Proc. 23rd Int. Workshop on Description Logics (DL2010), CEUR-WS 573, Waterloo, Canada, 2010.

# Towards Formal Comparison of Ontology Linking, Mapping and Importing

Martin Homola[1,2] and Luciano Serafini[1]

[1] Fondazione Bruno Kessler,
Via Sommarive 18, 38123 Trento, Italy

[2] Comenius University,
Faculty of Mathematics, Physics and Informatics,
Mlynská dolina, 84248 Bratislava, Slovakia

homola@fmph.uniba.sk, serafini@fbk.eu

**Abstract.** Multiple distributed and modular ontology representation frameworks have recently appeared. They typically extend Description Logics (DL), with new constructs to represent relations between entities across several ontologies. Three kinds of constructs appear in the literature: link properties, found in $\mathcal{E}$-connections, semantic mapping, found in Distributed Description Logics (DDL), and semantic imports, used in Package-based Description Logics (P-DL). In this work, we aim towards formal comparison of the expressive power of these frameworks, and thus also the ontology combination paradigms that they instantiate. Reduction from DDL to $\mathcal{E}$-connections is already known. We present two new reductions, from P-DL to DDL and vice versa. These results show that there are similarities between these frameworks. However, due to the fact that none of the reductions is unconditional, it cannot be claimed that any of the three approaches is strictly more expressive than another.

## 1 Introduction

There are multiple reasons behind the research in modular and distributed ontologies. Introduction of modularity into ontology engineering, inspired by modular software engineering, calls for organizing ontologies into modules which could then be reused and combined in novel ways and thus the whole ontology engineering process will be facilitated and simplified. Another motivation comes from the Semantic Web vision, where ontologies are seen as a central ingredient, but on the other hand, decentralization and duplicity of knowledge sources is seen as very important too [1]. The centralized, monolithic treatment of ontologies, predominant in the mainstream theoretical research on ontology representation [2], is not sufficient; a novel decentralized and distributed ontology representation approach is of demand, that would deal with duplicity, temporal unavailability and also occasional inconsistence of the various ontological data sources.

The effort to achieve such an ontological representation breaks down into two problems: what are the requirements for an encapsulated and reusable ontology module, and how to combine such modules and enable reasoning with the combined ontology. There are two main research directions. The first is to combine the modules simply by means of union and to find out the requirements under which such combination is feasible [3,4,5]. The second direction, on which we focus in this paper, imposes none or only very basic requirements on the modules and provides new constructs in order to combine the modules. Representation frameworks of this kind typically work with multiple DL [2] ontologies, and they provide novel constructs to interlink these ontologies. Each framework employs slightly different constructs; three ontology combination paradigms are distinguished: ontology linking, ontology mapping and ontology importing.

*Ontology linking* allows individuals from distinct ontologies to be coupled with *links*. A strict requirement is that the domains of the ontologies that are being combined are disjoint. See Fig. 1 a), where an ontology of companies is inter-linked with another ontology of products using the link produces. Links allow for complex concepts to be constructed, and they basically act as cross-ontology roles. The linking paradigm is employed by $\mathcal{E}$-connections [6].

*Ontology mapping*, in contrast, allows to relate ontologies on the same domain or on partially overlapping domains. Special *mapping* constructs indicate how elements from different ontologies are semantically related. Concepts, roles and individuals are possibly related. Mapping enables for knowledge reuse and also for resolution of semantic heterogeneity between ontologies that model the same domain but each from a different perspective. See Fig. 1 b), where mapping is expressed between two ontologies which possibly cover same entities but one models business relations and the other one models legal relations of these entities. In this paper we take a look at DDL [7], another notable instance of this paradigm are Integrated Distributed Description Logics [8].

*Ontology importing* allows a subset of concepts, relations and individuals defined in one ontology to be imported into another ontology where they are then reused. The importing takes care of propagating also the semantic relations that exist between these entities in their home ontology into the ontology that imports them. In Fig. 1 c), several roles are imported from a dedicated ontology module that deals with partonomy. Importing has been studied by Pan et al. [9]. The P-DL framework [10] falls under this paradigm.

This work aims towards comparison of the expressive power of these approaches. Some comparisons have already appeared in the literature. Some of them stay on qualitative or intuitive level [11,12]. Formal comparisons of the expressive power are less frequent. To our best knowledge, only reduction that is known is between DDL and $\mathcal{E}$-connections [13,6]. In this paper, we extend these efforts by producing two new reductions, from P-DL to DDL (with a specifically adjusted semantics), and vice versa. These results show, that there are certain similarities between P-DL and DDL, however, the semantics of P-DL is stronger, and it cannot be claimed that one of the frameworks is more expressive than the other. Similarly, one has to be careful when interpreting the reduction between DDL and $\mathcal{E}$-connections [13,6], because also this result assumes a DDL semantics which is not currently considered the standard one.

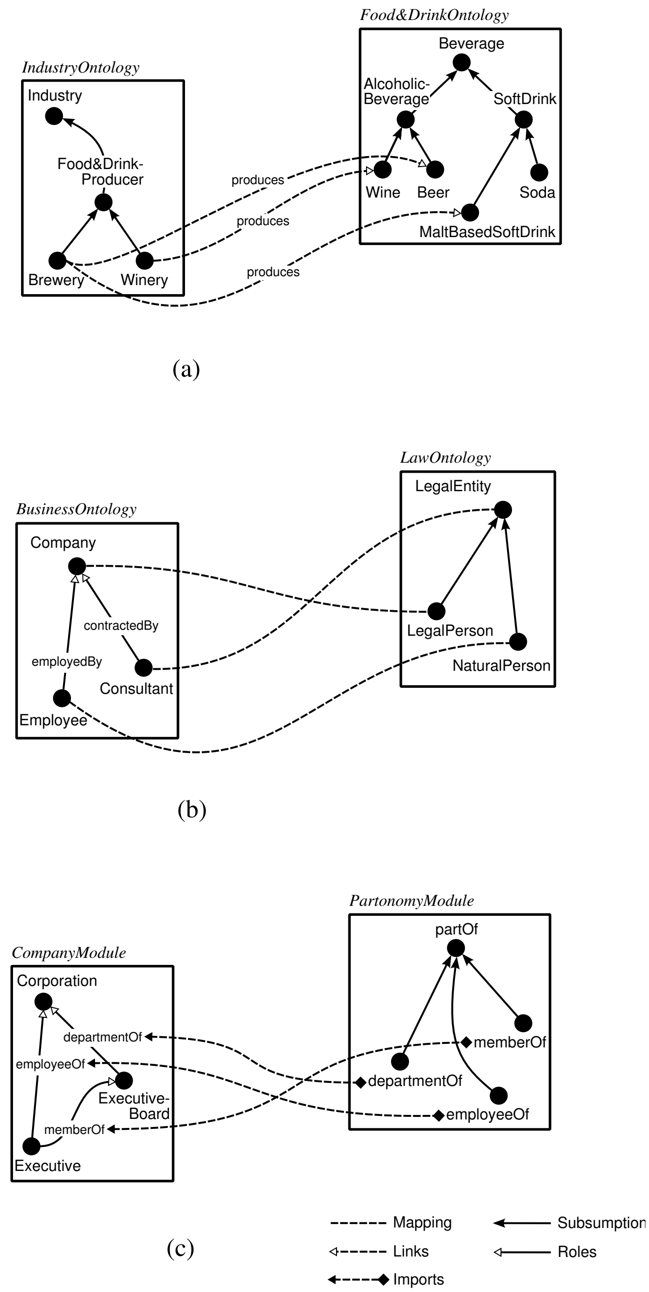


**Fig. 1.** Ontology combination paradigms: a) ontology linking; b) ontology mapping; c) ontology importing.

On the other hand, these results provide us with valuable insight on the relation between the paradigms of ontology linking, ontology mapping and ontology import, and can possibly guide the user that is to pick an appropriate formalism for a particular application.

## 2 Distributed Description Logics

DDL [7,14,15] follow the ontology mapping paradigm. They allow to connect multiple DL KB with bridge rules, a new kind of axioms that represents the mapping. An important point is that bridge rules are directed. Typically, DDL are built over $\mathcal{SHIQ}$. For the lack of space we cannot introduce $\mathcal{SHIQ}$ fully, please see the paper by Horrocks et al. [16].

Assume an index set $I$. Sets $N_{\mathrm{C}} = \{N_{\mathrm{C}i}\}_{i\in I}$, $N_{\mathrm{R}} = \{N_{\mathrm{R}i}\}_{i\in I}$ and $N_{\mathrm{I}} = \{N_{\mathrm{I}i}\}$ will be used for concept, role and individual names respectively. A distributed TBox is a family of TBoxes $\mathfrak{T} = \{\mathcal{T}_i\}_{i\in I}$, a distributed RBox is a family of RBoxes $\mathfrak{R} = \{\mathcal{R}_i\}_{i\in I}$, a distributed ABox is a family of ABoxes $\mathfrak{A} = \{\mathcal{A}_i\}_{i\in I}$, such that $\mathcal{K} = \langle \mathcal{T}_i, \mathcal{R}_i, \mathcal{A}_i\rangle$ is a $\mathcal{SHIQ}$ KB built over symbols from $N_{\mathrm{C}i}$, $N_{\mathrm{R}i}$, and $N_{\mathrm{I}i}$. By $i : \phi$ we denote that $\phi$ is a formula from $\mathcal{K}_i$. A bridge rule from $i$ to $j$ is an expression of one of the forms:

$$i : X \xrightarrow{\sqsubseteq} j : Y \ , \qquad i : X \xrightarrow{\sqsupseteq} j : Y \ , \qquad i : a \longmapsto j : b \ ,$$

where $X$ and $Y$ are either both concepts or both roles, and $a, b$ are two individuals, in the respective language. The expression $i : X \xrightarrow{\equiv} j : Y$ is a syntactic shorthand for the pair of bridge rules $i : X \xrightarrow{\sqsubseteq} j : Y$ and $i : X \xrightarrow{\sqsupseteq} j : Y$. The symbol $\mathfrak{B}$ stands for a set of bridge rules, such that $\mathfrak{B} = \bigcup_{i,j\in I, i\neq j} \mathfrak{B}_{ij}$ where $\mathfrak{B}_{ij}$ contains only bridge rules from $i$ to $j$. A DDL KB over $I$ is $\mathfrak{K} = \langle \mathfrak{T}, \mathfrak{R}, \mathfrak{A}, \mathfrak{B}\rangle$ with all four components ranging over $I$.

A distributed interpretation $\mathfrak{I} = \langle \{\mathcal{I}_i\}_{i\in I}, \{r_{ij}\}_{i,j\in I,i\neq j}\rangle$ consists of local interpretations $\{\mathcal{I}_i\}_{i\in I}$ and domain mappings $\{r_{ij}\}_{i,j\in I,i\neq j}$ (notation: $r_{ij}(d) = \{d' \mid \langle d, d'\rangle \in r_{ij}\}$ and $r_{ij}(D) = \bigcup_{d\in D} r_{ij}(d)$). For each $i \in I$ the local interpretation is of the form $\mathcal{I}_i = \left\langle \Delta^{\mathcal{I}_i}, \cdot^{\mathcal{I}_i} \right\rangle$. In DDL, $\Delta^{\mathcal{I}_i}$ may also be empty. In such a case we call $\mathcal{I}_i$ a hole and denote it by $\mathcal{I}_i = \mathcal{I}^\epsilon$. A distributed interpretation $\mathfrak{I}$ satisfies elements of $\mathfrak{K}$ (denoted by $\mathfrak{I} \models_\epsilon \cdot$) according to the following clauses:

1. $\mathfrak{I} \models_\epsilon i : \phi$ if $\mathcal{I}_i \models \phi$;
2. $\mathfrak{I} \models_\epsilon \mathcal{T}_i$ if $\mathfrak{I} \models_\epsilon i : \phi$ for each $\phi \in \mathcal{T}_i$;
3. $\mathfrak{I} \models_\epsilon \mathfrak{T}$ if $\mathfrak{I} \models_\epsilon \mathcal{T}_i$ for each $i \in I$;
4. $\mathfrak{I} \models_\epsilon \mathcal{R}_i$ if $\mathfrak{I} \models_\epsilon i : \phi$ for each $\phi \in \mathcal{R}_i$;
5. $\mathfrak{I} \models_\epsilon \mathfrak{R}$ if $\mathfrak{I} \models_\epsilon \mathcal{R}_i$ for each $i \in I$;
6. $\mathfrak{I} \models_\epsilon \mathcal{A}_i$ if $\mathfrak{I} \models_\epsilon \phi$ for each $\phi \in \mathcal{A}_i$;
7. $\mathfrak{I} \models_\epsilon \mathfrak{A}$ if $\mathfrak{I} \models_\epsilon \mathcal{A}_i$ for each $i \in I$;
8. $\mathfrak{I} \models_\epsilon i : X \xrightarrow{\sqsubseteq} j : Y$ if $r_{ij}\left(X^{\mathcal{I}_i}\right) \subseteq Y^{\mathcal{I}_j}$;
9. $\mathfrak{I} \models_\epsilon i : X \xrightarrow{\sqsupseteq} j : Y$ if $r_{ij}\left(X^{\mathcal{I}_i}\right) \supseteq Y^{\mathcal{I}_j}$;
10. $\mathfrak{I} \models_\epsilon i : a \longmapsto j : b$ if $b^{\mathcal{I}_j} \in r_{ij}\left(a^{\mathcal{I}_i}\right)$;
11. $\mathfrak{I} \models_\epsilon \mathfrak{B}$ if $\mathfrak{I} \models_\epsilon \phi$ for all axioms $\phi \in \mathfrak{B}$.

A distributed interpretation $\mathfrak{I}$ is a distributed model, also $\epsilon$-model, of $\mathfrak{K}$, if $\mathfrak{I} \models_\epsilon \mathfrak{T}$, $\mathfrak{I} \models_\epsilon \mathfrak{R}$, $\mathfrak{I} \models_\epsilon \mathfrak{A}$ and $\mathfrak{I} \models_\epsilon \mathfrak{B}$ (denoted $\mathfrak{I} \models_\epsilon \mathfrak{K}$). $\mathfrak{I}$ is a d-model of $\mathfrak{K}$, if $\mathfrak{I} \models_\epsilon \mathfrak{K}$ and it contains no hole (denoted $\mathfrak{I} \models_\mathrm{d} \mathfrak{K}$). A DDL KB $\mathfrak{K}$ is $i$-consistent, if it has an $\epsilon$-model with $\mathcal{I}_i \neq \mathcal{I}^\epsilon$; it is globally consistent if it has a d-model.

A concept $C$ is $\epsilon$-satisfiable (d-satisfiable) with respect to $\mathcal{K}_i$ of $\mathfrak{K}$, if there is an $\epsilon$-model (d-model) $\mathfrak{I}$ of $\mathfrak{K}$ such that $C^{\mathcal{I}_i} \neq \emptyset$. A formula $i : \phi$ is $\epsilon$-entailed (d-entailed) with respect to $\mathfrak{K}$, if in every $\epsilon$-model (d-model) of $\mathfrak{K}$ we have $\mathfrak{I}$ satisfies $i : \phi$. This is denoted by $\mathfrak{K} \models_\epsilon i : \phi$ ($\mathfrak{K} \models_\mathrm{d} i : \phi$).

The semantics corresponding to $\epsilon$-satisfiability and $\epsilon$-entailment is the actual state of the art in DDL. It enjoys many reasonable properties [7,14]. We will call this semantics $\mathrm{DDL}_\epsilon$. The notions of d-satisfiability and d-entailment (the semantics $\mathrm{DDL}_\mathrm{d}$) are rather auxiliary. As usual, entailment of subsumption formulae and (un)satisfiability are interreducible and both reducible into deciding $i$-consistence of a KB ($\mathrm{DDL}_\epsilon$) or global consistence of a KB ($\mathrm{DDL}_\mathrm{d}$).

$\mathrm{DDL}_\epsilon$ and $\mathrm{DDL}_\mathrm{d}$ permit any domain relations [7], but also alternate semantics with restricted domain relations have been investigated [17,18]. $\mathrm{DDL}_\epsilon$ which requires domain relations to be partial functions is denoted by $\mathrm{DDL}_\epsilon(\mathrm{F})$. $\mathrm{DDL}_\epsilon$ with injective domain relations is denoted by $\mathrm{DDL}_\epsilon(\mathrm{I})$. $\mathrm{DDL}_\epsilon$ with compositionally consistent domain relations ($r_{ij} \circ r_{jk} = r_{ik}$ for any distinct $i, j, k \in I$) is denoted by $\mathrm{DDL}_\epsilon(\mathrm{CC})$. $\mathrm{DDL}_\epsilon$ with restricted compositionality ($r_{ij} \circ r_{jk} = r_{ik}$ for any distinct $i, j, k \in I$, if there is a directed path of bridge rules from $i$ to $j$ and from $j$ to $k$) is denoted by $\mathrm{DDL}_\epsilon(\mathrm{RC})$. $\mathrm{DDL}_\epsilon$ with role-preserving domain relation (if $(x, y) \in R^{\mathcal{I}_i}$ than $r_{ij}(x) \neq \emptyset \implies r_{ij}(y) \neq \emptyset$), for each $R$ that appears on the left hand side of any bridge rule from $i$ to $j$, will be denoted by $\mathrm{DDL}_\epsilon(\mathrm{RP})$. Variants with $\mathrm{DDL}_\mathrm{d}$ and combinations are possible (e.g., later on we will discuss $\mathrm{DDL}_\epsilon(\mathrm{F,I,RC,RP})$, the semantics with partially functional, injective, restricted-compositional and role-preserving domain relations).

In addition, we will assume that local alphabets are mutually disjoint, and only atomic concepts are used in bridge rules. This is a normal form of DDL which is equivalent to the full version without these restrictions.

## 3 Package-based Description Logics

P-DL instantiate the ontology import paradigm. They allow a subset of terms to be imported from one DL KB to another (in P-DL these ontology modules are called packages). The review is based on the recent publication of Bao et al. [19] which builds on top of $\mathcal{SHOIQ}$ resulting into P-DL language $\mathcal{SHOIQP}$. The space does not permit us to introduce $\mathcal{SHOIQ}$ formally, please see the work of Horrocks [20].

A package based ontology is any $\mathcal{SHOIQ}$ ontology $\mathcal{P}$ which partitions into a finite set of packages $\{P_i\}_{i \in I}$, using an index set $I$. Each $P_i$ uses its own alphabet of terms $N_{\mathrm{C}i} \uplus N_{\mathrm{R}i} \uplus N_{\mathrm{I}i}$ (concept, role and individual names, respectively). The alphabets are not mutually disjoint, but for any term $t$ there is a unique home package of $t$, denoted by $\mathrm{home}\,(t)$. The set of home terms of a package $P_i \in \mathcal{P}$ is denoted by $\Delta_{S_i}$. A term $t$ occurring in $P_i$ is a *local term* in $P_i$ if $\mathrm{home}\,(t) = i$, otherwise it is a *foreign term* in $P_i$. If $t$ is a foreign term in $P_j$, and home $(t) = i$, then we write $P_i \xrightarrow{t} P_j$. If $P_i \xrightarrow{t} P_j$ for any term $t$, then also the package $P_j$ imports the package $P_i$ (denoted $P_i \rightarrow P_j$). By $\xrightarrow{*}$ we denote the transitive closure of $\rightarrow$ and by $P_j^*$ the set $P_j \cup \{P_i | i \xrightarrow{*} j\}$.

When $P_i \rightarrow P_j$, the symbol $\top_i$ occurring within $P_j$ represents the imported domain of $P_i$. Also in this case a novel *contextualized negation* constructor $\neg_i$ is applicable within $P_j$ $\neg_i$ (it is however a mere syntactic sugar and can be ruled out as we always have $\neg_i C \equiv \top_i \sqcap \neg_j C$).

A distributed interpretation of $\mathcal{P}$ is a pair $\mathcal{I} = \left\langle \{\mathcal{I}_i\}_{i \in I}, \{r_{ij}\}_{i \xrightarrow{*} j} \right\rangle$, such that each $\mathcal{I}_i = \langle \Delta^{\mathcal{I}_i}, \cdot^{\mathcal{I}_i} \rangle$ is an interpretation of the local package $P_i$ and each $r_{ij} \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$ is a domain relation between $\Delta^{\mathcal{I}_i}$ and $\Delta^{\mathcal{I}_j}$. A distributed interpretation $\mathcal{I}$ is a model of $\{P_i\}_{i \in I}$, if the following conditions hold:

1. there is at least one $i \in I$ such that $\Delta^{\mathcal{I}_i} \neq \emptyset$;
2. $\mathcal{I}_i \models P_i$;
3. $r_{ij}$ is an injective partial function, and $r_{ii}$ is the identity function;
4. if $i \xrightarrow{*} j$ and $j \xrightarrow{*} k$, then $r_{ik} = r_{ij} \circ r_{jk}$ (compositional consistency);
5. if $i \xrightarrow{t} j$, then $r_{ij}(t^{\mathcal{I}_i}) = t^{\mathcal{I}_j}$;
6. if $i \xrightarrow{R} j$ and $(x, y) \in R^{\mathcal{I}_i}$ than $r_{ij}(x) \neq \emptyset \implies r_{ij}(y) \neq \emptyset$ (role preserving).

The three main reasoning tasks for P-DL are consistency of KB, concept satisfiability and concept subsumption entailment with respect to a KB. These are always defined with respect to a so called witness package $P_w \in \mathcal{P}$. A package-based ontology $\mathcal{P}$ is consistent as witnessed by a package $P_w$ of $\mathcal{P}$, if there exists a model $\mathcal{I}$ of $P_w^*$ such that $\Delta^{\mathcal{I}_w} \neq \emptyset$. In this case we also say that $\mathcal{P}$ is $w$-consistent. A concept $C$ is satisfiable as witnessed by a package $P_w$ of $\mathcal{P}$, if there exists a model $\mathcal{I}$ of $P_w^*$ such that $C^{\mathcal{I}_w} \neq \emptyset$. A subsumption formula $C \sqsubseteq D$ is valid as witnessed by a package $P_w$ of $\mathcal{P}$ (denoted $\mathcal{P} \models C \sqsubseteq_w D$), if for every model $\mathcal{I}$ of $P_w^*$ we have $C^{\mathcal{I}_w} \subseteq D^{\mathcal{I}_w}$.

## 4 $\mathcal{E}$-connections

The $\mathcal{E}$-connections framework represents the ontology linking paradigm. Although many flavours of $\mathcal{E}$-connections are known, for sake of simplicity we introduce the language $\mathcal{C}^{\mathcal{E}}(\mathcal{SHIQ})$ [6,21,22]. This language allows to connect multiple ontologies expressed in $\mathcal{SHIQ}$ [16] with links.

Assume a finite index set $I$. For $i \in I$ let $N_{\mathrm{C}i}$ and $N_{\mathrm{I}i}$ be pairwise disjoint sets of concepts names and individual names respectively. For $i, j \in I$, $i$ and $j$ not necessarily distinct, let $\epsilon_{ij}$ be sets of properties, not necessarily mutually disjoint, but disjoint with respect to $N_{\mathrm{C}k}^{m_k}$ and $N_{\mathrm{I}k}^{m_k}$. An $ij$-property axiom is of the form $P_1 \sqsubseteq P_2$, where $P_1, P_2 \in \epsilon_{ij}$. An $ij$-property box $\mathcal{R}_{ij}$ is a finite set of $ij$-property axioms. The combined property box $\mathcal{R}$ contains all the property boxes for each $i, j \in I$.

Given some $i \in I$, $A \in N_{\mathrm{C}i}$, two $i$-concepts $C$ and $D$, and a $j$-concept $Z$, and $P, S \in \epsilon_{ij}$, $S$ is simple (see [16,22]), the following are also $i$-concepts:

$$\bot_i | \top_i | A | \neg C | C \sqcap D | C \sqcup D | \exists P.Z | \forall P.Z | \geqslant n\, S.Z | \leqslant n\, S.Z \enspace .$$

A combined TBox is a tuple $\mathcal{K} = \{\mathcal{K}_i\}_{i \in I}$ where each $\mathcal{K}_i$ is a finite set of $i$-local GCI axioms of the form $C \sqsubseteq D$, where $C, D$ are $i$-concepts. A combined ABox $\mathcal{A} = \{\mathcal{A}_i\}_{i \in I} \cup \mathcal{A}_{\mathcal{E}}$ is a set of local ABoxes $\mathcal{A}_i$, each comprising of a finite number of $i$-local concept assertions $C(a)$ and role assertions $R(a, b)$, where $C$ is an $i$-concept, $R \in \epsilon_{ii}$, $a, b \in N_{\mathrm{I}i}$. $\mathcal{A}_{\mathcal{E}}$ is a finite set of object assertions, each of the form $a \cdot E \cdot b$, where $E \in \epsilon_{ij}$, $a \in N_{\mathrm{I}i}$, $b \in N_{\mathrm{I}i}$. A combined KB is a triple $\Sigma = \langle \mathcal{K}, \mathcal{R}, \mathcal{A} \rangle$, where each component ranges over the same index set $I$.

Now we focus on the semantics. Given some KB $\Sigma = \langle \mathcal{K}, \mathcal{R}, \mathcal{A} \rangle$ with index set $I$, a combined interpretation is a triple $\mathcal{I} = \langle \{\Delta^{\mathcal{I}_i}\}_{i \in I}, \{\cdot^{\mathcal{I}_i}\}_{i \in I}, \{\cdot^{\mathcal{I}_{ij}}\}_{i,j \in I} \rangle$, where $\Delta^{\mathcal{I}_i} \neq \emptyset$, for $i \in I$, and $\Delta^{\mathcal{I}_i} \cap \Delta^{\mathcal{I}_j} = \emptyset$, for $i, j \in I, i \neq j$. The interpretation functions provide denotation for $i$-concepts ($\cdot^{\mathcal{I}_i}$) and for $ij$-properties ($\cdot^{\mathcal{I}_{ij}}$).

Each $ij$-property $P \in \epsilon_{ij}$ is interpreted by $P^{\mathcal{I}_{ij}} \subseteq \Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$. $\mathcal{I}$ satisfies an $ij$-property axiom $P_1 \sqsubseteq P_2$, if ${P_1}^{\mathcal{I}_{ij}} \subseteq {P_2}^{\mathcal{I}_{ij}}$. Each $i$-concept $C$ is interpreted by $C^{\mathcal{I}_i} \subset \Delta^{\mathcal{I}_i}$; $\top_i = \Delta^{\mathcal{I}_i}$ and $\bot_i = \emptyset$, and denotation of complex $i$-concepts must satisfy the constraints as given in Table 1. $\mathcal{I}$ satisfies an $i$-local GCI $C \sqsubseteq D$ (denoted by $\mathcal{I} \models C \sqsubseteq D$), if $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$. $\mathcal{I}$ satisfies an $i$-local concept assertion $C(a)$ (denoted $\mathcal{I} \models C(a)$), if $a^{\mathcal{I}_i} \in C^{\mathcal{I}_i}$; it satisfies an $i$-local role assertion $R(a, b)$ (denoted $\mathcal{I} \models R(a, b)$), if $\left\langle a^{\mathcal{I}_i}, b^{\mathcal{I}_i} \right\rangle \in R^{\mathcal{I}_{ii}}$; $\mathcal{I}$ satisfies an object assertion $a \cdot E \cdot b$ (denoted $\mathcal{I} \models a \cdot E \cdot b$), $a \in N_{\mathrm{I}i}$, $b \in N_{\mathrm{I}j}$, $E \in \rho_{ij}$, if $\left\langle a^{\mathcal{I}_i}, b^{\mathcal{I}_j} \right\rangle \in E^{\mathcal{I}_{ij}}$.

| $X$ | $X^{\mathcal{I}_i}$ |
|---|---|
| $\neg C$ | $\Delta^{\mathcal{I}_i} \setminus C^{\mathcal{I}_i}$ |
| $C \sqcap D$ | $C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}$ |
| $C \sqcup D$ | $C^{\mathcal{I}_i} \cup D^{\mathcal{I}_i}$ |
| $\forall P.Z$ | $\{x \in \Delta^{\mathcal{I}_i} \mid (\forall y)\ (x, y) \in P^{\mathcal{I}_{ij}} \implies y \in Z^{\mathcal{I}_j}\}$ |
| $\exists P.Z$ | $\{x \in \Delta^{\mathcal{I}_i} \mid (\exists y)\ (x, y) \in P^{\mathcal{I}_{ij}} \wedge y \in Z^{\mathcal{I}_j}\}$ |
| $\geqslant n\, S.Z$ | $\mathcal{I}_i = \{x \in \Delta^{\mathcal{I}_i} \mid \sharp\{y \mid (x, y) \in S^{\mathcal{I}_{ij}}\} \geq n\}$ |
| $\leqslant n\, S.Z$ | $\{x \in \Delta^{\mathcal{I}_i} \mid \sharp\{y \mid (x, y) \in S^{\mathcal{I}_{ij}}\} \leq n\}$ |

**Table 1.** Semantic constraints on complex $i$-concepts in $\mathcal{E}$-connections.

Finally, a combined interpretation $\mathcal{I}$ is a model of $\Sigma = \langle \mathcal{K}, \mathcal{R}, \mathcal{A} \rangle$ (denoted by $\mathcal{I} \models \Sigma$), if $\mathcal{I}$ satisfies every axiom in $\mathcal{K}$, $\mathcal{R}$ and $\mathcal{A}$. An $i$-concept is satisfiable with respect to $\Sigma$, if $\Sigma$ has a combined model $\mathcal{I}$, such that $C^{\mathcal{I}_i} \neq \emptyset$. We have $\Sigma \models C \sqsubseteq D$, for two $i$-concepts $C$ and $D$, if in each combined model $\mathcal{I}$ of $\Sigma$, $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$. Both reasoning tasks are inter-reducible, as usual.

## 5 Relating DDL to P-DL

In this section, we present two reductions. First is from P-DL into DDL, and later on also vice versa from DDL into P-DL. As we have learned, P-DL use a rather strongly restricted semantics, hence we will use $\mathrm{DDL}_\epsilon$(F,I,RC,RP) in order to make the reduction possible.

**Theorem 1.** *Given a $\mathcal{SHIQP}$ ontology $\mathcal{P} = \{P_i\}_{i\in I}$, with the importing relation $P_i \xrightarrow{t} P_j$. Let us construct a DDL KB $\mathfrak{K} = \langle \mathfrak{T}, \mathfrak{R}, \mathfrak{A}, \mathfrak{B}\rangle$ over $I$:*

- $\mathcal{T}_i := \{\phi \mid \phi \in P_i$ *is a GCI axiom*$\} \cup \{\top_i \equiv \top\}$*, for each* $i \in I$*;*
- $\mathcal{R}_i := \{\phi \mid \phi \in P_i$ *is a RIA axiom*$\}$*, for each* $i \in I$*;*
- $\mathcal{A}_i := \{\phi \mid \phi \in P_i$ *is an ABox assertion*$\}$*, for each* $i \in I$*;*
- $\mathfrak{B}_{ij} := \{i : C \xrightarrow{\equiv} j : C \mid P_i \xrightarrow{C} P_j\} \cup \{i : R \xrightarrow{\equiv} j : R \mid P_i \xrightarrow{R} P_j\} \cup \{i : a \longmapsto j : a \mid P_i \xrightarrow{a} P_j\}$*, for each* $i, j \in I$*.*

*Given any $i \in I$, $\mathcal{P}$ is $i$-consistent if and only if $\mathfrak{K}$ is $i$-consistent under $\mathrm{DDL}_\epsilon$(F,I,RC,RP).*

It trivially follows that also the decision problems of satisfiability and subsumption entailment are reducible, since they are reducible into $i$-consistence.

**Corollary 1.** *Deciding satisfiability and subsumption entailment in P-DL reduces into deciding i-consistence in DDL under the semantics $\mathrm{DDL}_\epsilon$(F,I,RC,RP).*

We will now show, that under this strong semantics of DDL, also a reduction in the opposite direction is possible.

**Theorem 2.** *Let $\mathfrak{K} = \langle \mathfrak{T}, \mathfrak{R}, \mathfrak{A}, \mathfrak{B}\rangle$ be a DDL KB over some index set $I$. We construct a package-based ontology $\mathcal{P} = \{P_i\}_{i\in I}$. Each package $P_j$ contains a union of the following components:*

1. $\mathcal{T}_j \cup \{C \sqsubseteq G \mid i : C \xrightarrow{\sqsubseteq} j : G \in \mathfrak{B}\} \cup \{G \sqsubseteq C \mid i : C \xrightarrow{\sqsupseteq} j : G \in \mathfrak{B}\}$;
2. $\mathcal{R}_j \cup \{R \sqsubseteq S \mid i : R \xrightarrow{\sqsubseteq} j : S \in \mathfrak{B}\} \cup \{S \sqsubseteq R \mid i : R \xrightarrow{\sqsupseteq} j : S \in \mathfrak{B}\}$;
3. $\mathcal{A}_j \cup \{a = b \mid i : a \longmapsto j : b \in \mathfrak{B}\}$.

*In addition, $\mathcal{P}$ uses the following imports:*

- $P_i \xrightarrow{C} P_j$ *if either* $i : C \xrightarrow{\sqsubseteq} j : G \in \mathfrak{B}$ *or* $i : C \xrightarrow{\sqsupseteq} j : G \in \mathfrak{B}$*, for any* $i, j \in I$*, for any $i$-concept $C$ and for any $j$-concept $G$;*
- $P_i \xrightarrow{R} P_j$ *if either* $i : R \xrightarrow{\sqsubseteq} j : S \in \mathfrak{B}$ *or* $i : R \xrightarrow{\sqsupseteq} j : S \in \mathfrak{B}$*, for any* $i, j \in I$*, for any $i$-role $R$ and for any $j$-role $S$;*
- $P_i \xrightarrow{a} P_j$ *if* $i : a \longmapsto j : b \in \mathfrak{B}$ *for any* $i, j \in I$*, for any $i$-individual $a$ and for any $j$-individual $b$.*

*Given any $i \in I$, $\mathcal{P}$ is $i$-consistent if and only if $\mathfrak{K}$ is $i$-consistent under $\mathrm{DDL}_\epsilon$(F,I,RC,RP).*

Also the remaining decision problems are reducible, because they are reducible into $i$-consistence in DDL.

**Corollary 2.** *Deciding satisfiability of concepts and subsumption entailment with respect in $DDL_\epsilon$(F,I,RC,RP) reduces into deciding i-consistence in P-DL.*

As we see, P-DL are closely related to $\mathrm{DDL}_\epsilon$(F,I,RC,RP). More precisely, these results show, that it is possible to implement importing with bridge rules, and vice versa, it is possible to simulate bridge rules with imports and additional local axioms, *if the requirements placed on domain relations are the same.*

These results do not imply, however, that the two frameworks have the same expressivity, for two reasons. First, in terms of expressive power, none of the reductions is complete. DDL does not handle nominals, hence P-DL ontologies with nominals cannot be reduced. On the other hand, we deal with simplified version of DDL in this paper. We did not provide any reduction for DDL ontologies with heterogeneous bridge rules [15], and as these bridge rules essentially require domain relations that are not functional, it is hard to imagine that any reduction is possible.

The second reason is the fact that the DDL semantics used by the reductions is considerably stronger than what is commonly understood as appropriate DDL semantics. $\mathrm{DDL}_\epsilon$(RC), a weaker version of $\mathrm{DDL}_\epsilon$(F,I,RC,RP) has been studied [18], and it has been showed that it does not satisfy all the desiderata commonly placed on DDL [7,14]. More specifically, this semantics may behave unexpectedly, in the presence of an accidental inconsistency in one of the ontologies that are combined [18]. The problem also occurs with the stronger semantics $\mathrm{DDL}_\epsilon$(F,I,RC,RP), and we will show that it also occurs in P-DL.

*Example 1.* Consider a package-based ontology $\mathcal{P}$ consisting of three packages $P_1$, $P_2$ and $P_3$ with the following imports:

$$P_1 \xrightarrow{C} P_2 \ , \qquad\qquad P_2 \xrightarrow{D} P_3 \ , \qquad\qquad P_1 \xrightarrow{E} P_3 \ .$$

This is a very basic P-DL setting, each of the three packages imports one concept. Furthermore, let us assume that all tree packages are empty (i.e., $P_1 = P_2 = P_3 = \emptyset$). This means, that the three concepts $C$, $D$ and $E$ are unrelated. It is easy to show, that if $P_2$ becomes inconsistent, for some reason, then the imported concept $D$ becomes unsatisfiable in $P_3$. This is quite intuitive, since $D$ is imported from $P_2$. However, as we will show, if $P_2$ becomes inconsistent, also $E$ becomes unsatisfiable in $P_3$ which we consider rather unintuitive, since this concept is imported from $P_1$ and is unrelated to $D$.

Let $P_2$ be inconsistent. For simplicity, let us assign $P_2 := \{\top \sqsubseteq \bot\}$. Due to the first two imports we get that $P_1 \xrightarrow{*} P_2$ and $P_2 \xrightarrow{*} P_3$, and so the semantics implies that in any model of $\mathcal{P}$, it must be the case that $r_{13} = r_{12} \circ r_{23}$. However, since $P_2$ is inconsistent, $\Delta^{\mathcal{I}_2} = \emptyset$ in every model, and hence $r_{12} \circ r_{23} = \emptyset$, and hence also $r_{13} = \emptyset$. And so, we also get $E^{\mathcal{I}_3} = r_{13}(E^{\mathcal{I}_1}) = \emptyset$.

The $\mathrm{DDL}_\epsilon$ semantics does not exhibit this problem [14]. On the other hand, $\mathrm{DDL}_\epsilon$ has problems with transitive propagation of the effects of bridge rules, which is not a problem for $\mathrm{DDL}_\epsilon$(RC), nor $\mathrm{DDL}_\epsilon$(F,I,RC,RP) [18]. In the P-DL setting this reformulates as the problem of transitive propagation of the imported semantic relations, and due to correspondence between P-DL and $\mathrm{DDL}_\epsilon$(F,I,RC,RP), we now have a formal proof that it never appears in P-DL. This clearly marks the difference between the two approaches.

## 6 On Relation of DDL and $\mathcal{E}$-connections

The relation of DDL and $\mathcal{E}$-connections has been studied in the literature. It is known, that under certain assumptions, it is possible to reduce a DDL KB into $\mathcal{E}$-connections and reason equivalently in the latter formalism. More specifically, for a DDL KB with bridge rules between concepts and between individuals, the reasoning problems associated with d-entailment are reducible [6,13].

**Theorem 3 ([6,13]).** *Assume a DDL KB $\mathfrak{K} = \langle \mathfrak{T}, \mathfrak{R}, \mathfrak{A}, \mathfrak{B} \rangle$ with index set $I$ and alphabet $N_\mathrm{C} = \{N_{\mathrm{C}i}\}_{i\in I}$, $N_\mathrm{R} = \{N_{\mathrm{R}i}\}_{i\in I}$ and $N_\mathrm{I} = \{N_{\mathrm{I}i}\}_{i\in I}$ which contains no bridge rules between roles. Let ${N_\mathrm{C}}' = N_\mathrm{C}$, ${N_\mathrm{I}}' = N_\mathrm{I}$, $\epsilon'_{ii} = N_{\mathrm{R}i}$, for each $i \in I$. Let $\epsilon'_{ij} = \{E_{ij}\}$, for each $i, j \in I$, $i \neq j$, where $E_{ij}$ is a new symbol.*

*Let us construct $\Sigma' = \langle \mathcal{K}', \mathcal{R}', \mathcal{A}' \rangle$, a $\mathcal{C}^\mathcal{E}(\mathcal{SHIQ})$ KB built over the index set $I$ and the vocabulary ${N_\mathrm{C}}'$, ${N_\mathrm{I}}'$ and $\{\epsilon'_{ij}\}_{i,j\in I}$, as follows:*

- $\mathcal{K}'_i := \mathcal{T}_i \cup \{\exists E_{ij}.C \sqsubseteq G \mid j : C \xrightarrow{\sqsubseteq} i : G \in \mathfrak{B}\} \cup \{H \sqsubseteq \exists E_{ij}.D \mid j : D \xrightarrow{\sqsupseteq} i : H \in \mathfrak{B}\}$*, for each* $i \in I$*;*
- $\mathcal{R}'_i := \mathcal{R}_i$*, for each* $i \in I$*;*
- $\mathcal{A}'_i := \mathcal{A}_i$*, for each* $i \in I$*;*
- $\mathcal{A}'_\mathcal{E} := \{a \cdot E_{ij} \cdot b \mid i : a \longmapsto j : b \in \mathfrak{B}\}$.

*It follows, that $\mathfrak{K}$ is globally consistent if and only if $\Sigma'$ is consistent.*

As a straightforward consequence of the theorem, also the remaining reasoning tasks related to global consistency are reducible.

**Corollary 3.** *Deciding d-satisfiability and d-entailment of subsumption with respect to a DDL KB reduces into deciding the consistence of KB in $\mathcal{E}$-connections.*

This shows, that there is some similarity between bridge rules in DDL and links in $\mathcal{E}$-connections. The reduction especially shows, what is the exact relation between bridge rules on one side, and links on the other one. However it cannot be claimed that $\mathcal{E}$-connections are more expressive than DDL based on this result. This is for two reasons.

First, the reduction is for the $\mathrm{DDL}_\mathrm{d}$ semantics. In DDL, however, "the semantics" is currently $\mathrm{DDL}_\epsilon$, which allows holes and satisfies a number of desired properties [7,14]. In particular, $\mathrm{DDL}_\epsilon$ offers effective means to deal with accidental inconsistency, and hence it is possible to reason with a DDL KB, even if some local ontologies are inconsistent. $\mathcal{E}$-connections offer no such features.

Second, to our best knowledge it is not possible to reduce the richer flavours of DDL that include either bridge rules between roles or heterogeneous bridge rules between concepts and roles [15].

These findings are well in line with the different purpose for which each of the formalisms was designed. $\mathcal{E}$-connections work with carefully crafted local modules with non-overlapping modeling domains, while DDL allow to connect overlapping ontologies in which part of the terminology is modeled on both sides although possibly differently.

## 7 Conclusion

The novel results of this paper are the reductions between P-DL and DDL (with a specifically adjusted semantics) and vice versa. This result suggests that these two frameworks have many similarities, and under certain assumptions, imports used in P-DL can be expressed by bridge rules used in DDL, and the other way around. On the other hand, these results also point out, that the actual difference between the two formalisms is in the fact that P-DL uses much stronger semantics than DDL. These semantics significantly differ, and hence it cannot be claimed that one of these frameworks is more expressive than the other.

Similarly, it cannot be claimed that $\mathcal{E}$-connections are more expressive than DDL, based on the reduction given by Kutz et al. [6,13]. This reduction is not given for the DDL semantics with holes, which is currently considered the most appropriate one, but for a simplified semantics instead. In addition, for some more complex DDL constructs no reduction is known in the literature. What the reduction does show, is how the concept of bridge rules and the concept of links (used in $\mathcal{E}$-connections) are related.

We conclude, that these results are well in line with the particular purpose for which each of the formalisms has been designed and is suitable for. $\mathcal{E}$-connections are particular well suited for combining multiple ontologies with separated local domains. This separation of domains, which has to be maintained, may also serve as a reasonable guide during modular ontology development.

On the other hand, sometimes we wish to combine and integrate also ontologies with partially overlapping domains. Especially in heterogeneous distributed knowledge environments, such as the Semantic Web, this is unavoidable. In such applications, DDL seems to be the most suitable, offering a versatile apparatus of ontology mapping, which allows concepts, roles and individuals to be associated freely according to the need of the application. DDL is robust enough to deal with accidental inconsistency and offers means to resolve possible heterogeneity in the modeling approaches used by different ontologies.

Finally, P-DL offers an ontology importing paradigm, which is very familiar to importing in software engineering. Thus P-DL is intuitive and easily understood also by users without deep understanding of ontology integration issues. Its strong semantics deals with many modeling issues, such as transitive propagation of imported relations, however, as the reduction suggests, it may possibly behave unexpectedly in the presence of accidental inconsistency in the system.

**Acknowledgement.** Martin Homola is supported from the project VEGA no. 1/0668/10 of Slovak Ministry of Education and Slovak Academy of Sciences. Extended version of this paper will shortly appear as a technical report.

## References


1. Berners-Lee, T., Hendler, J., Lassila, O.: The semantic web. Scientific American **284**(5) (2001) 34–43
2. Baader, F., Calvanese, D., McGuinness, D., Nardi, D., Patel-Schneider, P., eds.: The Description Logic Handbook. Cambridge University Press (2003)
3. Cuenca Grau, B., Horrocks, I., Kazakov, Y., Sattler, U.: A logical framework for modularity of ontologies. In: IJCAI 2007. (2007)
4. Ghilardi, S., Lutz, C., Wolter, F.: Did I damage my ontology? A case for conservative extensions in description logics. In: KR 2006, AAAI Press (2006)
5. Lutz, C., Walther, D., Wolter, F.: Conservative extensions in expressive description logics. In: IJCAI 2007. (2007)
6. Kutz, O., Lutz, C., Wolter, F., Zakharyaschev, M.: $\mathcal{E}$-connections of abstract description systems. Artificial Intelligence **156**(1) (2004) 1–73
7. Borgida, A., Serafini, L.: Distributed description logics: Assimilating information from peer sources. Journal of Data Semantics **1** (2003) 153–184
8. Zimmermann, A.: Integrated distributed description logics. In: DL-2007. Volume 250 of CEUR WS. (2007)
9. Pan, J.Z., Serafini, L., Zhao, Y.: Semantic import: An approach for partial ontology reuse. In: WoMo-06. Volume 232 of CEUR WS. (2006)
10. Bao, J., Honavar, V.: Ontology language extensions to support collaborative ontology building. In: ISWC2004. Volume 3298 of LNCS., Springer (2004)
11. Wang, Y., Haase, P., Bao, J.: A survey of formalisms for modular ontologies. In: IJCAI'07, SWeCKA Workshop. (2007)
12. Bao, J., Caragea, D., Honavar, V.: On the semantics of linking and importing in modular ontologies. In: ISWC 2006. Volume 4273 of LNCS., Springer (2006)
13. Cuenca Grau, B., Kutz, O.: Modular ontology languages revisited. In: IJCAI'07, SWeCKA Workshop. (2007)
14. Serafini, L., Borgida, A., Tamilin, A.: Aspects of distributed and modular ontology reasoning. In: IJCAI-05. (2005)
15. Ghidini, C., Serafini, L.: Mapping properties of heterogeneous ontologies. In: WoMo-06. Volume 232 of CEUR WS. (2006)
16. Horrocks, I., Sattler, U., Tobies, S.: Practical reasoning for expressive description logics. In: LPAR'99. Number 1705 in LNAI, Springer (1999) 161–180
17. Homola, M., Serafini, L.: Towards distributed tableaux reasoning procedure for DDL with increased subsumption propagation between remote ontologies. In: KR 2008 Ontology Workshop, Australian Computer Society (2008)
18. Homola, M., Serafini, L.: Augmenting subsumption propagation in distributed description logics. Applied Artificial Intelligence **24** (2010) 137–174
19. Bao, J., Voutsadakis, G., Slutzki, G., Honavar, V.: Package-based description logics. In: Modular Ontologies: Concepts, Theories and Techniques for Knowledge Modularization. Volume 5445 of LNCS. Springer (2009) 349–371
20. Horrocks, I., Sattler, U., Tobies, S.: Practical reasoning for very expressive description logics. J. Interest Group in Pure and Applied Logic **8**(3) (2000) 239–264
21. Cuenca Grau, B., Parsia, B., Sirin, E.: Working with multiple ontologies on the semantic web. In: ISWC2004. Volume 3298 of LNCS., Springer (2004)
22. Parsia, B., Cuenca Grau, B.: Generalized link properties for expressive E-connections of description logics. In: AAAI-2005, AAAI Press (2005)