes (aka context names) can be either a simple set, or a set equipped with some algebraic structure, like total or partial order, and operations on context indices. The relations and functions defined on I can be used to specify the organization of contexts in terms of an algebraic structure. For instance a partial order $\prec$ no I, can be used to represent that a context is wider (more general) than another context, e.g., $\mathsf{football} \prec \mathsf{sport}$, means that the context of sport is more specific that the context of football. To represent what is true in a scene in different time stamps, we can enrich I with a total order $\prec$ so that $\mathsf{CV\_Luciano\_2010} \prec \mathsf{CV\_Luciano\_2011}$ represents the fact that the context describing Luciano's curriculum vitæat 2010 precedes the context describing his CV at 2011.

A *model* for a multi context logic $\{L_i\}_{i\in I}$ is a pair $\langle M_I, C\rangle$ composed of a family of local models $M_I = \{M_i\}_{i\in I}$, where each M_i is a model of L_i, and a *compatibility relation* C among the local models. The formal structure of C

can vary depending on the type of local models and the type of constraints it is necessary to impose on the local models of different contexts. For this reason we don't give a general definition of C, which will be completely defined for each specific multi context logic. Satisfiability of formulas in L_i is defined w.r.t1. the local models. Namely If ϕ is a formula of the language L_i, then the a multi context model satisfies $i : \phi$ iff $M_i \models_i \phi$, where $\models_i$ is the satisfiability relation associated to the local logic L_i.

A *Multi context theory* in a multi context logic $L_I = \{L_i\}_{i \in I}$ is a family of theories $\{T_i\}_{i \in I}$, where each T_i is a set of statements in the logics L_i, and a set BR of *bridge rules*. Intuitively each T_i axiomatizes the constraints on the local models M_i, while the bridge rules BR axiomatizes the compatibility relations. Bridge rules are cross logical axioms and their syntactic form depends on the local logics, so as in the case of the compatibility relation their syntactic depends on the syntax of each L_i.

In my invited talk, I will go through the many possible examples of multi context logics, starting from the simplest one, the propositional multi context logics, going through hierarchical meta logics, multi context logics for beliefs and propositional attitudes, non-monotonic propositional context logics, distributed first order logics, distributed description logics, and logics for semantic import and contextualized knowledge repository for the semantic web.