A Decidable Temporal DL-Lite Logic with Undecidable First-Order and Datalog-rewritability of Ontology-Mediated Atomic Queries (Extended Abstract)

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Abstract
We design a logic in the temporal *DL-Lite* family (with non-Horn role inclusions and restricted temporalised roles), for which answering ontology-mediated atomic queries (OMAQs) can be done in ExpSpace and even in PSpace for ontologies without existential quantification in the rule heads but determining FO-rewritability or (linear) Datalog-rewritability of OMAQs is undecidable. On the other hand, we show (by reduction to monadic disjunctive Datalog) that deciding FO-rewritability of OMAQs in the non-temporal fragment of our logic can be done in 3NExpTime.

Keywords
Temporal description logics, DL-Lite, ontology-mediated query, first-order rewritability.

1. Introduction

Temporal description logics are used to reason about relational data that evolves in time. Along with the standard DL constructs on concepts and roles they admit temporal operators such as $\bigcirc_F$ (at the next moment), $\Box_F$ (always in the future), $\Diamond_F$ (sometime later) and their past-time counterparts, which give rise to *temporalised concepts* and *roles*. Being non-monodic [1], temporalised roles notoriously lead to high computational complexity even if coupled with a lightweight DL component [2, 3, 4]. Thus, any type of *DL-Lite* temporalised concept inclusions (CIs) with unguarded non-Horn temporalised role inclusions (RIs) (say, $P \sqsubseteq R \sqcup \bigcirc_F S$) result in an undecidable logic, while with Horn, Krom or core temporalised RIs the logic becomes decidable in ExpSpace or PSpace depending on the available temporal operators [3].

The proliferation of ontology-based data access [5] in the past decade has extended the list of traditional reasoning problems in DLs—such as satisfiability checking and answering ontology-mediated queries (OMQs)—with the *rewritability problem* into a target query language

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$\mathcal{L}$ [6, 7, 8, 9]: given an OMQ Q, decide whether there exists an *$\mathcal{L}$-rewriting* of Q, that is, an $\mathcal{L}$-query φ returning the same answers as Q over any data instance. Typical target query languages are the first-order logic (FO, possibly with various built-in predicates) and (linear) Datalog. For temporal OMQs given in the propositional temporal logic *LTL*, the FO-rewritability problem has recently been studied in [10]. Here, we present our initial observations on the FO- and Datalog-rewritability problems for OMQs with temporal *DL-Lite* ontologies. We design a logic with a restricted form of temporalised roles and non-Horn RIs whose syntax and potential usefulness are illustrated by the following example.

Example 1. Imagine that we are modelling the European transport network. Some passengers may like to go by plane in one direction but return by train (say, to safely bring back a selection of wines, cheeses and fine teas). To highlight such routes, one could use the following RI:

$$\mathit{flight} \sqcap \Diamond_F \mathit{train}^- \sqsubseteq \mathit{safeFlightConnection}, \tag{1}$$

where *train*$^-$ denotes the inverse of the role *train* (connection) from one city to another. (In our modelling, we do not regard the roles *flight* and *train* as 'global' because they depend on the season, day of a week, etc.) We may further partition the connections into certain classes, e.g., national and international, and infer properties of connected cities based on this classification. This is achieved by means of non-Horn RIs and CIs:

$$\begin{aligned} &\mathit{safeFlightConnection} \sqsubseteq \mathit{national} \sqcup \mathit{international}, \\ &\exists \mathit{international} \sqsubseteq \mathit{InternationalAirport}. \end{aligned} \tag{2}$$

Axiom (1) is an example of a *guarded $\Diamond$-RI*, where the temporalised role $\Diamond_F \mathit{train}^-$ is 'guarded' by the role name *flight*. Thus, *safeFlightConnection* may be inferred at a time instant only when there exists a *flight* from one city to another and, sometime later, there is a return *train*. $\dashv$

The temporal description logic $\textit{TDL-Lite}_{bool}^{[\Diamond]}$ we introduce in this paper admits arbitrary Boolean CIs with temporalised concepts, and only guarded $\Diamond$-RIs with temporalised roles that take the form

$$R_1 \sqcap \dots \sqcap R_n \sqcap \Diamond_{n+1} R_{n+1} \sqcap \dots \Diamond_k R_k \;\sqsubseteq\; R_{k+1} \sqcup \dots \sqcup R_m, \tag{3}$$

where the R_i are role names or their inverses, the $\Diamond_i$ are sequences of $\Diamond_F$ and $\Diamond_P$, and $n \geq 1$. We prove that answering ontology-mediated atomic queries (OMAQs) in $\textit{TDL-Lite}_{bool}^{[\Diamond]}$ can be done in ExpSpace for combined complexity, and in PSpace for the *flat* fragment of $\textit{TDL-Lite}_{bool}^{[\Diamond]}$, which disallows positive occurrences of $\exists R$ in ontology axioms (Th. 1). However, determining whether such a query can be rewritten into an FO- or a (linear) Datalog query is algorithmically undecidable (Th. 2). On the other hand, we observe that FO-rewritability of OMAQs in $\textit{DL-Lite}_{bool}$, i.e. in $\textit{TDL-Lite}_{bool}^{[\Diamond]}$ without temporalised concepts and roles, becomes decidable in 3NExpTime (Th. 3). For the proofs consult the full version of the paper at https://eprints.bbk.ac.uk/id/eprint/51763.

2. OMAQ Answering and FO-rewritability in $\textit{TDL-Lite}_{bool}^{[\Diamond]}$

The logic $\textit{TDL-Lite}_{bool}^{[\Diamond]}$ is a member of the *TDL-Lite* family [4, 3], which comprises temporal extensions of various languages in the atemporal *DL-Lite* family [11, 12, 13]. The alphabet of

TDL-Lite$^{[\Diamond]}_{bool}$ consists of *individual names* $a_0, a_1, \dots$, *concept names* $A_0, A_1, \dots$, and *role names* $P_0, P_1, \dots$. A *basic role* R is either a role name P_i or its inverse P_i^-. A *basic concept* C is either a concept name A_i or $\exists R$. *Temporalised concepts*, D, and *roles*, S, are defined by the grammar:

$$\begin{aligned} D \ &::= \ C \ \mid \ \bigcirc_F D \ \mid \ \bigcirc_P D \ \mid \ \Diamond_F D \ \mid \ \Diamond_P D \ \mid \ \Box_F D \ \mid \ \Box_P D, \\ S \ &::= \ R \ \mid \ \Diamond_F S \ \mid \ \Diamond_P S. \end{aligned} \tag{4}$$

A *concept inclusion* (CI) in *TDL-Lite*$^{[\Diamond]}_{bool}$ takes the form $D_1 \sqcap \dots \sqcap D_k \ \sqsubseteq \ D_{k+1} \sqcup \dots \sqcup D_{k+m}$, where the D_i are temporalised concepts. A *guarded* $\Diamond$*-role inclusion* (guarded $\Diamond$-RI) takes the form (3). As usual, the empty $\sqcap$ is $\top$ and the empty $\sqcup$ is $\bot$.

A *TBox*, $\mathcal{T}$, is a finite set of CIs, and an *RBox*, $\mathcal{R}$, is a finite set of guarded $\Diamond$-RIs. Taken together, they form an *ontology*, $\mathcal{O}$, in *TDL-Lite*$^{[\Diamond]}_{bool}$. An *ABox signature*, Σ, is a set of concept and role names. An *ABox*, $\mathcal{A}$, over Σ is a finite set of facts of the form $A_i(a, \ell)$ and $P_i(a, b, \ell)$, where $A_i, P_i \in \Sigma$ and $\ell \in \mathbb{Z}$ is a *timestamp*. We denote by $ind(\mathcal{A})$ the set of individual names in $\mathcal{A}$, by $\min(\mathcal{A})$ and $\max(\mathcal{A})$ the minimal and maximal timestamps in $\mathcal{A}$, respectively, and by $tem(\mathcal{A})$ the closed interval $[\min(\mathcal{A}), \max(\mathcal{A})]$.

An *ontology-mediated atomic query* (OMAQ) is a triple $Q(x, t) = (\mathcal{O}, \Sigma, A(x, t))$, where $\mathcal{O}$ is an ontology in *TDL-Lite*$^{[\Diamond]}_{bool}$, Σ is an ABox signature, A a concept name, x ranges over $ind(\mathcal{A})$, and t over $tem(\mathcal{A})$. (Note that the symbols in $\mathcal{O}$ and A do not have to be in Σ.) A pair $(a, \ell) \in ind(\mathcal{A}) \times tem(\mathcal{A})$ is a *certain answer* to $Q(x, t)$ over a Σ-ABox $\mathcal{A}$ if $A(a, \ell)$ is true in every model $\mathcal{M}$ of $(\mathcal{O}, \mathcal{A})$; for a detailed definition of a model the reader is referred to [3]. We denote by $ans_Q(\mathcal{A})$ the set of all certain answers to Q over $\mathcal{A}$. The *query answering problem* for Q over $\mathcal{A}$ is the decision problem for $ans_Q(\mathcal{A})$. We also consider *ontology-mediated Boolean atomic queries* (OMBAQs) of the form $Q = (\mathcal{O}, \Sigma, A)$ that require a 'yes/no' answer: a certain answer to an OMBAQ Q over $\mathcal{A}$ is 'yes' if, in every model $\mathcal{M}$ of $(\mathcal{O}, \mathcal{A})$, there exists a pair $(a, \ell) \in ind(\mathcal{A}) \times tem(\mathcal{A})$ such that $A(a, \ell)$ is true in $\mathcal{M}$, and 'no' otherwise.

With an ABox $\mathcal{A}$ we associate a temporal FO-structure $\mathfrak{S}_{\mathcal{A}}$ with domain $ind(\mathcal{A}) \times tem(\mathcal{A})$, over which we can evaluate FO($<$)- and Datalog-queries with atoms of the form $A(x, t)$, $P(x, y, t)$, $(t_1 < t_2)$. Let $\mathcal{L}$ be any relevant query language: FO($<$), linear or arbitrary Datalog queries. An OMAQ $Q(x, t) = (\mathcal{O}, \Sigma, A(x, t))$ is $\mathcal{L}$*-rewritable* if there exists an $\mathcal{L}$-query $\varphi(x, t)$, such that $ans_Q(\mathcal{A}) = \{ (a, \ell) \in ind(\mathcal{A}) \times tem(\mathcal{A}) \mid \mathfrak{S}_{\mathcal{A}} \models \varphi(a, \ell) \}$, for every Σ-ABox $\mathcal{A}$. An OMBAQ Q is $\mathcal{L}$*-rewritable* if there is an $\mathcal{L}$-query φ without answer variables such that $\mathfrak{S}_{\mathcal{A}} \models \varphi$ iff the certain answer to Q over $\mathcal{A}$ is 'yes'. It is known that FO($<$)-, linear Datalog-, and Datalog-rewritability guarantee answering OMAQs/OMBAQs in AC^0 [14], NL [15], and PTime [16] for data complexity, respectively. For the non-temporal fragment of *TDL-Lite*$^{[\Diamond]}_{bool}$, i.e. *DL-Lite*$_{bool}$, answering OMAQs/OMBAQs is ExpTime-complete for combined complexity [17]. We establish an ExpSpace upper bound for answering OMAQs/OMBAQs in full *TDL-Lite*$^{[\Diamond]}_{bool}$. We also consider the *flat TDL-Lite*$^{[\Diamond]}_{bool}$ that disallows concepts $\exists R$ on the right-hand side of CIs. This restriction reduces the complexity of answering OMAQs/OMBAQs to PSpace.

Theorem 1. *Answering TDL-Lite*$^{[\Diamond]}_{bool}$ *OMAQs/OMBAQs is in ExpSpace and ExpTime-hard for combined complexity. Answering flat TDL-Lite*$^{[\Diamond]}_{bool}$ *OMAQs/OMBAQs is PSpace-complete for combined complexity.*

However, checking whether a query is rewritable into FO($<$) or (linear) Datalog turns out to be undecidable even for flat ontologies.

Theorem 2. FO($<$)*-rewritability, linear Datalog-rewritability* (*if NL* $\neq$ *coNP*)*, and Datalog-rewritability* (*if PTime* $\neq$ *coNP*) *are undecidable for flat* $\textit{TDL-Lite}_{bool}^{[\Diamond]}$ *OMAQs/OMBAQs.*

The proof of Th. 2 makes use of the interaction between non-Horn RIs and temporal axioms. FO-rewritability of OMAQs/OMBAQs becomes decidable in the case when ontologies do not contain temporal operators. We obtain a positive result for the language $\textit{DL-Lite}_{bool}$ that disallows temporalised concepts and temporalised roles in $\textit{TDL-Lite}_{bool}^{[\Diamond]}$. The proof is via a translation to monadic disjunctive Datalog, adapting a similar technique for $\mathcal{ALCI}$ [7]. However, non-Horn RIs increase the complexity from 2NExpTime to 3NExpTime.

Theorem 3. *Checking* FO*-rewritability of* $\textit{DL-Lite}_{bool}$ *OMAQs/OMBAQs is in 3NExpTime.*

3. Related Work

The problem of deciding if a given OMQ is rewritable into a conventional query language $\mathcal{L}$ has been investigated for several important description logics. Bienvenu et al. [6] considered $\mathcal{ALC}$ and its extensions with OMAQs through the lenses of CSP and MMSNP. In particular, FO-rewritability of $\mathcal{ALCF}$ OMAQs is undecidable ($\mathcal{ALCF}$ is an extension of $\mathcal{ALC}$ with *functionality constraints* on roles). Feier et al. [7] proved 2NExpTime-completeness of FO-rewritability of OMQs with CQs in $\mathcal{ALCI}$. Lutz and Sabellek [8] established that every conjunctive OMQ in $\mathcal{EL}$ is either FO-rewritable, or linear Datalog-rewritable, or PTime-complete, and showed that each of the associated decision problems is ExpTime-complete. Gerasimova et al. [9] showed FO-rewritability to be in 2NExpTime for OMQs with CQs and the non-Horn ontology $\{A \sqsubseteq B \sqcup C\}$. Description logics with non-Horn RIs such as $\textit{DL-Lite}_{bool}$, however, have not been considered. Separately, rewritability to first-order languages was studied by Artale et al. [18] and Kurucz et al. [10] for pure temporal logics, using automata- and group-theoretic techniques.

In this paper, we take a first step towards understanding (temporal) *DL-Lite* with guarded non-Horn role inclusions by establishing two results: a decision procedure for FO-rewritability of OMQs in $\textit{DL-Lite}_{bool}$, and an undecidability result for FO($<$)- and Datalog-rewritability of OMQs in $\textit{TDL-Lite}_{bool}^{[\Diamond]}$, i.e., $\textit{DL-Lite}_{bool}$ with temporalised concepts and (guarded) roles. An open question is if rewritability is decidable for $\textit{DL-Lite}_{bool}$ with temporalised concepts only.

FO-rewritability (aka *boundedness*) has been studied for Datalog itself. Predicate boundedness is undecidable for binary programs [19], and even for linear programs with one binary IDB relation [20], while for linear *monadic* programs, it is in PSpace [21]. Uniform boundedness is undecidable for ternary programs [19], even if they are linear [22] (consult the latter for an explanation on different forms of Datalog boundedness). Temporalised RIs in $\textit{TDL-Lite}_{bool}^{[\Diamond]}$ can be viewed from the Datalog perspective as using binary, or even ternary IDBs. However, the undecidability proofs of [19], [20] and [22] make use of *chains*, i.e., Datalog rules where the right-hand part contains constructs like $R(X_1, X_2) \wedge R(X_2, X_3)$, which is inexpressible in *DL-Lite*. Compared to $\mathcal{ALCF}$, *DL-Lite* lacks negation and qualified existential restrictions.

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