Temporalising Unique Characterisability and Learnability of Ontology-Mediated Queries (Extended Abstract)

Jean Christoph Jung[1], Vladislav Ryzhikov[2], Frank Wolter[3] and
Michael Zakharyaschev[2]

[1]*TU Dortmund University, Otto-Hahn-Straße 12, 44227 Dortmund, Germany*
[2]*Birkbeck, University of London, Malet Street, London WC1E 7HX, UK*
[3]*University of Liverpool, Ashton Street, Liverpool L69 3BX, UK*

Abstract

Recently, the study of the unique characterisability and learnability of database queries by means of examples has been extended to ontology-mediated queries. Here, we study in how far the obtained results can be lifted to temporalised ontology-mediated queries. We provide a systematic introduction to the relevant approaches in the non-temporal case and then show general transfer results pinpointing under which conditions existing results can be lifted to temporalised queries.

Keywords

Ontology-mediated query, temporal data, query-by-example, unique characterisability, learnability.

1. Introduction

Motivated by the challenge of constructing logical expressions from data examples, the unique characterisability and learnability of queries, formulas, and concepts has been studied extensively by the database, logic, and KR communities [1, 2, 3, 4, 5, 6]. Recently, significant progress has been made for ontology-mediated queries, where one aims to characterise or learn a database query under background knowledge, both in the passive sense (where sets of positive and negative examples are given), see, e.g., [7], and in Dana Angluin's sense of exact learning with membership and/or equivalence queries [8], see, e.g., [9]. Also, rather general results have been obtained about the characterisation and learnability of temporal queries, but so far without ontologies [10]. Our aim here is to combine these two directions and study the temporalisation of unique characterisability and learnability under description logic (DL) ontologies.

Let $\mathcal{O}$ be an ontology and $\mathcal{Q}$ a class of conjunctive queries (CQs), which we assume for simplicity to have a single answer variable. We say that a query $\boldsymbol{q} \in \mathcal{Q}$ *fits* a pair $E = (E^+, E^-)$ of finite sets E^+ and E^- of pointed data instances $(\mathcal{D}, a)$ wrt $\mathcal{O}$ if $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}(a)$ for all

---

DL 2023: 36th International Workshop on Description Logics, September 2–4, 2023, Rhodes, Greece

jean.jung@tu-dortmund.de (J. C. Jung); vlad@dcs.bbk.ac.uk (V. Ryzhikov); wolter@liverpool.ac.uk (F. Wolter); michael@dcs.bbk.ac.uk (M. Zakharyaschev)

0000-0002-4159-2255 (J. C. Jung); 0000-0002-6847-6465 (V. Ryzhikov); 0000-0002-4470-606X (F. Wolter); 0000-0002-2210-5183 (M. Zakharyaschev)

CEUR Workshop Proceedings (CEUR-WS.org)

$(\mathcal{D}, a) \in E^+$, and $\mathcal{O}, \mathcal{D} \not\models \boldsymbol{q}(a)$ for all $(\mathcal{D}, a) \in E^-$. Then E *uniquely characterises* $\boldsymbol{q}$ wrt $\mathcal{O}$ within $\mathcal{Q}$ if $\boldsymbol{q}$ is the only (up to equivalence modulo $\mathcal{O}$) query in $\mathcal{Q}$ that fits E wrt $\mathcal{O}$. An ontology language $\mathcal{L}$ *admits (polysize) characterisations within* $\mathcal{Q}$ if every $\boldsymbol{q} \in \mathcal{Q}$ has a (polysize) characterisation wrt to any $\mathcal{L}$-ontology within $\mathcal{Q}$. Unique characterisations can be used to illustrate, explain, and construct queries. They are also a 'non-procedural' necessary condition for (polynomial) learnability using membership queries in Angluin's framework of exact learning, where membership queries to the oracle take the form 'does $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}(a)$ hold?' We focus our investigation on the class ELIQ of CQs that are equivalent to $\mathcal{ELI}$-concepts.

Examples, proofs, and further context can be found in the full version [11].

Non-temporal case. We begin by summarising the relevant results that will be used as a black box in our investigation of temporalised queries. Let $\mathcal{O}$ be an FO-ontology (typically in some DL). Given queries $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, we write $\boldsymbol{q}_1 \models_\mathcal{O} \boldsymbol{q}_2$ and say that $\boldsymbol{q}_1$ is *contained* in $\boldsymbol{q}_2$ wrt $\mathcal{O}$ if $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}_1(a)$ implies $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}_2(a)$, for any pointed data instance $(\mathcal{D}, a)$. We utilise a well-known reduction of containment to query entailment. An ontology $\mathcal{O}$ *admits containment reduction* if, for any CQ $\boldsymbol{q}(x)$, there is a pointed data instance $(\hat{\boldsymbol{q}}, a)$ such that the following conditions hold: $\boldsymbol{q}(x)$ is satisfiable wrt $\mathcal{O}$ iff $\mathcal{O}$ and $\hat{\boldsymbol{q}}$ are satisfiable; there is a surjective homomorphism $h\colon \boldsymbol{q} \to \hat{\boldsymbol{q}}$ with $h(x) = a$; and if $\boldsymbol{q}(x)$ is satisfiable wrt $\mathcal{O}$, then $\boldsymbol{q} \models_\mathcal{O} \boldsymbol{q}'$ iff $\mathcal{O}, \hat{\boldsymbol{q}} \models \boldsymbol{q}'(a)$, for any CQ $\boldsymbol{q}'(x)$. An ontology language $\mathcal{L}$ *admits containment reduction* if every $\mathcal{L}$-ontology does. For languages $\mathcal{L}$ that admit containment reduction, a unique characterization E of $\boldsymbol{q} \in \mathcal{Q}$ wrt $\mathcal{O}$ is called a *singular*$^+$*characterisation* if $E^+ = \{\hat{\boldsymbol{q}}\}$. It is easy to see that if $\mathcal{L}$ admits both (polysize) unique characterisations within $\mathcal{Q}$ and containment reduction, then every $\boldsymbol{q} \in \mathcal{Q}$ has a (polysize) singular$^+$characterisation. Containment reduction is a rather general condition: FO without equality including DLs such as $\mathcal{ALCHI}$ and *DL-Lite*$_\mathcal{H}$ [12] (aka *DL-Lite*$^\mathcal{H}_{\mathit{core}}$ [13]) and also some DLs with limited counting such as *DL-Lite*$_\mathcal{F}$ [12] (aka *DL-Lite*$^\mathcal{F}_{\mathit{core}}$ [13]) admit containment reduction but $\mathcal{ALCQ}$ does not.

The two main approaches to compute E^- and obtain singular$^+$characterisations for languages with containment reduction are based on frontiers and splittings (aka dualities) [6]. A *frontier of* $\boldsymbol{q}$ *wrt* $\mathcal{O}$ *within* $\mathcal{Q}$ is any set $\mathcal{F}_{\boldsymbol{q}} \subseteq \mathcal{Q}$ such that (a) $\boldsymbol{q} \models_\mathcal{O} \boldsymbol{q}'$ and $\boldsymbol{q}' \not\models_\mathcal{O} \boldsymbol{q}$, for all $\boldsymbol{q}' \in \mathcal{F}_{\boldsymbol{q}}$; and (b) if $\boldsymbol{q} \models_\mathcal{O} \boldsymbol{q}''$ for $\boldsymbol{q}'' \in \mathcal{Q}$, then $\boldsymbol{q}'' \models_\mathcal{O} \boldsymbol{q}$ or there is $\boldsymbol{q}' \in \mathcal{F}_{\boldsymbol{q}}$ with $\boldsymbol{q}' \models_\mathcal{O} \boldsymbol{q}''$. An ontology language $\mathcal{L}$ *admits* (*polysize*) *finite frontiers within* $\mathcal{Q}$ if every $\boldsymbol{q} \in \mathcal{Q}$ has a (polysize) finite frontier wrt to any $\mathcal{L}$-ontology within $\mathcal{Q}$.

Theorem 1. (i) *DL-Lite*$_\mathcal{H}$ *and the fragment DL-Lite*$^-_\mathcal{F}$ *of DL-Lite*$_\mathcal{F}$*, in which* R^- *is not functional for any* $B \sqsubseteq \exists R$*, admit polysize frontiers within ELIQ [14].* (ii) *DL-Lite*$_\mathcal{F}$ *does not admit finite frontiers within ELIQ [14].* (iii) $\mathcal{EL}$ *does not admit finite frontiers within ELIQ.*

The frontier of a query supplies the negative examples for a singular$^+$unique characterisation.

Theorem 2. *If* $\mathcal{L}$ *admits both* (*polysize*) *frontiers within* $\mathcal{Q}$ *and containment reduction, then* $\mathcal{L}$ *admits* (*polysize*) *singular*$^+$*characterisations within* $\mathcal{Q}$*, with* $E^- = \mathcal{F}_{\boldsymbol{q}}$*, for any* $\boldsymbol{q} \in \mathcal{Q}$*.*

The second path to singular$^+$characterisations is via finite splittings, which only exist if a finite signature σ of predicates is fixed. Let $\mathcal{Q}$ be a class of queries and $\mathcal{Q}^\sigma$ its restriction to σ, $Q \subseteq \mathcal{Q}^\sigma$ finite, and $\mathcal{O}$ a σ-ontology. A set $\mathcal{S}(Q)$ of pointed σ-data instances $(\mathcal{D}, a)$ is called a *split-partner* for Q wrt $\mathcal{O}$ within $\mathcal{Q}^\sigma$ if, for all $\boldsymbol{q}' \in \mathcal{Q}^\sigma$, we have $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}'(a)$ for some

$(\mathcal{D}, a) \in \mathcal{S}(Q)$ iff $\boldsymbol{q}' \not\models_{\mathcal{O}} \boldsymbol{q}$ for all $\boldsymbol{q} \in Q$. An ontology language $\mathcal{L}$ *has general split-partners within* $\mathcal{Q}^\sigma$ if all finite sets of $\mathcal{Q}^\sigma$-queries have split partners wrt any σ-ontology in $\mathcal{L}$.

Theorem 3. *(i) $\mathcal{ALCHI}$ has exponential-size general split-partners within σ-ELIQ, (ii) even wrt to the empty ontology, no polysize split-partners exist within σ-ELIQ [10].*

Thus, $\mathcal{EL}$ has finite general split-partners but no frontiers within ELIQ, and *DL-Lite*$_{\mathcal{F}}^{-}$ has finite frontiers but no finite general split-partners within ELIQ. This is in contrast to the ontology-free case where frontiers and splittings are more closely linked [6].

Theorem 4. *If $\mathcal{L}$ admits (polysize) general split-partners within $\mathcal{Q}^\sigma$ and containment reduction, then $\mathcal{L}$ admits (polysize) singular$^+$characterisations within $\mathcal{Q}$, with $E^- = \mathcal{S}(\{\boldsymbol{q}\})$, for any $\boldsymbol{q} \in \mathcal{Q}^\sigma$.*

Temporalisation. A *temporal data instance* is a sequence $\mathcal{A}_0, \dots, \mathcal{A}_n$ of domain data instances $\mathcal{A}_i$ with i regarded as a *timestamp*. To query temporal data, one can equip standard CQs with the operators of linear temporal logic *LTL* as proposed in [15, 16, 17, 18]. Within this framework, various query languages that admit (polysize) unique characterisations and learnability have been identified in the case when no background ontology is present [10]. Here, we assume that the temporal data is mediated by a standard (non-temporal) DL ontology whose axioms are supposed to be true at all times. We consider a few families of temporal queries defined in [10] that are built from domain queries in a given class $\mathcal{Q}$ (say, ELIQ or conjunctions of concept names, denoted $\mathcal{P}$) using $\land$ and the temporal operators $\bigcirc$ (at the next moment), $\Diamond$ (some time later), $\Diamond_r$ (now or later), and U (strict until): the family $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ of *path queries* of the form $\boldsymbol{q} = \boldsymbol{r}_0 \land \boldsymbol{o}_1(\boldsymbol{r}_1 \land \boldsymbol{o}_2(\boldsymbol{r}_2 \land \dots \land \boldsymbol{o}_n \boldsymbol{r}_n))$, where $\boldsymbol{o}_i \in \{\bigcirc, \Diamond, \Diamond_r\}$ and $\boldsymbol{r}_i \in \mathcal{Q}$; the family $\mathit{LTL}_p^{\mathsf{U}}(\mathcal{Q}^\sigma)$ of *path queries* $\boldsymbol{q} = \boldsymbol{r}_0 \land (\boldsymbol{l}_1 \,\mathsf{U}\, (\boldsymbol{r}_1 \land (\boldsymbol{l}_2 \,\mathsf{U}\, (\dots (\boldsymbol{l}_n \,\mathsf{U}\, \boldsymbol{r}_n) \dots))))$, where $\boldsymbol{r}_i \in \mathcal{Q}^\sigma$, $\boldsymbol{l}_i \in \mathcal{Q}^\sigma \cup \{\bot\}$; and its subfamily $\mathit{LTL}_{pp}^{\mathsf{U}}(\mathcal{Q}^\sigma)$ of *peerless queries* in which $\boldsymbol{r}_i \not\models_{\mathcal{O}} \boldsymbol{l}_i$ and $\boldsymbol{l}_i \not\models_{\mathcal{O}} \boldsymbol{r}_i$. The subfamily $\mathit{LTL}_p^{\bigcirc\Diamond}(\mathcal{Q})$ restricts $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ to the operators $\bigcirc$ and $\Diamond$; note that $\Diamond\boldsymbol{q} \equiv \bigcirc\Diamond_r\boldsymbol{q}$.

Temporal queries have a few essential differences from the domain ones. First, no example set can distinguish $\Diamond_r(A \land B)$ from $\Diamond_r(A \land (\Diamond_r B \land \Diamond_r(A \land \dots)))$ with sufficiently many alternating A, B. A syntactic criterion (excluding proper conjunctions that do not have a $\bigcirc$-neighbour) of unique characterisability of queries in $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{P})$, called *safety*, was found in [10]. Second, containment reduction does not work anymore since to characterise, say, $\Diamond A$ *two* positive examples are needed. By generalising safety in a natural way, we obtain our first transfer result:

Theorem 5. *Let $\mathcal{L}$ admit (polysize) singular$^+$characterisations within $\mathcal{Q}$ and $\mathcal{O}$ be a $\mathcal{L}$-ontology that admits containment reduction. Then $\boldsymbol{q} \in \mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ is (polysize) uniquely characterisable wrt $\mathcal{O}$ within $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ iff $\boldsymbol{q}$ is safe wrt $\mathcal{O}$; all $\boldsymbol{q} \in \mathit{LTL}_p^{\bigcirc\Diamond}(\mathcal{Q})$ are (polysize) uniquely characterisable wrt $\mathcal{O}$. If $\mathcal{O}$ admits polysize singular$^+$characterisations within $\mathcal{Q}$, then $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ is polynomially characterisable wrt $\mathcal{O}$ for bounded temporal depth.*

As a consequence of the above results, we obtain, e.g., that every safe query in $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) is polynomially characterisable wrt any *DL-Lite*$_{\mathcal{H}}$ or *DL-Lite*$_{\mathcal{F}}^{-}$ ontology and exponentially characterisable wrt any $\mathcal{ALCHI}$-ontology. Our second transfer result is as follows:

Theorem 6. *Let $\mathcal{L}$ have (exponential-size) general split-partners within $\mathcal{Q}^\sigma$ and let $\mathcal{O}$ be a σ-ontology in $\mathcal{L}$ that admits containment reduction. Then every $\boldsymbol{q} \in \mathit{LTL}_{pp}^{\mathsf{U}}(\mathcal{Q}^\sigma)$ is (exponential-size) uniquely characterisable within $\mathit{LTL}_p^{\mathsf{U}}(\mathcal{Q}^\sigma)$.*

As a consequence, we obtain that every query in $LTL^{\mathsf{U}}_{pp}(\mathcal{Q}^\sigma)$, where $\mathcal{Q}^\sigma$ is the class of σ-ELIQs, is exponentially uniquely characterisable within $LTL^{\mathsf{U}}_{p}(\mathcal{Q}^\sigma)$ wrt any $\mathcal{ALCHI}$ ontology.

Learning. We apply our results on characterisability to learnability of queries in $LTL_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) wrt ontologies in Angluin's framework of exact learning [8]. In the non-temporal case, exact learning of queries has recently been studied [6, 9, 14, 19]. Given some class $\mathcal{Q}$ of queries and an ontology $\mathcal{O}$, the *learner* aims to identify a *target query* $\boldsymbol{q}_T \in \mathcal{Q}$ using membership queries of the form 'does $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}(a)$ hold?' to the *teacher*. It is assumed that the target query $\boldsymbol{q}_T$ uses only symbols that occur in the ontology $\mathcal{O}$. We call $\mathcal{Q}$ *polynomial query* (*polynomial-time*) *learnable wrt $\mathcal{L}$-ontologies using membership queries* if there is a learning algorithm that receives an $\mathcal{L}$-ontology $\mathcal{O}$ and an example $(\mathcal{D}, a)$ with $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}_T(a)$ with $\mathcal{D}$ satisfiable under $\mathcal{O}$, and constructs $\boldsymbol{q}_T$ (up to equivalence wrt $\mathcal{O}$) using polynomially many queries of polynomial size (in time polynomial) in the size of $\boldsymbol{q}_T, \mathcal{O}, \mathcal{D}$.

As we always construct example sets effectively, our unique (exponential) characterisability results imply (exponential-time) learnability with membership queries. Obtaining polynomial-time learnability from polynomial characterisations is more challenging and, in fact, not always possible. We concentrate on ontologies formulated in fragments $\mathcal{L}$ of the DL $\mathcal{ELHIF}$ which are in normal form [20], but conjecture that our results continue to hold in general. $\mathcal{L}$ *admits polytime instance checking* if $\mathcal{O}, \mathcal{D} \models A(a)$, for a concept name A, can be decided in polynomial time. *Meet-reducibility* is in polytime if it can be checked in polytime whether an ELIQ is equivalent to a proper conjunction of ELIQs wrt to an $\mathcal{L}$-ontology. The following is shown by lifting the techniques for the non-temporal case developed in [19, 14] to the temporal case:

Theorem 7. *Let $\mathcal{L}$ be an ontology language that contains only $\mathcal{ELHIF}$-ontologies in normal form and that admits polysize frontiers within ELIQ that can be computed. Then:*

- (i) *The class of safe queries in $LTL_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) is polynomial query learnable wrt $\mathcal{L}$-ontologies using membership queries.*
- (ii) *The class $LTL_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) is polynomial query learnable wrt $\mathcal{L}$-ontologies using membership queries if the learner knows the temporal depth of the target query in advance.*
- (iii) *$LTL_p^{\bigcirc\Diamond}$(ELIQ) is polynomial query learnable wrt $\mathcal{L}$-ontologies using membership queries.*

If $\mathcal{L}$ further admits polynomial-time instance checking and polynomial-time computable frontiers within ELIQ, then in (ii) and (iii), polynomial query learnability can be replaced by polynomial-time learnability. If, in addition, meet-reducibility wrt $\mathcal{L}$-ontologies is in polynomial time, then also in (i) polynomial query learnability can be replaced by polynomial-time learnability.

Theorem 7 fully applies to *DL-Lite*$^{-}_{\mathcal{F}}$ as it enjoys all properties mentioned, while *DL-Lite*$_{\mathcal{H}}$ enjoys all properties mentioned except that meet-reducibility can be checked in poly-time. Most importantly, *DL-Lite*$^{-}_{\mathcal{F}}$ and *DL-Lite*$_{\mathcal{H}}$ admit polynomial time computable frontiers [14].

Outlook. Many interesting and challenging problems remain to be addressed. For instance, is it possible to overcome some of our negative results for unique characterisability by admitting some form of infinite (but finitely presentable) examples? Some results in this direction without ontologies are obtained in [21].

References

[1] D. M. L. Martins, Reverse engineering database queries from examples: State-of-the-art, challenges, and research opportunities, Inf. Syst. 83 (2019) 89–100.

[2] S. Staworko, P. Wieczorek, Characterizing XML twig queries with examples, in: Proc. of ICDT, 2015, pp. 144–160. URL: https://doi.org/10.4230/LIPIcs.ICDT.2015.144. doi:`10.4230/LIPIcs.ICDT.2015.144`.

[3] B. Konev, C. Lutz, A. Ozaki, F. Wolter, Exact learning of lightweight description logic ontologies, J. Mach. Learn. Res. 18 (2017) 201:1–201:63. URL: http://jmlr.org/papers/v18/16-256.html.

[4] B. ten Cate, V. Dalmau, P. G. Kolaitis, Learning schema mappings, ACM Trans. Database Syst. 38 (2013) 28:1–28:31. URL: https://doi.org/10.1145/2539032.2539035. doi:`10.1145/2539032.2539035`.

[5] A. Ozaki, Learning description logic ontologies: Five approaches. where do they stand?, Künstliche Intell. 34 (2020) 317–327. URL: https://doi.org/10.1007/s13218-020-00656-9. doi:`10.1007/s13218-020-00656-9`.

[6] B. ten Cate, V. Dalmau, Conjunctive queries: Unique characterizations and exact learnability, ACM Trans. Database Syst. 47 (2022) 14:1–14:41. URL: https://doi.org/10.1145/3559756. doi:`10.1145/3559756`.

[7] J. C. Jung, C. Lutz, H. Pulcini, F. Wolter, Logical separability of labeled data examples under ontologies, Artif. Intell. 313 (2022) 103785. URL: https://doi.org/10.1016/j.artint.2022.103785. doi:`10.1016/j.artint.2022.103785`.

[8] D. Angluin, Queries and concept learning, Mach. Learn. 2 (1987) 319–342. URL: https://doi.org/10.1007/BF00116828. doi:`10.1007/BF00116828`.

[9] M. Funk, J. C. Jung, C. Lutz, Actively learning concepts and conjunctive queries under ELr-ontologies, in: Proc. of IJCAI, ijcai.org, 2021, pp. 1887–1893. URL: https://doi.org/10.24963/ijcai.2021/260. doi:`10.24963/ijcai.2021/260`.

[10] M. Fortin, B. Konev, V. Ryzhikov, Y. Savateev, F. Wolter, M. Zakharyaschev, Unique characterisability and learnability of temporal instance queries, in: Proc. of KR, 2022. URL: https://proceedings.kr.org/2022/17/.

[11] J. C. Jung, V. Ryzhikov, F. Wolter, M. Zakharyaschev, Temporalising unique characterisability and learnability of ontology-mediated queries, CoRR abs/2306.07662 (2023). URL: https://doi.org/10.48550/arXiv.2306.07662. doi:`10.48550/arXiv.2306.07662`. `arXiv:2306.07662`.

[12] D. Calvanese, G. D. Giacomo, D. Lembo, M. Lenzerini, R. Rosati, Tractable reasoning and efficient query answering in description logics: The DL-Lite family, J. Autom. Reasoning 39 (2007) 385–429.

[13] A. Artale, D. Calvanese, R. Kontchakov, M. Zakharyaschev, The DL-Lite family and relations, J. Artif. Intell. Res. 36 (2009) 1–69.

[14] M. Funk, J. C. Jung, C. Lutz, Frontiers and exact learning of ELI queries under dl-lite ontologies, in: Proc. of IJCAI, 2022, pp. 2627–2633. URL: https://doi.org/10.24963/ijcai.2022/364. doi:`10.24963/ijcai.2022/364`.

[15] F. Baader, S. Borgwardt, M. Lippmann, Temporal query entailment in the description logic SHQ, J. Web Semant. 33 (2015) 71–93. URL: https://doi.org/10.1016/j.websem.2014.11.008.

doi:10.1016/j.websem.2014.11.008.

[16] S. Borgwardt, V. Thost, Temporal query answering in the description logic EL, in: Proc. of IJCAI, AAAI Press, 2015, pp. 2819–2825. URL: http://ijcai.org/Abstract/15/399.

[17] A. Artale, R. Kontchakov, A. Kovtunova, V. Ryzhikov, F. Wolter, M. Zakharyaschev, Ontology-mediated query answering over temporal data: A survey, in: Proc. of TIME, Schloss Dagstuhl, Leibniz-Zentrum für Informatik, 2017, pp. 1:1–1:37. URL: https://doi.org/10.4230/LIPIcs.TIME.2017.1. doi:10.4230/LIPIcs.TIME.2017.1.

[18] A. Artale, R. Kontchakov, A. Kovtunova, V. Ryzhikov, F. Wolter, M. Zakharyaschev, First-order rewritability and complexity of two-dimensional temporal ontology-mediated queries, J. Artif. Intell. Res. 75 (2022) 1223–1291. URL: https://doi.org/10.1613/jair.1.13511. doi:10.1613/jair.1.13511.

[19] M. Funk, J. C. Jung, C. Lutz, Exact learning of ELI queries in the presence of dl-lite-horn ontologies, in: Proc. of DL, CEUR-WS.org, 2022.

[20] F. Baader, I. Horrocks, C. Lutz, U. Sattler, An Introduction to Description Logics, Cambridge University Press, 2017.

[21] P. Sestic, Unique Characterisability of Linear Temporal Logic, Msc, University of Amsterdam, 2023.

[22] V. Gutiérrez-Basulto, J. C. Jung, R. Kontchakov, Temporalized EL ontologies for accessing temporal data: Complexity of atomic queries, in: Proc. of IJCAI, IJCAI/AAAI Press, 2016, pp. 1102–1108. URL: http://www.ijcai.org/Abstract/16/160.

[23] A. Artale, R. Kontchakov, V. Ryzhikov, M. Zakharyaschev, A cookbook for temporal conceptual data modelling with description logics, ACM Trans. Comput. Log. 15 (2014) 25:1–25:50. URL: https://doi.org/10.1145/2629565. doi:10.1145/2629565.

[24] P. A. Wałęga, B. Cuenca Grau, M. Kaminski, E. V. Kostylev, DatalogMTL over the Integer Timeline, in: Proc. of KR, 2020, pp. 768–777. URL: https://doi.org/10.24963/kr.2020/79. doi:10.24963/kr.2020/79.

[25] D. Angluin, Learning regular sets from queries and counterexamples, Inf. Comput. 75 (1987) 87–106. URL: https://doi.org/10.1016/0890-5401(87)90052-6. doi:10.1016/0890-5401(87)90052-6.

[26] M. Shahbaz, R. Groz, Inferring mealy machines, in: Proc. of FM, Springer, 2009, pp. 207–222.

[27] F. Aarts, F. Vaandrager, Learning i/o automata, in: Proc. of CONCUR, Springer, 2010, pp. 71–85.

[28] S. Cassel, F. Howar, B. Jonsson, B. Steffen, Active learning for extended finite state machines, Formal Aspects Comput. 28 (2016) 233–263. URL: https://doi.org/10.1007/s00165-016-0355-5. doi:10.1007/s00165-016-0355-5.

[29] F. Howar, B. Steffen, Active automata learning in practice - an annotated bibliography of the years 2011 to 2016, in: Machine Learning for Dynamic Software Analysis: Potentials and Limits, International Dagstuhl Seminar 16172, volume 11026 of *Lecture Notes in Computer Science*, Springer, 2018, pp. 123–148. URL: https://doi.org/10.1007/978-3-319-96562-8_5. doi:10.1007/978-3-319-96562-8_5.

[30] A. Camacho, S. A. McIlraith, Learning interpretable models expressed in linear temporal logic, in: Proc. of ICAPS, AAAI Press, 2019, pp. 621–630. URL: https://aaai.org/ojs/index.php/ICAPS/article/view/3529.

[31] C. Lemieux, D. Park, I. Beschastnikh, General ltl specification mining (t), in: Proc. of ASE, IEEE, 2015, pp. 81–92.

[32] D. Neider, I. Gavran, Learning linear temporal properties, in: Proc. of FMCAD, IEEE, 2018, pp. 1–10. URL: https://doi.org/10.23919/FMCAD.2018.8603016. doi:10.23919/FMCAD.2018.8603016.

[33] N. Fijalkow, G. Lagarde, The complexity of learning linear temporal formulas from examples, CoRR abs/2102.00876 (2021). URL: https://arxiv.org/abs/2102.00876. arXiv:2102.00876.

[34] M. Fortin, B. Konev, V. Ryzhikov, Y. Savateev, F. Wolter, M. Zakharyaschev, Reverse engineering of temporal queries mediated by LTL ontologies, in: Proceedings of IJCAI, 2023.

[35] M. Arenas, G. I. Diaz, The exact complexity of the first-order logic definability problem, ACM Trans. Database Syst. 41 (2016) 13:1–13:14.

[36] P. Barceló, M. Romero, The complexity of reverse engineering problems for conjunctive queries, in: Proc. of ICDT, 2017, pp. 7:1–7:17.

[37] J. Lehmann, P. Hitzler, Concept learning in description logics using refinement operators, Machine Learning 78 (2010) 203–250.

[38] V. Gutiérrez-Basulto, J. C. Jung, L. Sabellek, Reverse engineering queries in ontology-enriched systems: The case of expressive Horn description logic ontologies, in: Proc. of IJCAI-ECAI, 2018.

[39] M. Funk, J. C. Jung, C. Lutz, H. Pulcini, F. Wolter, Learning description logic concepts: When can positive and negative examples be separated?, in: Proc. of IJCAI, 2019, pp. 1682–1688.

[40] P. G. Kolaitis, Schema Mappings and Data Examples: Deriving Syntax from Semantics (Invited Talk), in: Proc. of FSTTCS, volume 13, Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, Dagstuhl, Germany, 2011, pp. 25–25. URL: http://drops.dagstuhl.de/opus/volltexte/2011/3359. doi:10.4230/LIPIcs.FSTTCS.2011.25.

[41] B. Alexe, B. ten Cate, P. G. Kolaitis, W. C. Tan, Characterizing schema mappings via data examples, ACM Trans. Database Syst. 36 (2011) 23.

[42] B. ten Cate, R. Koudijs, Characterising modal formulas with examples, CoRR abs/2304.06646 (2023). URL: https://doi.org/10.48550/arXiv.2304.06646. doi:10.48550/arXiv.2304.06646. arXiv:2304.06646.

[43] A. Artale, R. Kontchakov, A. Kovtunova, V. Ryzhikov, F. Wolter, M. Zakharyaschev, First-order rewritability of ontology-mediated queries in linear temporal logic, Artif. Intell. 299 (2021) 103536. URL: https://doi.org/10.1016/j.artint.2021.103536. doi:10.1016/j.artint.2021.103536.

[44] M. Bienvenu, B. ten Cate, C. Lutz, F. Wolter, Ontology-based data access: A study through disjunctive datalog, CSP, and MMSNP, ACM Trans. Database Syst. 39 (2014) 33:1–33:44.

[45] R. McKenzie, Equational bases and nonmodular lattice varieties, Transactions of the American Mathematical Society 174 (1972) 1–43.

[46] S. Kikot, R. Kontchakov, M. Zakharyaschev, On (in)tractability of OBDA with OWL 2 QL, in: R. Rosati, S. Rudolph, M. Zakharyaschev (Eds.), Proc. of DL, CEUR-WS.org, 2011. URL: https://ceur-ws.org/Vol-745/paper_7.pdf.

[47] C. Chang, H. J. Keisler, Model Theory, Elsevier, 1998.

[48] S. Tobies, Complexity results and practical algorithms for logics in knowledge representation, Ph.D. thesis, RWTH Aachen University, Germany, 2001. URL: http://sylvester.bth.rwth-aachen.de/dissertationen/2001/082/01_082.pdf.

[49] M. Bienvenu, P. Hansen, C. Lutz, F. Wolter, First order-rewritability and containment of conjunctive queries in Horn description logics, in: Proc. of IJCAI, 2016, pp. 965–971.