Towards Trajectory Planning for a 6-Degree-of-Freedom Robot Manipulator Considering the Orientation of the End-effector Using Computer Algebra*

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Abstract
We consider the trajectory planning of a 6-Degree-of-Freedom (DOF) robot manipulator using computer algebra, with controlling the orientation of the end-effector. As a first step towards the objective, we present a solution to the inverse kinematics problem of the manipulator such that the orientation of the end-effector remains constant using computer algebra.

Keywords
Trajectory planning, Inverse kinematic problem, Robot manipulator, Gröbner basis

1. Introduction

This paper discusses the trajectory planning of a 6-Degree-of-Freedom (DOF) robot manipulator. A manipulator is a robot resembling a human hand and comprises links that function as a human arm and joints that function as human joints. Each link is connected for movement relative to each other by a joint. The first link is connected to the ground, and the last link called end-effector, contains the hand, which can be moved freely. In this paper, we consider a manipulator called "myCobot 280" [1] (hereafter called "myCobot") that has six joints connected in series that can only rotate around a certain axis. Note that each joint has one degree of freedom. Therefore, myCobot has at most six degrees of freedom.

The inverse kinematics problem is a problem of determining the joint arrangement when the end-effector is placed in a specified direction on a certain coordinate in space. The trajectory planning problem of the manipulator is an inverse kinematics problem in which the position of the end-effector is expanded from a single coordinate to a trajectory. In other words, it can be regarded as a problem to find the displacement of the joint when the end-effector of the manipulator moves on a given trajectory from the initial position to the final position.

In computer algebra, methods for the inverse kinematics problem of a 6-DOF robot manipulator have been proposed for more than 30 years ([2], [3]). Furthermore, several methods have been proposed to solve the inverse kinematics problem of a manipulator using Gröbner basis computation ([4], [5], [6], [7]). Among them, two of the present authors have proposed methods for solving the inverse kinematic problem ([8], [9]) and trajectory planning problem ([10]) of a 3-DOF manipulator using computer algebra. In more detail, we have proposed a method for solving the inverse kinematic problem efficiently with the use of Comprehensive Gröbner

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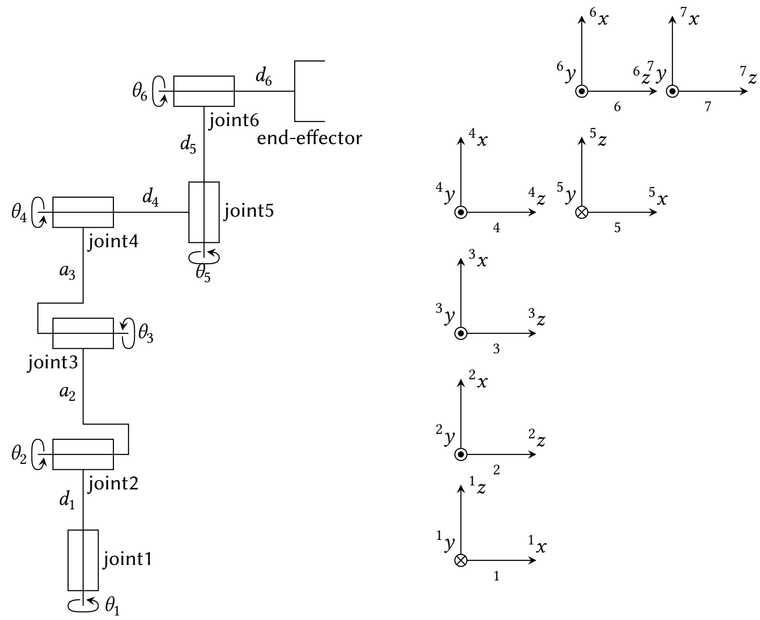

Figure 1: The schematic diagram and the local coordinate systems of each joint in myCobot.

Systems (CGS) and certifying the existence of a solution to the inverse kinematic problem using the CGS-QE, or the quantifier elimination (QE) based on the CGS computation.

In this paper, we consider the trajectory planning of a 6-DOF robot manipulator while controlling the orientation of the end-effector using computer algebra. As a first step towards the objective, we present a solution to the inverse kinematics problem such that the end-effector's orientation remains constant. More precisely, we present a solution to the inverse kinematics problem under the condition that the end-effector's local coordinate system overlaps the global coordinate system by translation.

The paper is organized as follows. In Section 2, the coordinate system and the notation of the manipulator are introduced. In Section 3, the analytical solution of general inverse kinematics problems is first described, followed by the solution in myCobot. In Section 4, future initiatives are described.

2. Preliminaries

2.1. Coordinate systems

For each joint in myCobot, the coordinate system is defined as follows (see Figure 1). Let each joint be numbered as joint i from the base to the end-effector in increasing order, and the end-effector be numbered as joint 7. Let $_i$ be the coordinate system of joint i. Here, $_1$ is the reference (global) coordinate system, while the other coordinate systems are local. The axes of each coordinate system are defined as follows: the ${}^iz$ axis in the direction of the joint axis (rotational axis in the case of a revolute joint), ${}^ix$ axis in the direction of the common normal of the ${}^iz$ and ${}^{i+1}z$ axes, and ${}^iy$ axis to be a right-handed system.

2.2. Notation

For the index i, the matrices are denoted by A_i, and vectors are denoted by ${}^{i+1}\underline{x}_n$, in which subscripts are used to distinguish points, and superscripts denote the local coordinate system to which they refer (vectors with no superscripts are referenced in $_1$). If the vector is referenced with respect to a different coordinate system, it is enclosed within brackets, and a separate superscript is added outside the brackets. (e.g. ${}^{i}[{}^{i+1}\underline{x}_n]$). Scalars are expressed in lowercase variables, and subscripts, such as a_i, are used where necessary. In $_i$, the origin is denoted by O_i and the unit vectors are denoted by ${}^{i}\underline{e}_x$, ${}^{i}\underline{e}_y$ and ${}^{i}\underline{e}_z$. In vector $\underline{v}$, the i-th component is denoted by $\underline{v}[i]$.

2.3. The Denavit-Hartenberg convention

The 'Denavit-Hartenberg parameters' [11] are used as a transformation method between coordinate systems. The following parameters are used in the transformations: a_i as the length of the common normal between the ${}^{i}z$ and ${}^{i+1}z$ axes, α_i as the angle from the ${}^{i}z$-axis towards the ${}^{i+1}z$-axis around the ${}^{i+1}x$-axis in the clockwise direction, d_i as the length between the common normal a_i and the origin of the coordinate system i, and θ_i as the angle between the common normal a_i and the ${}^{i}x$ axis.

Let A_i be the transformation matrix from $_{i+1}$ and $_i$. Then, as the product of matrices representing rotation and translation, A_i is expressed by using the above parameters as

$$A_i = \begin{pmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

For expressing the transformation of the coordinate system, affine coordinates are used as follows: if the coordinates of a point is expressed as ${}^{t}({}^{i}x, {}^{i}y, {}^{i}z)$ and also as ${}^{t+1}({}^{i+1}x, {}^{i+1}y, {}^{i+1}z)$, they are denoted by ${}^{i}\underline{X} = {}^{t}({}^{i}x, {}^{i}y, {}^{i}z, 1)$ and ${}^{i+1}\underline{X} = ({}^{i+1}x, {}^{i+1}y, {}^{i+1}z, 1)$, respectively, in which the last coordinates represent translation. Then, by the definition of A_i, we have ${}^{i}\underline{X} = A_i{}^{i+1}\underline{X}$ and ${}^{i+1}\underline{X} = A_i^{-1}{}^{i}\underline{X}$, where

$$A_i^{-1} = \begin{pmatrix} \cos\theta_i & \sin\theta_i & 0 & -a_i \\ -\sin\theta_i\cos\alpha_i & \cos\theta_i\cos\alpha_i & \sin\alpha_i & -d_i\sin\alpha_i \\ \sin\theta_i\sin\alpha_i & -\cos\theta_i\sin\alpha_i & \cos\alpha_i & -d_i\cos\alpha_i \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

If $n + 1$ coordinate systems are given, there exist n coordinate transformations between neighboring coordinate systems. Therefore, if the coordinates of a point are given as ${}^{7}\underline{X}$ with respect to $_7$ (the end-effector), the coordinates ${}^{1}\underline{X}$ of the same point with respect to $_1$ (the global coordinate system) is obtained by multiplying A_i in sequence, such that

$${}^{1}\underline{X} = \mathrm{Aeq}\,{}^{7}\underline{X}, \quad \mathrm{Aeq} = A_1A_2A_3A_4A_5A_6. \tag{2}$$

Note that the inverse transformation is given as $\mathrm{Aeq}^{-1} = A_6^{-1}A_5^{-1}A_4^{-1}A_3^{-1}A_2^{-1}A_1^{-1}$.

3. Solving the inverse kinematic problem

3.1. The forward kinematics

We first consider the forward kinematics problem of myCobot. Let $\underline{p} = \overrightarrow{O_1O_7}$ be the vector from the origin of $_1$ (position of the root of the manipulator) to the origin of $_7$ (position of the end-effector), and let $\underline{l} = {}^{t}(l_1, l_2, l_3) = {}^{1}[{}^{7}\underline{e}_x]$, $\underline{m} = {}^{t}(m_1, m_2, m_3) = {}^{1}[{}^{7}\underline{e}_y]$, $\underline{n} = {}^{t}(n_1, n_2, n_3) = {}^{1}[{}^{7}\underline{e}_z]$ be the vectors that are parallel to the unit vectors on the x, y, and z axes of $_7$, respectively. Then, $\underline{p}, \underline{l}, \underline{m}, \underline{n}$ is represented with respect to $_1$ as

$$\begin{pmatrix} l_1 \\ l_2 \\ l_3 \\ 0 \end{pmatrix} = \mathrm{Aeq}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ 0 \end{pmatrix} = \mathrm{Aeq}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ 0 \end{pmatrix} = \mathrm{Aeq}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{pmatrix} = \mathrm{Aeq}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

therefore, each component of Aeq and Aeq^{-1} is obtained as

$$\mathrm{Aeq} = \begin{pmatrix} l_1 & m_1 & n_1 & p_1 \\ l_2 & m_2 & n_2 & p_2 \\ l_3 & m_3 & n_3 & p_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathrm{Aeq}^{-1} = \begin{pmatrix} l_1 & l_2 & l_3 & -(l_1 p_1 + l_2 p_2 + l_3 p_3) \\ m_1 & m_2 & m_3 & -(m_1 p_1 + m_2 p_2 + m_3 p_3) \\ n_1 & n_2 & n_3 & -(n_1 p_1 + n_2 p_2 + n_3 p_3) \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

As seen in eq. (3), by putting the angles of the joints $\theta_1, \dots, \theta_6$ into the transformation matrix A_i, the position and orientation of the end-effector are obtained.

3.2. The inverse kinematic problem

To make use of eq. (3) in myCobot, we substitute some of the joint parameters $a_i, \alpha_i, d_i, \theta_i$ of myCobot into the transformation matrix A_i. As shown in the schematic diagram of myCobot in Figure 1, where the squares represent the revolute joints and the angle of rotation θ_i is given by taking a positive counterclockwise direction with respect to the ${}^iz$ axis, the joint parameters are given as

$$\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\} = \{\pi/2, 0, 0, \pi/2, \pi/2, 0\}, \quad a_1 = d_2 = d_3 = a_4 = a_5 = a_6 = 0. \tag{4}$$

By substituting the joint parameters in eq. (4) into the transformation matrix A_i in eq. (1), Aeq is calculated. Then, by comparing the components of Aeq with the components of Aeq in eq. (3), a system of 12 polynomial equations in the variables s_i and c_i ($i = 1, \dots, 6$) is obtained, where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. Each θ_i can be obtained by selecting the appropriate equation(s) from the above system and finding the solution. However, in the computation of Gröbner basis, the coefficients of the equations may expand, which makes it difficult to find the solution. Therefore, we focus on the structure of myCobot and solve its inverse kinematic problem from a different perspective as follows.

3.3. Solving the inverse kinematic problem with a fixed orientation

Inverse kinematics problems for robot manipulators of a certain structure can be solved analytically [12, 13]. Pieper [13] has pointed out that when the end effector of a 6-DOF manipulator has a spherical joint, the inverse kinematics problem can be separated into position and orientation problems of the end-effector. He has further noted that when the rotational axes of three consecutive rotational joints of a 6-DOF manipulator intersect at a single point, the inverse kinematics problem can also be separated into position and orientation problems of the end-effector.

Pieper's argument suggests that if there exists a combination of three consecutive rotational joints whose rotational axes intersect at a single point, it becomes possible to solve the inverse kinematics problem analytically. However, unfortunately, in the case of myCobot, there are no combinations of three consecutive rotational joints whose rotational axes intersect at a single point, although there are combinations of two consecutive joints whose rotational axes intersect at a single point. Therefore, following Pieper's approach, we solve the inverse kinematics problem by imposing constraints on the orientation of the end-effector.

For simplicity, we solve the inverse kinematic problem with the condition that the orientation of the end-effector remains constant, i.e. the axis in $_7$ is parallel to the axis in $_1$ preserving the same direction. Let $\underline{l} = {}^t(1, 0, 0), \underline{m} = {}^t(0, 1, 0), \underline{n} = {}^t(0, 0, 1)$, then Aeq and Aeq^{-1} in eq. (3) become as

$$\mathrm{Aeq} = \begin{pmatrix} 1 & 0 & 0 & p_1 \\ 0 & 1 & 0 & p_2 \\ 0 & 0 & 1 & p_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathrm{Aeq}^{-1} = \begin{pmatrix} 1 & 0 & 0 & -p_1 \\ 0 & 1 & 0 & -p_2 \\ 0 & 0 & 1 & -p_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Let P be the intersection point of axes ${}^4z$ and ${}^5z$, expressed as

$${}^1\underline{P} = \overrightarrow{O_1 P} = {}^t(x, y, z, 1). \tag{5}$$

Note that P is the position of Joint 5. We first express $\sin\theta_i$ and $\cos\theta_i \quad (i = 1, \ldots, 6)$ with the coordinate of the intersection P.

Let ${}^7\underline{P} = \overrightarrow{O_7P}$, then ${}^7\underline{P}$ is expressed in two ways as

$$ {}^7\underline{P} = A_6^{-1}A_5^{-1}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}-d_5\sin\theta_6\\-d_5\cos\theta_6\\-d_6\\1\end{pmatrix}, \quad {}^7\underline{P} = \text{Aeq}^{-1}\begin{pmatrix}x\\y\\z\\1\end{pmatrix} = \begin{pmatrix}x-p_1\\y-p_2\\z-p_3\\1\end{pmatrix}. $$

By equating two vectors ${}^7\underline{P}$ above, we have

$$ \sin\theta_6 = \frac{p_1 - x}{d_5}, \quad \cos\theta_6 = \frac{p_2 - y}{d_5}. \tag{6} $$

Let $\underline{w}_4$ be the unit vector in the direction of ${}^4z$-axis. Then, $\underline{w}_4$ and ${}^7\left[\underline{w}_4\right]$ are expressed as

$$ \underline{w}_4 = A_1A_2A_3\begin{pmatrix}\sin\theta_1\\-\cos\theta_1\\0\\0\end{pmatrix}, \quad {}^7\left[\underline{w}_4\right] = A_6^{-1}A_5^{-1}A_4^{-1}\begin{pmatrix}\cos\theta_6\sin\theta_5\\-\sin\theta_6\sin\theta_5\\-\cos\theta_5\\0\end{pmatrix}. \tag{7} $$

By using ${}^7\left[\underline{w}_4\right]$ above and Aeq, we obtain another expression for $\underline{w}_4$ as

$$ \underline{w}_4 = \text{Aeq}^7\left[\underline{w}_4\right] = {}^t(\cos\theta_6\sin\theta_5, -\sin\theta_6\sin\theta_5, -\cos\theta_5, 0). \tag{8} $$

By comparing each component of $\underline{w}_4$ in eqs. (7) and (8), respectively, we have

$$ \cos\theta_5 = 0, \quad \sin\theta_5 = \pm 1, \quad \cos\theta_1 = \pm\sin\theta_6 = \pm\frac{p_1 - x}{d_5}, \quad \sin\theta_1 = \pm\cos\theta_6 = \pm\frac{p_2 - y}{d_5}. \tag{9} $$

Next, $\underline{P}_3 = \overrightarrow{O_1P}$ is expressed as

$$ \underline{P}_3 = A_1A_2A_3\begin{pmatrix}0\\0\\d_4\\1\end{pmatrix} = \begin{pmatrix} d_4\sin\theta_1 + a_2\cos\theta_1\cos\theta_2 + a_3\cos\theta_1(\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3)\\ -d_4\cos\theta_1 + a_2\sin\theta_1\cos\theta_2 + a_3\sin\theta_1(\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3)\\ d_1 + a_2\sin\theta_2 + a_3(\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3)\\ 1\end{pmatrix}. \tag{10} $$

On the other hand, $\underline{P}_3$ is also expressed as ${}^1\underline{P} = {}^t(x, y, z, 1)$ as shown in eq. (5). Let $Q = {}^1\underline{P}[1]^2 + {}^1\underline{P}[2]^2 + ({}^1\underline{P}[3] - d_1)^2 = \underline{P}_3[1]^2 + \underline{P}_3[2]^2 + (\underline{P}_3[3] - d_1)^2$, then we have

$$ Q = x^2 + y^2 + (z - d_1)^2 = a_2^2 + a_3^2 + d_4^2 + 2a_2a_3\cos\theta_3. \tag{11} $$

Therefore,

$$ \cos\theta_3 = \frac{x^2 + y^2 + (z-d_1)^2 - a_2^2 - a_3^2 - d_4^2}{2a_2a_3}, \quad \sin\theta_3 = \pm\sqrt{1 - (\cos\theta_3)^2} $$

From the third component of $\underline{P}_3$ and the constraints,we have

$$ d_1 + a_2\sin\theta_2 + a_3(\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3) = z, \tag{12} $$
$$ (\sin\theta_2)^2 + (\cos\theta_2)^2 = 1. \tag{13} $$

If $\cos\theta_3, \sin\theta_3$ is obtained, $\cos\theta_2, \sin\theta_2$ is easily found from eq. (12).

Finally to find $\cos\theta_4$ and $\sin\theta_4$, let $\underline{w}_5$ be the unit vector in the direction of ${}^5z$-axis. Then, by using ${}^7\left[\underline{w}_5\right]$ expressed as

$$ {}^7[\underline{w}_5] = A_6^{-1}A_5^{-1}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \begin{pmatrix}\sin\theta_6\\\cos\theta_6\\0\\0\end{pmatrix}, $$

we have the expression for $\underline{w}_5$ in two ways as

$$\underline{w}_5 = A_1 A_2 A_3 A_4 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta_1 \sin(\theta_2+\theta_3+\theta_4) \\ \sin\theta_1 \sin(\theta_2+\theta_3+\theta_4) \\ -\cos(\theta_2+\theta_3+\theta_4) \\ 0 \end{pmatrix}, \quad \underline{w}_5 = \mathrm{Aeq}^7[\underline{w}_5] = \begin{pmatrix} \sin\theta_6 \\ \cos\theta_6 \\ 0 \\ 0 \end{pmatrix}.$$

Then, by comparing each component of $\underline{w}_5$ above, θ_4 is obtained as the one satisfying $\cos(\theta_2 + \theta_3 + \theta_4) = 0$ and $\sin(\theta_2 + \theta_3 + \theta_4) = \pm 1$, together with the use of the additivity theorem.

We could represent $\sin\theta_i$ and $\cos\theta_i \quad (i = 1, \dots, 6)$ using the coordinates of the intersection P. Next, we want to find x, y and z.

First, comparing the third component of ${}^7\underline{P}$, we have

$$-d_6 = z - p_3. \tag{14}$$

Next, from eq. (6) and the trigonometric identity, we have

$$\left(\frac{p_1 - x}{d_5}\right)^2 + \left(\frac{p_2 - y}{d_5}\right)^2 = 1. \tag{15}$$

Finally, by equating the first and the third components in the vector in the right-most-hand of eq. (10) with x and z, respectively, we have

$$\begin{aligned} d_4 \sin\theta_1 + a_2 \cos\theta_1 \cos\theta_2 + a_3 \cos\theta_1 (\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) &= x, \\ d_1 + a_2 \sin\theta_2 + a_3 (\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3) &= z. \end{aligned}$$

Then, by substituting $\sin\theta_1 = \frac{P_2 - y}{d_5}$ and $\cos\theta_1 = \frac{p_1 - x}{d_5}$ in eq. (9) into the above equations, assuming first that $\sin\theta_5 = 1$, we have the following system of equations in $\sin\theta_2, \cos\theta_2, \sin\theta_3, \cos\theta_3$:

$$\begin{aligned} d_4 \frac{p_2 - y}{d_5} + a_2 \frac{p_1 - x}{d_5} \cos\theta_2 + a_3 \frac{p_1 - x}{d_5} (\cos\theta_2 \cos\theta_3 - \sin\theta_2 \sin\theta_3) &= x, \\ d_1 + a_2 \sin\theta_2 + a_3 (\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3) &= z, \\ (\sin\theta_2)^2 + (\cos\theta_2)^2 &= 1, \\ (\sin\theta_3)^2 + (\cos\theta_3)^2 &= 1, \end{aligned} \tag{16}$$

in which the last two equations are added as trigonometric identities.

Then, the solution of the system in eq. (16) gives the value of $\cos\theta_3$ as

$$\begin{aligned} \cos\theta_3 = \frac{1}{2a_2 a_3 (p_1 - x)^2} \big(& -a_2^2 p_1^2 - a_3^2 p_1^2 + p_1^2 d_1^2 + p_2^2 d_4^2 + 2a_2^2 p_1 x + 2a_3^2 p_1 x - 2p_1 d_1^2 x - 2p_2 d_4 d_5 x \\ & - a_2^2 x^2 - a_3^2 x^2 + d_1^2 x^2 + d_5^2 x^2 - 2p_2 d_4^2 y + 2d_4 d_5 x y + d_4^2 y^2 - 2p_1^2 d_1 z + 4p_1 d_1 x z - 2d_1 x^2 z \\ & + p_1^2 z^2 - 2p_1 x z^2 + x^2 z^2 \big). \end{aligned} \tag{17}$$

By putting $\cos\theta_3$ in eq. (17) into the first equation in eq. (11) and multiplying both sides by $(p_1 - x)^2$, we have

$$\begin{aligned} & d_4^2 p_1^2 + d_4^2 p_2^2 - 2(d_4^2 p_1 + d_4 d_5 p_2) x + (d_4^2 + d_5^2 - p_1^2) x^2 + 2p_1 x^3 - x^4 \\ & \quad - 2d_4^2 p_2 y + 2d_4 d_5 x y + (d_4^2 - p_1^2) y^2 + 2p_1 x y^2 - x^2 y^2 = 0. \end{aligned} \tag{18}$$

In the case $\sin\theta_5 = -1$, perform the same calculation with $\sin\theta_1 = \frac{P_2 - y}{d_5}$ and $\cos\theta_1 = \frac{p_1 - x}{d_5}$. In this case, the sign in eqs. (17) and (18) changes in part, which gives the following equations.

$$\cos\theta_3 = \frac{1}{2a_2a_3(p_1-x)^2}\big(-a_2^2p_1^2 - a_3^2p_1^2 + p_1^2d_1^2 + p_2^2d_4^2 + 2a_2^2p_1x + 2a_3^2p_1x - 2p_1d_1^2x + 2p_2d_4d_5x - a_2^2x^2 - a_3^2x^2 + d_1^2x^2 + d_5^2x^2 - 2p_2d_4^2y - 2d_4d_5xy + d_4^2y^2 - 2p_1^2d_1z + 4p_1d_1xz - 2d_1x^2z + p_1^2z^2 - 2p_1xz^2 + x^2z^2\big), \quad (19)$$

$$d_4^2p_1^2 + d_4^2p_2^2 - 2(d_4^2p_1 - d_4d_5p_2)x + (d_4^2 + d_5^2 - p_1^2)x^2 + 2p_1x^3 - x^4 - 2d_4^2p_2y - 2d_4d_5xy + (d_4^2 - p_1^2)y^2 + 2p_1xy^2 - x^2y^2 = 0. \quad (20)$$

Furthermore, eq. (18) or eq. (20) together with eqs. (14) and (15), we obtain a system of polynomial equations in x, y, z. Solving the system (by using Gröbner basis computation, etc.) gives the position of the intersection point P.

4. Concluding remarks

In this paper, we have proposed a solution for the inverse kinematic problem of a 6-DOF manipulator under the condition that the orientation of the end-effector remains constant.

Our first task from here includes the verification of the solution with the CGS-QE method and efficiently solving the system of polynomial equations by using CGS, as we have proposed in the previous work. In addition, the orientation was specified this time for simplicity. However, orientation is not always constant in real-world manipulators. It is therefore necessary to develop the problem into an inverse kinematics problem for arbitrary orientations. There are several conditions on the geometry of the 6-DOF manipulator to be analytically solvable [13], and a method with computer algebra has been proposed for solving the inverse kinematic problem of the 6-DOF manipulator [3]. We will look for better solutions with reference to these methods.

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